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PSEUDO QUADRATURE MIRROR FILTERS

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This paper introduces a computationally efficient technique for splitting a signal into N equally spaced sub-bands subsampled by 1/N and for achieving near perfect reconstruction of the original signal from the sub-band signals. We show that the proposed scheme behaves in a way similar to that of conventional quadrature mirror filters, but that the implementation may be greatly simplified by using a multirate filtering technique which exploits the special structure of the sub-band filters.

1. INTRODUCTION

Conventional quadrature mirror filter (QMF) banks are now widely used in many digital signal processing applications, particularly in relation to speech compression [1]. These banks divide the signal into N adjacent sub-bands subsampled by 1/N, and allow the reconstruction of the original signal from the N sub-band signals with negligible distorsion, regardless of the order of band-pass filters. The alias-free reconstruction condition is met only when the equal bandwidth band-pass filters are derived by a radix-2 tree decomposition technique from a half-band low-pass filter prototype. With this particular filter structure, the filter bank can be implemented either as a radix-2 tree [1-2] or with direct realization of the band-pass filters defined by the tree decomposition technique [3].

In this paper, we expand on earlier work [4] to show that it is possible to devise a filter bank which has nearly the same alias-free reconstruction properties as the conventional QMF, but with the band-pass filters derived by frequency shifting from a prototype low-pass filter. It is then shown that the special structure of the band-pass filters yields a multirate implementation [5] which is more efficient than the conventional QMF approach or the polyphase technique proposed in [6].

2. THE PSEUDO-QMF FILTER BANK

We consider a signal $x(t)$ which is band-limited to a frequency $f_s/2$ and sampled at rate f_s, with $\omega_s = 2\pi f_s = 2\pi/T_s$. We want to split the sampled input signal into an even number N of sub-band signals $y_{2k}(Nn)$ and $y_{N-1-2k}(Nn)$ subsampled at rate f_s/N, by using N equally spaced adjacent band-pass filters with impulse responses $h_{2k}(n)$ and $h_{N-1-2k}(n)$, and with $k = 0, \ldots, N/2 - 1$. The signal splitting must be done in such a way that a near perfect approximation $\hat{x}(n)$ of the original signal $x(n)$ may be reconstructed from the subsampled channel signals.

In the following, we shall use a bank of N real band-pass filters where the even-numbered filters $h_{2k}(n)$ are derived by frequency translation (fig. 1) from a prototype low-pass filter $h(n)$ with cut-off frequency at $f_s/4N$, and with

$$h_{2k}(n) = h(n)\cos[2\pi(4k+1)(2n+1)/8N] \tag{1}$$

The odd-numbered band-pass filters are derived from the even-numbered band-pass filters by a modulation at frequency $f_s/2$, with

$$h_{N-1-2k}(n) = (-1)^n h_{2k}(n) \tag{2}$$

Under these conditions, the z-transforms of the band-pass filters are given as a function of the z-transform $H(z)$ of $h(n)$ by

$$H_{2k}(z) = \frac{W^{4k+1}}{2} H(W^{-2(4k+1)}z) + \frac{W^{-(4k+1)}}{2} H(W^{2(4k+1)}z) \tag{3}$$

$$H_{N-1-2k}(z) = H_{2k}(-z) \tag{4}$$

with

$$W = e^{-j2\pi/8N} \quad , \quad j = \sqrt{-1} \tag{5}$$

The subsampled channel signals, with z-transform $Y_{2k}(z)$ and $Y_{N-1-2k}(z)$ are derived from the input signal $x(n)$ with z-transform $X(z)$, by a filtering operation with the filter $H_{2k}(z)$, $H_{N-1-2k}(z)$, followed by a decimation [7] where N-1 out of every N consecutive channel samples are dropped. Thus, $Y_{2k}(z)$ and $Y_{N-1-2k}(z)$ are given by

$$Y_{2k}(z) = \frac{1}{N}\sum_{u=0}^{N-1} X(W^{8u}z^{1/N})H_{2k}(W^{8u}z^{1/N}) \quad (6)$$

$$Y_{N-1-2k}(z) = \frac{1}{N}\sum_{u=0}^{N-1} X(W^{8u}z^{1/N})H_{2k}(-W^{8u}z^{1/N}) \quad (7)$$

We show now that it is possible to reconstruct a signal $\hat{x}(n)$, with z-transform $\hat{X}(z)$, from the N channel signals, in such a way that $\hat{x}(n)$ is almost perfectly identical to x(n). This is done as shown in figure 2, by inserting N-1 zero-valued samples between successive samples of the channel signals, by filtering the resulting even sequences with the filters of z-transforms $H_{2k}(z)$, and the odd sequences with the filters of z-transforms $-H_{N-1-2k}(z)$, and finally by summing the resulting signals. Hence

$$\hat{X}(z) = \frac{1}{4N}\sum_{k=0}^{N/2-1}\sum_{u=0}^{N-1} [[[W^{4k+1}H(W^{-2(4k-4u+1)}z) + W^{-(4k+1)}H(W^{2(4k+4u+1)}z)] [W^{4k+1}H(W^{-2(4k+1)}z) + W^{-(4k+1)}H(W^{2(4k+1)}z)]] -[[W^{4k+1}H(-W^{-2(4k-4u+1)}z) + W^{-(4k+1)}H(-W^{2(4k+4u+1)}z)] [W^{4k+1}H(-W^{-2(4k+1)}z) + W^{-(4k+1)}H(-W^{2(4k+1)}z)]]]X(W^{8u}z) \quad (8)$$

In order to eliminate the aliasing terms in the reconstructed signal, we must insure that the coefficients of $X(W^{8u}z)$ in (8) are zero, except for u=o. With conventional QMF, the aliasing terms are completely eliminated. In our approach, we relax slightly this condition by insuring that the aliasing terms corresponding to adjacent filters are completely eliminated and by designing the prototype low-pass filter H(z) in such a way that non-adjacent band-pass filters do not overlap, with

$$H(W^{a}) = 0 \qquad \text{for} \quad |a| \geqslant 4 \quad (9)$$

Under these conditions, we have $H(W^{a}z)H(z) = 0$ for $|a| \geqslant 8$, and (8) reduces to

$$\hat{X}(z) = \frac{1}{N}[G_o(z)X(z) + \sum_{u=1}^{N-1} G_u(z)X(W^{8u}z)] \quad (10)$$

with

$$G_o(z) = \sum_{k=0}^{N/2-1} (H^2_{2k}(z) - H^2_{2k}(-z)) \quad (11)$$

and for $u \neq 0$,

$$G_u(z) = \frac{1}{4}\sum_{k=0}^{N/2-1} [H(W^{-2(4k-4u+1)}z)H(W^{2(4k+1)}z) + H(W^{2(4k+4u+1)}z)H(W^{-2(4k+1)}z) - H(-W^{-2(4k-4u+1)}z)H(-W^{2(4k+1)}z) - H(-W^{2(4k+4u+1)}z)H(-W^{-2(4k+1)}z)] \quad (12)$$

For u even, the only non-zero products in (12) appear for k = u/2 and k = (N-u)/2, thus, (12) reduces to

$$G_u(z) = \frac{1}{4}[H(W^{4u-2}z)H(N^{4u+2}z) + H(W^{4N+4u+2}z)H(W^{-4N+4u-2}z) - H(W^{-4N+4u-2}z)H(W^{4N+4u+2}z) - H(W^{4u+2}z)H(W^{4u-2}z)] = 0 \quad (13)$$

Similarily, the only non-zero products that appear in (12) for u odd correspond to k = (u-1)/2 and the corresponding terms cancel out. Therefore, the aliasing terms disappear completely provided the prototype low-pass filter has zero transmission above twice its ideal cut-off frequency, a condition which is easily met in practice, and the reconstructed signal reduces to

$$\hat{X}(z) = \frac{G_o(z)X(z)}{N} \quad (14)$$

In order to achieve perfect reconstruction, we must now insure that $G_o(z)$ has a flat frequency response.

We assume here that the prototype L-tap low-pass filter h(n) is a linear phase filter such that

$$L = 4pN \quad (15)$$

Then, it can be seen from (1) that the band-pass filters $h_{2k}(n)$ are also linear phase, since

$$h_{2k}(L-1-n) = h_{2k}(n) \quad (16)$$

Under these conditions, the evalutation of $G_o(z)$, defined by (11), on the unit circle yields

$$G_o(e^{j\omega T}) = e^{-j(L-1)\omega T_s}\sum_{k=0}^{N/2-1} [H^2_{2k}(\omega) + H^2_{2k}(\omega+\frac{\omega_s}{2})] \quad (17)$$

where $H_{2k}(\omega)$ is the magnitude of $H_{2k}(e^{j\omega T})$. In order to have a flat frequency response, the filters must be such that

$$\sum_{k=0}^{N/2-1} [H^2_{2k}(\omega) + H^2_{2k}(\omega + \frac{\omega_s}{2})] = \text{constant} \quad (18)$$

This condition is very similar to the usual requirement for flat response with conventional QMF filters. It implies that $H(\omega)$ is set at - 3 dB at frequency $f_s/4N$, and that

$$H^2(\omega + \frac{\omega_s}{4N}) + H^2(\omega - \frac{\omega_s}{4N}) = 1 \quad \text{for} \ |\omega| < \frac{\omega_s}{4N} \quad (19)$$

With this condition, the frequency response is flat except near f = 0 and $f = f_s/2$ where the two shifted spectra of the prototype filter overlap. Thus, with the exception of the two edges of the spectrum, the signal is perfectly reconstructed, with a total delay of L-1 samples and a multiplicative factor equal to 1/4N. In most applications such as speech compression, the imperfect reconstruction at both edges of the spectrum causes no problems because the lowest and the highest band-pass filters are not used.

3. MULTIRATE IMPLEMENTATION

Since the analysis and reconstruction process are nearly identical, we shall restrict here our discussion to the analysis filter bank. The subsampled even and odd channel signals are given respectively by

$$Y_{2k}(Nm) = \sum_{n=0}^{L-1} h(n)\cos[2\pi(4k+1)(2n+1)/8N]\, x(Nm-n) \quad (20)$$

$$Y_{N-1-2k}(Nm) = \sum_{n=0}^{L-1} (-1)^n h(n)\cos[2\pi(4k+1)(2n+1)/8N]\, x(Nm-n) \quad (21)$$

The computation of (20) and (21) is done by evaluating separately the terms which correspond to n even and odd, with

$$Y_{2k}(Nm) = Y^{(1)}_{2k}(Nm) + Y^{(2)}_{2k}(Nm) \quad (22)$$

$$Y_{N-1-2k}(Nm) = Y^{(1)}_{2k}(Nm) - Y^{(2)}_{2k}(Nm) \quad (23)$$

and

$$Y^{(1)}_{2k}(Nm) = \sum_{n=0}^{L/2-1} h(2n)\cos[2\pi(4k+1)(4n+1)/8N]\, x(Nm-2n) \quad (24)$$

$$Y^{(2)}_{2k}(Nm) = \sum_{n=0}^{L/2-1} h(2n+1)\cos[2\pi(4k+1)(4n+3)/8N]\, x(Nm-2n-1) \quad (25)$$

Since $L = 4 \cdot pN$, we can change index n in (24-25), with

$$n = Nn_1 + n_2 \quad , n_1 = 0, \ldots, 2p-1, \quad n_2 = 0, \ldots, N-1 \quad (26)$$

$$Y^{(1)}_{2k}(Nm) = \sum_{n_1=0}^{2p-1}\sum_{n_2=0}^{N-1} h(2Nm_1+2n_2)(-1)^{n_1} \cos[2\pi(4k+1)(4n_2+1)/8N]\, x[N(m-2n_1) - 2n_2] \quad (27)$$

$$Y^{(2)}_{2k}(Nm) = \sum_{n_1=0}^{2p-1}\sum_{n_2=0}^{N-1} h(2Nn_1+2n_2+1)(-1)^{n_1} \cos[2\pi(4k+1)(4n_2+3)/8N]\, x[N(m-2n_1) - 2n_2 - 1] \quad (28)$$

each of these expressions reduces into N filters of length 2p, plus one modified cosine transform with

$$Y^{(1)}_{2k}(Nm) = \sum_{n_2=0}^{N-1} a(n_2)\cos[2\pi(4k+1)(4n_2+1)/8N] \quad (29)$$

$$a(n_2) = \sum_{n_1=0}^{2p-1} h(2Nn_1+2n_2)(-1)^{n_1} x[N(m-2n_1)-2n_2] \quad (30)$$

$$Y^{(2)}_{2k}(Nm) = \sum_{n_2=0}^{N-1} b(n_2)\cos[2\pi(4k+1)(4n_2+3)/8N] \quad (31)$$

$$b(n_2) = \sum_{n_1=0}^{2p-1} h(2Nn_1+2n_2+1)(-1)^{n_1}\, x[N(m-2n_1)-2n_2-1] \quad (32)$$

With this approach, the N length-L band-pass filters are replaced by 2N filters of length L/2N, plus two modified cosine transforms.

The modified cosine transform (29) can be computed by taking the real part of a DFT (discrete Fourier transform) of size N/2, with

$$Y^{(1)}_{2k}(Nm) = \mathrm{Re}[\sum_{n_2=0}^{N-1} a(n_2) W_{8N}^{(4k+1)(4n_2+1)}] \quad (33)$$

$$Y^{(1)}_{2k}(Nm) = \mathrm{Re}[W_{8N}^{4k+1} \sum_{n_2=0}^{N/2-1} (a(n_2) - j\, a(n_2+N/2)) W_{2n}^{n_2} W_{N/2}^{n_2 k}] \quad (34)$$

$$W_{8N} = e^{-j2\pi/8N} \quad W_{2N} = e^{-j2\pi/2N} \quad W_{N/2} = e^{-j4\pi/N} \quad (35)$$

The modified cosine transform (31) is computed similarily. Thus, each modified cosine transform is computed with N/2 complex multiplications by $W_{2N}^{n_2}$, N/2 complex multiplications by W_{8N}^{4k+1} and one DFT of size N/2 (figure 3).

4. APPLICATION TO THE SUB-BAND CODING OF BASEBAND SPEECH

As an application of the pseudo-QMF technique, we consider here the coding of baseband speech into 8 sub-bands which span the frequency range 0-1000 Hz for use with a voice-excited vocoder. Using a suboptimum 64-tap prototype low-pass filter designed in the frequency domain, the frequency response is flat, with a ripple which does not exceed ±0,2 dB and a rejection of the main aliasing terms in excess of 40 dB.

The input signal is sampled at 2000 Hz. Since the 8 band-pass filters are reduced to 16 filters of 4 taps operating at the sampling rate 2000/8 Hz, the number of operations in the filter is 16000 multiplications per second and 12000 additions per second. Each modified discrete cosine transform is computed with one 4-point DFT, 4 complex premultiplications by W_{2N}^{n2}, and 4 complex postmultiplications by W_{8N}^{4k+1}. The multiplications by W_{8N}^{4k+1} are implemented with only 2 real multiplications and 1 real addition, since we take only the real part of the output. Using a complex multiplication algorithm with 3 real multiplications and 3 real additions, the premultiplications are computed with 8 real multiplications and 8 real additions. Thus, each DCT is evaluated with 16 real multiplications and 28 real additions, which gives for the two DCT's a computation rate of 8000 multiplications per second and 14000 additions per second. Including the 8 additions (22-23), this gives a total of 24000 real multiplications per second and 28000 real additions per second. This is less than half the number of operations required with a conventional implementation using the symmetries of the filters.

5. CONCLUDING REMARKS

We have shown that it is possible to reduce significantly the computational complexity of QMF filter banks by using an approach where the band-pass filters are derived by frequency translation from a prototype low-pass filter. This technique provides perfect cancellation of the aliasing terms only when the band-pass filters do not overlap over non adjacent filters. In practice, this restriction is not very severe, and we have shown on an example that one can achieve specifications which are comparable to those of conventional QMF filter banks.

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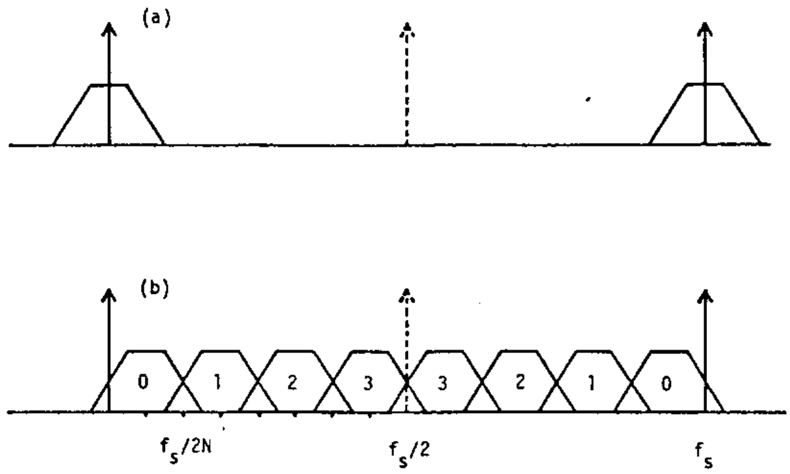

Figure 1: Frequency domain representation of the prototype low-pass filter (a) and of the band-pass filters (b).

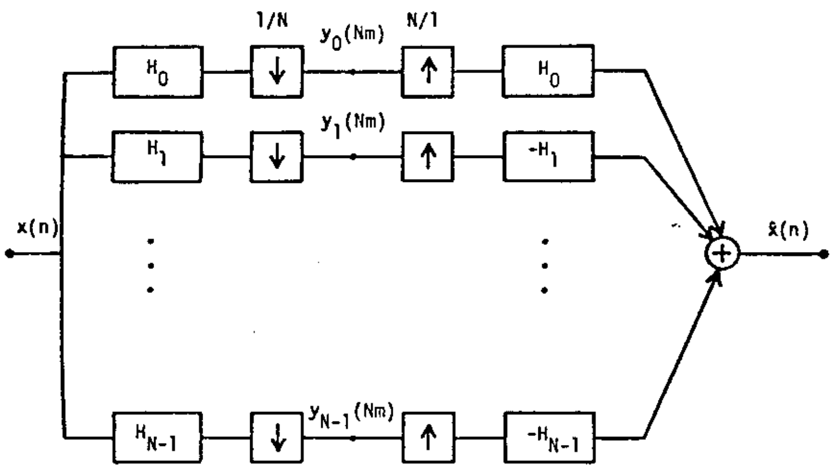

Figure 2: Block diagram of the splitting and reconstruction process.

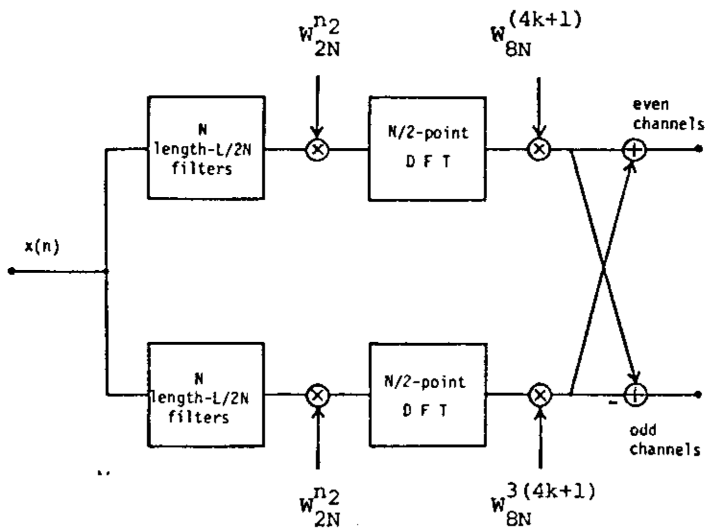

Figure 3: Implementation with reduced filters and FFT's.