A Radial Heat Flow Apparatus for Thermal Conductivity Characterisation of Cylindrical Samples

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INTRODUCTION

During the last two decades, silicon carbide based ceramic composites (SiC/SiC) have become a candidate material for nuclear fuel cladding, first for advanced nuclear systems, such as the GFR, and recently also as potential replacement material for nuclear fuel cladding in light water reactors. This work is contributing to the General Atomics/Westinghouse led CARAT project. In this frame, the thermal conductivity (TC) of cladding prototype sections are measured both before and after neutron irradiation with the purpose of better understanding the effects of neutron irradiation on the pyrolytic carbon interphase linking the SiC fibres to the SiC matrix.

To this end, an apparatus has been designed and built to measure the TC of short cylindrical samples, whose lengths range from 5 to 20 cm, by the radial heat flow method [1],[2],[3]. To avoid oxidation issues, the experiments are carried out in vacuum. Today the most common measurement method used to characterise the TC of materials is the so-called laser-flash technique [4], whereby a short high intensity laser pulse is shot on one surface of a thin flat sample and the energy transfer through the sample is analysed on the other side. Though much faster and easier to implement than the apparatus presented in this work, laser-flash equipment does have issues when dealing with highly anisotropic materials or capturing shape effects such as those present in large aspect ratio samples such as cladding tube sections.

The two aforementioned effects are exactly what the developed equipment has been built to capture. The radial heat flow method is based on the principle of measuring the temperature difference building up through a piece of material as this material acts as a thermal resistance against a known heat flow crossing it.

THEORETICAL CONSIDERATIONS

Though this technique is based on a very simple equation, i.e. solving the one-dimensional steady state heat equation within a hollow cylinder, it comes with a major drawback: the radiant heat flux on the inner and outer surfaces of the cylindrical sample have to be known to obtain the thermal conductivity from the heat equation. To determine these two quantities, a thorough analysis of the geometry and the thermal radiation configurations has to be carried out. The theory behind these calculations is that of radiosity [5].

Heat Equation Solution

The mathematical analysis of the apparatus is based on two assumptions: first, the system is considered in steady state and second, the heat equation is solved in one dimension at the mid-height of the sample. Both are agreeable, indeed, sufficient time is given to the measurements so that the temperature gradient across the sample wall does not vary in time. In addition to this, the circumferential and axial components of the temperature field can be neglected at the measurement position since the aspect ratio of the samples is high (up to 20).

Hence, the heat equation be states that $\alpha\Delta T = 0$, where $\alpha = {}^{k}/_{\rho C_p}$ is the thermal diffusivity of the sample and Δ is the Laplace operator. Another relevant relationship is the well-known Fourier's law: $\underline{\boldsymbol{q}} = -k\nabla T$. With these, the thermal conductivity of the sample is given by Equation 1.

$$k = -\frac{\boldsymbol{q}}{\nabla T} = -\frac{1}{T_{in}-T_{out}}\frac{\dot{Q}_{in}}{2\pi l}\ln\left(\frac{r_{out}}{r_{in}}\right) \quad (1)$$

With l, r_{in} and r_{out} being the physical dimensions of the tubular sample. The heat deposition rate $\dot{Q}_{in}$ is the quantity which requires a radiosity analysis.

Heat Flux Determination

The radiosity of a surface i, part of an enclosure comprising N surfaces, is noted $J_{e,i}$ and is defined as the radiant heat flux leaving any surface per unit area. Mathematically it is written as follows:

$$J_{e,i} = \varepsilon_i \sigma T_i^4 + (1-\varepsilon_i)\sum_{j=1}^{N} F_{ij} J_{e,j} \quad (2)$$

Another relevant quantity is the irradiance, $E_{e,i}$, of any of the aforementioned surfaces. It is the sum of all the incoming radiant heat on a given surface:

$$E_{e,i} = \sum_{j=1}^{N} F_{ij} J_{e,j} \quad (3)$$

In Equations 1 and 2, ε_i is the emissivity of a surface, T_i^4 its temperature to the fourth, F_{ij} is the view factor between two surfaces i and j, and σ is the Stefan-Boltzmann constant. Given the form of the equation above, it is best handled in its matrix form. Rearranging the terms appropriately yields the matrix equation below, where the left-hand side matrix is the so-called Gebhart factors matrix, the central vector it the radiosity vector, $\underline{\boldsymbol{J}}_{\boldsymbol{e}}$, and the vector on the right-hand side is the radiant emittance of the surfaces.

$$\underbrace{\begin{bmatrix} 1-\rho_1 F_{11} & \cdots & -\rho_1 F_{N1} \\ \vdots & \ddots & \vdots \\ -\rho_N F_{N1} & \cdots & 1-\rho_N F_{NN} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} J_{e,1} \\ \vdots \\ J_{e,N} \end{bmatrix}}_{\underline{J}_e} = \underbrace{\begin{bmatrix} \sigma\varepsilon_1 T_1^4 \\ \vdots \\ \sigma\varepsilon_N T_N^4 \end{bmatrix}}_{\underline{M}_e} \quad (4)$$

Solving for the various $J_{e,i}$'s is then a formality: $\underline{\boldsymbol{J}}_{\boldsymbol{e}} = \mathbf{A}^{-1}\underline{\boldsymbol{M}}_{\boldsymbol{e}}$

The equations necessary to calculate all of the view factors will not be given here, however, papers by Rea [6], Brockmann [7] as well as Leuenberger and Person [8] can be consulted for more details.

At this point, the rate of heat transfer at the surfaces forming the enclosure representing the apparatus can be calculated. Thus, the rate at which energy is deposited in within the sample will be given by the difference between the incoming and outgoing radiant fluxes at the inner and outer surfaces of the sample. Using the definitions of the irradiance and radiosity of the surface given in Equations 1 and 2, as well as the matrix relation from Equation 3, the heat transfer rates are calculated using Equation 4.

$$\dot{Q}_i = A_i\left(J_{e,i} - E_{e,i}\right) = A_i \frac{\varepsilon_i}{1-\varepsilon_i}\left(\sigma T_i^4 - J_{e,i}\right) \quad (5)$$

DESCRIPTION OF THE APPARATUS

The experimental setup is made of a vacuum chamber, a central positioned resistance heating rod made of graphite, a positioning system allowing for samples of diameters varying from 8 to 50 mm and length ranging from 4 to 20 cm. To reduce back-reflections of the thermal radiation from the sample to itself via the chamber walls, those have been covered with a carbon-based paint so that their emissivity would be as close as possible to that of a black body.

A GW-Instek PSW 30-108 power supply provides current ranging from 0 to 108 Amps at tensions ranging from 0 to 30 Volts. Thermocouples are placed at several positions to measure the temperatures needed for the analysis presented in the previous section.

Thermocouples positioned at the top of the sample surface as well as at the top of the resistance heater are necessary to account for the axial temperature distribution along the lengths of the sample and heating rod. These are taken into account when calculating the average temperature of the inner and outer surface of the sample as well as that of the heating rod. Indeed, at any other position than the mid-height of the sample, the temperature field is two-dimensional and described by Bessel-type functions. This effect is included in the processing of the experimental via a fitting procedure based on finite element modelling of the apparatus configuration for the sample specific case.

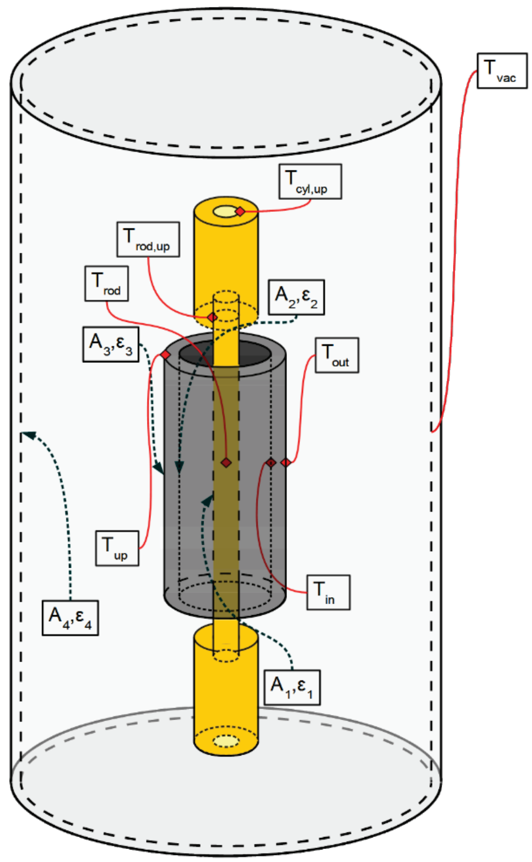

Fig. 1. Sketch of the experimental apparatus

RESULTS

The curves shown in Fig. 2 are the typical output obtained with the apparatus presented in this paper. The results obtained so far are in the range of values found in the literature. Younglood et al. [9] [10] have dealt with this topic in terms of modelling as well as measurements via the laser flash method.

We observe an increase of thermal conductivity with increasing temperature at temperatures lower than expected. This could be explained by a start of electronic heat transport at rather low temperatures.

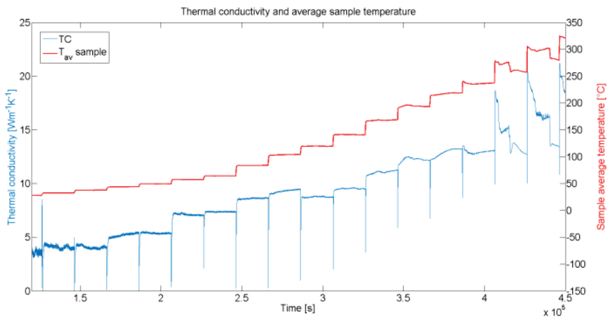

Fig. 2. Thermal conductivity curve (blue) and sample temperature (red) plotted against experiment running time

CONCLUDING REMARKS

This paper presents a new implementation of one of the classical thermal conductivity measurement techniques. Since the development of the laser-flash method in the sixties [4], steady-state measurements such as the radial heat flow have fallen in disuse. In this discussion, it is shown that a direct measurement method can be used and even be preferable in the case of highly anisotropic materials such as SiC/SiC composites. The advantage of this experimental setup is that the actual transverse conductivity is obtained, whereas laser-flash measurements yield a response in which both the transverse and axial conductivities are mixed.

Further steps of this study will include the measurements of samples irradiated with neutrons in the MITR test-reactor. The aim will be to evidence the role played by the pyrolytic carbon interphase by linking the TC measurements with energy filtered electron microscopy.

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