

*Implementation of variably
saturated flow into PHREEQC for
the simulation of biogeochemical
reactions in the vadose zone*

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1 **Abstract**

2 A software tool for the simulation of one-dimensional unsaturated flow and solute transport
3 together with biogeochemical reactions in the vadose zone was developed by integrating a
4 numerical solution of Richards' equation into the geochemical modelling framework
5 PHREEQC. Unlike existing software, the new simulation tool is fully based on existing capa-
6 bilities of PHREEQC without source code modifications or coupling to external software
7 packages. Because of the direct integration, the full set of PHREEQC's geochemical reactions
8 with connected databases for reaction constants is immediately accessible. Thus, unsatu-
9 rated flow and solute transport can be simulated together with complex solution speciation,
10 equilibrium and kinetic mineral reactions, redox reactions, ion exchange reactions and sur-
11 face adsorption including diffuse double layer calculations. Liquid phase flow is solved as a
12 result of element advection, where the Darcy flux is calculated according to Richards' equa-
13 tion. For accurate representation of advection-dominated transport, a total-variation-
14 diminishing scheme was implemented. Dispersion was simulated with a standard approach;
15 however, PHREEQC's multicomponent transport capabilities can be used to simulate spe-
16 cies-dependent diffusion. Since liquid phase saturation is recalculated after every reaction
17 step, biogeochemical processes that modify liquid phase saturation, such as dissolution or
18 precipitation of hydrated minerals are considered *a priori*. Implementation of the numerical
19 schemes for flow and solute transport have been described, along with examples of the ex-
20 tensive code verification, before illustrating more advanced application examples. These in-
21 clude (i) the simulation of infiltration with saturation-dependent cation exchange capacity,
22 (ii) changes in hydraulic properties due to mineral reactions and (iii) transport of mobile or-
23 ganic substances with complexation of heavy metals in varying geochemical conditions.

24 **Keywords:**

25 reactive transport; unsaturated flow; surface complexation; geochemical modelling; cation
26 exchange; hydraulic properties

27 **Software Availability**

Name of software:	RICH-PHREEQ
Developers:	L. Wissmeier and D.A. Barry
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Hardware requirements:	Platform supporting MS Windows Applications
Software requirements:	PHREEQC (http://www.falw.vu/~posv/phreeqc/download.html or http://wwwbrr.cr.usgs.gov/projects/GWC_coupled/phreeqc/index.html) Windows Script Host
Programming language:	VBScript
Availability and cost:	Available free of charge at http://infoscience.epfl.ch/record/133662

28 **1 Introduction**

29 The vadose zone poses a challenge to soil scientists, hydrologists and environmental engi-
30 neers in that it combines highly non-linear flow processes with complex biogeochemical re-
31 actions. Its vital role as the major life-supporting ecosystem on land is widely recognized.
32 Understanding the interaction of flow and reaction in the unsaturated zone is of importance
33 to quantify limits to its capacities for beneficial use, and to remediate contamination.

34 Over several decades, many numerical models with sophisticated and efficient algorithms
35 have been developed to solve the governing equations of flow and solute transport (e.g.,
36 FEFLOW: Koskinen et al., 1996; MODFLOW: Harbaugh et al., 2000; SUTRA: Voss and Provost,
37 2008; HYDRUS-1D: Šimůnek et al., 2009). At the same time, flexible software programs for
38 aqueous geochemical reactions evolved and now became standard tools in soil chemistry
39 (e.g., PHREEQC: Parkhurst and Appelo, 1999; ORCHESTRA: Meeussen, 2003; MINTEQ:
40 Gustafsson, 2008). Today's environmental problems of soil degradation, salinisation and
41 pollution with complex organic and inorganic reactants together with novel management
42 and remediation technologies, however, require explicit estimates of interactive flow and
43 reaction processes.

44 Particularly in the last decade, a new class of computer programs has emerged. These com-
45 bine sophisticated software packages for flow and transport with geochemical programs to
46 allow for the calculation of complex transient flow fields while preserving the flexibility to
47 simulate a wide variety of geochemical reactions (e.g., MIN3P: Mayer, 1999; LEHGC2.0: Yeh
48 et al., 2001; PHT3D: Prommer et al., 2003; PHAST: Parkhurst et al., 2004; HP1: Jacques and
49 Šimůnek, 2005, Šimůnek et al., 2006b; PHWAT: Mao et al., 2006; CRUNCHFLOW: Steefel,
50 2007). A disadvantage of coupling otherwise independent programs is the large amount of
51 information that has to be passed between modules. A deeper issue in practice, although
52 not in principle, is the loss of information between modules, leading to difficulties for the
53 transport of pH and redox potential as well as for the influence of reactions on liquid phase
54 saturation. In the following, a scheme is developed that avoids these issues through direct
55 implementation of variably saturated flow and transport into the geochemical modelling
56 software PHREEQC. The scheme and its implementation are referred to as RICH-PHREEQ in
57 this report.

58 RICH-PHREEQC is the latest development in a series of reactive solute transport models
59 (Prommer et al., 1999; Mao et al., 2006; Brovelli et al., 2009) , that couple liquid phase po-
60 rous media flow with a detailed description of hydro-geochemical processes on the molecu-
61 lar scale. Previously, we illustrated the implementation of a numerical solution for the mois-
62 ture-based Richards' equation into PHREEQC (Wissmeier and Barry, 2008). In the present
63 paper, a more versatile approach is used. The mixed form of Richards' equation is employed
64 (Celia et al., 1990), and the applicability of the code is extended to combined saturated (i.e.,
65 positive head)-unsaturated conditions and heterogeneous soil profiles, as well as more gen-
66 eral boundary conditions. For advection-dominated transport, the implementation of a to-
67 tal-variation-diminishing (TVD) scheme (Gupta et al., 1991) gives accurate results also with
68 sharp concentration fronts.

69 RICH-PHREEQ is a visual basic script file (VBScript) that creates an input file for simulating
70 saturated-unsaturated flow and geochemical reactions in PHREEQC. Users can edit the
71 VBScript and thereby rapidly develop saturated-unsaturated flow models, upon which arbi-

72 trary geochemical reaction models can be built. The structure of the VBScript is discussed in
73 the *User Notes*.

74 The generated PHREEQC input file typically starts with the definition of initial and boundary
75 solutions (SOLUTION) followed by the various user-defined geochemical reactions
76 (SURFACE, EXCHANGER, EQUILIBRIUM_PHASES etc.). Keywords for visualization
77 (USER_GRAPH) and output of results (SELECTED_OUTPUT, USER_PUNCH) follow. The com-
78 putations of variably saturated flow and advective-dispersive solute transport are per-
79 formed entirely in the subsequent USER_PRINT keyword. Changes in element amounts
80 through flow are imposed on the solutions via time-dependent reactions in the KINETICS
81 keywords. The TRANSPORT keyword not only adds the computation of species-dependent
82 diffusion but provides also the general space and time discretisation of the scheme. The im-
83 plementation at the script level ensures compatibility with future releases of PHREEQC
84 without the requirement for source code maintenance. In addition, the numerical methods
85 are transparent and can be extended by advanced PHREEQC users including those without
86 expert programming knowledge. Specifically, the advantages that arise from the implemen-
87 tation into PHREEQC are (i) direct access to its comprehensive set of geochemical reactions,
88 (ii) direct access to several distributed databases for species definitions, reaction constants
89 and rate definitions, and (iii) the convenience of PHREEQC's graphical user interfaces for the
90 flexible definition of geochemical reactions together with initial and boundary conditions.

91 The RICH-PHREEQ implementation has a wide range of potential applications. Among these
92 are:

- 93 • Evaluation of the sustainability and efficiency of management options for agricultural
94 soils (water holding capacity, nutrient status);
- 95 • Risk assessment for waste disposal facilities and drinking water suppliers;
- 96 • Evaluation of remediation strategies for contaminated soils such as:
 - 97 – Enhanced bioremediation;
 - 98 – Surfactant enhanced remediation;
- 99 • Simulation of vadose zone constructed wetlands and their treatment efficiency and
100 capacity;

- 101 • Evaluation of reactive barriers, their lifetime and potential failures.

102 In the following, we outline the theoretical background of RICH-PHREEQ before commenting
103 on its numerical implementation. After demonstrating the scheme's accuracy by means of
104 code verification, we will present example applications for (i) cation exchange with satura-
105 tion-dependent exchange capacity, and (ii) changes in hydraulic properties due to mineral
106 reactions and (iii) transport of a soluble humic substance with multi-site ion complexation
107 according to the diffuse double layer model.

108 User notes, the VBScript for PHREEQC input file generation, an example simulation and a
109 Matlab® script for result visualization are appended as supplemental online material to-
110 gether with the PHREEQC input files for the simulation example in this paper.

111 **2 Theory**

112 The direct implementation of RICH-PHREEQ without coupling to external software facilitates
113 the *compositional approach* (Panday and Corapcioglu, 1989; Liu et al., 1994) where, in
114 common with traditional approaches, the liquid phase advective flux is calculated from
115 volumetric liquid phase saturation. The liquid phase flux enters in the advection-dispersion
116 equation for solute transport. In contrast to common approaches, mass-conservative liquid
117 phase flow results from the application of the solute transport equation to every element in
118 the solution including oxygen and hydrogen. In addition, this approach leads to an accurate
119 representation of pH, thereby avoiding the need for an artificial "proton excess" master
120 species as suggested by Yeh and Tripathi (1991). At the same time, the distinction between
121 solute and solvent that is usually employed in reactive transport modelling (e.g., Yeh et al.,
122 2001; Steefel, 2007) is avoided. Unlike PHREEQC, RICH-PHREEQ requires strictly charge bal-
123 anced initial solutions and reactions, which is an acceptable constraint in natural environ-
124 mental systems.

125 2.1 Mass balance equation

126 The general mass balance equation in a one-dimensional soil column that contains multiple
 127 phases, which themselves consist of multiple species is given by (e.g., Panday and
 128 Corapcioglu, 1989; Lichtner, 1996; Miller et al., 1998; Barry et al., 2002)

$$\frac{\partial}{\partial t}(\theta_\alpha \rho_\alpha \omega_{i\alpha}) = -\frac{\partial}{\partial z}(\theta_\alpha \rho_\alpha \omega_{i\alpha} v_\alpha) - \frac{\partial}{\partial z} j_{i\alpha} + J_{i\alpha} + \mathcal{R}_{i\alpha} + \mathcal{S}_{i\alpha}, \quad (1)$$

129 with time t (T), distance downward from the soil surface z (L), volume fraction θ (-), density
 130 ρ (M L^{-3}), mass fraction ω , mean pore fluid velocity v (L T^{-1}), non-advective transport j (M L^{-2}
 131 T^{-1}), interphase mass exchange \mathcal{J} ($\text{M L}^{-3} \text{T}^{-1}$), intraphase reaction \mathcal{R} ($\text{M L}^{-3} \text{T}^{-1}$) and external
 132 source \mathcal{S} ($\text{M L}^{-3} \text{T}^{-1}$). Subscripts α denote phases and i species within phases.

133 Following Miller et al. (1998) the identities

$$\sum_\alpha \theta_\alpha = 1, \quad \sum_\alpha J_{i\alpha} = 0, \quad \sum_i \omega_{i\alpha} = 1, \quad \sum_i j_{i\alpha} = 0, \quad \sum_i \mathcal{R}_{i\alpha} = 0, \quad (2)$$

134 apply to Eq. (1).

135 Considering a single liquid phase l and an immobile phase s where solution species precipi-
 136 tate, dissolve and adsorb, the phase mass balance is obtained by summing over all species il
 137 and is ,

$$\frac{\partial}{\partial t}(\theta_l \rho_l) = -\frac{\partial}{\partial z}(\theta_l \rho_l v_l) + \sum_i J_{il} + \sum_i \mathcal{S}_{il}, \quad (3)$$

$$\frac{\partial}{\partial t}(\theta_s \rho_s) = \sum_i J_{is} + \sum_i \mathcal{S}_{is}. \quad (4)$$

138 If there are no external sources, then $\sum_i J_{il} = -\sum_i J_{is}$ and Eqs. (3) and (4) can be combined
 139 to yield

$$\frac{\partial}{\partial t}(\theta_l \rho_l) = -\frac{\partial}{\partial z}(\theta_l \rho_l v_l) - \frac{\partial}{\partial t}(\theta_s \rho_s). \quad (5)$$

140 As can be seen from Eq. (5), mineral reactions affect the flow directly by changing phase
 141 saturation, but also indirectly through altering constitutive relations (Wissmeier and Barry,
 142 2009).

143 2.2 Momentum balance/Flow equation for liquid phase

144 The volume flux q_l of the liquid phase is given by the Darcy equation as

$$\theta_l v_l = q_l = -\frac{k_l}{\mu_l} \left(\frac{\partial}{\partial z} p_l + \rho_l \right), \quad (6)$$

145 where k_l (L^2) is the effective permeability, μ_l ($M L^{-1} T^{-1}$) is the dynamic viscosity and p_l ($M L^{-1}$
 146 T^{-2}) is the fluid pressure (e.g., Hassanizadeh, 1986; Bear and Verruijt, 1987; Bear and
 147 Bachmat, 1998).

148 With θ_l being the volumetric liquid phase saturation, the combination of Eqs. (6) and (5) re-
 149 sults in the mixed form of Richards' equation (e.g., Richards, 1931; Klute, 1952; Philip, 1969,
 150 1970; Celia et al., 1990)

$$\frac{\partial \theta_l}{\partial t} = -\frac{\partial}{\partial z} \left[\frac{k_l}{\mu_l} \left(\frac{\partial}{\partial z} p_l + \rho_l \right) \right] - \frac{\partial}{\partial t} (\theta_s \rho_s), \quad (7)$$

151 where the final term on the right hand side accounts for the influence of mineral reactions
 152 on liquid phase saturation.

153 2.3 Transport equation

154 Species transport in the liquid phase is given by

$$\frac{\partial}{\partial t} (\theta_l C_{il}) = -\frac{\partial}{\partial z} (q_l C_{il}) + \frac{\partial}{\partial z} \left[(D_l^l |q_l| + \theta_l D_{il}^w \tau) \frac{\partial}{\partial z} C_{il} \right] + J_{il} + \mathcal{R}_{il} + \mathcal{S}_{il}, \quad (8)$$

155 with species concentration C (mol L^{-3}), longitudinal dispersivity D^l (L), molecular diffusion
 156 coefficient in free water D^w ($L^2 T^{-1}$), and tortuosity factor in the liquid phase τ (e.g., Bear,
 157 1972; Bear and Verruijt, 1987; Bear and Bachmat, 1998).

158 Reaction equations for solution speciation, equilibrium interphase reactions (mineral pre-
 159 cipitation/dissolution, ion exchange, surface complexation) and general kinetic reactions in
 160 PHREEQC are concisely described by Parkhurst and Appelo (1999). Detailed discussions on

161 the formulation of equations for geochemical reactions are given by Appelo and Postma
162 (2005), Langmuir (1997), Lichtner (1996), Mayer (1999), Steefel (2000, 2007), Stumm and
163 Morgan (1996) and Yeh and Tripathi (1989).

164 **3 Numerical methods**

165 A split-operator approach is employed for the decoupling of simultaneous processes of flow,
166 element transport and geochemical reactions in a discrete time domain. Operator splitting
167 techniques and their performance are described by Barry et al. (1996a; 1996b; 1997; 2000),
168 Kanney et al. (2003), Steefel and Lasaga (1994), Valocchi and Malmstead (1992), Yeh and
169 Tripathi (1989) and Yeh and Tripathi (1991). RICH-PHREEQ uses sequential operator split-
170 ting, which provides first-order accuracy with time ($\mathcal{O}(\Delta t)$). The dependency of the intro-
171 duced splitting error on grid size and time step size for reactions in variably saturated flow
172 conditions was discussed by Jacques et al. (2006).

173 *Fig. 1 near here*

174 RICH-PHREEQ's program flow is illustrated in Fig. 1. A PHREEQC input file is generated using
175 VBScript (Microsoft, 2009), which is run in PHREEQC. Within each time step during the
176 PHREEQC simulation, geochemical equilibrium reactions are calculated followed by kinetic
177 reactions for every cell in the simulation domain. The mass of H₂O and element concentra-
178 tions are saved. After reaction calculations for all cells, liquid phase flow and solute trans-
179 port is computed. The rates of changes in element concentrations due to transport are
180 saved and applied as kinetic reactions in the following reaction step.

181 **3.1 Liquid phase flow**

182 For the numerical solution of liquid phase flow the general mass-conservative scheme of
183 Celia et al. (1990) is used. Because flow of a single liquid phase is considered, we omit sub-
184 script l for the liquid phase from now on.

185 Picard iteration together with the Thomas algorithm (Press et al., 1990) was employed to
 186 solve the tridiagonal system of nonlinear algebraic equations that results from a second-
 187 order, central finite difference space discretisation of pressure heads with a fully implicit
 188 time-marching algorithm. Following Šimůnek et al. (2009), discretisation of Eq. (7) leads to

$$\frac{\theta_m^{j+1,k+1} - \theta_m^j}{\Delta t} = \frac{1}{\Delta z} \left(K_{m+\frac{1}{2}}^{j+1,k} \frac{h_{m+1}^{j+1,k+1} - h_m^{j+1,k+1}}{\Delta z} - K_{m-\frac{1}{2}}^{j+1,k} \frac{h_m^{j+1,k+1} - h_{m-1}^{j+1,k+1}}{\Delta z} \right) + \frac{K_{m+\frac{1}{2}}^{j+1,k} - K_{m-\frac{1}{2}}^{j+1,k}}{\Delta z}, \quad (9)$$

189 where K ($L T^{-1}$) is the unsaturated hydraulic conductivity defined as

$$K = \frac{k}{\mu} \rho g \quad (10)$$

190 and h (L) is the liquid phase pressure head defined as

$$h = \frac{p}{\rho g}. \quad (11)$$

191 g is the magnitude of gravitational acceleration ($L T^{-2}$) and Δz and Δt are numerical cell sizes
 192 and time steps, respectively. Subscript m is the cell index and superscripts j and k are indi-
 193 ces for time steps and Picard iterations.

194 Rewriting the left hand side of Eq. (9) as a truncated Taylor series with respect to h around
 195 the expansion point $h^{j+1,k}$ leads to

$$\frac{\theta_m^{j+1,k+1} - \theta_m^j}{\Delta t} = c_m^{j+1,k} \frac{h_m^{j+1,k+1} - h_m^{j+1,k}}{\Delta t} + \frac{\theta_m^{j+1,k} - \theta_m^j}{\Delta t}, \quad (12)$$

196 where c (L^{-1}) is the specific moisture capacity function defined as

$$c(h) = \frac{d\theta}{dh}. \quad (13)$$

197 In order to improve convergence in near-saturated conditions our scheme applies an under-
 198 relaxation technique to the nodal values of hydraulic conductivity (Paniconi and Putti, 1994;
 199 Phoon et al., 2007). Conductivity is therefore calculated as a function of h according to

$$K_m^{j+1,k} = K \left(h_m^{j+1,k} \varpi + h_m^{j+1,k-1} (1 - \varpi) \right), \quad (14)$$

200 where $0 \leq \varpi \leq 1$ is the relaxation factor.

201 In Eq. (9) the gradient $\frac{\partial K}{\partial z}$ is discretised using internodal conductivities $K_{m+\frac{1}{2}}$ and $K_{m-\frac{1}{2}}$. For
 202 most common analytical conductivity models, the use of the geometric mean for the inter-
 203 polation of internodal conductivities from nodal values K_{i+1}, K_i and K_i, K_{i-1} is superior in
 204 resolving sharp moisture fronts compared to other simple weighting methods such as the
 205 arithmetic and harmonic mean (Warrick, 1991; Belfort and Lehmann, 2005). Haverkamp and
 206 Vauclin (1979) found that the model using the geometric mean as weighting relation is pref-
 207 erable in terms of flexibility, precision, and feasibility for simulating transient water flow in
 208 partially saturated soil (see also Zaidel and Russo (1992) for discussion). The influence of
 209 conductivity interpolation on the resolution of steep infiltration fronts is illustrated in sec-
 210 tion 4.1.2. In RICH-PHREEQ, the user can choose between arithmetic, geometric and har-
 211 monic mean for the computation of internodal conductivities. The geometric mean is used
 212 by default.

213 For the calculation of water retention and unsaturated hydraulic conductivity the user has
 214 the choice between the analytical models by van Genuchten/Mualem (Mualem, 1976; van
 215 Genuchten, 1980), Brooks/Corey (Brooks and Corey, 1966) and Fujita/Rogers (Fujita, 1952;
 216 Rogers et al., 1983). Other weighting functions for internodal conductivities as well as hy-
 217 draulic property models can be easily integrated.

218 As boundary conditions the constant moisture content/constant head, constant flux and
 219 free drainage boundaries are implemented. System-dependent boundaries like atmospheric
 220 conditions with time series of precipitation and potential evaporation can be simulated as a
 221 conditional combination of flux and head boundaries (Šimůnek et al., 2009).

222 The singularity in q that results from a step-change of the moisture content on a constant
 223 moisture boundary leads to false positioning of moisture fronts (Bajracharya and Barry,
 224 1994). The numerical accuracy is improved by using $\theta_0^{j_1}/\theta_0^j = 0.5$ (Hildebrand and Cliffs,

225 1968), where the subscript $m = 0$ denotes the boundary node and j_1 is the first time step
 226 after the change in moisture content on the boundary.

227 3.2 Solute transport

228 With the definition of chemical elements e as the components of solution species i and con-
 229 stant, species-independent hydrodynamic dispersion, non-reactive solute transport for the
 230 liquid phase is described by

$$\frac{\partial M_e}{\partial t} = -\frac{\partial}{\partial z}(qC_e) + \frac{\partial}{\partial z}\left(D^l|q|\frac{\partial}{\partial z}C_e + \theta D_i^w \tau \cdot \frac{\partial}{\partial z}C_i\right). \quad (15)$$

231 Because C is given in molar concentration M is the moles of an element in the liquid phase.
 232 The change in moles of an element with time can be treated as a kinetic geochemical reac-
 233 tion that is conveniently solved by the PHREEQC implemented rate integrator. During the
 234 integration of a single time step, the right-hand side of Eq. (15) is treated as a constant. This
 235 results in linear convergence of the scheme with Δt , which is consistent with the accuracy of
 236 the operator splitting.

237 Traditionally, solute transport is applied to selected solution species in the aqueous phase
 238 only. However, it is recognized that Eq. (15) solves Richards' equation if it is calculated for all
 239 aqueous elements since the net non-advective transport is zero according to Eq. (2). With
 240 the assumption of constant density, liquid phase flow results from

$$\frac{\partial \theta}{\partial t} = \frac{1}{\rho V_c} \left\{ \sum_e \epsilon_e \left[-\frac{\partial}{\partial z}(qC_e) + \frac{\partial}{\partial z}\left(D^l|q|\frac{\partial}{\partial z}C_e\theta\right) \right] + \sum_i \epsilon_i \frac{\partial}{\partial z}\left(D_i^w \tau \cdot \frac{\partial}{\partial z}C_i\right) \right\}, \quad (16)$$

241 where ϵ is the molar mass (M mol^{-1}) and V_c is the volume of a computational cell.

242 For the numerical evaluation of the advective flux in Eq. (16), the user has the choice be-
 243 tween a first-order upstream weighting scheme and a TVD scheme. In discretised form the
 244 first-order scheme is given by

$$\begin{aligned} \left[-\frac{\partial}{\partial z}(qC) \right]_m^{j+1} &= q_{m-\frac{1}{2}}^{j+1} \left[H\left(q_{m-\frac{1}{2}}^{j+1}\right) C_m^j + H\left(-q_{m-\frac{1}{2}}^{j+1}\right) C_{m-1}^j \right] \\ &\quad - q_{m+\frac{1}{2}}^{j+1} \left[H\left(q_{m+\frac{1}{2}}^{j+1}\right) C_{m+1}^j + H\left(-q_{m+\frac{1}{2}}^{j+1}\right) C_m^j \right], \end{aligned} \quad (17)$$

245 where H is the Heaviside function used to distinguish between upward and downward flow
 246 (e.g., Wolfram MathWorld, 2008). Indices for elements have been omitted. Internodal fluxes
 247 at cell faces are calculated from the liquid phase flow scheme according to

$$q_{m+\frac{1}{2}}^{j+1} = -K_{m+\frac{1}{2}}^{j+1} \left(\frac{h_{m+1}^{j+1} - h_m^{j+1}}{\Delta z} + 1 \right). \quad (18)$$

248 The internodal flux provides the connection between solute transport and the numerical
 249 solution of Richards' equation.

250 Due to explicit time marching for solute transport, the stability of the scheme is restricted to
 251 $Cr \leq 1$ (Bear, 1972), where the Courant number Cr is defined by

$$Cr = \frac{q\Delta t}{\theta\Delta z}. \quad (19)$$

252 Compared to upstream weighting schemes using nodal fluxes (Šimůnek et al., 2009), the
 253 present scheme gives more accurate solutions especially in cases where the influent con-
 254 centration at the boundary is lower than the initial concentration in the column. It is noted,
 255 however, that ordinary first-order schemes suffer from severe numerical dispersion, which
 256 leads to flawed simulation results for steep concentration gradients in advection-dominated
 257 systems (see section 4.2) (Bajracharya and Barry, 1993b, a; Barry and Bajracharya, 1993;
 258 Bajracharya and Barry, 1994). Ordinary second- and higher-order schemes perform better in
 259 resolving sharp concentration fronts but introduce spurious oscillations, which may result in
 260 negative concentrations (Bresler, 1973; Bear and Verruijt, 1987; Rausch, 2005; Bear, 2007).
 261 Since PHREEQC simulations fail with negative element concentrations, we abstained from
 262 the implementation of ordinary higher-order schemes.

263 In addition to the upstream weighting scheme, the TVD scheme of Gupta et al. (1991) was
 264 implemented. The TVD scheme preserves sharp concentration fronts while maintaining

265 monotonicity of the solution, which is especially relevant in flow situations with high grid
 266 Péclet numbers (see section 4.2) (Yee et al., 1985; Cox and Nishikawa, 1991; Moldrup et al.,
 267 1994; Ataie-Ashtiani et al., 1996; Oldenburg and Pruess, 2000; Gottlieb et al., 2001;
 268 Kadalbajoo and Kumar, 2006). Despite the neglect of the local Courant number in comput-
 269 ing the flux limiter, the accuracy of the TVD scheme greatly exceeds the accuracy of com-
 270 monly applied first- and second-order schemes. The stability of the scheme is restricted to
 271 $Cr \leq 0.5$ (Blunt and Rubin, 1992). Fully implicit, unconditionally stable flux limiter schemes
 272 are available but necessitate an additional iterative process to evaluate the flux limiter at
 273 the new time level (Blunt and Rubin, 1992). This was not followed here as the additional
 274 computational burden outweighs the advantage of a further increased accuracy. Further-
 275 more, the time step requirement for the numerical solution of unsaturated flow is usually
 276 more restrictive than the grid Courant number.

277 For the spatial discretisation of the dispersive element flux, given by $\frac{\partial}{\partial z} \left(D^l |q| \cdot \frac{\partial}{\partial z} C_e \right)$ in Eq.
 278 (17) a standard second-order finite difference discretisation is employed.

279 Charge balanced, species-dependent diffusive transport modelled by $\frac{\partial}{\partial z} \left(\theta D_i^w \tau \cdot \frac{\partial}{\partial z} C_i \right)$ can
 280 be added to advective-dispersive transport using PHREEQC's multicomponent diffusion ca-
 281 pabilities. Tortuosity is then restricted to

$$\theta \tau = \theta^\zeta, \quad (20)$$

282 where the exponent ζ is a free tortuosity parameter (Parkhurst, 2007). Note that for disper-
 283 sion, elements are transported using the same dispersivity. However, diffusion acts on spe-
 284 cies with individual diffusion coefficients, which are defined in one of PHREEQC's databases
 285 (phreeqd.dat). Obviously, elements need be speciated prior to diffusion calculations.

286 In a strict sense, water is defined as the solution species H_2O . The volumetric moisture con-
 287 tent, θ , however is equal to the sum of the mass of all elements in the liquid phase divided
 288 by the liquid phase density and the volume of soil under consideration. For applications
 289 where the density of the solution is close to 1 kg dm^{-3} and the contribution of species other
 290 than H_2O to the solution volume is negligible, the liquid phase saturation θ can be taken as
 291 the mass of H_2O divided by the cell volume. In the present scheme this interpretation of θ is

292 used. However, together with appropriate models for the liquid phase density (e.g., Monnin,
 293 1994), the PHREEQC output of moles of elements as well as species concentrations can be
 294 conveniently used to calculate liquid phase density and therefore θ more accurately.

295 4 Code verification

296 Because reactions in PHREEQC are well tested, we focus on verification of unsaturated flow
 297 and non-reactive solute transport.

298 4.1 Unsaturated flow

299 4.1.1 Comparison to the exact solution of Sander et al. (1988)

300 The simulation of liquid phase flow was validated for a constant flux boundary using the ex-
 301 act solution of Sander et al. (1988). Since the solution is restricted to a certain class of hy-
 302 draulic models, the hydraulic property functions of Fujita/Rogers (Fujita, 1952; Rogers et al.,
 303 1983) were introduced into RICH-PHREEQC. Formulas for water retention and hydraulic con-
 304 ductivity are (Watson et al., 1995):

$$h(\Theta) = h_{air} + \frac{1}{\alpha} \log \left[\frac{(1 - \nu)\Theta}{1 - \nu\Theta} \right], \quad (21)$$

$$K(h) = K_{sat} \exp \left[\frac{K_{sat}(1 - \nu)}{D_0} (h - h_{air}) \right], \quad (22)$$

305 with the reduced moisture content Θ defined as

$$\Theta = \frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}}, \quad (23)$$

306 where θ_{sat} and θ_{res} are, respectively, saturated and residual moisture content, h_{air} is the
 307 air-entry pressure (L), K_{sat} is the saturated hydraulic conductivity ($L T^{-1}$), while D_0 ($L T^{-1}$) and
 308 ν are free parameters.

309 In the exact solution, the initial soil moisture is set to the residual moisture content,
 310 $\theta_{ini} = \theta_{res}$. Due to the singularity of $h(\Theta)$ in Eq. (21), the numerical scheme cannot solve
 311 for this condition. Therefore, we chose $\theta_{ini} = \theta_{res} + \epsilon$ with $\epsilon > 0$ being small enough to

312 avoid a significant influence on simulation results (Watson et al., 1995). The parameters of
313 the verification are summarized in Table 1, where q_{top} is the constant flux at the top bound-
314 ary.

315 *Table 1 near here*

316 *Fig. 2 near here*

317 Fig. 2 shows excellent agreement between RICH-PHREEQ and the exact solution even for the
318 relatively coarse spatial discretisation and therefore verifies the implementation of the con-
319 stant flux boundary condition.

320 4.1.2 Comparison to HYDRUS-1D (Šimůnek et al., 2009)

321 Verification of infiltration with the Brooks and Corey (Brooks and Corey, 1966) hydraulic
322 model was performed by comparison to HYDRUS-1D for loamy sand (Leij et al., 1997). The
323 hydraulic model is given by:

$$K(\theta) = K_{sat} \theta^{\frac{2}{n_{BC}} + l_{BC} + 2}, \quad (24)$$

$$\theta = \begin{cases} |\alpha_{BC} h|^{-n}, & h < -\frac{1}{\alpha} \\ 1, & h \geq -\frac{1}{\alpha} \end{cases} \quad (25)$$

324 where α_{BC} (L^{-1}), l_{BC} and n_{BC} are soil parameters. The properties of the simulation are
325 summarized in Table 2.

326 *Table 2 near here*

327 *Fig. 3 near here*

328 The results of the verification example in Fig. 3 show that a finer spatial discretisation is
 329 necessary in HYDRUS-1D in order to achieve a similar resolution of the sharp moisture
 330 fronts. The better performance of RICH-PHREEQ in this special case is due to the use of the
 331 geometric mean for the computation of internodal conductivities as opposed to the arith-
 332 metic mean used by HYDRUS-1D (see section 3.1). The figure illustrates the general ten-
 333 dency of the arithmetic mean to smooth out sharp infiltration fronts, whereas the geometric
 334 mean tends to over-sharpen them. Using the arithmetic average in RICH-PHREEQ, HYDRUS-
 335 1D and RICH-PHREEQC give identical results (see section 4.2).

336 4.2 Solute transport

337 The performance of the TVD scheme for transport of sharp concentration fronts in transient
 338 unsaturated flow conditions was evaluated by comparison to the implicit Galerkin and
 339 Crank-Nicholson upstream weighting finite element schemes in HYDRUS-1D.

340 Flow was calculated using the van Genuchten/Mualem (Mualem, 1976; van Genuchten,
 341 1980) hydraulic model, with

$$\theta(h) = \begin{cases} \theta_{res} + \frac{\theta_{sat} - \theta_{res}}{[1 + (\alpha_{GM}h)^{n_{GM}}]^{m_{GM}}}, & h < 0 \\ \theta_{sat}, & h \geq 0 \end{cases}, \quad (26)$$

$$K(h) = K_{sat} \Theta^{l_{GM}} \left[1 - \left(1 - \Theta^{\frac{1}{m_{GM}}} \right)^{m_{GM}} \right]^2, \quad (27)$$

342 where α_{GM} (L^{-1}), n_{GM} , m_{GM} and l_{GM} are soil parameters. The top boundary was set to a con-
 343 stant moisture content/constant concentration boundary with θ_{top} and C_{top} , respectively.
 344 The initial solute concentration is C_{ini} . For better comparability with HYDRUS-1D, the com-
 345 putation of internodal conductivity in RICH-PHREEQ was set to arithmetic averages. The pa-
 346 rameters of the simulation are summarized in Table 3.

347 *Table 3 near here*

348 *Fig. 4 near here*

349 The perfect agreement of moisture contents computed with HYDRUS-1D and RICH-PHREEQ
 350 is shown in Fig. 4a. The location of the plane of separation, z_{ps} , between event and pre-
 351 event water is calculated according to (Katou et al., 2001)

$$\int_0^{z_{ps}} \theta dz = \int_0^{\infty} \theta - \theta_{ini} dz. \quad (28)$$

352 The plane of separation gives the theoretical position of an influent concentration front for
 353 a non-reactive solute where the initial pore water is completely replaced by the influent so-
 354 lution without mixing or hydrodynamic dispersion. In Fig. 4a it is denoted by a dashed verti-
 355 cal line at $z_{ps} = 3.5$. The right hand side of Eq. (28) is represented by the upward hatched
 356 area (/) whereas the left hand side is represented by the downward hatched area (\). The
 357 upward and downward hatched areas overlap in the crossed area.

358 Fig. 4b shows the concentration profiles as calculated by HYDRUS-1D together with the TVD
 359 scheme in RICH-PHREEQ. The explicit upstream weighting finite difference scheme in
 360 RICH-PHREEQ gives very similar results to the upstream weighting finite element scheme in
 361 HYDRUS-1D and is therefore not displayed. The figure illustrates the superior performance
 362 of the TVD scheme compared to the second- and first-order schemes, by reproducing the
 363 theoretical position of the half concentration $z\left(\frac{C_{top}-C_{ini}}{2}\right) = z_{ps}$, avoiding spurious oscilla-
 364 tions and significantly reducing numerical dispersion. Therefore, the TVD scheme accurately
 365 simulates the sharp interface between event and pre-event water in advection-dominated
 366 flow conditions. Due to the simplifications of the scheme (see section 3.2 for details), the
 367 concentration gradient shows a limited amount of artificial dispersion. However, in contrast
 368 to the upstream weighting scheme, concentration gradients are not further reduced at later
 369 times but remain constant.

370 5 Example applications

371 The application examples in this section are simulated with small adaptations of
 372 RICH-PHREEQ, which can be easily followed by the users familiar with PHREEQC.

373 5.1 Saturation-dependent exchange capacity

374 In this example, we investigate a cation exchange process in a soil with variable exchange
375 capacity, in particular its influence on the solution composition of an infiltrating moisture
376 front. The simulation is motivated by the consideration that exchange sites in the dry pore-
377 space do not participate in the exchange process until the liquid phase saturation rises, the
378 pore-space becomes saturated and the sites are activated. The simulation was carried out
379 for an initially dry soil column of sandy clay loam (Leij et al., 1997) where the residual soil
380 moisture contains a sodium carbonate (Na_2CO_3) solution. The exchange sites, both active
381 and inactive, are thus entirely filled by sodium. From the top end, a calcium chloride (CaCl_2)
382 solution infiltrates the soil column at low constant flux, thereby increasing the moisture con-
383 tent and cation exchange capacity. The properties of the simulation are summarised in Table
384 4.

385 *Table 4 near here*

386 For simplicity, a linear relation between moisture content and the amount of exchange sites
387 was used. A direct relation between the number of exchange sites and the wetted surface
388 area can be implemented through models that compute wetted surface area with moisture
389 saturation (e.g., Taylor and Jaffe, 1990). However, the linear relation to moisture content
390 can be used as a first approximation and guideline for more sophisticated model develop-
391 ment. The amount of accessible exchange sites was set to $5.55 \times 10^{-2} \text{ meq kg}_{\text{H}_2\text{O}}^{-1}$.

392 *Fig. 5 near here*

393 Fig. 5a illustrates the progressing moisture front after 10, 25 and 50 h. Fig. 5b shows the ex-
394 changer composition in milliequivalents. As expected the active exchange capacity (sum of
395 equivalents of exchange species) closely follows the behaviour of moisture content profiles.
396 Solution concentrations of cations are displayed in Fig. 5c. Because anion transport is unaf-
397 fected by cation exchange anion concentrations are omitted. Profiles of sodium concentra-

398 tions clearly show a snowplough effect, where previously adsorbed sodium is exchanged for
399 the infiltrating calcium that has a higher concentration and higher preference for the ex-
400 change sites. Consequently, concentrations of desorbed sodium develop a characteristic
401 peak, which by far exceeds the initial and influent concentrations (Barry et al., 1983;
402 Bajracharya and Barry, 1993a, 1995).

403 With a 50-fold higher input of equivalents of calcium compared to the total exchange capac-
404 ity the retardation of the calcium concentrations is insignificant. From Fig. 5b it can be seen
405 that the exchange front, where sodium is released from the exchanger and replaced by cal-
406 cium, coincides with the arrival of the dispersed calcium front. Nevertheless, the moisture
407 front is still ahead of the exchange front. Accelerated moisture movement is due to the ini-
408 tial soil moisture that is mobilized by the infiltrating solution and pushed ahead of the event
409 water, as demonstrated in section 4.2. In this case, the rise of active exchange capacity be-
410 hind the exchange front accompanied by continued desorption of sodium causes the
411 skewed shape of the sodium peak. For steep moisture fronts and high initial moisture con-
412 tents the retardation of the exchange front behind the moisture front results in an insignifi-
413 cant rise of exchange capacity behind the exchange front, which is why under these condi-
414 tions moisture-dependent exchange can be ignored.

415 5.2 Dynamic hydraulic properties

416 This example illustrates an extension of RICH-PHREEQ for simulating the effect of mineral
417 reactions on unsaturated hydraulic properties in variably saturated flow conditions. The
418 model equations for the dynamic adjustment of parameters in the hydraulic functions were
419 derived in Wissmeier and Barry (2009). The model is based on the conversion between wa-
420 ter retention and pore size distribution via the Young-Laplace equation and implements a
421 thermodynamically sound adjustment of pore-size distributions due to precipita-
422 tion/dissolution reactions. In order to avoid further complications, the change in mineral
423 volume ΔV_M ($L^3 L^{-3}$) is translated to the entire pore-size distribution without respecting the
424 fact that part of the pores in the distribution may be dry and therefore unaffected by min-
425 eral reactions. Together with the van Genuchten/Mualem hydraulic model, the following
426 model equations apply:

$$\theta_{sat}^* = \theta_{sat} - \Delta V_M \quad (29)$$

$$\alpha_{GM}^* = \alpha_{GM} \left(\frac{\theta_{sat}^* - \theta_{res}}{\theta_{sat} - \theta_{res}} \right)^{\frac{1}{3}} \quad (30)$$

$$K_{sat}^* = K_{sat} \frac{\sqrt{\theta_{sat}^*} (\alpha_{GM}^*)^2 (\theta_{sat}^* - \theta_{res})^2}{\sqrt{\theta_{sat}} \alpha_{GM}^2 (\theta_{sat} - \theta_{res})^2}, \quad (31)$$

427 where * denotes adjusted hydraulic parameters after mineral reactions. Wissmeier and
 428 Barry (2009) provide a detailed discussion of the effect of mineral reactions on the hydraulic
 429 parameters of the van Genuchten/Mualem model in variably saturated conditions.

430 The following simulation is motivated by the frequently observed permeability reduction in
 431 reactive barriers, where minerals precipitate due to adsorption-desorption processes (Li et
 432 al., 2006; Parbs et al., 2007). In this case, a natron-saturated (Na_2CO_3) solution infiltrates
 433 into a calcite-saturated (CaCO_3) sandy clay loam (Leij et al., 1997) that contains a high-
 434 capacity cation exchanger at 2 cm depth whose sites are initially filled with calcium. Upon
 435 arrival of the infiltration front at the exchange layer, sodium ions exchange with adsorbed
 436 calcium due to their high concentration and increase calcium concentrations in solution. Be-
 437 cause of the large abundance of carbonate, calcite becomes supersaturated and precipitates
 438 following the exchange process. The precipitation of calcite leads to a reduction in the satu-
 439 rated moisture content and saturated conductivity but at the same time in a decrease of the
 440 van Genuchten parameter α_{GM} . The properties of the simulation are summarized in Table 5.

441 *Table 5 near here*

442 *Fig. 6 near here*

443 The results in Fig. 6 show the combined effect of cation exchange and mineral precipitation
 444 on the moisture profile of the infiltration front. The growing discontinuity in moisture con-
 445 tent results from the discontinuous hydraulic properties that follow calcite precipitation.

446 Solution concentrations show strong retardation compared to the moisture front caused by
447 (i) the exchange process with extremely high exchange capacity and (ii) the significant initial
448 soil moisture. The pH profiles illustrate the high pH of the infiltrating (pH 11.5) and initial
449 (pH 9.9) solutions but also the decline due to calcite precipitation. Sodium and carbonate
450 have similar concentration profiles. However, sodium is taken up from the free solution by
451 exchange whereas carbonate is precipitated as calcite as a consequence of exchange.

452 *Fig. 7 near here*

453 The results in Fig. 7 illustrate the development of the hydraulic parameters that accompany
454 the precipitation of calcite. The decrease in saturated moisture content θ_{sat} reflects directly
455 the volume of precipitated calcite according to Eq. (29). Together with decreasing pore vol-
456 ume, saturated conductivity as well as the van Genuchten parameter α_{GM} are decreasing.
457 The exchanger compositions that results from Na/Ca exchange are displayed in the two
458 lower graphs. Their profiles show that despite the larger preference for divalent ions, cal-
459 cium is completely removed from the exchanger due to the large abundance of sodium in
460 the infiltrating solution.

461 This simulation demonstrates the flexibility of RICH-PHREEQ to incorporate changes in flow
462 properties due to mineral reactions. The simulation shows the importance of the mutual
463 influence of flow and reaction, where flow sustains equilibrium reactions that lead to
464 changes in hydraulic properties and therefore influence back on flow dynamics.

465 An implication for the design of reactive barriers that results from this simulation could be
466 that the adsorbing substrate is pre-washed with a solution of similar major ion content
467 compared to those expected in field conditions in order to avoid large concentration con-
468 trast between the ions in the infiltrating solution and the exchanger.

469 5.3 Transport of organic complexes

470 Humic substances act as important chelating agents in natural environments. Due to their
471 complexing properties, soluble humic substances retain and transport heavy metals in the

472 unsaturated zone via colloid-facilitated solute transport (Carrillo-González et al., 2006;
473 Šimůnek et al., 2006a). In addition, they facilitate the biological uptake of trace metals and
474 other cations by several mechanisms, one of which is preventing their precipitation; another
475 seems to be a direct and positive influence on their bioavailability (Senesi, 1992; von
476 Wandruszka, 2000). In this example, we illustrate the release of complexed cadmium from a
477 soluble humic substance due to changing geochemical conditions during an infiltration
478 event. The organic substance is conceptualized as an assembly of mobile surfaces that con-
479 tains 20 different site types for mono and bidentate metal binding according to the WHAM
480 submodel V (Tipping, 1992, 1993). By including protonation/deprotonation reactions and
481 electrostatic forces the pH and ionic strength dependence of association reactions is consid-
482 ered (Dzombak and Morel, 1990). In addition to complexation, ions in the solution are
483 bound to the sorption sites' diffuse layer whose composition is calculated explicitly using a
484 fixed Donnan volume (Parkhurst and Appelo, 1999). The explicit calculation of ions in the
485 diffuse layer is required by the charge balance constraint. In contrast to more sophisticated
486 colloid transport models (Penrose et al., 1990; Šimůnek et al., 2006a; Bradford and
487 Torkzaban, 2008; Flury and Qiu, 2008), we assume conservative transport of the organic
488 substance.

489 The influent solution that enters the domain with a constant flux of 0.01 cm min^{-1} contains 1
490 $\mu\text{molal CdCl}_2$ together with 0.0466 kg m^{-3} of the humic substance. The humic substance is
491 composed of 3.97456×10^{-5} moles of sorption sites with a site density of $46,500 \text{ m}^2 \text{ g}^{-1}$
492 (Appelo, 2009). The surface assembly is initially in equilibrium with the influent solution. At
493 a depth of 2 - 6 cm the sandy clay loam (Leij et al., 1997) is amended with gypsum as a solid
494 phase, which is allowed to equilibrate with the soil solution. The parameters of the simula-
495 tion are given in Table 6.

496 *Table 6 near here*

497 *Fig. 8 near here*

498 The column profiles in Fig. 8a show the progressing moisture front together with pH. In the
499 profile, after 40 min a pronounced pH peak has developed directly ahead of the solute front.
500 The influence of the gypsum layer at 2 cm is still negligible. Since there is pure pre-event wa-
501 ter in the upper 2 cm, the pH peak is purely a result of diffusion/dispersion of the influent
502 solution. A pH change of similar magnitude is observed when diluting the initial humic solu-
503 tion with pure water in a simple batch simulation. This demonstrates the effect on ionic
504 strength and absolute solution concentration of surface complexes. When the solution
505 reaches the gypsum layer, its pH drops because free calcium is adsorbed to the humics. The
506 latter process is accompanied by releases of protons from surface functional hydroxyl
507 groups. Fig. 8b displays concentrations of calcium and cadmium in the solution. At the initial
508 moisture content, water fluxes are negligible and thus the solutes are immobile. As a result,
509 increased calcium concentrations appear behind the gypsum layer only at the latest profile
510 after 200 min when the moisture front has passed. When the humic substance enters the
511 gypsum layer, the competition with calcium for the adsorption sites leads to a strong release
512 of previously complexed cadmium. Calcium is provided by the dissolution of gypsum at con-
513 centrations that are 1.5×10^4 times larger than the initial cadmium concentration. Fig. 8c
514 shows concentrations of chloride and the humic substance, which are both unaffected by
515 complexation and mineral dissolution. Fig. 8d displays amounts of cadmium and calcite that
516 are bound to the mobile organic substance. The graph clearly shows the change from purely
517 cadmium-filled sites to calcium-dominated sites despite the sites' larger preference for
518 cadmium. However, the total moles of complexed ions decrease because of the lower pH.
519 Fig. 8e shows the large amount of calcium in the electric double layer (EDL) compared to
520 complexed calcium. Due to the large abundance of calcium in the gypsum layer, the fraction
521 of cadmium in the diffuse layer declines. Together with ionic strength, Fig. 8f shows the
522 negative charge in the EDL that counterbalances the overall positive charge of the com-
523 plexes. With increasing calcium concentration in the gypsum layer, the charge in the diffuse
524 layer decreases and indicates an increasing number of positively charged complexes.

525 This example illustrates the capability of RICH-PHREEQ to simulate complexation according
526 to the diffuse double layer model by Dzombak and Morel (1990) together with unsaturated
527 flow. In addition, it shows the scheme's flexibility in defining mobile adsorbents such as sur-

528 faces and ion exchangers. The simulation illustrates how heavy metals can be complexed by
529 organic substances and re-enter the free solution through changing geochemical conditions,
530 where they pose a potential threat to the aquatic ecosystem. It is shown that the humic
531 substance carries a positive charge in the given geochemical setting. Therefore, electrostatic
532 attraction may lead to additional reactions such as colloid adsorption in soils with high im-
533 mobile cation exchange capacity. The adsorption process can be included in the simulation
534 by defining additional reactions between the mobile adsorption sites and the immobile ex-
535 changer. In addition, processes such as attachment/detachment and straining/wedging also
536 result in non-conservative transport of the humic substance (McGechan and Lewis, 2002;
537 Flury and Qiu, 2008). In principle, RICH-PHREEQ allows for the extension to a full colloid-
538 facilitated transport similar to that published by Šimůnek et al. (2006a). However, this ex-
539 ample is intended to indicate the capabilities of RICH-PHREEQ without large modifications of
540 the original scheme.

541 **6 Concluding remarks**

542 RICH-PHREEQ solves for advective-dispersive transport of chemical elements (rather than
543 species), where the Darcy flux is calculated from a standard numerical solution of the mixed
544 form of Richards' equation. The verification examples demonstrate the accurate simulation
545 of liquid phase flow that results from transporting the entire ensemble of elements in the
546 solution. The explicit TVD scheme for advective solute transport increases the accuracy of
547 scheme at high grid Péclet numbers by transporting elements with negligible numerical dis-
548 persion. Species-dependent diffusion can be included by making use of PHREEQC's multi-
549 component transport capabilities and associated databases for diffusion coefficients.

550 Before every transport step, the moisture saturation of the soil is recalculated from the
551 mass of the solution constituents, which may change during the course of reaction calcula-
552 tions. Unlike existing software, our implementation considers moisture content-modifying
553 reactions, such as dissolution and precipitation of hydrated minerals. The influence of bio-
554 logical activity on the liquid phase saturation and composition can be incorporated using
555 appropriate stoichiometric reaction expressions. Since PHREEQC for Windows (Post, 2009)

556 explicitly calculates the solution density from the ion size database by Millero (2000), the
557 extension of RICH-PHREEQ to density-dependent unsaturated flow where the solution den-
558 sity changes due to geochemical reactions is envisaged.

559 The scope of RICH-PHREEQ is similar to HP1, which is a coupling of HYDRUS-1D and
560 PHREEQC (Jacques and Šimůnek, 2005; Šimůnek et al., 2006b). However, the presented
561 scheme is based on a different approach and thereby produces some beneficial features by
562 comparison:

- 563 – Easy extendibility due to implementation on a script level;
- 564 – Choice of standard weighting schemes for the computation of internodal conductivi-
565 ties;
- 566 – *A priori* consideration of moisture content modifying reactions;
- 567 – Accurate simulation of advection-dominated transport; and
- 568 – Compatibility with future releases of PHREEQC without additional maintenance.

569 Compared to other integrated codes for unsaturated reactive flow RICH-PHREEQ has the key
570 advantage of direct access to all PHREEQC reactions.

571 Disadvantages of RICH-PHREEQC are long computation times that could be significantly re-
572 duced by implanting an adaptive time-stepping procedure, and the lack of a graphical user
573 interface for user-friendly setup of liquid phase flow.

574 Nevertheless, RICH-PHREEQ offers a valuable tool for the simulation of non-standard prob-
575 lems of flow and reaction in the vadose zone. The examples show cases that can be simu-
576 lated with RICH-PHREEQ with only minor modifications but are beyond the scope of HP1 or,
577 to the authors' knowledge, any other model for flow and reaction in the vadose zone.

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580

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