WARPING-INCLUSIVE KINEMATIC COUPLING IN MIXED-DIMENSION MACRO MODELS FOR STEEL WIDE-FLANGE BEAM-COLUMNS

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ABSTRACT

A warping-inclusive kinematic coupling method to be used in finite element analysis of members featuring wide-flange cross-sections is proposed in this paper. This coupling method is used in mixed-dimension macro models that combine continuum and beam-column elements to reduce the computational cost of purely continuum finite element models. The proposed coupling method, utilizing either linear or nonlinear constraint equations, is implemented and validated in a commercial finite element software; the source code is made publicly available. Case studies indicate that including warping in the coupling formulation is critical for components that may experience coupled local and lateral-torsional buckling. Also highlighted is the potential of macro models to reduce the total degrees-of-freedom by up-to about 60 %, and computational memory use by up-to around 80 %, while retaining solution fidelity for beam, column, and panel zone components in
steel moment-resting frames. The case studies show that the linear constraint equation formulation may not be suitable for all problems, however, it may still yield acceptable results as long as the level of twisting is insignificant and lateral-torsional buckling is not critical.

Keywords: Continuum finite element analysis; Steel structures; Nonlinear geometric instabilities; Kinematic coupling; Beam-columns.

INTRODUCTION

Background

Simulation-based infrastructure design against natural hazards, such as earthquakes, relies heavily on the development of representative models of various fidelities (Krawinkler et al. 2006). Additionally, numerical models are necessary to evaluate the vulnerability of buildings and bridges and the social vulnerability of populations in at-risk communities (Deierlein and Zsarnóczay 2019). Within the scope of steel structures subjected to earthquake hazards, geometric instabilities are a significant source of structural damage, therefore, modeling this effect is a necessity. Models that simulate nonlinear geometric instabilities in steel structures under cyclic loading can be divided into three main categories of increasing fidelity and complexity: (a) point hinge models (concentrated plasticity); (b) distributed plasticity models (commonly using fiber beam-column elements); and (c) continuum finite element (CFE) models. Although fracture can be critical for steel components, the focus of this paper is mainly on models simulating component-level strength and stiffness deterioration due to local and/or member instabilities (member and lateral torsional buckling), as well as their interaction.

Point hinge models are placed at predefined locations of anticipated plasticity along a member. Concentrated plasticity models are known to be computationally efficient for parametric nonlinear analyses of metal structures (e.g., Ibarra and Krawinkler (2005)), and do not suffer from spurious localization issues during component deterioration. Sivaselvan and Reinhorn (2000) and Ibarra et al. (2005) proposed hysteretic models that capture the basic modes of cyclic deterioration in steel components, namely strength, post-peak strength and unloading stiffness deterioration. Others (Lignos and Krawinkler 2011; Lignos et al. 2019) have proposed ad hoc modifications to the Ibarra
et al. model’s hysteretic rules in an effort to properly describe the hysteretic behavior of steel members.

Point hinge models have become the main workhorse in collapse-risk assessment studies (FEMA 2009; NIST 2010), however, they exhibit considerable limitations. First, their calibration requires large component datasets (Lignos and Krawinkler 2013) that may not be always available. Second, point hinge models do not capture coupled geometric instabilities, and the effects of residual stresses on steel structures (Mathur et al. 2012). Third, the above approaches do not consider varying axial loads and bi-directional loading effects, nor do they capture axial shortening that may be influential in estimating earthquake induced losses (Elkady et al. 2020). These are major drawbacks, considering the findings from recent full-scale experimental studies (Suzuki and Lignos 2015; Ozkula et al. 2017; Elkady and Lignos 2018a; Cravero et al. 2020), along with corroborating CFE analyses (Elkady and Lignos 2015a; Fogarty and El-Tawil 2015; Elkady and Lignos 2018b; Wu et al. 2018).

The capability of standard flexibility- (Spacone et al. 1996) and displacement-based (Mari 1984) fiber-based beam-column elements to simulate the inelastic behavior of steel and composite-steel structures is well established (Hajjar et al. 1998; Sivaselvan and Reinhorn 2002). Numerical accuracy, for a viable computational expense, can be achieved by properly selecting the number of fibers to discretize the cross-section (Kostic and Filippou 2012). Standard fiber beam-column elements employ the plane-sections-remain-plane assumption that is violated by torsion warping present in wide-flange steel profiles. Theoretical developments have been proposed to address this challenge for fiber-based beam-column elements (Le Corvec 2012; Di Re et al. 2018). Inclusion of the warping component is critical for evaluating wide-flange steel components subjected to torsion and those that are susceptible to lateral-torsional buckling.

Continuum finite element component models are currently the only viable way to provide insights into complex instability interactions prior to global and partial structural collapse (Miyamura et al. 2015; Stoakes and Fahnstock 2016; Wu et al. 2018). CFE models are also suitable for detailed studies of critical regions of metal structures (Kalochairetis and Gantes 2011; Elkady and Lignos...
Furthermore, it is acknowledged that CFE simulations can serve as benchmarks for theoretical developments of advanced fiber beam-column elements, and for the further development of advanced experimental techniques (Whyte et al. 2016).

A current difficulty in employing CFE models in structural analysis is that they are computationally expensive for multiple nonlinear dynamic analyses of structures. Reducing the computational expense of simulation models in aforementioned studies (Miyamura et al. 2015; Wu et al. 2018) would be beneficial to increase the number of structural configurations and ground motions that can be investigated within any time-frame. Such a consideration is particularly important in, e.g., incremental dynamic analysis (Vamvatsikos and Cornell 2002), in which the computational complexity is further increased through the number of intensity levels analyzed for each ground motion. There is a clear need to reduce the computational demands from CFE component models while retaining the solution fidelity of this modeling approach. To achieve this goal, methods to reduce the associated computational cost in CFE simulations should be further developed.

The approach advocated in this paper is to employ a mixed-dimension component macro model, that combines domains of 1D beam-column elements with continuum domains comprised of 2D shell or 3D solid elements. Here, the dimensionality of the element refers to the number of dimensions used to parametrize the element geometry: 1D elements are lines, 2D elements are surfaces, and 3D elements are volumes. The mixed-dimension macro model idea is illustrated in Fig. 1a that shows the beam-column ($\Omega_1$) and continuum ($\Omega_2$) element domains for a wide-flange cross-section. Macro models can capture material plasticity and local instabilities in the continuum domains, while maintaining the computational efficiency of beam-column elements. A chief issue in mixed-dimension macro models, and the focus of this paper, is that there is a need to select an accurate coupling method between the beam-column and continuum domains that is also computationally friendly.
Coupling between the beam-column and continuum domains is typically accomplished through one of two means: the transition element approach, or the multipoint constraint (MPC) approach. One-dimension-to-continuum transition elements have been developed by Wagner and Gruttmann (2002), and were later extended by Chavan and Wriggers (2004) to include warping, and Koczubieć and Cichoń (2014) further extended these elements to the total Lagrangian formulation. More recently, Sadeghian et al. (2018) developed transition elements between 1D beam-column elements and 2D membrane elements for the analysis of reinforced concrete members. One limitation of transition elements is that they are typically less efficient than MPC approaches due to the proliferation of elements that connect all the nodes across both domains at the interface. Further limitations of transition elements include a proclivity to locking, and the need to treat such effects (Ho et al. 2010).

The MPC coupling approach specifies constraint equations among the relevant degrees of freedom (DOFs) between the two domains on the beam-column/continuum interface. The MPCs that form the coupling between domains can be implemented through any common constraint method (e.g., elimination, Lagrange multipliers, and the penalty method). Mixed-dimension MPC coupling methods have been developed by Monaghan et al. (1998) for coupling beam-column and solid domains by balancing the work done on the interface of each region. McCune et al. (2000, Shim et al. (2002) expanded on Monaghan et al. (1998)’s formulations to generalize for 1, 2, and 3D coupling. The notable limitations of this method are that linear constraint equations are enforced (i.e., the dependent DOFs have a linear relation to the independent DOFs), and that deformations due to warping are not transferred between the beam-column and continuum domains. Ho et al. (2010) proposed a multi-dimensional coupling for use with explicit time integration methods. Warping and shear deformations are not included in this method, and the reliance on explicit time integration limits the applicability of this approach. Song (2010) developed coupling between Timoshenko beam-column elements and 3D solid elements using transformation matrices. Again, warping was not included, and only linear material behavior and small deformations are considered. The lack
of warping transfer between the two domains limits the applicability of any coupling method for components experiencing to nonuniform torsion or susceptible to lateral-torsional buckling.

Kinematic coupling constitutes another class of MPC methods. In this method, displacement and/or rotation continuity of a dependent region of continuum element nodes is enforced based on the displacements and rotations of an independent beam-column node. Existing formulations (Dassault Systèmes 2014; Liu 2016) fall under the category of kinematic coupling as they enforce displacement continuity in static problems, and a mixture of velocity and displacement continuity in dynamic problems. Liu (2016) showed the capability of kinematic coupling utilizing nonlinear constraint equations for solving problems involving finite-rotations. However, torsion warping is again absent in these formulations. Incorporating warping into kinematic coupling, and studying the effect of doing so, are two aims of this paper.

Paper Objectives

Component models that mix elements of varying fidelity have been used previously in the assessment of steel structures (see e.g., Tada et al. (2008), Krishnan (2010), Sreenath et al. (2011), Imanpoure et al. (2016)), however, such studies have not focused on the effect of the coupling method between the different domains. Further investigation into this matter is necessary to develop accurate and efficient modeling recommendations for a wide range of steel beam-column components. The main issues apparent in the reviewed coupling methods are that transition elements that include warping may not be computationally efficient when compared with the MPC approach, and that existing MPC approaches do not consider warping that can be critical for beam-columns susceptible to coupled local and lateral-torsional buckling. Furthermore, nonlinear constraint equations should be employed for modeling collapse limit states often of interest in earthquake engineering.

An MPC kinematic coupling method that includes torsion warping is proposed to address the aforementioned considerations. This paper focuses on answering the following questions: What is the effect of warping-inclusive coupling on simulation results? When are formulations with linear constraint equations acceptable? What are the computational benefits of the component macro model approach? Linear and nonlinear versions of the proposed coupling method are
implemented; the proposed coupling method is found to be easily compatible with existing finite
element analysis software typically used in practice. A series of computational case studies are
then used to evaluate each of questions posed above using the proposed MPC formulation and an
existing coupling method that is available in commercial software.

PROPOSED COUPLING METHOD

Beam-Column Element Kinematics

A coupling strategy for beam-column and continuum elements requires an understanding of
the governing element kinematics. The beam-column kinematics used herein are based on a line
of centroids and a set of cross-section planes that rotate about each of the centroid points, see
e.g., Simo and Vu-Quoc (1991). Following this reference, a warping function is also defined on
the cross-section to incorporate out-of-plane torsion warping. Each beam-column element node is
assumed to have three displacement DOFs, three rotation DOFs, and one torsion warping DOF.

Beam kinematics are defined by material points, denoted $x_{mp}^b$, where the subscript indicates
that this is a material point and the superscript indicates the beam-column element. The position
of any material point in the cross-section defined by the beam element formulation is as follows:

$$x_{mp}^b(\xi, \eta, \zeta) = x(\zeta) + \xi n_1(\zeta) + \eta n_2(\zeta) + w(\zeta)\psi(\xi, \eta)t(\zeta), \quad (1)$$

where $\zeta$ is the coordinate along the element centerline, $\xi$ and $\eta$ are the distances measured along
the $n_1$, $n_2$ axes in the undeformed configuration, $x(\zeta)$ is the position of the centroid along the
centerline in the undeformed configuration, $w(\zeta)$ is the warping amplitude, $\psi(\xi, \eta)$ is the warping
function, and $t(\zeta)$ is the axis in the direction of the cross-section normal. The representation of
these parameters is shown in Fig. 2a for a wide-flange profile, and leads to the limits of $0 \leq \zeta \leq L,$
$-b_f/2 \leq \xi \leq b_f/2$ and $-d/2 \leq \eta \leq d/2$, where $L$ is the member length; $b_f$ is the flange width;
and $d$ is the full cross-section depth. The first term in Eqn. 1 is the position of the cross-section
with respect to the centerline, the second and third terms represent the cross-section plane spanned
by $n_1$ and $n_2$, and the fourth term is the out-of-plane deformation due to torsion warping.
Only wide-flange sections are considered within the scope of this paper, however, the proposed coupling method may be used with other cross-sections provided that the warping function is readily available at each point on the cross-section. The warping function used in this paper is based on elastic homogeneous thin-walled open cross-sections, and for wide-flange cross-sections can be simply defined as (Chen and Atsuta 2008)

\[ \psi(\xi, \eta) = \xi \eta. \]  

(2)

**Nonlinear Definition of the Constraint Equations**

The kinematic coupling method proposed in this paper is composed of constraint equations that relate the beam-column and continuum domains, as defined in Fig. 2b. The beam-column domain is \( \Omega_1 \), the continuum domain is \( \Omega_2 \), and the interface between these two domains is \( \Gamma \). The proposed coupling method stipulates that the beam-column node is coupled to the three displacement DOFs of the continuum nodes on the interface, therefore, coupling of rotation DOFs in the continuum domain are neglected (e.g., in shell elements). This method is referred to as the Warping-Inclusive Kinematic Coupling, or WIKC, herein, and can be identified as an enhancement to kinematic coupling methods, for instance, in Abaqus (Dassault Systèmes 2014). Each of the constraint equations are a function of seven variables owing to the seven DOFs of the beam-column node, and in total \( 3N \) equations are defined for \( N \) continuum nodes.

First, the constraints for a single continuum node are presented without warping, and afterwards the warping term is included. The nonlinear, rigid-body coupling constraint equations without warping are written succinctly in matrix-vector form for a single node as

\[ \mathbf{u}^c = \mathbf{u}^b + \mathbf{R}^b l - l, \]  

(3)

where \( \mathbf{u}^c \) is the vector of continuum displacements, \( \mathbf{u}^b \) is the vector of beam-column node displacements, \( l \) is the link vector that relates the continuum node initial position to the beam-column node initial position, \( (l = \mathbf{x}^c - \mathbf{x}^b, \mathbf{x}^c \) is the initial position of the continuum element node, and
The initial position of the beam-column node, as shown in Fig. 2a. The term $R^b l$ is the link rotated into the deformed configuration by the rotation matrix $R^b$. The definition of each of these variables is shown in Fig. 2b, and the deformed configuration is shown in Fig. 2c.

The rotation matrix, $R^b$, is formed from the rotation vector of the beam-column node, $\phi^b$, using the well-known Rodrigues formula (Rodrigues 1840):

$$r = \|\phi^b\|, \quad r = \phi^b / r,$$

$$R^b = \cos[r]I + (1 - \cos[r])rr^T + \sin[r][r]_\times,$$ (4b)

where $I$ is the $3 \times 3$ identity matrix, and $[r]_\times$ is the skew-symmetric matrix formed from $r$, i.e.,

$$[r]_\times = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix}.$$ (5)

Including warping, the constraint equations that constitute the warping-inclusive kinematic coupling method become

$$u^c = u^b + R^b l - l + \psi w^b R^b t,$$ (6)

where $\psi$ is the warping function evaluated at $\xi$ and $\eta$ of the node, $w^b$ is the value of the warping DOF at the beam-column node, and the term $R^b t$ is the orientation of the local $t$-axis in the deformed configuration. Finally, the standard, or homogenous, form of the constraint equations is

$$f := u^c - u^b - R^b l + l - \psi w^b R^b t = 0.$$ (7)

**Linearized Constraint Equations**

Linearized constraint equations are necessary for implementation of the coupling in a nonlinear incremental-iterative analysis (e.g., Newton-Raphson). The key components of the linearized constraint equations are the constraint coefficient matrices, $A_1$ and $A_2$, that are based on the
standard form of the constraint equations. These two matrices are the coefficients of the beam-
column and continuum nodes, respectively, from the linearized constraint equation. Linearizing
Eqn. 7, $A_1$ and $A_2$ are defined in Eqn. 8 as follows:

$$
\delta f = A_1\delta u^e + A_2 \begin{bmatrix} \delta u^b \\ \delta \theta^b \\ \delta w^b \end{bmatrix} = 0,
$$

(8)

where $\delta \theta^b$ is the linearized rotation vector. Carrying out the linearization, the following expression
is obtained:

$$
\delta f = \delta u^e - \delta u^b - R^b l \times \delta \theta - \psi w^b R^b t \times \delta \theta - \psi R^b t \delta w^b,
$$

(9)

where, for any rotation field, $\delta (R^b l) = \delta \theta \times R^b l$ (see, e.g., Sec. 1.3.1 of Dassault Systèmes (2014)).
Additionally in Eqn. 9, the term $\delta l = 0$ because this vector is considered to be constant throughout
the analysis (i.e., deformation of the cross-section due to Poisson’s effect is neglected). From
Eqn. 9, the $A_1$ matrix related to the displacement $s$ of the beam-column node is equal to the identity
matrix, $I$, since all the coefficients are equal to unity. Collecting the remaining terms in Eqn. 9,
and using the matrix definition of the cross-product, the $A_2$ matrix of dimension $3 \times 7$ becomes

$$
A_2 = \begin{bmatrix} A_2^{disp} \\ A_2^{rot} \\ A_2^{warp} \end{bmatrix} = \begin{bmatrix} -I, -[R^b l + \psi w^b R^b t]_x, -\psi R^b t \end{bmatrix}.
$$

(10)

To close, a few remarks are made regarding $A_2$:

- The $[R^b l]_x$ term can be interpreted as forces acting at the continuum node due to bending/(pure)torsion acting on the beam-column node in the current deformed configuration.
- The $[\psi w^b R^b t]_x$ term can be interpreted as a nonlinear effect of warping that arises because
of the displacement component in the local $t$-axis between the continuum and beam-column
nodes.
- The coupling method is nonlinear because the $A_2^{rot}$ and $A_2^{warp}$ matrices depend on the
current rotation $R^b$, therefore, the $A_2$ matrix needs to be re-computed at each iteration in the analysis.

### Linear Coupling Method

A linear version of the WIKC method is derived for use in evaluating linear coupling methods. The linear constraint coefficient matrix, $A^{lin}$, can be recovered from Eqn. 10 by considering the current configuration always equal to the initial configuration. This is done by replacing $-\left[R^b l + \psi w^b R^b t\right]_x$ with $-[l]_x$ in $A^{rot}$, and replacing $-\psi R^b t$ with $-\psi t$ in $A^{warp}$. With these considerations, the linear version of the $A_2$ matrix becomes

$$A^{lin}_2 = \begin{bmatrix} -I, & -[l]_x, & -\psi t \end{bmatrix}.$$  \hfill (11)

The governing linear constraint equations can be recovered by applying $A_2$ to the vector of beam-column DOFs, and considering that $l_3 = 0$ as the continuum and beam-column nodes are assumed to be in the same plane initially:

$$u_c = A^{lin}_2 \begin{bmatrix} u^b \\ \phi^b \\ w^b \end{bmatrix} \Rightarrow u_c = \begin{bmatrix} u_1^b - \phi_3^b \eta, \\ u_2^b + \phi_3^b \xi, \\ u_3^b + \phi_1^b \eta - \phi_2^b \xi + \xi \eta w^b \end{bmatrix}.$$  \hfill (12)

### CASE STUDIES

A series of case studies are investigated in this paper to: (a) validate the proposed WIKC method, illustrate the importance of including warping in the constraint equations, (b) assess the applicability of the linear coupling method, (c) and highlight the computational efficiency of the proposed macro models. To achieve these objectives, four computational models are compared for each of the cases described in subsequent sections: (1) a full-shell model that represents the continuum “benchmark” analysis (denoted as Full-shell); (2) a beam-shell macro model using a “built-in” kinematic coupling formulation that does not transfer warping across the interface (denoted as Macro Built-in); (3) a beam-shell macro model using the proposed nonlinear WIKC (denoted as Macro Warping); and
(4) a beam-shell macro model using the proposed linear WIKC (denoted as Macro Lin. Warping).

All of the computational work in this paper is carried out using Abaqus v6.14 (Dassault Systèmes 2014). Both the linear and nonlinear versions of the WIKC method are implemented in Abaqus using MPC user subroutines; the source code, instructions for use, and examples are available in Hartloper (2020). The following sections provide a description of the case studies, their links with practical applications, and a set of general modeling recommendations. Afterwards, the results for each case are presented and discussed.

Case Studies Overview

The case studies investigated in this paper are summarized in Table 1. This table gives a brief description of the primary mode of deformation, the components and their respective cross-sections, and the stability of the anticipated equilibrium path. The objectives of Case Study 1 are to demonstrate the error in the elastic torsional stiffness of mixed-dimension macro models when warping is not included in the coupling formulation, and to identify the limits of the proposed macro model approach when twisting is significant. Case Studies 2–4 demonstrate the implications of this issue in the context of steel components subjected to cyclic loading. These examples are based on structural components in steel moment-resisting frames (MRFs), subjected to both quasi-static and dynamic loading, as shown schematically in Fig. 3. Through these four case studies, the proposed macro model approach and coupling method are evaluated for main structural components in steel MRFs.

Case Study 1: Nonuniform Torsion

Case Study 1 is based on a seminal experiment conducted by Farwell and Galambos (1969) using a 6 × 6-25 beam of ASTM-A36 (nominal $f_y = 250$ MPa) steel subjected to a central torque. The geometric properties of the 6 × 6-25 beam are provided in Table 2, and a schematic of the test set-up is provided in Fig. 4. The test consists of a steel beam fixed against rotation at the ends but free to warp. A torque is applied at the center of the beam through the circular loading plate. The beam ends are assumed to be restricted in axial displacement due to friction at the beam end supports, as this condition provides the best match with the test data. For this reason the centroid
of the beam is considered as axially fixed at the ends. The equilibrium path in Case Study 1 is stable (i.e., no buckling occurs) as the beam is only subjected to torsion. The continuum lengths are nominally chosen as $L_{\Omega_2} = 0.25L = 482.5$ mm at both member ends for the macro models.

**Case Study 2: Interior Subassembly**

Case Study 2 focuses on an interior subassembly, in this case represented by the DBBW specimen tested by Engelhardt et al. (2000) as a part of the SAC\(^1\) Program (FEMA 2000). The beams in the subassembly have a W36X150 cross-section, and the column is a W14X398, dimensions of the components are provided in Table 2, and a schematic of the subassembly is shown in Fig. 5b. This table also includes the normalized LTB slenderness per CEN (2005), the ratio of unbraced length to weak-axis radius of gyration ($L_b/r_y$), and the limiting length for inelastic LTB ($L_r$) per ANSI/AISC 360-16 (AISC 2016). The material for all components is ASTM A572 Gr. 50 steel ($f_y = 345$ MPa). The beams have roller supports near the ends and lateral supports at the ends, and a cyclic displacement-controlled load is applied at the column top. The column bottom is pinned, and the top is fixed against out-of-plane displacements and has a flexible torsional support that is modeled with a stiffness equivalent to two 1.5 m W14X150 beams. Both beam-to-column connections include radius-cut reduced beam sections (RBS), and the subassembly was designed with a balanced panel zone concept that leads to yielding in the panel zone and buckling in the RBS region, therefore, the equilibrium path is designated as unstable. Based on observations of the Full-shell model, the continuum lengths are chosen to be approximately 400 mm greater than the extend of the RBS cut-out in the beams ($L_{\Omega_2} = 1300$ mm), and approximately 400 mm greater than the beams on both sides for the column ($L_{\Omega_2} = 1700$ mm).

**Case Study 3: Quasi-Static Collapse-Consistent Loading of First-story Column**

Case Study 3 focuses on a typical first-story column with a relatively compact cross-section ($b_f/2t_f = 5.92$, $h/t_w = 33.2$), in this case represented by the C5 test carried out by Elkady and Lignos (2018a). The column cross-section is a W24X146, with a length of 3.9 m, the geometric

\(^{1}\)Joint venture between the Structural Engineers Association of California (SEAOC), the Applied Technology Council (ATC), and California Universities for Research in Earthquake Engineering (CUREe)
properties are summarized in Table 2. The column is not critical for LTB since $\tilde{\lambda}_{LT} < 0.4$. A unidirectional collapse consistent load protocol (Suzuki and Lignos 2020) is applied in the strong axis of the column ($u_y$), and a constant compressive 20 % of the measured axial yield load is applied at the column top ($-F_z$), as illustrated in Fig. 5c. Rotations and displacements are fixed at the column base, and a rotationally flexible boundary in the strong axis is assumed at the column top by synchronizing $\phi_x$ with $u_y$. The basis of the flexible boundary is to obtain an inflection point at $z = 3/4L$ when the column is elastic, representative of first-story columns in steel moment-resisting frames Elkady and Lignos (2018a). The equilibrium path is classified as unstable because local buckling is observed in the test, thereby causing cyclic deterioration in strength and stiffness. The continuum lengths are chosen as $L_{\Omega_2} = 0.5L = 900$ mm at both member ends for the macro models to capture the local buckling at the member ends.

*Case Study 4: Column Nonlinear Response History Analysis*

Case Study 4 focuses on an equivalent single-degree-of-freedom type representation of a typical first story column. The column selected for this example is based on the interior first-story column in a prototype four-story steel MRF designed for urban California (Elkady and Lignos 2015b). The column cross-section is a W24X94 with a length of 5500 mm, the geometric properties are summarized in Table 2, and a schematic of the model is provided in Fig. 5d. The criticality of including warping in the coupling formulation is demonstrated in this case since the column has a typical cross-section and is susceptible to inelastic lateral-torsional buckling ($\tilde{\lambda}_{LT} > 0.4$).

All displacements and rotations are fixed at the column bottom, and out-of-plane displacements and all rotations are fixed at the column top. A constant axial load corresponding to 20 % of the expected axial yield strength of the cross-section ($F_z = 1300$ kN) is applied at the top of the column, and the Northridge 1994 Canoga Park record is applied to the base of the column ($a_{g,y}$). The first-mode period of the target prototype structure is about 1.5 s (Elkady 2016), therefore, the mass of the column is chosen such that the first-mode period is equal to 1.5 s. This procedure results in a mass of 766.5 s$^2$N/mm, and 2 % mass proportional damping is assumed. A frequency analysis is conducted to validate the computed mass and the results are presented later. The shell lengths
are chosen as \( L_{\Omega_2} = 0.3L = 1650 \text{ mm} \) at both member ends for the macro models to capture the coupled buckling along the member length.

**Nonlinear Finite Element Modeling Approach**

Modeling guidelines for the Full-shell models are based on those of Elkady and Lignos (2018b), and the macro models are then adapted from the full-shell models. A later section also includes a methodology for generating and imposing imperfections for continuum mechanics problems containing an unstable equilibrium path, and afterwards a summary of the model and imperfection properties is provided for all the case studies.

Numerical details of the analyses carried out in Abaqus v6.14 are outlined for clarity. Implicit time integration is used for both the quasi-static and dynamic problems. The system of equilibrium equations is solved using Newton’s method with double precision. The default convergence criteria of 0.5 \% on the relative force and moment residuals is used for all analyses.

Based on Elkady and Lignos (2018b), four-node reduced integration shell elements (S4R) are used in the continuum domains. The Abaqus enhanced hourglass stiffness control is used (Dassault Systèmes 2014), and five Simpson integration points are used throughout the shell thickness. A mesh size of \(<25 \text{ mm}\) is used in the continuum domains where buckling may occur, and the element size is relaxed in the column of Case Study 2 since only yielding of the panel zone occurs. Linear, two-node beam-column elements with torsion warping (B31OS) are used in the beam-column element domains. These are displacement-based beam-column elements with a single integration point at the center of the element, and 7-DOFs at each node. Cross-section properties and forces are evaluated in these beam-column elements at five integration points in each flange, and five integration points along the web (Dassault Systèmes 2014). A minimum of four beam-column elements are used in each \( \Omega_1 \) domain so that possible member buckling can be captured in the macro model. Extents of the beam-column and shell element domains, i.e., the length \( L_{\Omega_2} \) in Fig. 1a, are provided for all the cases in Table 3.

The Voce-Chaboche (VC) nonlinear kinematic/isotropic hardening material model native to Abaqus v6.14 is used in all the beam-column and shell domains to represent the ASTM A992 Gr. 50
and ASTM A572 Gr. 50 materials subjected to cyclic straining. Consistent model parameters for the A992 Gr. 50 steel are taken from de Castro e Sousa et al. (2020b). VC model parameters for the A572 Gr. 50 steel are calibrated using the tensile-only approach of de Castro e Sousa et al. (2020a) using the SAC average stress-strain curve (Engelhardt et al. 2000). Employed parameters for both materials are provided in Table 4, where a Poisson’s Ratio of $\nu = 0.3$ is always assumed. One limitation of the employed Abaqus v6.14 beam-column elements is that only one material may be specified for the element. The flange material properties are assumed throughout the beam-column domains because yielding is primarily expected to occur in the flanges due to the axial stress gradient from bending. Residual stresses are not considered in any of the numerical models because the beam-column elements in Abaqus v6.14 do not allow for residual stress variations through the cross-section, this issue is discussed later in the Limitations section.

**Modeling Exceptions for Case Study 1**

The following exceptions to the aforementioned guidelines are made for Case Study 1: the mesh size is slightly smaller than that recommended by Elkady and Lignos (2018b) due to the smaller cross-section dimensions, and an increased number of quadratic beam-column elements (B32OS) are used in $\Omega_1$. The increased elements and additional integration point along the length are required to capture the yielding throughout the member, whereas this is not necessary for the other case studies since yielding is focused at the member ends in the continuum domains. Furthermore, a beam-column element only model (denoted “Full-beam”) is used in this case study to compare a beam-column element only model with a shell element only model. An exception is also made for the ASTM-A36 material properties in this case study: a piece-wise isotropic hardening model is used to represent this steel material since only monotonic loading is applied. The parameters for the ASTM-A36 material are $E = 213400$ MPa, $\nu = 0.3$, $\sigma_{y,0} = 285$ MPa and hardening defined by 289 MPa at $\varepsilon^p = 0.01329$ and 876.2 MPa at $\varepsilon^p = 0.094$ as assumed in Pi and Trahair (1995).

**Imperfection Methodology**

Imperfections are critical when geometric instabilities are expected in nonlinear analysis problems (Galambos 1998; Ziemian 2010; AISC 2016), for this reason, care is taken in describing the
approach used to apply imperfections. Member out-of-straightness and out-of-plumbness imperfections are deemed critical for flexural buckling modes, and local flange and web imperfections are deemed critical for local buckling modes. Twisting imperfections are also considered to be critical for modes associated with lateral-torsional buckling. Flexural buckling modes are not critical in the analyses examined herein, and therefore, out-of-straightness and out-of-plumbness imperfections are not considered.

Typically, imperfections may be included by applying scaled buckling modes from elastic eigenvalue analysis to the geometrically perfect model (Fogarty and El-Tawil 2015; Elkady and Lignos 2018b; Cravero et al. 2020). One issue with this typical method is that the similitude between the full-continuum and macro models is not guaranteed because the presence of the beam-continuum interface will influence the computed buckling modes. Using the exact same imperfections in the full-shell and macro models is considered important to remove a potential source of bias when comparing the two modeling approaches. An alternative method is proposed in this study to derive local imperfection geometries that are applied directly to the shell domains. This proposed method also addresses an anticipated challenge of applying geometric imperfections to members in system-level simulation studies where the member buckling modes may not be available for each individual component.

The proposed imperfection method, implemented in the Python package pywikc (Hartloper 2020), defines a local imperfection geometry according to elastic plate buckling theory. The shape of the imperfection is defined as a function of the buckling wavelength, $L_{bw}$, and the maximum imperfection amplitude. The buckling wavelength is determined using elastic eigenvalue buckling analysis of the component, and the maximum imperfection amplitude is based on previous studies and measurements of wide-flange cross-sections (Hartloper and Lignos 2019). First, the imperfection shapes are defined, then methods for obtaining the maximum amplitudes and $L_{bw}$ are defined.

Equations describing the buckled shape along the width of the flanges and web are adapted from Hill (1940) and Haaijer (1956). The mathematical model for the half-flange plate is shown in
Fig. 6. A uniform compressive stress, $\sigma$, is assumed to act along the two fixed edges, while one edge is free and the other edge has a rotational restraint (RR) from the web. The shape along the flange width in the $n_1$ direction is described by

$$v_\xi(\tilde{\xi}) = \tilde{\xi} + \frac{\varepsilon}{2a_3}\left(\tilde{\xi}^3 + a_1\tilde{\xi}^4 + a_2\tilde{\xi}^3 + a_3\tilde{\xi}^2\right),$$  

(13)

where $a_1 = -4.963$, $a_2 = 9.852$, $a_3 = -9.778$, $\varepsilon$ is the relative restraint per unit width provided by the web, and $\tilde{\xi} = \xi/(b_f/2)$. The model for the web plate is similar to that of the flange, but both unloaded edges have a rotational restraint due to the flanges. The shape along the width of the web in the $n_2$ direction is

$$v_\eta(\tilde{\eta}) = \frac{\pi\varepsilon}{2}\left(\tilde{\eta}^2 - 0.25\right) + (1 + \varepsilon/2)\cos(\pi\tilde{\eta}),$$  

(14)

where $\varepsilon$ is the relative restraint per unit width provided by the flanges, and $\tilde{\eta} = \eta/(d - t_f)$. The shape of both the flange and web plates in the $t$ direction is needed to complete the imperfection. A function is defined to represent a single buckling wave that satisfies the fixed boundary conditions at the loaded edges, a suitable function is

$$v_\xi(\zeta) = \sin^2[\pi\zeta/L_{bw}].$$  

(15)

The final buckled shape is obtained by multiplying $v_\xi$ with $v_\xi$ for the flange, and $v_\eta$ with $v_\zeta$ for the web. The complete buckled shape for the flange is shown in Fig. 6b. Contours of the two functions $v_\xi$ and $v_\zeta$ are shown projected onto the axes to give a sense of the effect of these functions on the imperfection geometry. The normalized vector of the flange imperfection is obtained from Eqns. 13 and 15:

$$v_f(\tilde{\xi}, \zeta) = \frac{v_\xi(\tilde{\xi})v_\zeta(\zeta)}{\max[v_\xi(\tilde{\xi})v_\zeta(\zeta)]}n_2.$$  

(16)

The normalized vector of the web imperfection is obtained from Eqns. 14 and 15:

$$v_w(\tilde{\eta}, \zeta) = \frac{v_\eta(\tilde{\eta})v_\zeta(\zeta)}{\max[v_\eta(\tilde{\eta})v_\zeta(\zeta)]}n_1.$$  

(17)
The max is used in these equations so that the maximum imperfection amplitude can easily be made equal to a pre-specified value.

Finally, local imperfection fields for the entire member are defined through $v_{flange}$ and $v_{web}$:

$$v_{flange} = \begin{cases} 
    a_f v_f(\tilde{\xi}, \zeta) & \text{if } 0 \leq \zeta \leq L_{bw} \\
    -a_f v_f(\tilde{\xi}, \zeta - (L - L_{bw})) & \text{if } L - L_{bw} \leq \zeta \leq L 
\end{cases}$$

$$v_{web} = \begin{cases} 
    a_w v_w(\tilde{\eta}, \zeta) & \text{if } 0 \leq \zeta \leq L_{bw} \\
    -a_w v_w(\tilde{\eta}, \zeta - (L - L_{bw})) & \text{if } L - L_{bw} \leq \zeta \leq L 
\end{cases}$$

where $a_f$ is the maximum flange imperfection amplitude, and $a_w$ is the maximum web imperfection amplitude. The different cases in these equations lead to an anti-symmetric in-plane local imperfection pattern at the bottom and top of the member, i.e., the imperfection is “in-wards” at one end and “out-wards” at the other. This mode is chosen on the basis of recommendations validated with beam-column experiments (Suzuki and Lignos 2015; Elkady and Lignos 2018a; Cravero et al. 2020), and also agrees with expected cross-section imperfections due to the hot-rolling process and manufacturing (e.g., welding) (CEN 1993; ASTM 2016). Fig. 7b illustrates the anti-symmetric buckling pattern applied to an analysis model, and Fig. 7c shows the local buckling pattern present at the base of a W24x84 column subjected to 20% axial load and a symmetric cyclic lateral loading history.

The local buckling wavelength, $L_{bw}$, of the local imperfections is unknown through the aforementioned procedure, and is determined in this study by matching with elastic eigenvalue buckling analysis in Abaqus. The length $L_{bw}$ is chosen as the wavelength of the end-most local buckle present in a local buckling dominated mode. This end-most wavelength is deemed to be critical because local buckling is known to initiate at the member ends due to the moment gradient in columns subjected to lateral loads (Elkady and Lignos 2018a). A local buckling dominated mode from eigenvalue analysis is shown in Fig. 7a with the proposed method for applying local imperfections in Fig. 7b.
The last ingredient of the local imperfections are the maximum amplitudes $a_f$ and $a_w$. The local imperfection amplitudes are based on the compatible minimum of $d/300$ and $b_f/250$ that have been used in prior studies based on measured imperfections (Hartloper and Lignos 2019), and are similar to those in Elkady and Lignos (2018b). Compatibility in this proposed method is defined by maintaining the 90 degree angle between the flange and web plate centerlines, therefore, only one of the maximum amplitudes $a_f$ or $a_w$ will be reached depending on the section geometry. Note that these imperfection amplitudes are below the tolerances set forth in standards such as (CEN 1993; ASTM 2016).

Local imperfections are incorporated into the simulation models by applying the imperfection fields, Equations 18a and 18b, to the continuum domains. This method generates a set of nodal perturbations that can be easily applied to the initially perfect geometry. The computation of the local imperfection geometries has been implemented by the authors in the Python package pywiki for wide-flange cross-sections (Hartloper 2020).

Twisting imperfections, occurring due to base plate welding, have typically been applied similarly to local imperfections by applying a scaled buckling mode, or the nodal perturbation could be specified directly. In the context of beam-shell macro models, however, the twisting imperfection cannot be applied as an initial geometric imperfection in Abaqus because only initial displacements can be specified to the nodes. This is an issue because the twisting imperfection is specified about the member centerline, and therefore, the beam-column element centerline only has a rotation, and does not have a displacement.

Similitude between the full-shell and macro models is obtained in this case by applying a torque at the center of the model that leads to a twist equivalent to those measured in full-scale experiments, similar to the notional load concept (AISC 2016). Twist magnitudes measured in Elkady (2016) are used as a basis of expected amplitudes. The average twist magnitude measured at the column top was 0.6 %rad, the minimum was 0.0 %rad, and the maximum was 1.5 %rad. Torques are applied that lead to a mid-height twist equal to half the top twist assuming that initial angle of twist is constant along the member. Concentrated torques are simple to apply to the beam-column
elements by ensuring a node exists at the center of the beam-column domain, and a distributed coupling in Abaqus v6.14 is used to apply the concentrated torque to the full-shell models in a region around the mid-height of the member.

The imperfection parameters for all the components included in the case studies are provided in Table 3. For the beams in Case Study 2, $v_{flange}$ and $v_{web}$ are applied to the entire RBS region ($L_{bw} = 686$ mm) with an offset equal to the distance from the beam end to the start of the radius cut. For Case Studies 3 and 4, the length of the local buckling wave was found to be around 1.2$d$.

The twisting imperfection in Case Study 3 is based on the measured value, while the twisting imperfection in Case Study 4 is chosen near the maximum of the measured values in Elkady and Lignos (2018a). Finally, no imperfections are included in components with stable equilibrium paths because geometric instabilities are not expected, thereby diminishing the importance of imperfections (Galambos 1998; Gantes and Fragkopoulos 2010; Ziemian 2010).

**CASE STUDY RESULTS**

Results in the form of component behavior and computational effort are reported for each of the case studies. Computational effort in each case is measured in three possible ways: the total number of DOFs in the model, the amount of memory required to run the analysis, and the number of iterations required throughout the analysis. The total number of DOFs is representative of the size of the problem being solved, and is correlated to the effort required to solve each iteration of the Newton-Raphson method. The memory required represents a base-line requirement to run the numerical model. The number of iterations required is indicative of the rate of convergence; the rate of convergence can be used to validate the implementation of the WIKC method, and is a major factor in the total analysis time. Direct measurements of the analysis time are not given herein because they are highly dependent on the computational system used to solve the problem, whereas the included metrics are independent of the utilized system.

**Case Study 1: Nonuniform Torsion Results**

Torque-twist responses of the finite element models along with the physical data is shown in Fig. 8. From Fig. 8a, it is clear that the Full-shell model captures the component behavior
throughout the entire load path, while the other modeling options may only be accurate in the initial loading stages (see Fig. 8b). The Macro (Warping) model with nonlinear WIKC begins to lose accuracy with respect to the Full-shell model after a twist of around 0.25 rad because the Abaqus beam-column elements are only valid for small angles of twist (Dassault Systèmes 2014), as seen in the Full-beam model response as well. Results in Fig. 8b also indicate that the linear WIKC method is viable for components that experience twists of up-to around 0.05 rad. In the context of steel MRF components, the maximum observed twist in full-scale experiments on wide-flange columns is in the neighborhood of 0.10 rad (Elkady 2016; Elkady and Lignos 2018a), therefore, the Macro (Warping) model employing the nonlinear WIKC at beam-continuum interfaces should be suitable for most applications, while the linear version presents a limitation in this regard.

A notable divergence is found in the elastic torsional stiffness of the Macro (Built-in) model compared with the other options, as the elastic torsional stiffness is computed to be 226 kN-m/rad and this quantity is around 90 kN-m/rad for the Full-shell model. Differences in the elastic stiffnesses is clearly visible in Fig. 8b. The built-in kinematic coupling in Abaqus, that does not transfer warping, creates a warping-fixed boundary in the continuum domains at the interface, this in-turn leads to the increase in the torsional stiffness. This issue will have later implications in the response of steel MRF components subjected to cyclic loading.

Accuracy of the coupling method can also be assessed in this case by comparing the von Mises equivalent stress distributions at the interface shell elements in Fig. 9 near the point of yielding \( M_c = 11.25 \text{ kN-m} \). The stress distribution of the Macro (Warping) model is closer to the Full-shell model when compared with the Macro (Built-in) model, particularly in the web elements. The rate of convergence is also important for validating the implementation of the proposed WIKC method and to assess the overall computational effort required. To this end, a plot of the total number of iterations in the analysis with respect to the applied torque is provided in Fig. 10. This figure shows that the convergence rate of the Macro (Warping) model is comparable to the Full-shell model up to the point of divergence between these models at around 12 kN-m.
Case Study 2: Interior Subassembly Results

The overall subassembly behavior in terms of story drift ratio (SDR = column tip displacement / initial column length) and column tip load is shown in Fig. 11. The Full-shell model is able to capture reasonably well the pre- and post-peak behavior of the subassembly, note that the relative error in the maximum absolute column tip load is around 2 % between the Full-shell and Test Data. There is also good agreement between the Full-shell model unloading stiffness and the test data prior to the last cycle of loading, indicating that the Full-shell model is able to capture the progression of local buckling in the RBS region and twisting of the beams and column. This observation is reflected by the apparent match between the post-test deformed shapes found in the Full-shell model and the experiment itself, as shown in Fig. 12. Certain features of the test data are not replicated by the model (e.g., the apparent stiffening of the test response in cycles after 2 %SDR), although, this is attributed to boundary condition effects that are not fully captured by the model, as this particular test was stopped after the cycles to 5 %SDR due to damage in the lateral support system (Engelhardt et al. 2000). The angle of twist predicted by the Full-shell model at the column bottom is approximately 0.12 rad at the final excursion to -5 %SDR, and the beam angle of twist is approximately 0.22 rad (see also Fig. 12), indicating that torsional effects are expected to be significant. The Full-shell model is also able to capture the panel zone behavior of the subassembly, as show in Fig. 13a, although the peak shear rotation is slightly overestimated.

The Macro (Built-in) and Macro (Warping) models are compared with the Full-shell model in Figs. 11b and 11c. Both models predict reasonably well the Full-shell model’s response, although, the Macro (Built-in) model overestimates the overall unloading/reloading stiffness after the initiation of beam twisting due to the increased torsional stiffness of the beams and column. Additionally, the Macro (Warping) model is able to represent the Full-shell model panel zone behavior, as indicated in Fig. 13b. Note that results for the Macro (Lin. Warping) model are not shown for this case, as convergence issues were experienced for this model due to the linear coupling formulation.

The memory and DOF requirements for Case Study 2 are provided in Fig. 14. This figure plots the memory required by the analyses normalized by the requirement of the Full-shell model, and
similarly with the total DOFs in the models. Results are shown for the Macro (Warping) model only, however, they are very similar for the Macro (Built-in) model barring differences of around 1 % in the DOF and memory requirements that may be attributed to the use of a user subroutine in Abaqus v6.14. For Case 2, a reduction in the number of DOFs required is around 55 %, and the reduction in memory required is around 48 %.

**Case Study 3: Quasi-Static Collapse-Consistent Loading of First-story Column Results**

Column reaction moments at the base without the second-order contribution, $M_{col}$, are reported herein. For clarity, $M_{col} = M_{total} - M_{PD}$, where $M_{total}$ is the total base reaction moment, and $M_{PD} = u_y F_z$ is the second-order moment about the column base. The SDR is computed as $SDR = (u_{y,\text{top}} - u_{y,\text{base}})/L_0$, and axial shortening is reported as a percent of the initial length, $u_z/L_0$.

The column moment vs. story drift ratio (SDR) is plotted in Fig. 15 including the test data and comparisons between the Full-shell model and different macro models. Relative error between the maximum absolute moment of the Full-shell and test data is around 7 %, indicating the models’ ability to capture the initial flexural yielding behavior of the steel material and initiation of local buckling in the column. Furthermore, agreement of the loading/unloading stiffness between the Full-shell results and Test Data indicates that the progression of local buckling is reasonably well captured by the Full-shell model. Computation of the column response is also validated through the deformed shape of the column models at the excursion at +10 %SDR with a photograph from the test at the same point in Figures 16a and 16b. Agreement between the deformed shapes and the test results is notable. The maximum angle of twist observed in this test and finite element analyses is around 0.10 rad in the last loading excursion, although the effect of torsion is not significant in this case as the problem is dominated by local buckling (Elkady and Lignos 2018a).

Each of the macro modeling options are also compared individually with the Full-shell model in Fig. 15. In this case that the strength/stiffness deterioration is dominated by local buckling, there are no significant differences between the Full-shell and macro model responses in terms of moment-rotation throughout the entire loading history, regardless of the coupling method. Despite the agreement in overall component behavior, the stress at the interface may be significantly in error.
with the linear coupling method, as noted in Fig. 16e. The number of DOFs in the model and the
computer memory demands are extracted from information provided by the Abaqus analysis and
presented in Fig. 14 above the label “Case 3”. A savings of around 60% in the number of DOFs is
obtained by the macro models since around 2/3 of the total length is modeled with beam-column
elements, and the memory savings is around 80% in this case (see Fig. 14).

Case Study 4: Column Nonlinear Response History Analysis Results

Modal frequencies determined through eigenvalue analysis of the four W24X94 models are
provided in Table 5, Mode 1 is the primary lateral mode of deformation, and Mode 3 is the twisting
mode provided for comparison purposes. In all cases the first mode period is approximately equal
to 1.5 s validating the assumed mass. The Mode 3 frequency of the Macro (Built-in) model is
around 50% larger than the Full-shell frequency because of the difference in elastic torsional
stiffness. Note that the linear WIKC model’s Mode 1 and Mode 3 frequencies are consistent with
the Full-shell model.

The SDR, column base moment, and axial displacement as a percentage of the initial column
length are provided with respect to the applied ground motion time in Fig. 17. The point at which
the column loses load carrying capacity is evident as the slopes of the SDR and axial displacement
approach $-\infty$. The Full-shell model point of collapse is reasonably well computed by the Macro
(Warping) model, as evident in Fig. 17, however, the same cannot be said for the Macro (Built-in)
model.

In contrast with Case Study 3, divergence in the response of the Macro (Built-in) and Macro
(Lin. Warping) models from the Full-shell model is found in Case Study 4. Divergence at the
point indicated in Fig. 17 stems from the computation of different buckling modes: the W24X94
Macro (Built-in) and Macro (Lin. Warping) models deforms in a primarily local buckling mode,
while the Full-shell and Macro (Warping) model have a coupled buckling response, as shown in
Fig. 18. Differences in the buckling modes subsequently leads to differences in the column base
moment responses, as highlighted in Fig. 17, while the Macro (Warping) model agrees well with
the Full-shell model until near the point of the loss of load carrying capacity. The W24X94 column
Full-shell model stress distribution is also reasonably well predicted by the Macro (Warping) model as indicated by comparing the von Mises stress contours in Fig. 18. Computational resource demands of the column models are summarized in Fig. 14 with respect to the Full-shell model. The number of DOFs are reduced by around 40 % are obtained for the W24X94 macro models compared to the Full-shell model, and the memory requirements are reduced by around 75 %.

**DISCUSSION OF RESULTS**

**Macro Model Accuracy**

Case studies investigated in this paper show that, as long as the coupling between beam-column and continuum elements can account for all relevant mechanical phenomena, the component macro models represent well the Full-shell model. This statement is made with regards to parameters that have been identified as critical in collapse simulations of steel frame buildings (Ibarra and Krawinkler 2005), such as a component’s: pre-peak stiffness, peak strength, and post-peak stiffness. These results indicate that the macro models can replicate the Full-shell model’s initiation and progression of local and lateral-torsional buckling, as well as the progression of axial shortening. This is true even in the complex case of coupled local and lateral-torsional buckling. Therefore, component macro models can be used to address the limitations outlined in the introduction for concentrated and distributed plasticity approaches provided that a suitable coupling method is used, and the component deformation is within the range of its constitutive elements.

More specifically, from the cases investigated, there is little-to-no difference between the different coupling methods when computing the pre-peak response and initiation of local instabilities in wide-flange beam-column components subjected to axial load and/or lateral drift demands. However, the influence of warping in the constraint formulation is paramount in the case of nonuniform torsion and coupled buckling mechanisms due to the increase in torsional stiffness found using the Abaqus v6.14 built-in coupling. The stiffness increase is a result of the warping-fixed boundary enforced on the continuum domains, as seen in the results of case studies, as well as the Mode 3 frequency in Table 5 for the W24X94 column. The increase in torsional stiffness decreases the member’s proclivity to LTB, this in-turn leads to the divergence in buckling modes between the
Macro (Built-in) and Macro (Warping) models in the W24X94 dynamic analyses, as shown in Fig. 7. Differences of stiffness in the descending portion of the post-peak response, such as the ones shown in Fig. 11b for the Macro (Built-in) model, are critical in collapse simulations (Ibarra and Krawinkler 2005), alternatively, this stiffness is well predicted by the Macro (Warping) model in Fig. 11c. An initial recommendation based on the results presented in this paper is that a warping-inclusive coupling method should be used for beam-column components that have a significant torsional load, or that have a normalized LTB slenderness $\bar{\lambda}_{LT} > 0.4$ that is considered susceptible to inelastic LTB according to CEN (2005).

From an accuracy point of view, it appears that if torsion is not dominant in the component response and there is no coupled buckling, the choice of coupling method is not influential in the column post-peak region. This result suggests that including warping in the coupling can be neglected for beam-column components if $\bar{\lambda}_{LT} \leq 0.4$. Furthermore, linear coupling methods are found to be acceptable under the following conditions: local buckling is dominant in the member response, the lateral deformation is below 10% SDR, and the angle of twist does not exceed 0.10 rad. If torsion is dominant, the linear coupling method may be used as long as the angle of twist does not exceed 0.05 rad (see Fig. 8b). The linear WIKC method can also be employed in eigenvalue analyses, as supported by the results of the frequency analysis in Table 5.

**Macro Model Computational Efficiency**

Results of the computational resource metrics shown in Fig. 14 illustrate that the number of DOFs in the macro models are reduced in approximate proportion to the ratio $L_{\Omega_i}/L$ for single components. For instance, the number of DOFs are approximately 2/3 for the W24X94 macro models compared to the Full-shell model in Fig. 14 since 1/3 of the length is modeled with beam-column elements. This ratio of 2/3 is increased for the W24X94 analyses owing to the increased beam-column domain length. The memory savings are expected to be larger than the DOF savings, although the relationship is not clear, as elements of the square matrices need to be saved in addition to state variables for each element. For these reasons, the memory reduction using component macro models in Case Study 2 is less pronounced than the other cases, potentially due to the increased
complexity and number of constraints present in this model to form the subassembly.

In the context of frame analysis, additional computational savings can be gained if the macro model approach is applied selectively depending on the desired level of modeling fidelity and available computational resources. For instance, the macro model approach could be useful to model the torsion induced on columns due to buckling of beams with RBS connections, as well as first-story columns that experience deterioration in capacity-designed MRFs. This selective approach focuses the computational effort where it is most useful, while lower fidelity component models (i.e., concentrated and distributed plasticity approaches) can be used elsewhere in the structural model. Furthermore, the number of iterations required for convergence of the Macro (Warping) model appears similar to the Full-shell and Macro (Built-in) models, see Fig. 10. Therefore, the reduced number of DOFs in macro models is expected to also reduce the required CPU time compared to full-shell models. Finally, the linear WIKC method may lead to issues with global convergence in certain nonlinear problems. The linear WIKC method is, therefore, not computationally advantageous in these cases, and is recommended only if the nonlinear method is unavailable.

Limitations

The following limitations uncovered through this study are stated:

- Residual stresses have not been included in the beam-column elements used in this study. This limitation could be addressed using a beam-column element formulation such as in Lamarche and Tremblay (2011). Residual stresses may influence the Wagner constant (Trahair 1993; de Castro e Sousa and Lignos 2017), that in-turn, could substantially reduce the torsional stiffness of beam-columns. This reduction may be critical for slender members as it further increases their susceptibility to LTB.

- The limitation of using beam-column elements is that local deformations cannot be captured in the beam-column domains, therefore, the length of the continuum domains need to be chosen appropriately a priori. This challenge has yet to be addressed because the extent
of local instabilities in the member depends on the primary mode of deterioration (e.g.,
local buckling, lateral-torsional buckling, flexural buckling) that is ultimately influenced
by the member geometry, material properties, initial imperfections, loading, and boundary
conditions.

- The results presented herein are for shell elements in the continuum domain only, solid
elements have not yet been investigated by the authors. However, the proposed coupling
method is valid for these elements since only the displacement DOFs of the continuum
domain are coupled with the beam-column element domain.

CONCLUSIONS

Continuum finite element models address the limitations in available concentrated and dis-
dtributed plasticity component models for steel wide-flange beam-columns, however, they may be
computationally expensive in the context of earthquake engineering analysis. Mixed-dimension
macro models employing 1D beam-column and 2D/3D continuum elements are promising to re-
duce this computational expense. A review of existing coupling methods highlights that there is a
need for a multi-point constraint method that also incorporates torsion warping. In this paper, we
propose a warping-inclusive kinematic coupling (WIKC) formulation to transfer warping deforma-
tions between the beam-column and continuum element domains. The beam-shell macro models
used in this study reduce the model degrees-of-freedom by up-to around 60 % and the computa-
tional memory requirement by up-to around 80 % compared to their full-shell counterparts, while
retaining the solution fidelity of full-continuum finite element models even for coupled local and
lateral-torsional buckling.

The main conclusions drawn from this paper with respect to steel wide-flange beam-columns
and subassemblies are as follows:

- Including warping in the coupling formulation may be critical in accurately estimating the
full-continuum model failure mode and, therefore, should be included for general use. This
is especially true for components subjected to nonuniform torsion, or that are susceptible to
coupled local and lateral-torsional instabilities, i.e., if $\bar{\lambda}_{LT} > 0.4$.

- A nonlinear, finite-rotation constraint formulation appears to be necessary for nonlinear problems, and there does not appear to be any computational advantages offered by the linear coupling method. Linear, small-rotation coupling methods appear to be acceptable for eigenvalue analyses, as well as problems not dominated by torsion if: (i) LTB is not critical, (ii) the angle of twist is less than 0.10 rad, and (iii) the column drift is below 10 %.

If twisting is the dominant mode of deformation, the angle of twist should remain below 0.05 rad if a linear coupling method is employed.

- The macro model approach leads to a reduction in the DOFs and memory usage proportional to the ratio of the beam-column element domain length to the total component length. The macro model approach does not appear to negatively affect the rate of convergence compared to the full-continuum model, therefore, the computational time savings are expected be a function of the DOF reduction.

Macro model parameters (e.g., proportion of continuum domain, mesh discretization, imperfections, etc.) need to be calibrated in a future study based on the member geometry, boundary conditions, material properties, and loading. Such guidelines will allow for the practical use of component macro models in frame simulations.

**DATA AVAILABILITY**

- Some or all data, models, or code generated or used during the study are available in a repository or online in accordance with funder data retention policies. This includes: the code for the proposed coupling method and imperfection generation (Hartloper 2020); and the code used to generate the Voce-Chaboche material model properties (de Castro e Sousa et al. 2019).

- Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request. This includes: the finite element models used in the case studies.
ACKNOWLEDGEMENTS

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The following symbols are used in this paper:

- $A_1, A_2$: constraint coefficient matrices;
- $b_f$: flange width;
- $(\cdot)^b$: beam quantity;
- $(\cdot)^c$: continuum quantity;
- $d$: wide-flange section depth;
- $f$: constraint equations for a continuum node;
- $I$: $3 \times 3$ identity matrix;
- $L$: member length;
- $l$: link vector;
- $n_1, n_2$: cross-sectional plane vectors;
- $R$: rotation matrix;
- $t$: cross-sectional normal vector;
- $t_f$: flange thickness;
- $t_w$: web thickness;
- $u$: nodal displacement vector;
- $v$: nodal initial imperfection vector;
- $w$: warping DOF;
- $x$: nodal position vector;
- $\delta(\cdot)$: infinitesimal quantity;
- $\Gamma$: beam-continuum interface;
- $\xi$: distance along $n_1$ axis;
- $\theta$: infinitesimal nodal rotation vector;
- $\eta$: distance along $n_2$ axis;
- $\psi$: warping function;
\( \phi \) = nodal rotation vector;
\( \zeta \) = distance along \( n_3 \) axis;
\( \Omega_1 \) = beam-column element domain;
\( \Omega_2 \) = continuum element domain; and
\( [(\cdot)]_x \) = matrix representation of cross-product.
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### TABLE 1. Overview of the case studies.

<table>
<thead>
<tr>
<th>Case Study ID</th>
<th>Description</th>
<th>Component(s)</th>
<th>Material</th>
<th>Equilibrium Path</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Nonuniform torsion</td>
<td>Beam: $6 \times 6-25$</td>
<td>A36</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>Interior subassembly</td>
<td>Column: W14X398, Beam: W36X150</td>
<td>A572 Gr. 50</td>
<td>Unstable</td>
</tr>
<tr>
<td>Case 2</td>
<td>First-story column, quasi-static loading</td>
<td>Column: W24X146</td>
<td>A992 Gr. 50</td>
<td>Unstable</td>
</tr>
<tr>
<td>Case 3</td>
<td>First-story column, dynamic loading</td>
<td>Column: W24X94</td>
<td>A992 Gr. 50</td>
<td>Unstable</td>
</tr>
</tbody>
</table>
TABLE 2. Wide-flange member geometry used in the analysis cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Section</th>
<th>$d$ [mm]</th>
<th>$b_f$ [mm]</th>
<th>$t_f$ [mm]</th>
<th>$t_w$ [mm]</th>
<th>$L_b$ [mm]</th>
<th>$b_f/2t_f$</th>
<th>$h/t_w$</th>
<th>$L_b/r_y$</th>
<th>$\tilde{\lambda}_{LT}$</th>
<th>$L_r$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6 × 6-25</td>
<td>152</td>
<td>151</td>
<td>12.2</td>
<td>8.0</td>
<td>1930</td>
<td>6.20</td>
<td>16.0</td>
<td>50</td>
<td>0.20</td>
<td>10.7</td>
</tr>
<tr>
<td>2</td>
<td>W14X398</td>
<td>465</td>
<td>442</td>
<td>72.4</td>
<td>45.0</td>
<td>3708</td>
<td>2.92</td>
<td>6.44</td>
<td>34</td>
<td>0.15</td>
<td>41.8</td>
</tr>
<tr>
<td>3</td>
<td>W24X146</td>
<td>627</td>
<td>325</td>
<td>27.1</td>
<td>17.6</td>
<td>3900</td>
<td>5.92</td>
<td>33.2</td>
<td>51</td>
<td>0.29</td>
<td>10.2</td>
</tr>
<tr>
<td>4</td>
<td>W24X94</td>
<td>617</td>
<td>230</td>
<td>22.2</td>
<td>13.1</td>
<td>5500</td>
<td>5.18</td>
<td>41.9</td>
<td>109</td>
<td>0.50</td>
<td>6.4</td>
</tr>
</tbody>
</table>

$a$: measured $d, b_f, t_f, t_w$. $b$: $\tilde{\lambda}_{LT} = \sqrt{Z_x f_y / M_{cr}}$

$M_{cr} = C_1 \pi^2 E I_y / (k_y L)^2 \sqrt{(k_y / k_w)^2 I_w / I_y + (k_y L)^2 G I_t / (\pi^2 E I_y)}$, $C_1 = 2.55$, $k_y = 0.5$, $k_w = 1.0$
### TABLE 3. Case study macro model and imperfection parameters.

<table>
<thead>
<tr>
<th>Case Study ID</th>
<th>$L_{\Omega_2}$ [mm]</th>
<th>$\Delta_{shell}$ [mm]</th>
<th>Shell Type</th>
<th>$N_{beam}$</th>
<th>Beam Type</th>
<th>$L_{bw}$ [mm]</th>
<th>$\theta_{twist}$ [%rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>482.5</td>
<td>&lt;20</td>
<td>S4R</td>
<td>8</td>
<td>B32OS</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 2, column</td>
<td>1700</td>
<td>&lt;50</td>
<td>S4R</td>
<td>4</td>
<td>B31OS</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 2, beams</td>
<td>1300</td>
<td>&lt;25$^a$</td>
<td>S4R</td>
<td>6</td>
<td>B31OS</td>
<td>686</td>
<td>-</td>
</tr>
<tr>
<td>Case 3</td>
<td>800</td>
<td>&lt;25</td>
<td>S4R</td>
<td>4</td>
<td>B31OS</td>
<td>752</td>
<td>0.05</td>
</tr>
<tr>
<td>Case 4</td>
<td>1650</td>
<td>&lt;25</td>
<td>S4R</td>
<td>4</td>
<td>B31OS</td>
<td>740</td>
<td>0.6</td>
</tr>
</tbody>
</table>

$^a$: <25 mm in the RBS region, <50 mm elsewhere.
TABLE 4. Case study Voce-Chaboche material model parameters.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ [GPa]</th>
<th>$\sigma_{y,0}$ [MPa]</th>
<th>$Q_\infty$ [MPa]</th>
<th>$b$ [MPa]</th>
<th>$C_1$ [MPa]</th>
<th>$\gamma_1$</th>
<th>$C_2$ [MPa]</th>
<th>$\gamma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A572 Gr.50&lt;sup&gt;a&lt;/sup&gt;</td>
<td>207.34</td>
<td>352.01</td>
<td>61.60</td>
<td>8.96</td>
<td>2305.55</td>
<td>20.15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A992 Gr.50 Flange&lt;sup&gt;b&lt;/sup&gt;</td>
<td>179.75</td>
<td>318.47</td>
<td>100.72</td>
<td>8.00</td>
<td>11608.17</td>
<td>145.22</td>
<td>1026.33</td>
<td>4.68</td>
</tr>
<tr>
<td>A992 Gr.50 Web&lt;sup&gt;b&lt;/sup&gt;</td>
<td>182.97</td>
<td>339.18</td>
<td>77.52</td>
<td>9.29</td>
<td>8716.08</td>
<td>118.47</td>
<td>1182.05</td>
<td>5.22</td>
</tr>
</tbody>
</table>

<sup>a</sup>: calibrated as a part of this study, <sup>b</sup>: from de Castro e Sousa et al. (2020).
Table 5. Frequencies for selected modes of the W24X94 column.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mode 1 (Lateral)</th>
<th>Mode 3 (Twisting)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. [Hz]</td>
<td>%</td>
</tr>
<tr>
<td>Full-shell</td>
<td>0.667</td>
<td>-</td>
</tr>
<tr>
<td>Macro (Built-in)</td>
<td>0.668</td>
<td>0.15</td>
</tr>
<tr>
<td>Macro (Warping)</td>
<td>0.668</td>
<td>0.15</td>
</tr>
<tr>
<td>Macro (Lin. Warping)</td>
<td>0.668</td>
<td>0.15</td>
</tr>
</tbody>
</table>

\[ E_{fs} = \frac{(\text{Full-shell} - \text{Macro})}{\text{Full-shell}}. \]
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Fig. 2. Schematic wide-flange cross-section and mixed-dimension domains.
Fig. 3. Schematic view of prototype frame and overview of Case Studies 2, 3, and 4.
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Twisting restraint, axial displacement restraint, warping free at ends

(a) Nonuniform torsion

(b) DBBW subassembly

(c) Quasi-Static column analysis

(d) Dynamic column analysis

Fig. 5. Schematic representations of the illustrative examples (extruded view of shell domains shown).
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Fig. 17. Results from the W24X94 column nonlinear response history analyses.
Fig. 18. Deformed shapes from the W24X94 column nonlinear response history analyses at \( t = 4.5 \) s. von Mises equivalent stress profile shown on the deformed shape, red regions indicate yielding.