Robust Learning Model Predictive Control for Periodically Correlated Building Control

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***Abstract*— Accounting for more than 40% of global energy consumption, residential and commercial buildings will be key players in any future green energy systems. To fully exploit their potential while ensuring occupant comfort, a robust control scheme is required to handle various uncertainties, such as external weather and occupant behaviour. However, prominent patterns, especially periodicity, are widely seen in most sources of uncertainty. This paper incorporates this correlated structure into the learning model predictive control framework, in order to learn a global optimal robust control scheme for building operations.**

I. INTRODUCTION

Around 40% of global energy use comes from residential and commercial buildings [1], which drives research interest in building control. Maximizing operational efficiency while maintaining occupant comfort is the key objective therein. However, various sources of uncertainty, such as internal heat gain and outdoor temperature, pose significant challenges to building operation. Even though uncertain, most of them reveal prominent patterns, especially periodicity. For example, the campus load is shown to evolve within a periodic envelope in [2]. Moreover, the alternation between days and nights endows internal heat gain and external temperature periodic pattern on a daily basis [3].

Besides uncertainty, most buildings are also operated under a periodic scheme. Such periodicity has been widely adopted in building control applications [4], [5], where iterative learning control (ILC) is the key tool enabling efficient performance refinement [6]. On the other hand, model predictive control (MPC) is a receding horizon control scheme that optimally computes its control inputs by recurrent forecast into the future. Its natural integration of optimization objective and constraints populates its applications in building control [2], [3], [7]. Taking advantages of both ILC and MPC [8], both control schemes deal with optimality and robustness separately. Instead of splitting the control task, learning model predictive control (LMPC) is an optimization-based control scheme that unifies monotonic performance improvement and safety/robustness [9]–[11].

In this work, we incorporate the periodically correlated uncertainty into the LMPC framework, which enables LMPC to handle time-varying dynamics. Moreover, owing to a priori knowledge of periodic correlation, the proposed scheme shows higher data efficiency and lower conservativeness. The detailed contribution of this paper is concluded as follows:

- Explore a parametric decomposition scheme to handle correlated noise.
- Propose a novel less conservative robust LMPC scheme for periodically correlated process noise, which is designed for periodic tasks.
- Demonstrate the convergence and optimality of the proposed LMPC scheme.

In the following, we will first introduce the building control problem and the classic LMPC control law in Section II. In Section III, we introduce a decomposition approach of the periodically correlated disturbance and the novel LMPC is illustrated. The recursive feasibility and performance guarantee of the proposed LMPC is discussed in IV. In Section V and VI, we describe how to adapt different initial states and model uncertainty in the proposed framework and validate the proposed scheme with a single zone building system. An extensive version with more elaborated numerical results can be found at `https://arxiv.org/abs/2011.13781`.

Notation

Set of consecutive integers $\{a, a+1, \dots, b\}$ is denoted by $\mathbb{N}_a^b$. $\mathbb{A} \ominus \mathbb{B}$ denotes Pontryagin set difference. Let η^j denote the value of η at jth iteration. Given value of η at time t as η_t, its prediction at k is denoted by $\eta_{k|t}$, similarly, we have $\eta_{t|t} := \eta_t$. $\{a_i\}_{i=1}^N$ is a countable set of cardinality N, whose elements a_i are indexed by i. $\vee$ denotes the logic "or".

II. SET UP THE STAGE

A. Problem setting

In this work, we consider a building operation on a daily basis, where a discrete-time periodic time-varying linear building model [12] with period T,

$$x_{t+1} = A_t x_t + B_t u_t + C_t w_t, \ \forall t \ \in \mathbb{N}_0^T , \tag{1}$$

where states, control inputs and the bounded process noise are denoted by $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $w \in \mathbb{R}^d$ separately.

This system is manipulated to execute an iterative task, which means at jth iteration, it starts from $x_0^j = x_s$. The states and inputs are required to satisfy the following periodic, convex polytopic constraints:

$$F_t x_t + G_t u_t \le f_t, \forall \, t \in \mathbb{N}_0^T . \tag{2}$$

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The optimal building operation problem of the jth day is concluded as follows:

$$J^{j,*} = \min_{\{u_t^j\}_{t=0}^T} \sum_{t=0}^{T} l_t(x_t^j, u_t^j)$$
$$\text{s.t. } \forall t \in \mathbb{N}_0^T,\ x_0^j = x_s$$
$$x_{t+1}^j = A_t x_t^j + B_t u_t^j + C_t w_t^j, \quad (3a)$$
$$F_t x_t^j + G_t u_t^j \leq f_t, \quad (3b)$$

where l_t denotes stage cost at time t and w_t^j represents the uncertainty occurred within the jth day. T in Problem (3) is in general large in building control. For example, if the control law changes every 10 minutes, then T reaches 144.

B. Learning Model Predictive Control

Learning model predictive control (LMPC) is an iterative control scheme proposed to learn infinite/long horizon optimal control law, where a relatively short horizon problem is solved in a moving horizon scheme [9]. For the sake of clarity, we elaborate LMPC with a deterministic system (*i.e.* $w = 0$ in (3a)). At time $t \in \mathbb{N}_0^T$, the following problem is solved:

$$\min_{\{u_{k|t}^j\}_{k=t}^{t+N-1}} \sum_{k=t}^{t+N-1} l_k(x_{k|t}^j, u_{k|t}^j) + Q^j(x_{t+N|t}^j) \quad (4a)$$
$$\text{s.t. } \forall k \in \mathbb{N}_t^{t+N-1},\ x_{t|t}^j = x_t^j$$
$$x_{k+1|t}^j = A x_{k|t}^j + B_k u_{k|t}^j$$
$$F x_{k|t}^j + G u_{k|t}^j \leq f,$$
$$x_{t+N|t}^j \in \mathbb{SS}^j\ . \quad (4b)$$

$Q^j(\cdot)$ in (4a) and set $\mathbb{SS}^j$ in (4b) are two main components which makes ensure the safety and monotonic improvement of LMPC. In particular, $\mathbb{SS}^j$ denotes the safe set within which there is at least one control law ensuring system safety. This set is mainly constructed by the convex hull of all trajectories before current iteration j. Meanwhile, $Q^j(\cdot)$ is an overestimate of optimal cost-to-go, which ensures the cost calculated in (4a) overestimate the optimal cost in Problem (3). In particular, $Q^j(\cdot)$ is modelled by parametric quadratic programming in standard LMPC [13].

The LMPC control scheme guarantees convergence to infinite/long horizon solution [14] and has wide extension to robust control with additive noise [10], [11] and deterministic periodic control [15].

III. Main Results

In this section, the incorporation of correlation information is first introduced by finite order approximation in Section III-A. The adapted LMPC algorithm for the resulting problem is then introduced in Section III-B.

A. Process Noise Decomposition

Most sources of uncertainty in building control reveal significant periodic patterns, such as external temperature and internal heat gain. The main idea behind our approach is to decompose the information of uncertainty into the periodically correlated part and the uncorrelated part (*i.e.* white noise). To proceed, we first make the following assumption.

Assumption 1: $w_t,\ \ t \in \mathbb{N}_0^T$ is a bounded stochastic process and $\mathbb{E}(w_t) = a_0,\ \forall\, t \in \mathbb{N}_0^T$.

w_t is a stochastic process with finite end time T, then the w_t^j is a realization of this process. More specifically, if w_t is the process of external temperature, then w_t^j is the temperature trajectory in the jth day. Assumption 1 ensures that the process noise in the jth day is square integrable with respect to its probability space [16]. By Karhunen–Loève theorem [17], w_t^j is decomposed based on Fourier series as

$$w_t^j = a_0^j + \sum_{q=1}^{\infty} a_q^j \sin(\frac{2\pi qt}{T}) + b_q^j \cos(\frac{2\pi qt}{T})\ , \quad (5)$$

where $a_q = \frac{2}{T}\int_0^T w(t)\sin(\frac{2\pi qt}{T})dt$ and b_q is defined accordingly. To only preserve the low frequency information, Equation (5) is further approximated by

$$w_t^j = a_0^j + \sum_{k=1}^{M} [a_q^j \sin(\frac{2\pi qt}{T}) + b_q^j \cos(\frac{2\pi qt}{T})] + w_{r,t}^j \quad (6)$$
$$= a_0^j + w_{\theta^j,t}^j + w_{r,t}^j\ ,$$

where $w_{r,t}^j$ models the truncation error caused by the finite order approximation $w_{\theta^j,t}^j$. In particular, the collection of parameters $\theta^j := \{a_q^j, b_q^j\}_{q=1}^M$ captures the periodic correlation within the jth day, which is bounded as $\theta^j \in \mathbb{W}_\theta,\ \forall\ j$. The residue $w_{r,t}$ is a zero-mean bounded white noise whose variance is $\text{var}(w_{r,t}) = \mathbb{E}(\sum_{q=M+1}^{\infty}(||a_q^j||_2^2 + ||b_q^j||_2^2))$, which is well defined by Assumption 1 and that preserves the energy of the process noise(*i.e.* Parseval theorem [18]). To explain (6) more specifically, one can consider w_t as external temperature. In the jth day, a_0^j is the averaged temperature, $\{a_q^j,\ b_q^j\}_{q=1}^M$ models the daily evolution of the temperature, while $w_{r,t}^j$ models the highly stochastic fast fluctuation. Regarding this interpretation, a_0^j and θ^j varies among days. Similar to (5), other orthogonal basis functions can be used to approximate specific noise patterns, such as Haar Wavelet basis [19] for internal heating gains. For the sake of simplicity, we elaborate our method with a simpler model as

$$w_t^j = a_0^j + a_1^j sin(2\pi t/T) + w_{r,t}^j$$
$$= w_{\theta^j,t}^j + w_{r,t}^j,\ \theta^j = \{a_0^j, a_1^j\}\ . \quad (7)$$

Remark 1: Notice that $\{a_q^j, b_q^j\}_{q=1}^\infty$ are realizations of random variables according to Karhunen–Loève theorem [17], which means that they are fixed in w_t^j. In practice, within each iteration, these parameters can be effectively estimated by different methods, such as Bayesian learning [20].

Remark 2: As one might notice that a stochastic process decomposition is mainly applied to a continuous time stochastic process, the decomposition procedure discussed above implicitly discretize the process by stochastic integration. In particular, the closeness of the Fourier basis under linear dynamics leads to (7).

B. LMPC for correlated noise

As noise are decomposed into the correlated part and the uncorrelated part in (6), they can be handled separately in the robust control problem. In particular, the white noise $w_{r,t}^j$ are handled by standard robust model predictive control method [21] (details in Appendix VIII-A). The resulting robust form of the long horizon Problem (3) is

$$J^{j,*} = \min_{\{v_t^j\}_{t=0}^T} \sum_{t=0}^{T} l_t(z_t^j, v_t^j)$$
$$\text{s.t. } \forall t \in \mathbb{N}_0^T,\ z_0^j = x_s$$
$$z_{t+1}^j = A_t z_t^j + B_t v_t^j + C_t w_{\theta^j,t}^j, \tag{8a}$$
$$\bar{F}_t z_t + \bar{G}_t v_t \leq \bar{f}_t, \tag{8b}$$

where z_t^j, v_t^j denote the state and input of a nominal system, and (8b) is the tightened constraint (Appendix VIII-A).

Correspondingly, the robust form of LMPC problem (4) is:

$$J_{LMPC}^{j,*} = \min_{\{v_{k|t}^j\}_{k=t}^{t+N-1}} \sum_{k=t}^{t+N-1} l_k(z_{k|t}^j, v_{k|t}^j) + Q_{t+N}^j(z_{t+N|t}^j)$$
$$\text{s.t. } \forall k \in \mathbb{N}_t^{t+N-1},\ z_{t|t}^j = z_t^j$$
$$z_{k+1|t}^j = A_k z_{k|t}^j + B_k v_{k|t}^j + C_k w_{\theta^j,k}^j$$
$$\bar{F}_k z_{k|t}^j + \bar{G}_k v_{k|t}^j \leq \bar{f}_k,$$
$$z_{t+N|t}^j \in \mathbb{SS}_{t+N}^j\ . \tag{9}$$

The daily changed disturbances included in the dynamics and periodic tasks make classic LMPC not applicable, which requires new adaptive algorithms to calculate $\mathbb{SS}_t^j$ and $Q_t^j(\cdot)$. In the following, we show the strategy of constructing these two main components accordingly. To proceed, we first define the following notation for a more compact layout.

At time t of jth iteration, denote by the vectors

$$\mathbf{v}_t^{j,*} = [v_{t|t}^{j,*}, v_{t+1|t}^{j,*}, ..., v_{t+N-1|t}^{j,*}], \tag{10}$$
$$\mathbf{z}_t^{j,*} = [z_{t|t}^{j,*}, z_{t+1|t}^{j,*}, ..., z_{t+N|t}^{j,*}]. \tag{11}$$

the optimal input sequence and the resulted state sequence. Then at time t, the input applied to the closed-loop system is

$$v_t^j = \begin{cases} v_{t|t}^{j,*}, & t+N \leq T, \\ v_{t|T-N}^{j,*}, & t+N > T. \end{cases} \tag{12}$$

In the following, the idea of historical trajectory shifting will enable us to define the adapted safe sets $\mathbb{SS}_t^j$ and Q function $Q_t^j(\cdot)$. Consider at a historical ith iteration, the vectors

$$\mathbf{z}^i = [z_0^i, z_1^i, ..., z_T^i]$$
$$\mathbf{v}^i = [v_0^i, v_1^i, ..., v_T^i] \tag{13}$$

record the historical states and inputs in the closed-loop trajectories. When building a safe set for jth iteration, a shifting method is applied on the historical data, $\mathbf{z}^i$ and $\mathbf{v}^i$. For a shifting starting from time step t of ith historical trajectory, denote by $v_{k|t}^{i,j}$ the shifted input, by $z_{k|t}^{i,j}$ the shifted state, by $e_{k|t}^{i,j} = z_{k|t}^{i,j} - z_k^i$ the error state, the shifting follows a procedure:

$$e_{k+1|t}^{i,j} = \Phi_k e_{k|t}^{i,j} + C_k(w_{\theta^j,k}^j - w_{\theta^i,k}^i)$$
$$v_{k|t}^{i,j} = v_k^i + K_k e_{k|t}^{i,j}$$
$$z_{k|t}^{i,j} = z_k^i + e_{k|t}^{i,j}, \forall k \in \mathbb{N}_t^T \tag{14}$$

and $e_{t|t}^{i,j} = 0$, where K_k is chosen to stabilize $\Phi_k = A_k + B_k K_k$. As a result, $z_{k|t}^{i,j}$ and $v_{k|t}^{i,j}$ satisfy jth dynamics:

$$z_{k+1|t}^{i,j} = A_k z_{k|t}^{i,j} + B_k v_{k|t}^{i,j} + C_k w_{\theta^i,k}^i + C_k(w_{\theta^j,k}^j - w_{\theta^i,k}^i)$$
$$= A_k z_{k|t}^{i,j} + B_k v_{k|t}^{i,j} + C_k w_{\theta^j,k}^j$$

Note that the shifted states and inputs may result in infeasible shifted data due to the constraints violation. The elimination of these infeasible shifted data leads us to the concept of *Feasible Disturbance Set*.

Definition 1 (Feasible Disturbance Set): at time t in a historical iteration, the Feasible Disturbance set $\mathbb{W}_t^i$ is defined as:

$$\mathbb{W}_t^i = \{\theta | \bar{F}_k(z_k^i + e_{k|t}^{i,\cdot}) + \bar{G}_k(v_k^i + K_k e_{k|t}^{i,\cdot}) \leq \bar{f}_k, e_{t|t}^{i,\cdot} = 0$$
$$e_{k+1|t}^{i,\cdot} = \Phi_k e_{k|t}^{i,\cdot} + C_k(w_{\theta,k} - w_{\theta^i,k}^i), \forall k \in \mathbb{N}_t^T\}$$

After finishing jth iteration and recording closed-loop states $\mathbf{z}^j$, inputs $\mathbf{v}^j$, the feasible disturbance set at each time is computed and recorded.

Algorithm 1 Safe set

Given historical closed loop states $\mathbf{z}^i$, inputs $\mathbf{v}^i$, feasible disturbance set $\mathbb{W}_t^i$, $\forall t \in \mathbb{N}_0^T, i \in \mathbb{N}_0^{j-1}$

1) For $i \in \mathbb{N}_0^{j-1}$, $t \in \mathbb{N}_0^T$
   a) If $\theta^j \in \mathbb{W}_t^i$
      i) Compute the shifting from time t $[z_{t|t}^{i,j}, ..., z_{T|t}^{i,j}]$, $[v_{t|t}^{i,j}, ..., v_{T|t}^{i,j}]$
      ii) Add state $z_{k|t}^{i,j}$ to $\mathbb{SS}_k^j$, $\forall k \in \mathbb{N}_t^T$
      iii) Compute and record shifted cumulative cost $J_{k|t}^{i,j}(z_{k|t}^{i,j}) = \sum_{r=k}^{T} l(z_{r|t}^{i,j}, v_{r|t}^{i,j})\ \forall k \in \mathbb{N}_t^T$

Now we build the safe set $\mathbb{SS}_t^j$ for jth iteration by the Algorithm 1. Note in the shifting starting from time t, it computes the shifted states from t to T and each shifted state $z_{k|t}^{i,j}$ is added to $\mathbb{SS}_k^j$ correspondingly. Meanwhile, the estimated cost-to-go (*i.e.* $Q_k^j(\cdot)$ in (9)) are updated by shifted cumulative costs $J_{k|t}^{i,j}$ as

$$Q_k^j(z) = \begin{cases} \min\limits_{(i,t)\in F_k^j(z)} J_{k|t}^{i,j}(z), & \text{if } z \in \mathbb{SS}_k^j \\ +\infty, & \text{if } z \notin \mathbb{SS}_k^j \end{cases} \tag{15}$$

where $F_k^j(z) = \{(i,t) : i \in [0, j-1], t \in [0,k]$ with $z_{k|t}^{i,j} = z$, for $z_{k|t}^{i,j} \in \mathbb{SS}_k^j\}$. Note different from [9], at jth iteration, $\mathbb{SS}_t^j$ and $Q_t^j(z)$ are built for each time step t.

Remark 3: At each time step t and each shifted state z in $\mathbb{SS}_t$, $Q_t^j(z)$ is assigned a value, $J_{k|t^*}^{i^*,j}$, which is the minimal shifted cumulative cost starting from $z_{k|t^*}^{i^*,j} = z$. (i^*, t^*) is chosen by the minimizer in (15):

$$(i^*, t^*) = \underset{(i,t)\in F_k^j(z)}{argmin} \; J_{k|t}^{i,j}(z), \forall z \in \mathbb{SS}_k^j$$

Assumption 2: Assume a feasible trajectory at 0th iteration, $\{\mathbf{z}^0, \mathbf{v}^0\}$, is given and all the disturbance feasible sets are subject to, $\mathbb{W}_t^0 \supseteq \mathbb{W}_\theta$.

Assumption 2 is standard under the LMPC control scheme. It results in an non-empty safe set $\mathbb{SS}_t^j$, $\forall\, t \in \mathbb{N}_0^T, j \in \mathbb{N}_+$. In practice, Assumption 2 is not restrictive as it essentially requires a default feasible control law. It is also noteworthy to point out that neither historical nor shifted trajectory is required to achieve a steady state, while this convergence requirement is necessary for classic LMPC.

Remark 4: The online computation increase of the proposed scheme is fair, as feasible disturbance sets $\mathbb{W}_t^i$, safe set $\mathbb{SS}_t^j$ and Q function $Q_t^j(\cdot)$ only update at the beginning of each iteration.

Remark 5: Even though this work has a special focus on building control, the proposed scheme can be adopted to most time-varying periodic tasks such as airborne wind energy harvest [22].

IV. Properties

In this section, the properties of the proposed LMPC method are presented, including feasibility and performance.

A. Recursive Feasibility

Theorem 1 (Recursive Feasibility): Suppose Assumption 2 is satisfied, then the problem (9) is feasible for any time step t at any jth iteration.

Proof: The proof can be found in the extensive version. ■

B. Performance

In this section, we present 2 results regarding to the controller performance. At jth iteration, denote the optimal value of the objective function of the problem (9) at time step t by $J_{LMPC}^{j,*}(z_t^j) = \sum_{k=t}^{N} l_k(z_{k|t}^{j,*}, v_{k|t}^{j,*}) + Q_{t+N}^j(z_{t+N|t}^{j,*})$, the closed-loop cumulative cost starting from time t by $J^j(z_t^j) = \sum_{k=t}^{T} l_k(z_k^j, v_k^j)$.

Assumption 3: Consider a continuous, semi-positive and convex stage cost function $l_t(z, v) \geq 0$

Different from [9], the stage cost is not limited to a tracking error. Some economic cost can be used, like the electricity cost in the building control.

Theorem 2: Under Assumption 3, for each $t \in \mathbb{N}_0^{T-N}$ of the jth iteration, the cumulative trajectory cost $J^j(z_t^j)$ is upper bounded by the shifted trajectory cost $J_{t|t'}^{i,j}(z_{t|t'}^{i,j})$, starting from any $z_{t|t'}^{i,j} = z_t^j \in \mathbb{SS}_t^j$. Specially, if $\theta^j \in \mathbb{W}_0^i$, $J^j(z_0^j) \leq J_{0|0}^{i,j}(z_0^j)$.

Proof: The proof can be found in the extensive version. ■

After execution of jth iteration, if in a new iteration j', it happens to perform the same disturbance parameters $\theta^{j'} = \theta^j$, z_t^j can be added in $\mathbb{S}_t^j$ without shifting. Then by Theorem 2, $J^{j'}(x_s) \leq J_{0|0}^{j,j'}(x_s) = J^j(x_s)$, which means the closed-loop iteration cost does not increase.

Corollary 1: Under Assumption 3, considering that the system 1 is controlled by the proposed periodic LMPC (9) and (12), if at jth iteration, it achieves a steady-state solution $\{\mathbf{z}^{j,ss}, \mathbf{v}^{j,ss}\}$ w.r.t θ^j, then $\{\mathbf{z}^{j,ss}, \mathbf{v}^{j,ss}\}$ is the optimal solution of (8).

Proof: It follows a similar procedure of proof in [14, Theorem 1] as (8) is strictly convex. ■

V. Practical Issues

In practice, the initial state of each iteration are not necessarily the same, i.e. $\exists\; i < j, z_0^i \neq z_0^j$. For example, even the building controller is idle in the evening and the system state converges to a steady state due to the dissipative nature, the resulting steady state also varies due to external temperature.

A trick is involving initial state deviation as part of the disturbance function w_t. By defining a nominal initial state $x_{s,n}$ and the deviation between it and initial state at jth iteration $w_s^j = z_0^j - x_{s,n}$, an extension of the disturbance function is

$$w_{\theta^j,t}^j(w_s^j) = \begin{cases} w_s^j, & t = -1 \\ w_{\theta^j,t}^j, & o.w. \end{cases} \tag{16}$$

It has an influence on the shifting procedure (14) starting from time 0,

$$\begin{aligned} e_{k+1|0}^{i,j} &= (A_k + B_k K_k) e_{k|0}^j + C_k(w_{\theta^j,k}^j(w_s^j) - w_{\theta^i,k}^i(w_s^i)) \\ v_{k|0}^{i,j} &= v_k^i + K_k e_{k|0}^{i,j} \\ z_{k|0}^{i,j} &= z_k^i + e_{k|0}^{i,j}, \forall k \in \mathbb{N}_0^T \end{aligned} \tag{17}$$

and $e_{0|0}^{i,j} = w_s^j - w_s^i$, and and the feasible disturbance set $\mathbb{W}_0^i$ for $\{z_0, \theta\}$ at time 0 is recomputed by the above error dynamics.

Similarly, if dynamics of system (1) varies from iteration to iteration. Define nominal dynamics matrices $\overline{A}_t, \overline{B}_t$, the dynamics deviation $dA_t^j = A_t^j - \overline{A}_t$. Assume K_t stabilize all the possible $A_t^j + B_t^j$. A new shifting procedure starting from time t is,

$$\begin{aligned} e_{k+1|t}^{i,j} =& (A_k^j + B_k^j K_k) e_{k|t}^j + C_k(w_{\theta^j,k}^j - w_{\theta^i,k}^i) \\ &+ (dA_k^j - dA_k^i) * z_k^i + (dB_k^j - dB_k^i) * v_k^i \\ v_{k|t}^{i,j} =& v_k^i + K_k e_{k|t}^{i,j} \\ z_{k|t}^{i,j} =& z_k^i + e_{k|t}^{i,j}, \forall k \in \mathbb{N}_t^T \end{aligned} \tag{18}$$

and $e_{t|t}^{i,j} = 0$, and the new feasible disturbance set $\mathbb{W}_t^i$ for $\{A_t, B_t, \theta\}$ is computed based on that.

VI. Simulation and Results

In this section, the proposed LMPC is tested on a single zone building model,where we consider a periodic tracking task, where scheduled comfort conditions on temperatures and three different correlated real-world disturbances decomposition are considered.

A. A single zone building system

A small scale linear time invariant building model [23] with $x_{t+1} = Ax_t + Bu_t + Cw_t$ is considered, where

$$A = \begin{bmatrix} 0.8511 & 0.0541 & 0.0707 \\ 0.1293 & 0.8635 & 0.0055 \\ 0.0989 & 0.0032 & 0.7541 \end{bmatrix}, B = \begin{bmatrix} 0.0035 \\ 0.0003 \\ 0.0002 \end{bmatrix}$$

$$C = 10^{-3} * \begin{bmatrix} 22.2170 & 1.7912 & 42.2123 \\ 1.5376 & 0.6944 & 2.9214 \\ 103.1813 & 0.1032 & 196.0444 \end{bmatrix}.$$

The states $x = [x_1, x_2, x_3]^T$ represent the temperatures of the room, the wall connected with another room, and the wall connected to the outside respectively. The single input is heating and cooling. Suppose the sampling rate of the system is 10 minutes, an one-day iteration consists of 144 time steps.

In this test, the disturbances of internal heat-gain, solar-radiation and external temperature are considered, which are denoted by $w = [w_1, w_2, w_3]^T$ accordingly. These disturbances all reveal daily repetitive patterns and can be predicted by some well-built systems [7]. For the sake of simplicity, We use the combination of sinusoidal, triangular and square wave functions and white noises to approximate the decomposition of disturbances in (6):

$$w_{1,t} = a_1 + a_2 sin(2\pi t/T) + w_{r,1,t}$$

$$w_{2,t} = \begin{cases} a_3(4t - T)/T + w_{r,2,t}, & T/4 \le t < T/2 \\ a_3(3T - 4t)/T + w_{r,2,t}, & T/2 \le t < 3T/4 \\ w_{r,2,t}, & t < T/4 \lor t \ge 3T/4 \end{cases}$$

$$w_{3,t} = \begin{cases} a_4 + a_5 + w_{r,3,t}, & T/3 \le t < 3T/4 \\ a_4 + w_{r,3,t}, & t < T/3 \lor t \ge 3T/4 \end{cases}$$

, where the parameters and white noises are bounded by:

$$a_1 \in [10, 14], a_2 \in [-6, -2]$$
$$a_3 \in [0, 16], a_4 \in [0, 2], a_5 \in [6, 7]$$
$$w_{r,1} \in [-3, 3], w_{r,2} \in [-5, 5], w_{r,3} \in [-2, 2]$$

The room temperature is supposed to satisfy a comfort constraint during work time and the constraint is relaxed at night. The constraints are modeled as:

$$u \in [-30, 30], \begin{cases} 18 \le x_1 \le 30, & t < T/3 \lor t \ge 3T/4 \\ 22 \le x_1 \le 26, & T/3 \le t \le 3T/4 \end{cases}$$

Then consider a control objective to regulate the room temperature to a time-varying reference

$$x_{1,ref,t} = \begin{cases} 20, & t < T/3 \lor t \ge 3T/4 \\ 24, & T/3 \le t < 3T/4 \end{cases}$$

while minimizing the energy cost. The stage cost is $l_t(x_t, u_t) = ||x_{1,t} - x_{1,ref,t}||_2^2 + ||cp_t u_t||_1$, in which cp_t denotes the electricity price and there are periodic high price and low price periods:

$$cp_t = \begin{cases} 1, & t < 5T/12 \lor t \ge 2T/3 \\ 2, & 5T/12 \le T < 2T/3 \end{cases}$$

The experiment is carried out with an initial state $x_s = [19; 19; 15]^T$ and prediction horizon $N = 16$. The feedback gain K in (14) and (19) is computed by the optimal LQR gain choosing parameters $Q = 10I$ and $R = 1$. The constraints are tightened by robust positive invariant ε in (20), which is computed by an approximation method in [24]. The noise parameters $\{a_1, a_2, a_3, a_4, a_5\}$ and the white noise $w_{r,1}, w_{r,2}, w_{r,3}$ are uniformly sampled from their domain.

In **Figure** 1, the cumulative cost by LMPC converges to the optimal cumulative cost. In particular, the optimal cost refers to the optimal solution of problem (8). Note the cost difference between J_{LMPC} and J^* does not decrease monotonically due to the shifted trajectories. However, **Figure** 2 shows that the closed-loop cumulative cost from $t = 0$ is upper bounded by any shifted cumulative cost from $t = 0$, guaranteed by Theorem 2. And the final convergent state trajecotry is shown in **Figure**. 3. The trajectory evolution along the iterations can be found in the extensive version.

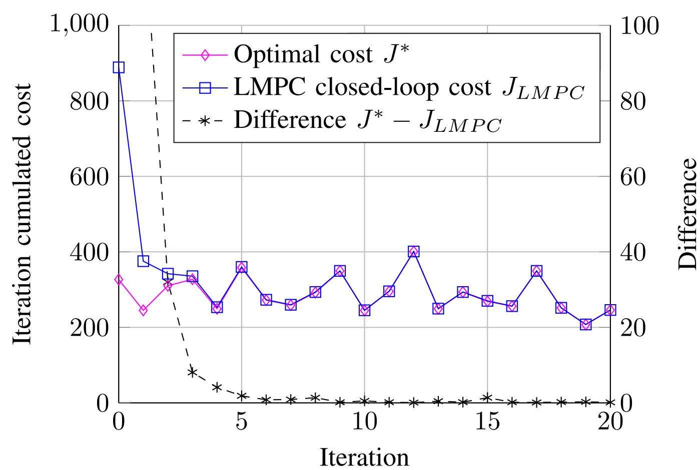

Fig. 1. Cumulative cost of each iteration

VII. Conclusion

We presented a novel less conservative robust LMPC scheme for periodically correlated process noise in the building control. The framework is specified for time-varying iterative tasks with periodicity in system dynamics, stage cost and constraints. The feasibility and performance convergence are verified by a single zone building system.

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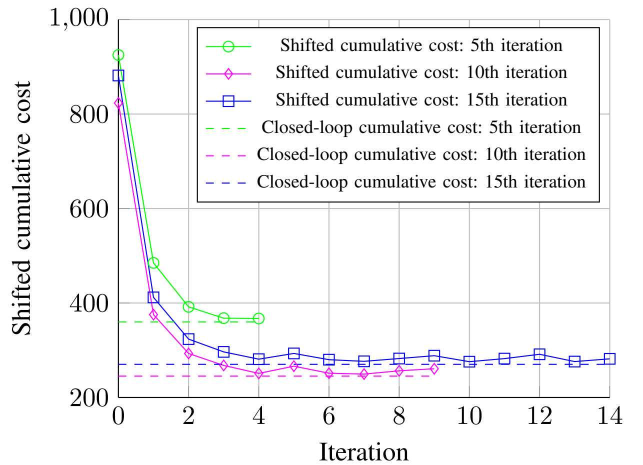

Fig. 2. Comparison of shifted and closed-loop cumulative cost starting from time 0

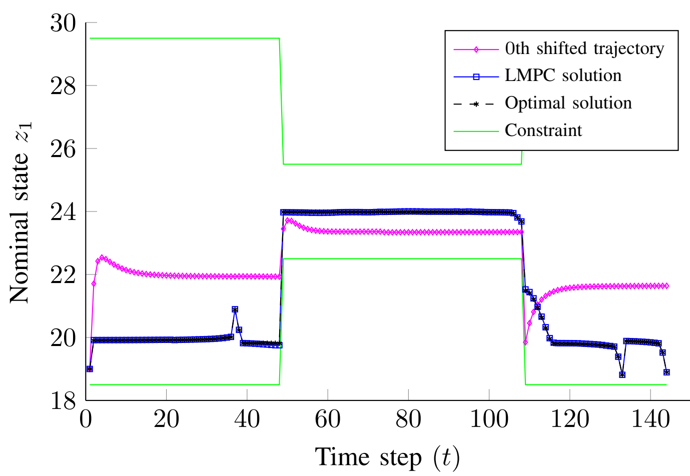

Fig. 3. Building system:x_1 at iteration 20

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VIII. Appendix

A. Robust and stochastic LMPC

The long-horizon optimal problem (3) is difficult to solve because the stochastic $w^j_{r,t}$ leads to a stochastic optimization objective and it optimizes over all possible control policy. A possible approach to deal with the problem is the tube method with a nominal optimization objective [21]. Denote by z^j_t the nominal state, by $e^j_t = x^j_t - z^j_t$ the error state, by v^j_t the nominal input, and by $Ke(k)$ the tube controller, where K stabilize all different $A_t + B_tK$. Then the tube controller is defined as

$$\begin{aligned} z^j_{t+1} &= A_t z^j_t + B_t v^j_t + C_t w^j_{\theta^j,t}, \\ e^j_{t+1} &= (A_t + B_tK)e^j_t + C_t w^j_{r,t} \\ u^j_t &= Ke^j_t + v^j_t \end{aligned} \tag{19}$$

and $z^j_0 = x_s$. Compute the *Robust Positive Invariant set* ε of e_t with dynamics (19). Then a constraint tightening is applied on the nominal system:

$$F_t z_t + G_t v_t \leq f_t - (F_t + G_tK)e_t, \forall e_t \in \varepsilon. \tag{20}$$

Thus, optimize the problem over the nominal stage cost $l_t(z_t, v_t)$ with the constraints (20), a robust problem in (8) is derived.