Correlation-Based Tuning of Linear Multivariable Decoupling Controllers

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***Abstract*— The recently-proposed method for iterative correlation-based controller tuning is considered in this paper for the tuning of multivariable Linear Time-Invariant (LTI) controllers. The parameters of the controller are updated directly using the data acquired in closed-loop operation. This approach allows one to tune some elements of the controller transfer function matrix to satisfy the desired closed-loop performance, while the other elements are tuned to mutually decouple the closed-loop outputs. The controller parameters are calculated by minimization of the cross-correlation function involving instrumental variables. A very simple choice of the instruments is proposed. The approach is applied to a simulation model of a gas turbine engine, and excellent results are obtained in terms of decoupling and performance.**

***Index Terms*— Controller tuning, correlation-based tuning, multivariable control, instrumental variables, decoupling.**

I. Introduction

The essential ingredients of any control design procedure include the acquisition of process knowledge and its efficient integration into the controller. Reliable models of industrial plants are often difficult or impossible to obtain mainly due to the high complexity of the plants and/or the excessive cost of modeling. The controllers designed on the basis of reduced-order models might well lead to unsatisfactory performance when applied to real plants due to modeling errors.

An alternative to model-based control design is to use the information collected on the plant *directly* for controller update. This idea stems from the area of direct adaptive control, in particular from self tuning regulation (STR) and model reference adaptive control (MRAC) [1]. Recently, several methods have appeared in the field of data-driven controller tuning such as controller unfalsification [2], simultaneous perturbation stochastic approximation control [3], iterative feedback tuning [4] and virtual reference feedback tuning [5]. An important question that arises in this research area is how to cope with the noise that necessarily corrupts the measurements and therefore also affects the closed-loop performance.

In the recently-proposed Correlation-based Tuning (CbT) approach, the problem of measurement noise is addressed differently [6]. The underlying idea is inspired from the correlation approach that uses instrumental variables and is well known in the system identification community [7]. The controller parameters are tuned to make the closed-loop output error between the designed and achieved closed-loop systems uncorrelated with the external reference signal. This way, the closed-loop output error ideally only contains the contribution of the noise, while the achieved closed-loop system captures the dynamics of the designed one. Moreover, the calculated controller parameters are asymptotically insensitive to measurement noise. The iterative correlation-based tuning approach has been successfully applied to a laboratory-scale magnetic suspension system in [6]. The controller parameters are calculated iteratively as the solution of a cross-correlation equation involving instrumental variables. In [8], the tuning objective is reformulated as the minimization of the 2-norm of the correlation function between the closed-loop output error and the reference signal. Also, a frequency-domain interpretation of the criterion shows that the algorithm minimizes the integral of the squared difference between the achieved and designed output sensitivity functions weighted by the square of the reference signal spectrum. In [9], a generalized correlation criterion is proposed that allows tuning the controller parameters so as to decorrelate as much as possible the reference signal with both the input and output closed-loop errors. It has been shown that, by minimizing this generalized criterion, the desired closed-loop output can be approached while taking into account some penalty on the control action, i.e. it is possible to handle the mixed sensitivity specifications. In [10], an adaptation of this approach to the disturbance rejection problem has been treated. A theoretical survey of this method can be found in [11].

The application of data-driven methods to the control of LTI multivariable systems has a few drawbacks. One of the main difficulties is the calculation of the gradient of the

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criterion. Typically, the number of experiments needed to estimate the gradient increases with the number of plant inputs, n_u, and outputs, n_y. An example is the IFT approach where $n_y n_u + 1$ experiments per iteration are necessary [12]. However, some efforts have been done recently to reduce the number of experiments with this approach (for more details see [13], [14], [15]). Another problem is the design of decouplers. For a method that minimizes a norm of an error signal, it is not possible to incorporate the decoupler design into the criterion if all references are excited simultaneously. Instead, for eliminating the influence of a reference on a particular output, it is necessary to excite that reference while keeping the other references constant and minimize an error signal norm related to that output. For MIMO systems with large n_u and n_y, this requires a large number of experiments.

In this work, the tuning of LTI multivariable controllers using the correlation approach is proposed. Assuming that the number of inputs and outputs is equal, the off-diagonal elements of the controller transfer function matrix are tuned to eliminate any interaction between the controlled outputs (in the sequel this will be called "diagonalization of the closed-loop system"), while the elements on the main diagonal are tuned to provide the desired closed-loop performance. The fact that the decoupling is done in a natural way by decorrelating a certain input from an output without the need for additional experiments makes CbT particularly appealing for the tuning of MIMO controllers. The controllers on the main diagonal feature the same characteristics as those for SISO systems. The parameters of the resulting decouplers and controllers are asymptotically not affected by the noise. Only one experiment per iteration is needed for the tuning of all controllers and decouplers regardless of the number of inputs and outputs since all reference inputs can be excited simultaneously.

The design of decouplers using standard methods may be very sensitive to modelling errors and uncertainties. The proposed method is model free and, if the data are acquired under sufficient persistency of excitation, the decouplers are most accurate where the output sensitivity is large [8].

The remainder of the paper is organized as follows. Some notations and the basic idea of the CbT approach are given in Section II. Section III deals with the tuning of LTI multivariable controllers. The simulation results are presented in Section IV. Finally, some concluding remarks are given in Section V.

II. Preliminaries

A. Notations

Consider an unknown LTI multivariable discrete-time system described by:

$$\mathbf{y}(t) = \mathbf{G}(q^{-1})\mathbf{u}(t) + \mathbf{v}(t) \tag{1}$$

where $\mathbf{y}(t) \in \mathcal{R}^{n_y}$ denotes the outputs of the true plant at time t, $\mathbf{u}(t) \in \mathcal{R}^{n_u}$ the control signals, $\mathbf{v}(t) \in \mathcal{R}^{n_y}$ the disturbances acting at the output and $\mathbf{G}(q^{-1}) \in \mathcal{R}^{n_y \times n_u}$ a transfer function matrix with q^{-1} being the backward-shift operator. It is assumed that $\mathbf{v}(t)$ is a zero-mean quasi-stationary stochastic process.

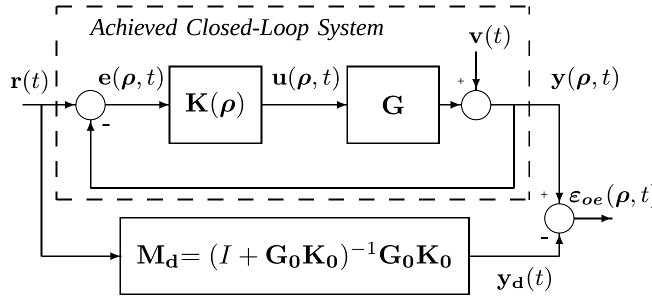

Fig. 1. Achieved multivariable closed-loop system and its reference model

Consider the block diagram of the model-following problem presented in Fig. 1. The upper part shows the achieved closed-loop system with the true plant, where $\mathbf{K}(\boldsymbol{\rho}) \in \mathcal{R}^{n_u \times n_y}$ is a transfer function matrix parameterized by some parameter vector $\boldsymbol{\rho} \in \mathcal{R}^{n_\rho}$, and $\mathbf{r}(t) \in \mathcal{R}^{n_y}$ represents external reference signals. The reference signals $\mathbf{r}(t)$ are assumed to be uncorrelated with the disturbances $\mathbf{v}(t)$. Furthermore, the elements of the reference signal vector $\mathbf{r}(t)$ are assumed to be mutually independent.

The lower part in Fig. 1 shows the reference model $\mathbf{M_d}$ defining the desired behavior of the closed-loop outputs $\mathbf{y_d}(t)$. The reference model can be constructed, for example, as the desired closed-loop system containing a model of the plant $\mathbf{G_0}$ and the controller $\mathbf{K_0}$. It is assumed that the reference model $\mathbf{M_d}$ has a diagonal structure. In this way, the controller $\mathbf{K_0}$ meets the control and decoupling specifications with respect to $\mathbf{G_0}$.

The closed-loop output error is defined as:

$$\boldsymbol{\varepsilon_{oe}}(\boldsymbol{\rho},t) = \mathbf{y}(\boldsymbol{\rho},t) - \mathbf{y_d}(t). \tag{2}$$

Let the following sensitivity functions be defined:

- Output sensitivity function:

$$\boldsymbol{\mathcal{S}} = (I + \mathbf{GK})^{-1} \tag{3}$$

- Complementary sensitivity function:

$$\boldsymbol{\mathcal{T}} = (I + \mathbf{GK})^{-1}\mathbf{GK} \tag{4}$$

where $I \in \mathcal{R}^{n_y \times n_y}$ is the identity matrix. The closed-loop response can be written as:

$$\mathbf{y}(\boldsymbol{\rho},t) = \boldsymbol{\mathcal{T}}\mathbf{r}(t) + \boldsymbol{\mathcal{S}}\mathbf{v}(t), \tag{5}$$

and the control error is:

$$\mathbf{e}(\boldsymbol{\rho},t) = \mathbf{r}(t) - \mathbf{y}(\boldsymbol{\rho},t) = \boldsymbol{\mathcal{S}}\left(\mathbf{r}(t) - \mathbf{v}(t)\right). \tag{6}$$

The $(i,j)^{th}$ element of the controller transfer function matrix is in the form of following one-degree-of-freedom controller:

$$K^{(ij)}(q^{-1},\boldsymbol{\rho}) = \frac{S^{(ij)}(q^{-1},\boldsymbol{\rho})}{R^{(ij)}(q^{-1},\boldsymbol{\rho})} \tag{7}$$

where

$$R^{(ij)}(q^{-1},\boldsymbol{\rho}) = 1 + r_1^{(i,j)}q^{-1} + \cdots + r_{n_r}^{(ij)}q^{-n_r}$$
$$S^{(ij)}(q^{-1},\boldsymbol{\rho}) = s_0^{(ij)} + s_1^{(ij)}q^{-1} + \cdots + s_{n_s-1}^{(ij)}q^{-n_s+1}$$

For simplicity, it is assumed that all controllers $K^{(ij)}(q^{-1},\boldsymbol{\rho})$, $i = 1,\ldots,n_u$, $j = 1,\ldots,n_y$ have the same order. The controller parameter vector $\boldsymbol{\rho}$ is written as follows:

$$\boldsymbol{\rho}^T = [\boldsymbol{\rho}^{(11)^T}, \boldsymbol{\rho}^{(12)^T}, \ldots, \boldsymbol{\rho}^{(1n_y)^T}, \boldsymbol{\rho}^{(21)^T}, \ldots, \boldsymbol{\rho}^{(n_u n_y)^T}]$$

where

$$\boldsymbol{\rho}^{(ij)^T} = [r_1^{(ij)}, r_2^{(ij)}, \ldots, r_{n_r}^{(ij)}, s_0^{(ij)}, s_1^{(ij)}, \ldots, s_{n_s-1}^{(ij)}]$$

Note that $n_\rho = (n_r + n_s)n_u n_y$. It will be assumed in the sequel that $\boldsymbol{\rho}$ is expressed as:

$$\boldsymbol{\rho}^T = [\rho^{(1)}, \rho^{(2)}, \ldots, \rho^{(n_\rho)}] \quad (8)$$

As far as the notations are concerned, the signals collected under closed-loop operation using the controller $\mathbf{K}(\boldsymbol{\rho})$ will carry the argument $\boldsymbol{\rho}$. The elements of vector signals and transfer function matrices will carry the position as a superscript in the parenthesis and will not be in bold. For example, $y^{(i)}(\boldsymbol{\rho},t)$ will denote the i^{th} component of the output vector $\mathbf{y}(\boldsymbol{\rho},t)$. Furthermore, the backward-shift operator q^{-1} will be omitted whenever appropriate.

B. Idea of Multivariable Correlation-Based Tuning

Consider again the model-following problem shown in Fig. 1, and assume for simplicity that $n_y = n_u = 2$ with the controller given in Fig. 2 operating in the loop. Let the design specification be as follows: Controllers $K^{(21)}(\boldsymbol{\rho})$ and $K^{(12)}(\boldsymbol{\rho})$ decouple the outputs $y^{(2)}(\boldsymbol{\rho},t)$ and $y^{(1)}(\boldsymbol{\rho},t)$ from $r^{(1)}(t)$ and $r^{(2)}(t)$, respectively; controllers $K^{(11)}(\boldsymbol{\rho})$ and $K^{(22)}(\boldsymbol{\rho})$ provide satisfactory tracking of $y_d^{(1)}(t)$ by $y^{(1)}(\boldsymbol{\rho},t)$ and $y_d^{(2)}(t)$ by $y^{(2)}(\boldsymbol{\rho},t)$, respectively.

Consider the output $y^{(1)}(\boldsymbol{\rho},t)$. When applying the controller $\mathbf{K_0}$ to the true plant excited by the reference signals $\mathbf{r}(t)$, the output $y^{(1)}(\boldsymbol{\rho},t)$ contains the contributions due to the disturbances $\mathbf{v}(t)$ and the reference signals $r^{(1)}(t)$ and $r^{(2)}(t)$. The reference signals $r^{(1)}(t)$ and $r^{(2)}(t)$ are mutually independent and uncorrelated with $\mathbf{v}(t)$. Hence, the idea of adjusting the parameters of $K^{(12)}(\boldsymbol{\rho})$ is to make the output $y^{(1)}(\boldsymbol{\rho},t)$ uncorrelated with the reference signal $r^{(2)}(t)$. The resulting decoupler provides $y^{(1)}(\boldsymbol{\rho},t)$ that contains only the contributions due to $v^{(1)}(t)$ and $r^{(1)}(t)$, i.e. the influence of $r^{(2)}(t)$ and $v^{(2)}(t)$ on $y^{(1)}(\boldsymbol{\rho},t)$ is eliminated.

Now consider the tuning of $K^{(11)}(\rho)$. Again, with $\mathbf{K_0}$ operating in the loop, the observed closed-loop output error $\varepsilon_{oe}^{(1)}(\boldsymbol{\rho},t)$ contains a contribution due to the disturbances $\mathbf{v}(t)$ and another contribution stemming from the difference between $\mathbf{G}$ and $\mathbf{G_0}$ that, in turn, has two parts originating from $r^{(1)}(t)$ and $r^{(2)}(t)$. The idea is to adjust the parameters of $K^{(11)}(\boldsymbol{\rho})$ so as to make $\varepsilon_{oe}^{(1)}(\boldsymbol{\rho},t)$ uncorrelated with $r^{(1)}(t)$. The controller updated in such a way compensates the effect of modeling errors to the extent that the closed-loop error $\varepsilon_{oe}^{(1)}(\boldsymbol{\rho},t)$ contains only $v^{(1)}(t)$ filtered by the closed-loop system. This way, the output $y^{(1)}(\rho,t)$ will achieve the desired output $y_d^{(1)}(t)$. Note that the effect of modeling errors due to $r^{(2)}(t)$ is eliminated by the decoupler $K^{(12)}(\boldsymbol{\rho})$.

A similar reasoning follows for the output $y^{(2)}(\boldsymbol{\rho},t)$.

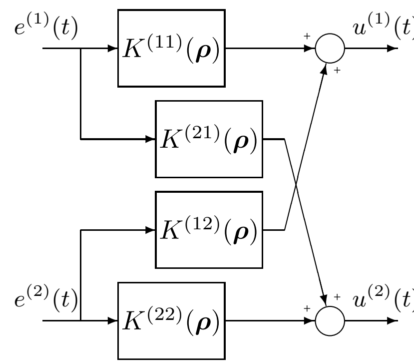

Fig. 2. Multivariable 2×2 controller

III. Correlation Reduction for MIMO Systems

A. Control Design Criterion

Without loss of generality, it is assumed that the plant has an equal number of inputs and outputs $n_y = n_u$. In this work, the elements of the controller transfer matrix $\mathbf{K}(\boldsymbol{\rho})$ will be tuned in the following way:

- The diagonal elements $K^{(ii)}(\boldsymbol{\rho})$, $i = 1,\ldots,n_u$ are tuned to satisfy the control specifications defined by $\mathbf{M_d}$.
- The off-diagonal elements $K^{(ij)}(\boldsymbol{\rho})$, $i \neq j, i,j = 1,\ldots,n_u$ are tuned to be decouplers. That is, controller $K^{(ij)}(\boldsymbol{\rho})$ is tuned to eliminate the influence of the reference signal $r^{(j)}(t)$ on the output $y^{(i)}(\boldsymbol{\rho},t)$. This in turn means that, if the decoupler $K^{(ji)}(\boldsymbol{\rho})$ has been tuned similarly, the mutual influence of $y^{(j)}(\boldsymbol{\rho},t)$ and $y^{(i)}(\boldsymbol{\rho},t)$ is suppressed.

Introduce the following control design criterion:

$$J(\boldsymbol{\rho}) = F^T(\boldsymbol{\rho})F(\boldsymbol{\rho}) \quad (9)$$

with the cross-correlation function $F(\boldsymbol{\rho})$ defined as:

$$F(\boldsymbol{\rho}) = E\left\{\bar{F}(\boldsymbol{\rho})\right\} \quad (10)$$

where $E\{\cdot\}$ is the mathematical expectation, and the vector $\bar{F}(\boldsymbol{\rho}) \in \mathcal{R}^{n_z n_u n_u}$ reads:

$$\bar{F}^T(\boldsymbol{\rho}) = [\,\bar{f}_{11}^T(\boldsymbol{\rho}), \bar{f}_{12}^T(\boldsymbol{\rho}), \ldots, \bar{f}_{1n_u}^T(\boldsymbol{\rho}), \bar{f}_{21}^T(\boldsymbol{\rho}), \ldots, \bar{f}_{n_u n_u}^T(\boldsymbol{\rho})] \quad (11)$$

with

$$\bar{f}_{ij}(\boldsymbol{\rho}) = \frac{1}{N}\sum_{t=1}^{N}\zeta_{ij}(t)\eta_{ij}(\boldsymbol{\rho},t) \quad (12)$$

where N is the number of data, and n_z the dimension of the instrumental variable vector ζ_{ij}. The component $\bar{f}_{ij}(\boldsymbol{\rho}) \in \mathcal{R}^{n_z}$ corresponds to the controller $K^{(ij)}(\boldsymbol{\rho})$. The way the instrumental variables $\zeta_{ij}(t)$ and the variable $\eta_{ij}(\boldsymbol{\rho},t) \in \mathcal{R}$

are constructed depends on whether or not the controller $K^{(ij)}(\boldsymbol{\rho})$ is on the main diagonal of $\mathbf{K}(\boldsymbol{\rho})$:

- $\underline{i=j}$: $K^{(i,i)}(\boldsymbol{\rho})$ is tuned so as to reduce the correlation between $\varepsilon_{oe}^{(i)}(\boldsymbol{\rho},t)$ and $r^{(i)}(t)$. Taking into account that the tuning of the controllers $K^{(i,i)}(\boldsymbol{\rho})$ and the decouplers $K^{(i,j)}(\boldsymbol{\rho})$ is done simultaneously, the output $y^{(i)}(\boldsymbol{\rho},t)$ will follow $y_d^{(i)}(t)$ up to the effect of the disturbance error in the case of perfect decorrelation. Thus, the vector of instrumental variables $\zeta_{ii}(t)\in\mathcal{R}^{n_z}$ should be chosen to be correlated with the reference signal $r^{(i)}(t)$ and independent of the disturbance $v^{(i)}(t)$. Here, a shifted version of the reference signal $r^{(i)}(t)$ is adopted:

$$\zeta_{ii}^T(t)=[r^{(i)}(t+l),\ldots,r^{(i)}(t),\ldots,r^{(i)}(t-l)] \quad (13)$$

where l is large enough with respect to the number of controller parameters, i.e. $n_z=2l+1\geq n_r+n_s$. Furthermore:

$$\eta_{ii}(\boldsymbol{\rho},t)=\varepsilon_{oe}^{(i)}(\boldsymbol{\rho},t) \quad (14)$$

The reader is referred to [11] for a discussion on the properties and implementation aspects of the CbT approach.

- $\underline{i\neq j}$: To eliminate the influence of $r^{(j)}(t)$ on $y^{(i)}(t)$, it is sufficient to decorrelate these two signals. Therefore, $\zeta_{ij}(t)$ and $\eta_{ij}(\boldsymbol{\rho},t)$ can be chosen as:

$$\zeta_{ij}^T(t)=[r^{(j)}(t+l),\ldots,r^{(j)}(t),\ldots,r^{(j)}(t-l)] \quad (15)$$

and

$$\eta_{ij}(\boldsymbol{\rho},t)=y^{(i)}(\boldsymbol{\rho},t). \quad (16)$$

To tune the decoupler $K^{(i,j)}(\boldsymbol{\rho})$ by a method that minimizes the 2-norm of the closed-loop output error (such as IFT), it is necessary to excite the component j of the reference signal $\mathbf{r}(t)$ while keeping the other components equal to zero and then minimize the 2-norm of $y^{(i)}(\boldsymbol{\rho},t)$. For MIMO systems with a large number of inputs and outputs, this requires a large number of experiments per iteration to tune all decouplers $K^{(i,j)}(\boldsymbol{\rho})$, $i\neq j$. Moreover, by minimizing the 2-norm of $y^{(i)}(\boldsymbol{\rho},t)$, one makes a trade-off between noise attenuation and elimination of the influence of $r^{(j)}(t)$ on $y^{(i)}(\boldsymbol{\rho},t)$. In contrast, with the CbT approach, only one experiment per iteration is needed for the tuning of all controllers $K^{(i,j)}(\boldsymbol{\rho})$. Furthermore, the criterion (9)-(12) is not influenced by the noise. All this makes the correlation-based approach particularly adequate for the tuning of decouplers.

Note that $n_z\geq n_r+n_s$. When $n_z=n_r+n_s$, the parameters of $\mathbf{K}(\boldsymbol{\rho})$ are the solution of a cross-correlation equation $F(\boldsymbol{\rho})=0$. Note also that the vector $\bar{F}(\boldsymbol{\rho})$ can be expressed in compact form as:

$$\bar{F}(\boldsymbol{\rho})=\frac{1}{N}\sum_{t=1}^{N}Z(t)\Delta(\boldsymbol{\rho},t) \quad (17)$$

where $Z(t)\in\mathcal{R}^{n_z n_u n_u\times n_u n_u}$ is a matrix of instrumental variables in block diagonal form:

$$\begin{aligned}Z(t)= \;& \mathrm{diag}\,(\,\zeta_{11}(t),\zeta_{12}(t),\ldots,\zeta_{1n_u}(t),\\ & \zeta_{21}(t),\ldots,\zeta_{n_u n_u}(t))\end{aligned} \quad (18)$$

and vector $\Delta(\boldsymbol{\rho},t)\in\mathcal{R}^{n_u n_u}$ follows as:

$$\begin{aligned}\Delta^T(\boldsymbol{\rho},t)= \;& [\,\eta_{11}(\boldsymbol{\rho},t),\eta_{12}(\boldsymbol{\rho},t),\ldots,\eta_{1n_u}(\boldsymbol{\rho},t),\\ & \eta_{21}(\boldsymbol{\rho},t),\ldots,\eta_{n_u n_u}(\boldsymbol{\rho},t)]\,.\end{aligned} \quad (19)$$

The minimization of criterion (9) is intractable since it involves the product of expectations that are unknown. Therefore, let us define the following criterion:

$$\bar{J}(\boldsymbol{\rho})=E\left\{\bar{F}^T(\boldsymbol{\rho})\bar{F}(\boldsymbol{\rho})\right\} \quad (20)$$

This new criterion can be minimized using the stochastic approximation method. It can be shown that $J(\boldsymbol{\rho})\leq\bar{J}(\boldsymbol{\rho})$, i.e. by minimizing (20) one minimizes, in fact, an upper bound on (9) [11].

B. Iterative Solution

A local minimum of (20) can be found as the solution of:

$$\bar{J}'(\boldsymbol{\rho})=E\left\{\frac{\partial\bar{F}(\boldsymbol{\rho})}{\partial\boldsymbol{\rho}}\bar{F}(\boldsymbol{\rho})\right\}=0 \quad (21)$$

which can be obtained using the following iterative formula [17]:

$$\boldsymbol{\rho}_{i+1}=\boldsymbol{\rho}_i-\gamma_i\left.\frac{\partial\bar{F}(\boldsymbol{\rho})}{\partial\boldsymbol{\rho}}\right|_{\boldsymbol{\rho}_i}\bar{F}(\boldsymbol{\rho}_i) \quad (22)$$

where γ_i is a scalar step size. Under the standard assumptions of the stochastic approximation methods, this scheme converges to a local minimum of the criterion as the number of iterations goes to infinity.

Since the matrix of instruments $Z(t)$ is independent of $\boldsymbol{\rho}$, the derivative of $\bar{F}(\boldsymbol{\rho})$ is determined as follows:

$$\begin{aligned}\left.\frac{\partial\bar{F}(\boldsymbol{\rho})}{\partial\boldsymbol{\rho}}\right|_{\boldsymbol{\rho}=\boldsymbol{\rho}_i} &= \frac{1}{N}\sum_{i=1}^{N}\left.\frac{\partial\Delta(\boldsymbol{\rho},t)}{\partial\boldsymbol{\rho}}\right|_{\boldsymbol{\rho}_i}Z^T(t)\\ &= \frac{1}{N}\sum_{i=1}^{N}\left.\frac{\partial\mathbf{y}(\boldsymbol{\rho},t)}{\partial\boldsymbol{\rho}}\right|_{\boldsymbol{\rho}_i}\Gamma\,Z^T(t)\end{aligned} \quad (23)$$

The latter equality in (23) follows due to the fact that (i) the elements of the vector $\Delta(\boldsymbol{\rho},t)$ are either the components of $\boldsymbol{\varepsilon_{oe}}(\boldsymbol{\rho},t)$ or the components of $\mathbf{y}(\boldsymbol{\rho},t)$, and (ii) $\partial\boldsymbol{\varepsilon_{oe}}(\boldsymbol{\rho},t)/\partial\boldsymbol{\rho}=\partial\mathbf{y}(\boldsymbol{\rho},t)/\partial\boldsymbol{\rho}$. The linear transformation matrix $\Gamma\in\mathcal{R}^{n_u\times n_u n_u}$ is in block diagonal form:

$$\Gamma=\mathrm{diag}\left(g^T,g^T,\ldots,g^T\right) \quad (24)$$

with the vector $g\in\mathcal{R}^{n_u}$ being $g^T=[1,1,\ldots,1]$.

An accurate value of the derivative (23) cannot be computed because the derivative of $\mathbf{y}(\boldsymbol{\rho},t)$ with respect to $\boldsymbol{\rho}$ is unknown. Nevertheless, one can write formally:

$$\left.\frac{\partial\mathbf{y}^T(\boldsymbol{\rho},t)}{\partial\boldsymbol{\rho}}\right|_{\boldsymbol{\rho}_i}=\left[\psi_1(\rho_i^{(1)},t),\psi_2(\rho_i^{(2)},t),\ldots,\psi_{n_\rho}(\rho_i^{(n_\rho)},t)\right]$$

with

$$\psi_k(\rho_i^{(k)}, t) = \left.\frac{\partial \mathbf{y}(\boldsymbol{\rho}, t)}{\partial \rho^{(k)}}\right|_{\rho_i^{(k)}} = \boldsymbol{\mathcal{S}}\mathbf{G} \left.\frac{\partial \mathbf{K}(\boldsymbol{\rho})}{\partial \rho^{(k)}}\right|_{\rho_i^{(k)}} \mathbf{e}(\boldsymbol{\rho}_i, t)$$

Although the transfer function matrices $\boldsymbol{\mathcal{S}}$ and $\mathbf{G}$ are typically unknown, they can be identified and replaced by their estimates $\hat{\boldsymbol{\mathcal{S}}}$ and $\hat{\mathbf{G}}$. Note that an unbiased estimate of $\frac{\partial \mathbf{y}^T(\boldsymbol{\rho}, t)}{\partial \boldsymbol{\rho}}$ can alternatively be obtained at the cost of $n_u n_u$ additional closed-loop experiments, as is done in the IFT approach for MIMO systems [12].

For N sufficiently large, the criterion (20) can be considered as deterministic and minimized using the much faster Gauss-Newton iterative algorithm:

$$\boldsymbol{\rho}_{i+1} = \boldsymbol{\rho}_i - Q^{-1} \left.\frac{\partial \bar{F}(\boldsymbol{\rho})}{\partial \boldsymbol{\rho}}\right|_{\boldsymbol{\rho}_i} \bar{F}(\boldsymbol{\rho}_i) \qquad (25)$$

where Q is chosen as:

$$Q(\boldsymbol{\rho}_i) = \left.\frac{\partial \bar{F}(\boldsymbol{\rho})}{\partial \boldsymbol{\rho}}\right|_{\boldsymbol{\rho}_i} \left(\left.\frac{\partial \bar{F}(\boldsymbol{\rho})}{\partial \boldsymbol{\rho}}\right|_{\boldsymbol{\rho}_i} \right)^T \qquad (26)$$

This section has presented the principles of iterative correlation-based tuning of multivariable LTI controllers. The vector of parameters is found by minimizing the cross-correlation criterion (20) using the stochastic approximation method. This algorithm converges to a stationary point provided an unbiased estimate of the gradient is available. However, obtaining an unbiased estimate of the gradient for MIMO systems is very costly. It is proposed here to compute the gradient using an identified MIMO model, which requires only one experiment with the closed-loop system regardless of the number of inputs and outputs. However, in this case, local convergence of the algorithm is guaranteed only if an unbiased model can be identified.

IV. Simulation Example

In this section, the basic features of the proposed algorithm are investigated and compared to the ones of IFT for MIMO systems. The aim is to tune a multivariable PI controller for a LV100 gas turbine engine [18]. The initial controller and simulation conditions are taken from [12]. The plant is represented by a continuous-time state-space model with five states, two inputs and two outputs. The model is discretized using Tustin approximation with the sampling period $T_s = 0.1s$. Each experiment is performed with a different realization of the measurement noise $\mathbf{v}(t)$ that is generated as a zero-mean, stationary, white Gaussian sequence with variance $0.0025I$.

The initial 2×2 controller $\mathbf{K_0}$ (see Fig. 2) is given as:

$$\mathbf{K_0} = \begin{pmatrix} \frac{1-0.99q^{-1}}{1-q^{-1}} & \frac{0.1-0.099q^{-1}}{1-q^{-1}} \\ -\frac{1-0.99q^{-1}}{1-q^{-1}} & \frac{1-0.99q^{-1}}{1-q^{-1}} \end{pmatrix} \qquad (27)$$

Eight numerator coefficients are tuned (two for each transfer function element), while the denominators are kept fixed at $1 - q^{-1}$. The following reference model is given:

$$\mathbf{M_d} = \begin{pmatrix} \frac{0.4q^{-1}}{1-0.6q^{-1}} & 0 \\ 0 & \frac{0.4q^{-1}}{1-0.6q^{-1}} \end{pmatrix} \qquad (28)$$

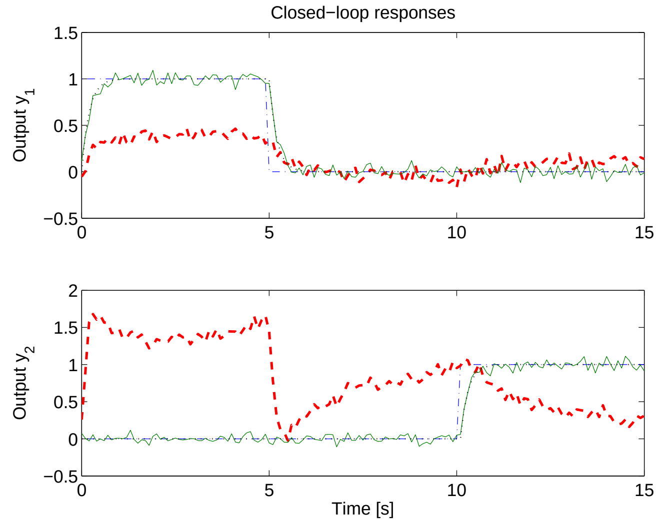

Fig. 3. Closed-loop responses in a noisy enviroment. Reference signals (dash-dot), desired responses (dotted), achieved responses with the initial controller (dashed) and final controller (solid)

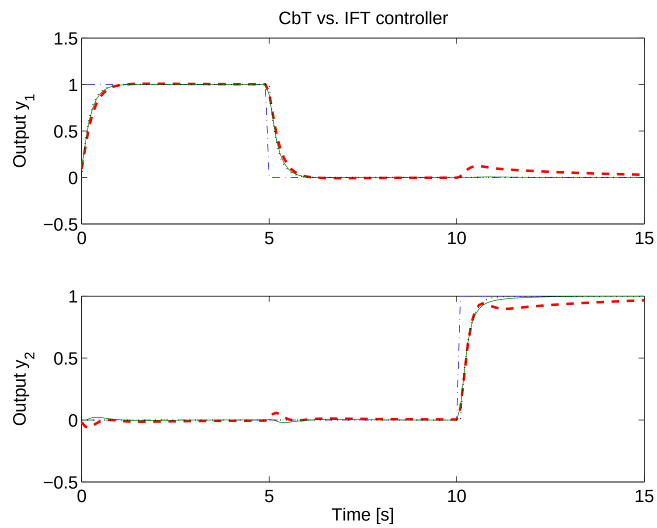

Fig. 4. CbT vs. IFT controller without noises: Reference signals (dash-dot), desired responses (dotted), closed-loop response with the CbT controller (solid) and IFT controller (dashed)

The reference signals $\mathbf{r}(t)$ are given in Fig. 3. The algorithm (25) is used to calculate the controller parameters.

A discrete-time state-space model with three states is identified to calculate the estimate $\hat{\mathbf{G}}$. After eight iterations, this procedure provides the closed-loop response shown in Fig. 3. A comparison with the desired response (dotted line) shows that these two curves are nearly superposed except for the effect of noise. In addition, the change of the reference signals $r^{(1)}(t)$ and $r^{(2)}(t)$ does not provoke any visible change on the outputs $y^{(2)}(t)$ and $y^{(1)}(t)$, respectively. In other words, the closed-loop system is almost fully diagonalized. The value of the tuning criterion is reduced by more than 99%. The CbT controller is given as follows:

$$\mathbf{K}_{CbT} = \begin{pmatrix} \frac{0.3636-0.09866q^{-1}}{1-q^{-1}} & \frac{0.3653-0.2691q^{-1}}{1-q^{-1}} \\ \frac{18.69-18.16q^{-1}}{1-q^{-1}} & \frac{-3.453+2.652q^{-1}}{1-q^{-1}} \end{pmatrix} \quad (29)$$

In order to compare the IFT controller provided in [12],

$$\mathbf{K}_{IFT} = \begin{pmatrix} \frac{0.248-0.03q^{-1}}{1-q^{-1}} & \frac{0.38-0.199q^{-1}}{1-q^{-1}} \\ \frac{16.47-15.91q^{-1}}{1-q^{-1}} & \frac{0.063+0.054q^{-1}}{1-q^{-1}} \end{pmatrix} \quad (30)$$

with the CbT controller, an experiment is performed with the simulation conditions mentioned above. Define the sum of squared output errors as:

$$SSOE = \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{\varepsilon}_{\mathbf{oe}}^{T}(\boldsymbol{\rho}, t)\boldsymbol{\varepsilon}_{\boldsymbol{oe}}(\boldsymbol{\rho}, t)$$

where $N = 151$. The observed $SSOE$ with the CbT controller is 0.0050, while that with the IFT controller is 0.0082. Since IFT contains a noise-rejection objective, while CbT does not, one would expect IFT to perform better in a noisy situation. However, $SSOE$ obtained with CbT is smaller. This is due to the fact that IFT did not succeed in (i) fully decoupling the closed-loop system, and (ii) completely satisfying the model-following specification. To illustrate this, an additional experiment without noise is performed. The results are shown in Fig. 4. The closed-loop response obtained with the CbT controller follows almost perfectly the desired response. In contrast, the closed-loop response obtained with the IFT controller shows some discrepancy in the last 5 seconds of the response. In addition, the influence of the change in the reference signal $r^{(1)}(t)$ at the instants 0s and 5s is visible on $y^{(2)}(t)$.

In terms of experimental cost, the IFT controller is obtained after 6 iterations (and a total of 30 experiments) compared to 8 iterations (and a total of 8 experiments) with the CbT controller.

V. Conclusions

In this contribution, the parameters of a linear time invariant multivariable controller have been tuned by minimizing a cross-correlation function. The diagonal controllers are tuned to fulfill the desired output specifications, while the off-diagonal controllers are tuned to decouple the various outputs. In contrast to the approaches where decouplers are designed first and diagonal controllers second, the design of the controllers and decouplers is done simultaneously here.

The cross-correlation criterion is minimized iteratively using the stochastic approximation method. An unbiased estimate of the gradient of the output is necessary to guarantee convergence of the algorithm to a stationary point. It has been proposed here to compute the gradient using an identified MIMO model, which requires only one experiment with the closed-loop system regardless of the number of inputs and outputs.

Simulation results illustrate the features and the applicability of this tuning approach to a LTI MIMO system. Comparison of the proposed tuning method with IFT on the simulation model of a LV100 gas turbine engine shows that the correlation-based controller tuning provides better tracking performance in fewer experiments. In addition, CbT controller diagonalizes almost perfectly the closed-loop system.

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