Characterizing the Optimal Incentives for Pervasive Demand Response

Tri Kurniawan Wijaya, Matteo Vasirani,
Karl Aberer
School of Computer and Communication
Sciences, EPFL, Switzerland
{tri-kurniawan.wijaya, matteo.vasirani,
karl.aberer}@epfl.ch

Jonas Christoffer Villumsen
IBM Research - Ireland
jonasvil@ie.ibm.com

ABSTRACT

While Demand Response (DR) has been focused on large and industrial consumers, pervasive implementation (by including residential consumers) is needed to maximize its potential. This paper presents analyses from the economics perspective of pervasive, incentive-based DR, and consider cases whether (1) DR is used to encourage consumers to decrease or increase their demand, and (2) utility companies have access to a single or multiple energy sources. We derive the necessary conditions and the optimal incentives to benefit from DR events.

Categories and Subject Descriptors

G.0 [**Mathematics of Computing**]: General; I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Control theory*; H.4.2 [**Information Systems Applications**]: Types of Systems—*Decision support*

Keywords

demand response, smart grid, electricity market, incentive-based demand response, net benefit analysis

1. INTRODUCTION

The ever-growing energy demand, increasing penetration of electric vehicles, and integration of renewable energy sources raise important challenges to electricity grids in matching supply and demand. To this end, demand response (DR) emerges as one of the cheapest, greenest, and sustainable solutions. DR is defined as changes in electric usage by end-use consumers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments.

While there are two types of DR, i.e., price- and incentive-based [3, 4, 14], this paper focuses on incentive-based DR, where under flat-rate price, consumers are offered some incentives to change their energy consumption. The incentives can be, for example, movie tickets, bill rebates, redeemable vouchers, or virtual currencies which can be converted into real products or other benefits (as in the concept of *miles* in airline marketing). In addition, the messages sent to communicate upcoming DR events can also be seen as a trigger as to *when to act* [16]. From consumer psychological point of view, incentive-based DR also provides a positive impression. While in price-based DR consumers suffer from price increase, in incentive-based DR consumers enjoy incentive offers. More specifically, in price-based DR, cheaper price periods are considered as normal time, whereas more expensive, peak price periods considered as *DR events.* On the other hand, incentive-based DR uses flat-rate price, where periods with no incentive offer is considered as normal time, and periods with incentive offers considered as DR events.

Additionally, DR has been focused on large, industrial consumers. To maximize its potential, however, DR implementation needs to be pervasive by attracting residential consumers as well. To this end, the vision of smart grids and deployment of smart meters has provided a wide-open opportunity to include their participation. Consequently, a deep insights about pervasive DR is necessary. For example, *what are the necessary conditions to benefit from DR? How much incentives should be given to consumers?*

However, this task is very challenging due to at least three reasons. First, utility companies acquire their energy supply from various sources. Some of them buy energy from the market, some own coal power plant, some use natural gas (or other sources), and some own several generators powered by different sources, which contribute differently to their profit. Second, DR can be used not only to induce lower energy consumption, but also to encourage higher consumption, especially when there is an energy surplus in the grid. This surplus can happen, for example, when the sun shines (for area with high penetration of solar power) or when the wind blows strongly (for area with high penetration of wind power) but the demand is low. Third, there are a lot of uncertainties in consumer responses, since they are influenced by numerous factors, such as the incentives, time of day, day of the week, outdoor temperature, weather, holidays, or guests at home.

Overview of Contributions. This paper provides economical analyses of pervasive, incentive-based DR. Our analysis focuses more on the commodity subsystem rather than the physical subsystem.[1] We consider cases where DR is used as a mechanism to reduce and increase energy consumption. We determine the lower bound of the generation cost and *consumer reduction (or increase) rate*[2] such that DR is still beneficial. While a common belief states that "DR is best to be carried out when the market price is greater than the retail price," there is more to it than that. Consumers *willingness to accept* should be taken into account as well. Additionally, we derive the optimal incentives to maximize gains from DR events. We present our analyses in cases where utility companies have access to both, single and multiple energy sources.

The rest of the paper is organized as follows. In Section 2, we present a brief review of the literature. In Section 3, we introduce our key assumptions. We analyze when pervasive DR is used to reduce energy demand in Section 4, and when it is used to increase energy demand in Section 5. We conclude our work in Section 6.

2. RELATED WORK

Albadi and El-Saadany provided a good overview of Demand Response [3, 4]. In addition, The U.S. Department of Energy and Federal Energy Regulatory Commission outlined the benefit of DR and its relation to the electricity market [14, 33]. In contrast to our work, Borenstein and Holland analyzed the economics of real-time pricing (RTP) [7]. In particular, they assessed the market efficiency where both, RTP and flat-pricing scheme, coexist in the consumer base. They showed that increasing the number of consumers who adopt RTP scheme might harm the other consumers who are already on RTP, but could bring advantage to consumers who (stay) in the flat-rate. Joskow and Tirole also analyzed the efficiency of RTP, assuming different types of consumers: price-sensitive consumers with real-time (or smart-) meters, price-insensitive and partially-sensitive consumers with real-time meters, and price-insensitive consumers with traditional meters (whose meters are read, for example, only once a month) [23]. While the energy consumption of price-insensitive consumers with smart meters can be charged according to the real-time price, the energy consumption of consumers with traditional meters cannot. Since the energy bill of consumers with traditional meters is calculated based on the average energy price, thus they are not exposed to price fluctuation in the market. Consequently, in case of price-insensitive consumers, efficient pricing could still exist only if the consumers are equipped with smart meters.

There are also a number of studies which aim to foster the emergence of pervasive DR. However, most of them focused on price-based mechanisms [8, 26, 36, 29, 30, 34, 35]. These mechanisms require consumers to tirelessly track price fluctuation and adjust their consumption schedule accordingly to maximize their benefit (achieving electricity bill as low as possible with least inconvenience). To this end, some studies propose to use software agents (or energy management systems), for automatic price monitoring and schedule optimization, to make the entire process seamless from the consumers' perspective [36, 29, 30, 34, 35]. Holland and Mansur studied the environmental impact of implementing RTP throughout the United States [21]. They found that, in contrary to public belief, real-time pricing does not always reduce pollution. Pollutant reduction strongly depends on the type of generators used to meet peak demand.

While we focus on a more holistic view of incentive-based DR, there are also some studies dedicated to consumer baseline. In incentive-based DR, *consumer baseline* (or DR baseline) is an estimate of what consumers would have consumed in the absence of a DR event. It also plays an important role to determine the amount of incentives that consumers should receive [32]. Analyses of consumer baseline applied to large and commercial consumers have been discussed in [10, 9, 1, 2, 28], while its application to residential consumers has been discussed in [37].

3. KEY ASSUMPTIONS

3.1 Load Generation

Let us assume that a utility company has access to a set of generators or energy sources G. For a particular time period, let:

- L_{g_i} be the load assigned to generator $g_i \in G$, and $L = \sum_{g_i \in G} L_{g_i}$,
- $cap(g_i)$ be the capacity of generator g_i, and
- $C_{g_i}(L')$ be the total cost of meeting load demand L' using generator g_i.

To meet the load demand L, a utility company assign L to one or more generators[3] depending on the capacity and generation cost. We assume that the utility company assigns (or, the market implicitly assigns) the load to the cheapest generator first, up to its capacity, before using the more expensive generators. Thus, for simplicity, we assume that the set of generators G is ordered by its generation cost, i.e., $C_{g_i}(L') \leq C_{g_j}(L')$ for $i \leq j$. Thus, for $j = i + 1$, we have $L_{g_j} > 0$ iff $L_{g_i} = cap(g_i)$ and $L > \sum\limits_{1 \leq k \leq i} L_{g_k}$. We also assume that there are no temporal constraints (such as generator ramp constraints and start-up constraints) and no appropriate energy storage solutions available to the utility company.

3.2 Consumer Responses

One of the main challenges faced by utility companies to carry out DR events for residential consumers is that only little is known about how consumers will respond to them. Consumer responses can be affected by many factors, such as the amount of incentives, time of the day, day of the week, weather, outdoor temperature, holidays, or guests at home. Understanding the influence of these factors requires real implementation. Utility companies, however, need to have a deep understanding and holistic view about DR before

---

[1] See the difference between commodity and physical subsystem in [25], Section 3.2. In the future, it would be possible to extend our work by considering constraints from the physical subsystem.

[2] It indicates how sensitive a consumer is to incentives (see also Section 3.2). The higher the consumer reduction (or increase) rate, the more sensitive she is.

[3] We use the terms *energy sources* and *generators* interchangeably.

starting any real implementation. Hence, we have a *chicken and egg* problem.

In this paper, inspired by prior research on consumer response to dynamic pricing, which confirmed that consumer energy reduction increases as the electricity prices increases [12], we assume that consumer responses are affected by the amount of incentives while other factors are held constant. More specifically, we model consumer responses to grow linearly with the incentives. As we will also show later, this allows us to understand the economics of DR, to benefit from it, while keeping things as simple as possible. For simplicity, below we specify consumer response as demand reduction (see Section 5 for consumer response in the context of DR to increase demand). Let $\{x_1, x_2, \ldots, x_n\}$ be the set of factors that (possibly) influence consumer responses, and $x_1 = \mathcal{I}$ be the incentives (unit: \$).[4] We define the consumer response (or the demand reduction) during a DR event as:

$$r(x_1, x_2, \ldots, x_n) = r(\mathcal{I}, x_2, \ldots, x_n) = m\mathcal{I} + c, \qquad (1)$$

where m is the *consumer reduction rate* (unit: $kWh/\$$), and c is a constant. We assume $m \geq 0$ and $\mathcal{I} \geq 0$. We also assume that any reduction during a DR event require some incentives, i.e., $c = 0$. Thus, we have:

$$r(\mathcal{I}, x_2, \ldots, x_n) = m\mathcal{I}. \qquad (2)$$

Intuitively, consumer reduction rate m is the amount of demand reduction that can be obtained using a unit of incentive. It can also be thought of as $\frac{dr}{d\mathcal{I}}$, the rate of reduction per unit incentive. Our response model can also be seen as an alternative and first step towards defining a more realistic response model due to the absence of real response data. In the sequel, we often use R instead of $r(\mathcal{I}, f_1, \ldots, f_n)$ to simplify notation.

4. DR TO REDUCE ENERGY CONSUMPTION

4.1 Without DR (Business As Usual)

Cost We define the cost to meet load demand L as $\sum_{g_i \in G} C_{g_i}(L_{g_i})$, where $L = \sum_{g_i \in G} L_{g_i}$.

Revenue Let P_{ret} be the retail price (unit: $\$/kWh$) paid by the consumers, and L be the total load demanded by the consumers. Thus, company revenue = $P_{ret}L$.

Profit We define company's profit as its revenue minus cost:

$$P_{ret}L - \sum_{g_i \in G} C_{g_i}(L_{g_i}) \qquad (3)$$

4.2 With DR

Let $\mathcal{I}$ be the incentives that the utility company gives to the consumers, and R denotes the consumers' demand reduction in a particular DR event. Thus, the total demand in the presence of the DR event is $L^{DR} = L - R$.

Cost The cost of meeting the load demand L^{DR} is $\sum\limits_{g_i \in G} C_{g_i}(L^{DR}_{g_i})$, where $L^{DR} = \sum\limits_{g_i \in G} L^{DR}_{g_i}$.

Revenue Company revenue = $P_{ret}L^{DR}$.

Profit Company's profit = revenue - cost - incentives:

$$P_{ret}L^{DR} - \sum_{g_i \in G} C_{g_i}(L^{DR}_{g_i}) - \mathcal{I} \qquad (4)$$

Gain We define the gain of a utility company from a DR event as the difference between its profit with and without the DR event, i.e., by substracting Eq. 3 from Eq. 4:

$$\begin{aligned} gain = \; & P_{ret}L^{DR} - \textstyle\sum_{g_i \in G} C_{g_i}(L^{DR}_{g_i}) - \mathcal{I} - P_{ret}L + \\ & \textstyle\sum_{g_i \in G} C_{g_i}(L_{g_i}) \\ = \; & \textstyle\sum_{g_i \in G} \left(C_{g_i}(L_{g_i}) - C_{g_i}(L^{DR}_{g_i})\right) + P_{ret}L^{DR} - \\ & P_{ret}L - \mathcal{I} \\ = \; & \textstyle\sum_{g_i \in G} \left(C_{g_i}(L_{g_i}) - C_{g_i}(L^{DR}_{g_i})\right) - P_{ret}R - \mathcal{I}. \end{aligned} \qquad (5)$$

If $L^{DR} \leq L$, then there are some generators that are unused or do not run at their full capacity. Let us denote this set of generators as G_{red}, i.e., the *reduced* generators. And, we define the rest of the generators as $G_{base} = G \setminus G_{red}$. Formally:

- $g_i \in G_{red}$, iff $L^{DR}_{g_i} < L_{g_i}$, and
- $g_i \in G_{base}$, iff $L^{DR}_{g_i} = L_{g_i}$.

For instance, when there is a demand reduction during a DR event, then by definition, the most expensive generator belongs to G_{red}. We can rewrite the gain computation in Eq. 5 by separating the set of generators in G_{base} and G_{red} as:

$$\begin{aligned} gain = \; & \textstyle\sum_{g_i \in G} \left(C_{g_i}(L_{g_i}) - C_{g_i}(L^{DR}_{g_i})\right) - P_{ret}R - \mathcal{I} \\ = \; & \textstyle\sum_{g_i \in G_{base}} \left(C_{g_i}(L_{g_i}) - C_{g_i}(L^{DR}_{g_i})\right) + \\ & \textstyle\sum_{g_i \in G_{red}} \left(C_{g_i}(L_{g_i}) - C_{g_i}(L^{DR}_{g_i})\right) - P_{ret}R - \mathcal{I} \\ = \; & \textstyle\sum_{g_i \in G_{red}} \left(C_{g_i}(L_{g_i}) - C_{g_i}(L^{DR}_{g_i})\right) - P_{ret}R - \mathcal{I} \end{aligned} \qquad (6)$$

Thus, the gain of a DR event depends on the (i) difference in the total cost of meeting the load demand, (ii) reduction in the revenue, and (iii) total incentives.

The next challenge is to determine the right incentives to obtain *positive gain* (or benefit) from DR. Once we have identified necessary conditions to obtain positive gain, we aim to find the optimal incentives to maximize gain. To this end, we divide the problem into two cases. First, where a utility company has access to a single energy source. Second, in a more general setting, where the utility company has access to multiple energy sources.

4.3 Single Energy Source

---

[4] In practice, the incentives can be of form movie tickets, bill rebates, redeemable vouchers, etc. For simplicity, we quantify $\mathcal{I}$ with its monetary value (unit: \$) – the cost of the DR provider to provide the incentives.

In this section, we study more deeply the setting where a utility company has access to only one energy source, i.e., $|G| = 1$. The energy source can be, for example, an energy market. Let P_{mkt} be the unit cost to meet the load demand, or the market price if the energy source is an energy market. Then, we can rewrite the gain computation in Eq. 6:

$$\begin{aligned} gain = \; & \textstyle\sum_{g_i \in G_{red}} \left(C_{g_i}(L_{g_i}) - C_{g_i}(L^{DR}_{g_i})\right) - P_{ret}R - \mathcal{I} \\ = \; & P_{mkt}L - P_{mkt}L^{DR} - P_{ret}R - \mathcal{I} \\ = \; & P_{mkt}R - P_{ret}R - \mathcal{I} \\ = \; & (P_{mkt} - P_{ret})R - \mathcal{I}. \end{aligned} \tag{7}$$

Note that, when the energy source considered is indeed an energy market, then the market price before and after DR events might be different due to the changes in the demand. In this case, however, we assume that demand reduction R is much smaller compared to the overall demand in the market. Thus, it does not influence the market price. Next, without loss of generality, we assume that the energy source is the market.

4.3.1 *The lower bound of market price*

By substituting R from Eq. 2 into Eq. 7, we obtain:

$$\begin{aligned} gain = \; & (P_{mkt} - P_{ret})R - \mathcal{I} \\ = \; & (P_{mkt} - P_{ret})m\mathcal{I} - \mathcal{I} \\ = \; & \big((P_{mkt} - P_{ret})m - 1\big)\mathcal{I}. \end{aligned} \tag{8}$$

To have positive gain, $(P_{mkt} - P_{ret})m - 1$ should be positive:

$$\begin{aligned} (P_{mkt} - P_{ret})m - 1 \; & > \; 0 \\ P_{mkt} \; & > \; \tfrac{1}{m} + P_{ret}. \end{aligned} \tag{9}$$

Recall that the consumer reduction rate, m (unit: $kWh/\$$), expresses the amount of energy a consumer willing to sacrifice for a unit of incentive during a DR event. In economics, m is also known as consumer's *willingness to pay*, that is, the maximum amount of energy a consumer is willing to sacrifice from her normal consumption level for a unit incentive. While $\frac{1}{m}$ (unit: $\$/kWh$) is consumer *willingness to accept*, i.e., the minimum amount of incentive a consumer is willing to accept for a unit of energy she sacrifices from her normal consumption level during a DR event. Inequality in Eq. 9 states that DR can bring an advantage to utility companies when the market price is greater than the retail price plus the consumer *willingness to accept*. This offers a deeper insight to the common belief that DR should be carried out when $P_{mkt} > P_{ret}$. The customer willingness to accept should be taken into account as well, i.e., $P_{mkt} > \frac{1}{m} + P_{ret}$.

4.3.2 *The lower bound of consumer reduction rate*

From Eq. 9, we can also derive the lowest consumer reduction rate required to obtain positive gain:

$$m \; > \; \tfrac{1}{P_{mkt} - P_{ret}}. \tag{10}$$

That is, the higher the difference between the market and the retail price, the less sensitive the consumers that a company need in its portfolio for the DR event, i.e., having consumers with low m works just fine. However, when there is only a marginal difference between the market and the retail price, then consumers that are highly sensitive to incentives (or having high m) is needed.

4.3.3 *The optimal incentives*

When Eq. 9 or 10 is satisfied, setting larger $\mathcal{I}$ leads to higher gain. However, large $\mathcal{I}$ also causes large R. Thus, when $R > L$, it means that consumers give some energy back to the grid (or producing energy, i.e., becoming *prosumers*). Then, the (positive) gain is due to consumers' energy price, $\frac{1}{m}$, is cheaper than $P_{mkt} - P_{ret}$.

Demand reduction, however, is typically limited. This can be, for example, because the grids accept only limited bidirectional energy flow, or the consumers (distributed) energy generation capacity are limited. Let R_{max} denotes the consumers' maximum reduction, where $R \leq R_{max}$, i.e.,

$$R = \begin{cases} m\mathcal{I}, & \text{if } \mathcal{I} \leq \dfrac{R_{max}}{m}, \\ R_{max}, & \text{if } \mathcal{I} > \dfrac{R_{max}}{m}. \end{cases} \tag{11}$$

In most cases, it is easier to estimate the base load L_{min}, the minimum amount of electricity that the consumers cannot live without. Thus, one can estimate R_{max} by computing $R_{max} = L - L_{min}$.

THEOREM 1. *In case of single energy source, when consumer reduction is bounded by R_{max} and Eq. 9 or 10 is satisfied, then the incentives that maximize company's gain is:*

$$\mathcal{I}_{opt} = \frac{R_{max}}{m}.$$

PROOF. Let $gain(\star)$ be the company's gain using incentives $\star$. Then, there are two cases:

Case 1. The company gives incentives $\mathcal{I}' < \mathcal{I}_{opt}$. Let $R' = m\mathcal{I}'$. We show that $gain(\mathcal{I}') < gain(\mathcal{I}_{opt})$:

$$\begin{aligned} gain(\mathcal{I}') = \; & (P_{mkt} - P_{ret})R' - \mathcal{I}' \\ = \; & (P_{mkt} - P_{ret})m\mathcal{I}' - \mathcal{I}' \\ < \; & \big((P_{mkt} - P_{ret})m\mathcal{I}' - \mathcal{I}'\big)\tfrac{\mathcal{I}_{opt}}{\mathcal{I}'} \\ = \; & (P_{mkt} - P_{ret})m\mathcal{I}_{opt} - \mathcal{I}_{opt} \\ = \; & gain(\mathcal{I}_{opt}). \end{aligned}$$

Case 2. The company gives incentives $\mathcal{I}' > \mathcal{I}_{opt}$. Since $\mathcal{I}' > \frac{R_{max}}{m}$, then according to Eq. 11, the demand reduction $R' = R_{max}$. We show that $gain(\mathcal{I}') < gain(\mathcal{I}_{opt})$:

$$\begin{aligned} gain(\mathcal{I}') = \; & (P_{mkt} - P_{ret})R' - \mathcal{I}' \\ = \; & (P_{mkt} - P_{ret})R_{max} - \mathcal{I}' \\ < \; & (P_{mkt} - P_{ret})R_{max} - \mathcal{I}_{opt} \\ = \; & (P_{mkt} - P_{ret})m\mathcal{I}_{opt} - \mathcal{I}_{opt} \\ = \; & gain(\mathcal{I}_{opt}). \end{aligned}$$

□

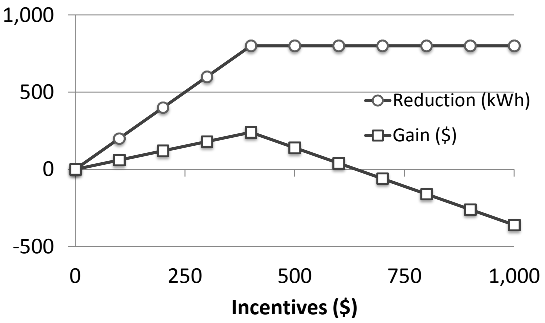

Figure 1: Company's gain for different incentives, where $P_{ret} = 0.2\$/kWh$, $P_{mkt} = 1\$/kWh$, $m = 2kWh/\$$, and $R_{max} = 800kWh$. The optimal incentive is 400$, while the highest incentives such that the company still experiences positive gain is 640$.

4.3.4 *The upper bound of incentives*

To obtain the highest incentive which still gives us positive gain, we require the gain to be positive while considering the maximum reduction R_{max}. That is, we replace R in Eq. 8 with R_{max}:

$$\begin{array}{rcl} (P_{mkt} - P_{ret})R_{max} - \mathcal{I} & > & 0 \\ (P_{mkt} - P_{ret})R_{max} & > & \mathcal{I}. \end{array} \tag{12}$$

The maximum incentives a utility company can give to consumers in order to have positive gain from DR should be smaller than the difference between the market price and the retail price times the maximum reduction R_{max}.

Example 1. *While typically the market price is lower than the retail price, we consider in this example a period where the market price is higher than the retail price. Let the market price* $P_{mkt} = 1\$/kWh$*, the retail price* $P_{ret} = 0.2\$/kWh$*, and* $m = 2kWh/\$$*. Thus, inequalities in Eq. 9 and 10 are met. Suppose that the total load demand is* $1000kWh$ *and* $L_{min} = 200kWh$*. Then, we have* $R_{max} = 1000 - 200 = 800kWh$*. Figure 1 illustrates company's gain for different amount of incentives given to the customers. Maximum gain is obtained where incentives is equal to* $\mathcal{I}_{opt} = R_{max}/m = 400\$$*. And, the upper bound of the incentives such that the company still experiences positive gain is* $(P_{mkt} - P_{ret})R_{max} = 640\$$*.*

4.4 Multiple Energy Sources

In this section, we discuss the case of utility companies who have access to multiple energy sources.[5] Let us define R_{g_i} as the demand reduction for generator g_i, that is, $R_{g_i} = L_{g_i} - L^{DR}_{g_i}$, and $\mathcal{I}_i = R_{g_i}/m$. Recall that, using the definition of G_{red} and G_{base}, we can rewrite the demand reduction as $R = \sum_{g_i \in G} R_{g_i} = \sum_{g_i \in G_{red}} R_{g_i}$. Similarly, we can also rewrite the incentives as:

$$\begin{array}{rcl} R & = & \sum_{g_i \in G_{red}} R_{g_i} \\ m\mathcal{I} & = & \sum_{g_i \in G_{red}} (m\mathcal{I}_i) \\ \mathcal{I} & = & \sum_{g_i \in G_{red}} \mathcal{I}_i. \end{array} \tag{13}$$

Next, suppose that the unit cost to meet load demand using generator g_i is P_{g_i}, that is, $C_{g_i}(L_{g_i}) = P_{g_i} L_{g_i}$. This assumption makes the cost function linear. However, sometimes we also would like to express the cost function as a step function. In this case, we can formulate the step function by considering each step as a distinct generator. Additionally, some studies also consider quadratic cost function[29, 17, 31]. Since a quadratic function can be approximated by a step function, our formulation above also allows us to approximate the quadratic cost function as well. Then, the gain computation in Eq. 6 can be rewritten as:

$$\begin{array}{rcl} gain & = & \sum_{g_i \in G_{red}} \left(C_{g_i}(L_{g_i}) - C_{g_i}(L^{DR}_{g_i}) \right) - P_{ret}R - \mathcal{I} \\ & = & \sum_{g_i \in G_{red}} \left(P_{g_i} L_{g_i} - P_{g_i} L^{DR}_{g_i} \right) - P_{ret}R - \mathcal{I} \\ & = & \sum_{g_i \in G_{red}} (P_{g_i} R_{g_i}) - P_{ret}R - \mathcal{I} \\ & = & \sum\limits_{g_i \in G_{red}} (P_{g_i} R_{g_i}) - \sum\limits_{g_i \in G_{red}} (P_{ret} R_{g_i}) - \sum\limits_{g_i \in G_{red}} \mathcal{I}_i \\ & = & \sum_{g_i \in G_{red}} \left((P_{g_i} - P_{ret}) R_{g_i} - \mathcal{I}_i \right) \end{array} \tag{14}$$

Further, we denote $gain_i = (P_{g_i} - P_{ret})R_{g_i} - \mathcal{I}_i$ as the gain contributed by generator g_i.

4.4.1 *Meaningful DR events*

We define *meaningful* demand reduction as reduction which yield positive gain, whereas demand reduction which yield zero or negative gain, is *meaningless*. Additionally, a DR event is *meaningful* if and only if demand reduction of each generator is meaningful.[6] This also implies that every demand reduction to each generator, or incentives given to customers should eventually increase company's gain:

$$\begin{array}{rl} \forall g_i \in G_{red}, & gain_i > 0, \\ & (P_{g_i} - P_{ret})R_{g_i} - \mathcal{I}_i > 0, \\ & (P_{g_i} - P_{ret})m\mathcal{I}_i - \mathcal{I}_i > 0, \\ & \left((P_{g_i} - P_{ret})m - 1 \right)\mathcal{I}_i > 0. \end{array} \tag{15}$$

To have positive gain, $\left((P_{g_i} - P_{ret})m - 1 \right)$ should be positive:

$$\begin{array}{ll} \forall g_i \in G_{red}, & (P_{g_i} - P_{ret})m - 1 > 0, \\ \forall g_i \in G_{red}, & P_{g_i} > \frac{1}{m} + P_{ret}, \text{ or } m > \frac{1}{P_{g_i} - P_{ret}}. \end{array} \tag{16}$$

4.4.2 *The optimal incentives*

Providing that the inequalities in Eq. 16 are satisfied, we can derive the optimal incentive.

Lemma 2. *In case of utility companies with multiple energy sources, if demand reduction* R *is unbounded and Eq. 16 is satisfied, then higher incentives* $\mathcal{I}$ *leads to higher demand reduction* R*, and eventually higher gain.*

---

[5] Note that, an energy market can also be seen as one of them.

[6] Note that, we could also define meaningful DR events in a *weaker* sense, i.e., if only if the event yield a positive gain. If we define it that way, it means that there could be a set of generators G^{+}_{red} where $gain_i > 0$ for all $g_i \in G^{+}_{red}$ and a set of generators G^{-}_{red} where $gain_i \leq 0$ for all $g_i \in G^{-}_{red}$. Since our goal is maximizing the gain, then there is no point in considering (reducing the load of) G^{-}_{red}. This is the reason we define meaningful DR events in its *stronger* sense, considering only G^{+}_{red}.

PROOF. Let $gain(\mathcal{I})$ be the gain using incentives $\mathcal{I}$ and $gain_i(\mathcal{I}_i)$ be the gain contributed by generator g_i using incentives $\mathcal{I}_i$. Therefore, we have $gain(\mathcal{I}) = \sum_{g_i \in G_{red}} gain_i(\mathcal{I}_i)$ (see Eq. 14).

We need to show that whenever $\mathcal{I} > \mathcal{I}'$, then $gain(\mathcal{I}) > gain(\mathcal{I}')$. We assume that whether the company gives $\mathcal{I}$ or $\mathcal{I}'$, Eq. 16 is satisfied. Let $R = m\mathcal{I}$ and $R' = m\mathcal{I}'$ be the demand reduction and G_{red} and G'_{red} be the set of *reduced* generators when the utility company gives incentives $\mathcal{I}$ and $\mathcal{I}'$, respectively. Because $R > R'$, then $G_{red} \supseteq G'_{red}$. Then, there are two cases:

Case 1. $G_{red} = G'_{red}$. This means that we have the same amount of reduction in every generators, in G_{red} and G'_{red}, up to their capacity, except for the cheapest one (since $R > R'$).

Let g_ω be the cheapest generator in G_{red} (and in G'_{red}). Then, $\forall g_i \in G_{red} \setminus g_\omega$, we have $R_{g_i} = R'_{g_i} = cap(g_i)$. Consequently, since $R > R'$, then $R_{g_\omega} > R'_{g_\omega}$.

Let $\mathcal{I}_i = R_{g_i}/m$ and $\mathcal{I}'_i = R'_{g_i}/m$. Then, we have $\mathcal{I}_\omega > \mathcal{I}'_\omega$ and $gain_{g_\omega}(\mathcal{I}_\omega) > gain_{g_\omega}(\mathcal{I}'_\omega)$. Thus:

$$\begin{aligned}
gain(\mathcal{I}) &= \textstyle\sum_{g_i \in G_{red} \setminus g_\omega} gain_i(\mathcal{I}_i) + gain_{g_\omega}(\mathcal{I}_\omega) \\
&> \textstyle\sum_{g_i \in G_{red} \setminus g_\omega} gain_i(\mathcal{I}_i) + gain_{g_\omega}(\mathcal{I}'_\omega) \\
&= \textstyle\sum_{g_i \in G'_{red} \setminus g_\omega} gain_i(\mathcal{I}'_i) + gain_{g_\omega}(\mathcal{I}'_\omega) \\
&= gain(\mathcal{I}')
\end{aligned}$$

Case 2. $G_{red} \supset G'_{red}$. Since the inequalities in Eq. 16 is satisfied, $\forall g_i \in G_{red} \setminus G'_{red}$, we have $gain_i(\mathcal{I}') > 0$. Thus:

$$\begin{aligned}
gain(\mathcal{I}) &= \textstyle\sum_{g_i \in G_{red}} gain_i(\mathcal{I}_i) \\
&= \sum_{g_i \in G'_{red}} gain_i(\mathcal{I}_i) + \sum_{g_i \in G_{red} \setminus G'_{red}} gain_i(\mathcal{I}_i) \\
&> \textstyle\sum_{g_i \in G'_{red}} gain_i(\mathcal{I}_i) \\
&= gain(\mathcal{I}')
\end{aligned}$$

□

Similar to the case of the single energy source, however, consumer demand reduction is typically not unbounded. Let R_{max} denote the upper bound of the reduction. Then, R_{max} upper bounding consumer demand reduction as described in Eq. 11.

THEOREM 3. *When demand reduction is bounded by R_{max} and Eq. 16 is satisfied, then the optimal incentives that maximize company's gain is: $\mathcal{I}_{opt} = R_{max}/m$.*

PROOF. Let $gain(\mathcal{I})$ denotes the gain by giving incentives $\mathcal{I}$. We show that when the utility company gives incentives $\mathcal{I}' \neq \mathcal{I}_{opt}$, then $gain(\mathcal{I}') < gain(\mathcal{I}_{opt})$.

Let $R' = m\mathcal{I}'$ and $R_{opt} = m\mathcal{I}_{opt}$ be the demand reduction and G'_{red} and G_{red} be the set of reduced generators when the utility company gives incentives $\mathcal{I}'$ and $\mathcal{I}_{opt}$, respectively. Then, there are two cases:

Case 1. The company gives incentives $\mathcal{I}_{opt} > \mathcal{I}'$. We need to show that $gain(\mathcal{I}_{opt}) > gain(\mathcal{I}')$. Note that, in this case, we have $R > R'$, which also implies $G_{red} \supseteq G'_{red}$. Then, we proceed as in the proof of Lemma 2, *Case 1* and *Case 2*, by substituting $\mathcal{I}$ with $\mathcal{I}_{opt}$ and R with R_{opt}.

Case 2. The company gives incentives $\mathcal{I}_{opt} < \mathcal{I}'$. We need show that $gain(\mathcal{I}_{opt}) > gain(\mathcal{I}')$. From Eq. 11, we have $R' = R_{opt} = R_{max}$. This also implies that $G'_{red} = G_{red}$.

$$\begin{aligned}
gain(\mathcal{I}') &= \textstyle\sum_{g_i \in G'_{red}} (P_{g_i} - P_{ret}) R' - \mathcal{I}' \\
&= \textstyle\sum_{g_i \in G'_{red}} (P_{g_i} - P_{ret}) R_{max} - \mathcal{I}' \\
&< \textstyle\sum_{g_i \in G'_{red}} (P_{g_i} - P_{ret}) R_{max} - \mathcal{I}_{opt} \\
&= \textstyle\sum_{g_i \in G_{red}} (P_{g_i} - P_{ret}) R_{max} - \mathcal{I}_{opt} \\
&= gain(\mathcal{I}_{opt}).
\end{aligned}$$

□

4.4.3 The upper bound of incentives

When the demand reduction is maximum, i.e., $R_{max} = L - L^{DR}$, we denote $R^{max}_{g_i} = L_{g_i} - L^{DR}_{g_i}$ as the demand reduction for generator g_i. Consequently, we have $R_{max} = \sum_{g_i \in G_{red}} R^{max}_{g_i}$. Then, by replacing R_{g_i} in Eq. 14 with $R^{max}_{g_i}$, we derive the highest incentive a company can provide while still maintaining positive gain:

$$\begin{aligned}
\textstyle\sum_{g_i \in G_{red}} \left[(P_{g_i} - P_{ret}) R^{max}_{g_i} - \mathcal{I}_i \right] &> 0 \\
\textstyle\sum_{g_i \in G_{red}} \left[(P_{g_i} - P_{ret}) R^{max}_{g_i} \right] - \mathcal{I} &> 0 \qquad (17) \\
\textstyle\sum_{g_i \in G_{red}} (P_{g_i} - P_{ret}) R^{max}_{g_i} &> \mathcal{I}.
\end{aligned}$$

EXAMPLE 2. *Suppose that a utility company has access to two different energy sources, g_1 and g_2, where $cap(g_1) = 5000kW$ and $cap(g_2) = 2000kW$. Assume that for a specific time period, we have load demand $L = 6500kWh$, $P_{g_1} = 0.1\$/kWh$ (base generator), and $P_{g_2} = 1\$/kWh$ (peak generator). Additionally, let $P_{ret} = 0.2\$/kWh$, $m = 2kWh/\$$, and $R_{max} = 1200kWh$. Figure 2 shows company's gain for different amount of incentives given to the customers. Maximum gain is obtained when incentives is equal to $\mathcal{I}_{opt} = R_{max}/m = 600\$$. The highest incentives such that the company still experiences positive gain is $(P_{g_2} - P_{ret})R^{max}_{g_2} = (1 - 0.2) \cdot 1200 = 960\$$.*[7]

5. DR TO INCREASE ENERGY CONSUMPTION

In the previous section, we have discussed about DR for energy reduction. However, DR could also be used to incentivize consumers to increase their energy consumption, especially when there is a surplus energy and balancing the surplus is costly. This can be, for example, when (the output of) some generators cannot be turned off (or reduced) easily without additional cost. Or, when the demand is surprisingly low, while we have bought much more energy in the

---

[7] Note that, to compute $R^{max}_{g_2}$, we need to first compute L^{DR} as if the reduction is R_{max}. Thus, $L^{DR} = L - R_{max} = 5300kWh$. Using the *load generation* assumption in Section 3.1, we have $L_{g_1} = 5000kWh$, $L_{g_2} = 1500kWh$, $L^{DR}_{g_1} = 5000kWh$, and $L^{DR}_{g_2} = 300kWh$. Therefore, we have $R^{max}_{g_1} = 0kWh$ and $R^{max}_{g_2} = 1200kWh$.

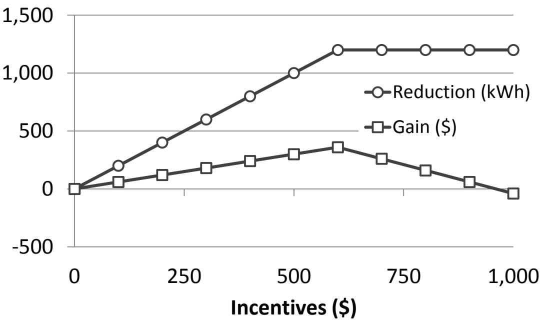

Figure 2: Company's gain for different incentives where the company has two energy sources, g_1 and g_2, with $cap(g_1) = 500kW$, $cap(g_2) = 5000kW$. For this time period, we have $L = 2000kWh$, $P_{g_1} = 0.1\$/kWh$, $P_{g_2} = 1\$/kWh$, $P_{ret} = 0.2\$/kWh$, $m = 2kWh/\$$, and $R_{max} = 1200kWh$. The optimal incentive is 600\$, while the highest incentive such that company still experiences positive gain is 960\$.

day-ahead market. If the grid operator balances the surplus, then we need to pay the penalty for the imbalances.

Together with its ability to reduce demand, DR's ability to increase demand form a framework for load shaping strategy. In this section, we discuss the economics of pervasive, incentive-based DR, when it is used as a mechanism to encourage demand increase. We present only the case where utility companies have access to multiple energy sources, since the case of single energy source can be regarded as a special case of that.

Let L^+ be the total energy produced in the surplus/abundance period, where $L^+ > L$, and $S_{max} = L^+ - L$ be the excess energy. Additionally, let $L^+_{g_i}$ be the energy produced by generator g_i. We define $C_B(S_{max})$ as the balancing cost, e.g., the cost to balance the grid due to the excess energy (this task is generally performed in a balancing market). Note that, in Section 4, the role of the balancing market to generate additional electricity when the supply is short can also be thought as one of (typically more expensive) energy sources.

5.1 Without DR (Business As Usual)

Cost We define company's total cost as the sum of generation and balancing cost: $\sum_{g_i \in G} C_{g_i}(L^+_{g_i}) + C_B(S_{max})$.

Revenue Company's revenue: $P_{ret}L$.

Profit Company's profit = revenue - cost:

$$P_{ret}L - \sum_{g_i \in G} C_{g_i}(L^+_{g_i}) - C_B(S_{max}). \tag{18}$$

5.2 With DR

Note that, in this case, there is a surplus of energy, S_{max}, which is ready to be consumed. Encouraging consumers to consume this surplus energy not only results in increasing company's revenue (because consumers' bill increases), but also acts as an energy balancing mechanisms, when there is a shortage of energy in the future. That is, it is potentially easier for the people to consume less energy in the future, when the supply is short, when some activities has been done or shifted to the previous (energy abundance) period. Therefore, we would like to use DR to incentivize consumers to increase their consumption during this surplus period. Let us assume that the consumers increase their demand from L to $L + S$, where $0 \leq S \leq S_{max}$.

Cost Compared to the previous case (without DR), company's cost is lower due to the decrease in the balancing cost: $\sum_{g_i \in G} C_{g_i}(L^+_{g_i}) + C_B(S_{max} - S)$.

Revenue Company's revenue increases due to the increase in consumers' demand: $P_{ret}(L + S)$.

Profit Company's profit = revenue - cost - incentive:

$$P_{ret}(L + S) - \sum_{g_i \in G} C_{g_i}(L^+_{g_i}) - C_B(S_{max} - S) - \mathcal{I}. \tag{19}$$

Gain We define the gain of a utility company during a DR event in this abundance period as the difference between its profit with and without the DR event, i.e., Eq. 19 - Eq 18:

$$\begin{aligned} gain = \; & P_{ret}(L + S) - \sum_{g_i \in G} C_{g_i}(L^+_{g_i}) - C_B(S_{max} - S) - \\ & \mathcal{I} - P_{ret}L + \sum_{g_i \in G} C_{g_i}(L^+_{g_i}) + C_B(S_{max}) \\ = \; & P_{ret}S - \mathcal{I} - C_B(S_{max} - S) + C_B(S_{max}) \end{aligned} \tag{20}$$

As in the case of the generation cost (see Section 4.4), let us assume that the balancing cost has a unit cost P_B. Then, we have:

$$\begin{aligned} gain &= P_{ret}S - \mathcal{I} + P_B(S_{max} - S) + P_B \cdot S_{max} \\ &= P_{ret}S - \mathcal{I} + P_B S \\ &= (P_{ret} + P_B)S - \mathcal{I} \end{aligned} \tag{21}$$

Similar to the case of DR to reduce energy consumption, we assume that consumer response grows linearly with the incentives offered, i.e., $S = m^+ \cdot \mathcal{I}$, where m^+ is the consumer increase rate (unit: $kWh/\$$). The higher the consumer increase rate, the more sensitive she is to incentives.[8] Since S is bounded by S_{max}, consequently,

$$S = \begin{cases} m^+ \cdot \mathcal{I}, & \text{if } \mathcal{I} \leq \dfrac{S_{max}}{m^+} \\ S_{max}, & \text{if } \mathcal{I} > \dfrac{S_{max}}{m^+}. \end{cases} \tag{22}$$

5.2.1 The lower bound of consumer increase rate

To have positive gain:

$$\begin{aligned} (P_{ret} + P_B)S - \mathcal{I} &> 0 \\ (P_{ret} + P_B)m^+ \cdot \mathcal{I} - \mathcal{I} &> 0 \\ \big((P_{ret} + P_B)m^+ - 1\big)\mathcal{I} &> 0 \end{aligned} \tag{23}$$

---

[8] In practice, consumers can increase their demand by shifting their later activities to the DR event period. If they do not have any activities to be shifted, however, rational consumers would have $1/m^+ > P_{ret}$, i.e., they respond (or increase their demand) iff the incentive per unit energy is greater than the retail price.

Thus, $(P_{ret}+P_B)m^+ - 1$ should be positive:

$$\begin{aligned}(P_{ret}+P_B)m^+ - 1 &> 0\\ m^+ &> \tfrac{1}{P_{ret}+P_B},\end{aligned}\tag{24}$$

If we assume that P_{ret} is fixed, then the sensitivity of the consumers required for a DR event is inversely related with the balancing prices. While highly sensitive consumers are generally preferred for DR, however when the balancing price is high, having less sensitive consumers (consumers with low m^+) are also fine.

5.2.2 The lower bound of the balancing price

Using Eq. 24, we can also derive the lower bound of the balancing price:

$$P_B > \tfrac{1}{m^+} - P_{ret} \tag{25}$$

5.2.3 The optimal incentives

When Eq. 24 or 25 is satisfied, we can derive the optimal incentives that maximize company's gain.

THEOREM 4. *In the case of DR to increase energy consumption, given that Eq. 24 or 25 is satisfied, the optimal incentives that maximize company's gain is:* $\mathcal{I}_{opt} = S_{max}/m^+$.

PROOF. Let $gain(\mathcal{I})$ denotes company's gain by providing incentives $\mathcal{I}$. If Eq. 24 or 25 is satisfied, then for $\mathcal{I}' \neq \mathcal{I}_{opt}$, we show that $gain(\mathcal{I}') < gain(\mathcal{I}_{opt})$. Let $S' = m^+ \cdot \mathcal{I}'$ be the increase in energy consumption due to incentive $\mathcal{I}'$. Then, there are two cases:

Case 1. $\mathcal{I}' < \mathcal{I}_{opt}$

$$\begin{aligned}gain(\mathcal{I}') &= (P_{ret}+P_B)S' - \mathcal{I}'\\ &= (P_{ret}+P_B)m^+ \cdot \mathcal{I}' - \mathcal{I}'\\ &< \left((P_{ret}+P_B)m^+ \cdot \mathcal{I}' - \mathcal{I}'\right)\tfrac{\mathcal{I}_{opt}}{\mathcal{I}'}\\ &= (P_{ret}+P_B)m^+ \cdot \mathcal{I}_{opt} - \mathcal{I}_{opt}\\ &= gain(\mathcal{I}_{opt}).\end{aligned}$$

Case 2. $\mathcal{I}' > \mathcal{I}_{opt}$. Since $\mathcal{I}' > S_{max}/m^+$, we have $S' = S_{max}$ (see Eq. 22). Then,

$$\begin{aligned}gain(\mathcal{I}') &= (P_{ret}+P_B)S' - \mathcal{I}'\\ &= (P_{ret}+P_B)S_{max} - \mathcal{I}'\\ &< (P_{mkt}+P_B)S_{max} - \mathcal{I}_{opt}\\ &= (P_{mkt}+P_B)m^+ \cdot \mathcal{I}_{opt} - \mathcal{I}_{opt}\\ &= gain(\mathcal{I}_{opt}).\end{aligned}$$

$\square$

5.2.4 The upper bound of incentives

The highest incentives that a utility company can give to consumers while still experiences positive gain can be obtained by assuming the maximum consumption increase:

$$\begin{aligned}(P_{ret}+P_B)S_{max} - \mathcal{I} &> 0\\ (P_{ret}+P_B)S_{max} &> \mathcal{I},\end{aligned}\tag{26}$$

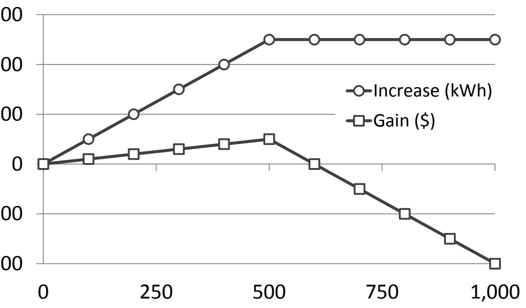

Figure 3: Increase in energy consumption and gain for different incentives given to consumers in the case of DR to increase energy consumption, where $L = 1000kWh$, $L^+ = 1500kWh$, $P_{ret} = 0.2\$/kWh$, $P_B = 1\$/kWh$, and $m^+ = 1kWh/\$$. The optimal incentive is 500\$, while the highest incentive such that the company still experiences positive gain is 600\$.

EXAMPLE 3. *Let us consider a case where the load demand* $L = 1000kWh$ *and the total energy generated in the abundance period* $L^+ = 1500kWh$. *In addition, let us assume that* $P_{ret} = 0.2\$/kWh$, $P_B = 1\$/kWh$, *and* $m^+ = 1kWh/\$$. *Figure 3 shows company's gain for different amount of incentives provided to consumers. Note that, we have* $S_{max} = 1500kWh - 1000kWh = 500kWh$. *Maximum gain is obtained when the incentive is equal to* $\mathcal{I}_{opt} = S_{max}/m^+ = 500\$$. *The highest incentives such that the company still experiences positive gain is given by* $(P_{ret}+P_B)S_{max} = 600\$$.

6. CONCLUSION AND FUTURE WORK

In this work, we identified the necessary conditions to benefit from pervasive DR. These conditions include: the lower bound of consumer reduction/increase rate, the lower bound of generation cost (including market price), the lower bound of balancing price, and the upper bound of incentives. Furthermore, we determine the optimal incentives to maximize gain. To derive these results, we presented a number of simplifying assumptions, such as the linearity of consumer response on the incentives, and perfectly known maximum reduction/increase that should be relaxed in the future.

Understanding consumer reduction/increase rate is of utmost important, since it is a key to successful DR programs. Therefore, when exploratory DR events are needed to better understand and learn consumer responses, our results can serve as the boundary to guarantee positive gain. In addition, we believe there are numerous other factors that could influence consumer responses in real world, such as time of the day, day of the week, weather, outdoor temperature, holidays, or guests at home. We also have not considered consumer fatigue; for some consumers, receiving too many DR signals could be annoying, and consequently, deter their participation in the next DR events. Thus, the availability of real data from real deployment in the future could be used to validate and refine our results.

7. REFERENCES

[1] The Demand Response Baseline. White Paper, 2011.
[2] PJM Empirical Analysis of Demand Response Baseline Methods. White Paper, Apr. 2011.
[3] M. H. Albadi and E. El-Saadany. Demand response in electricity markets: An overview. In *Power*

Engineering Society General Meeting, 2007. IEEE, pages 1–5, June 2007.

[4] M. H. Albadi and E. El-Saadany. A summary of demand response in electricity markets. *Electric Power Systems Research*, 78(11):1989 – 1996, 2008.

[5] B. R. Alexander. Dynamic Pricing? Not So Fast! A Residential Consumer Perspective. *The Electricity Journal*, 23(6):39 – 49, 2010.

[6] I. M. L. Azevedo, M. G. Morgan, and L. Lave. Residential and Regional Electricity Consumption in the U.S. and EU: How Much Will Higher Prices Reduce CO2 Emissions? *The Electricity Journal*, 24(1):21 – 29, 2011.

[7] S. Borenstein and S. Holland. On the Efficiency of Competitive Electricity Markets with Time-Invariant Retail Prices. *RAND Journal of Economics*, 36(3):469–493, Autumn 2005.

[8] Z. Chen, L. Wu, and Y. Fu. Real-time price-based demand response management for residential appliances via stochastic optimization and robust optimization. *Smart Grid, IEEE Transactions on*, 3(4):1822–1831, Dec 2012.

[9] K. Coughlin, M. A. Piette, C. Goldman, and S. Kiliccote. Statistical Analysis of Baseline Load Models for Non-Residential Buildings. *Energy and Buildings*, 41(4):374 – 381, 2009.

[10] K. Coughlin, M. A. Piette, C. A. Goldman, and S. Kiliccote. Estimating Demand Response Load Impacts: Evaluation of Baseline Load Models for Non-Residential Buildings in California. Technical Report LBNL-63728, Demand Response Research Center, CA, Jan. 2008.

[11] S. Fan and R. J. Hyndman. The price elasticity of electricity demand in South Australia. *Energy Policy*, 39(6):3709 – 3719, 2011.

[12] A. Faruqui and S. George. Quantifying Customer Response to Dynamic Pricing. *The Electricity Journal*, 18(4):53 – 63, 2005.

[13] A. Faruqui and S. Sergici. Household response to dynamic pricing of electricity: A survey of 15 experiments. *Journal of Regulatory Economics*, 38(2):193–225, 2010.

[14] Federal Energy Regulatory Commission. Assessment of Demand Response & Advanced Metering, Dec. 2010.

[15] M. Filippini. Short- and long-run time-of-use price elasticities in Swiss residential electricity demand. *Energy Policy*, 39(10):5811 – 5817, 2011. Sustainability of biofuels.

[16] B. Fogg. A behavior model for persuasive design. In *Proceedings of the 4th International Conference on Persuasive Technology*, Persuasive '09, pages 40:1–40:7, New York, NY, USA, 2009. ACM.

[17] M. Gonzalez Chapa and J. Vega Galaz. An economic dispatch algorithm for cogeneration systems. In *IEEE Power Engineering Society General Meeting*, pages 989–994 Vol.1, 2004.

[18] S. Gyamfi, S. Krumdieck, and T. Urmee. Residential peak electricity demand response – Highlights of some behavioural issues. *Renewable and Sustainable Energy Reviews*, 25(0):71 – 77, 2013.

[19] K. Herter, P. McAuliffe, and A. Rosenfeld. An exploratory analysis of California residential customer response to critical peak pricing of electricity. *Energy*, 32(1):25 – 34, 2007.

[20] K. Herter and S. Wayland. Residential response to critical-peak pricing of electricity: California evidence. *Energy*, 35(4):1561 – 1567, 2010.

[21] S. P. Holland and E. T. Mansur. Is Real-Time Pricing Green? The Environmental Impacts of Electricity Demand Variance. *The Review of Economics and Statistics*, 90(3):pp. 550–561, 2008.

[22] F. Jamil and E. Ahmad. Income and price elasticities of electricity demand: Aggregate and sector-wise analyses. *Energy Policy*, 39(9):5519 – 5527, 2011.

[23] P. Joskow and J. Tirole. Retail electricity competition. *The RAND Journal of Economics*, 37(4):799–815, 2006.

[24] E. Kahn, J. Sathaye, and D. Robbins. An engineering-economic approach to estimating the price elasticity of residential electricity demand. *Energy Economics*, 8(2):118 – 126, 1986.

[25] K. Kok. *The PowerMatcher: Smart Coordination for the Smart Electricity Grid*. PhD thesis, Vrije Universiteit, Amsterdam, Germany, May 2013.

[26] N. Li, L. Chen, and S. H. Low. Optimal demand response based on utility maximization in power networks. In *Power and Energy Society General Meeting, 2011 IEEE*, pages 1–8. IEEE, 2011.

[27] M. G. Lijesen. The real-time price elasticity of electricity. *Energy Economics*, 29(2):249 – 258, 2007.

[28] J. Mathieu, D. Callaway, and S. Kiliccote. Examining uncertainty in demand response baseline models and variability in automated responses to dynamic pricing. In *The 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, pages 4332–4339, 2011.

[29] A.-H. Mohsenian-Rad, V. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia. Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid. *Smart Grid, IEEE Transactions on*, 1(3):320–331, Dec 2010.

[30] S. D. Ramchurn, P. Vytelingum, A. Rogers, and N. Jennings. Agent-based control for decentralised demand side management in the smart grid. In *The 10th International Conference on Autonomous Agents and Multiagent Systems - Volume 1*, AAMAS '11, pages 5–12, Richland, SC, 2011. International Foundation for Autonomous Agents and Multiagent Systems.

[31] P. Rao and S. Nagendra. Combined heat and power economic dispatch: A direct solution. *Electric Power Components and Systems*, 34(9):1043–1056, 2006.

[32] S. Schnittger and S. Beare. Economic implications of the proposed demand response mechanism. Technical report, Australian Energy Market Commission, 2012.

[33] U.S. Department of Energy. Benefits of Demand Response in Electricity Markets and Recommendations for Achieving Them, Feb. 2006.

[34] M. Vasirani and S. Ossowski. Smart consumer load balancing: state of the art and an empirical evaluation in the spanish electricity market. *Artificial Intelligence Review*, 39(1):81–95, 2013.

[35] T. K. Wijaya, K. Larson, and K. Aberer. Matching demand with supply in the smart grid using agent-based multiunit auction. In *Communication Systems and Networks (COMSNETS), 2013 Fifth International Conference on*, pages 1–6, Jan 2013.

[36] T. K. Wijaya, T. G. Papaioannou, X. Liu, and K. Aberer. Effective consumption scheduling for demand-side management in the smart grid using non-uniform participation rate. In *Sustainable Internet and ICT for Sustainability (SustainIT), 2013*, pages 1–8, Oct 2013.

[37] T. K. Wijaya, M. Vasirani, and K. Aberer. When Bias Matters: An Economic Assessment of Demand Response Baselines for Residential Customers. *IEEE Transactions on Smart Grid*, 2014.