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SAWTOOTH PERIOD SIMULATIONS OF TCV DISCHARGES

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Abstract

The 1-D transport code PRETOR[1] is used to simulate TCV discharges. The discharges studied in this work are all ohmic L-modes and cover a very wide range of plasma and shape parameters. The code PRETOR also has a sawtooth crash model which was used to predict ITER sawtooth period[2]. It turns out that TCV is in the same collisionality regimes as ITER, with regard to the sawtooth crash criterion. Therefore PRETOR can be used to model TCV sawtooth periods in order to obtain more accurate q profiles and better transport simulations. In doing so, the limits and range of validity of the sawtooth model are also tested. The crash model involves several conditions, but for TCV ohmic discharges the decisive criterion is that the effective growth rate of the internal kink must be larger than some fraction of the diamagnetic frequencies. It is shown that the crash criterion, which can be written as $s_1 \geq s_{1crit}$ with s_1 the shear at q=1, allows one to model the sawtooth activity for all the ohmic L-modes shots considered. For transport analysis, setting $s_{1crit} = 0.2$ is sufficient as then only the sawtooth period is not correctly determined, but the inversion radius and the profiles are good.

I. Introduction

In all tokamak plasmas, the temperature and density profiles are strongly influenced by the presence (or absence) of sawtooth activity. It tends to flatten the profiles within a given radius related to the mixing radius defined in the Kadomtsev complete reconnection model[2-3]. Recently it has been shown that the width of the profiles in TCV can be directly related to the q = 1 radius using such a simple argument for current and pressure profiles[4]. Therefore if one wants to simulate and eventually predict the profiles in an experiment, using a 1-D transport model, one has to have a good sawtooth model. It is shown in another paper in this conference[5] that we can obtain the correct temperature and density profiles for most of the wide variety of the ohmic L-modes discharges in TCV using fixed transport coefficients. In this study, we want to go one step further: simulate the time evolution of the profiles including the sawtooth activity. In this way we can also simulate the sawtooth period.

As this was used to predict the sawtooth period in ITER, this study can be seen as the first benchmark of the model with respect to experimental data. Of course, as TCV does not have alpha particles, this is only a first step towards a reliable complete model valid for reactor-like parameters.

We have studied 17 ohmic L-modes shots which cover well the following range of parameters: $2.3 \leq q_{edge} \leq 4.6$; $0.1\ MA \leq I_p \leq 1\ MA$; $2 \leq n_{e19} \leq 12$; $0.1 \leq \delta$ (triangularity) ≤ 0.6; $1 \leq \kappa \leq 1.9$. The sawtooth period ranges from 2ms to 8ms and the inversion radius from 20 to 60% of the minor radius.

In Section II we describe the model, or more precisely the modification of the model with respect to the one in Ref. [2], and then present the results in Section III. All the variables are defined in the appendix of Ref.[2], except if specified here.

II. Sawtooth crash model

The aim of the model is to be able to predict when a sawtooth crash should occur and how to determine the current (or q), density and pressure profiles after the crash. In this way it can be coupled to a 1-D transport code, like PRETOR[1], to simulate the time evolution of these profiles including the sawtooth activity. As the crash time is much shorter than the transport time-scale, we are not interested in simulating the crash itself which moreover is a nonlinear phenomenon. Therefore the crash in our model is assumed to be instantaneous and all is required to know are the profiles after the crash in order to be able to simulate the profiles until the next crash is triggered.

First we need to consider the main scale lengths which play an important role in the magnetic reconnection process. We assume that the crash is triggered by a m=1/n=1 internal kink mode which starts to reconnect in a thin layer around the q=1 surface ρ_1. The layer width depends on the values of the ion Larmor radius ρ_i, the resistive layer width δ_η and the inertial skin depth d_e (note that $\delta_\eta \sim B^{-1/3}$ instead of B^{-1} in app. of Ref.[2]). In TCV we have $\delta_\eta \sim \rho_i$ ($\approx$0.3cm) >> d_e ($\approx$0.06cm). Therefore we are slightly more collisional than ITER, for which $\rho_i \gg \delta_\eta \gg d_e$, but the ion Larmor radius is of the order of the layer width and therefore also influences the expected growth rate.

Second we have to know in what regime we are with respect to the ideal internal kink mode. As TCV is also in the semi-collisional regime, we expect that the layer physics will determine the growth rate if, using the notations defined in Ref.[2]:

$$-\hat{\rho} < -\delta\hat{W} < 0.5\,\omega_{diai}\cdot\tau_A \qquad (1)$$

where $\hat{\rho} = \rho_i/\rho_1$. Otherwise the growth rate is given by the ideal internal kink $\gamma = -\delta\hat{W}/\tau_A$. The potential energy $\delta\hat{W}$ is determined by the destabilizing ideal MHD potential energy $\delta\hat{W}_{mhd}$ and the stabilizing potential energy contribution from the thermal trapped ions $\delta\hat{W}_{KO}$ [7]:

$$\delta\hat{W} = \delta\hat{W}_{mhd} + \delta\hat{W}_{KO} \qquad (2)$$

In TCV ohmic L-modes, the poloidal beta is just above the critical value, prior to the crash, and therefore $\delta\hat{W}_{mhd}$ is relatively small of the order of (-10^{-4}). On the other hand $\delta\hat{W}_{KO}$, due to its $1/s_1$ dependence, is not as small and is typically of the order of 10^{-3}. However, one can expect this term to be smaller if the ions are collisional, in particular if $\gamma < \nu_{ii}$ as less trapped particles can contribute to $\delta\hat{W}_{KO}$. A first estimate based on Ref. [8] suggests that the effective value of $\delta\hat{W}_{KO}$ is modified as

$$\delta\hat{W}_{KO}\,/\,(1+\nu_{ii}/\gamma) \quad \text{or} \quad \delta\hat{W}_{KO}\,/\,[1+(\nu_{ii}/\gamma)^2] \qquad (3)$$

As $\nu_{ii} \approx 10^4 s^{-1}$ and $\gamma \approx 3\cdot 10^3$ s, $\delta\hat{W}_{KO}/(1+(\nu_{ii}/\gamma)^2) \approx 10^{-4}$ is still of the order of $|\delta\hat{W}_{mhd}|$ and has a stabilizing effect. However as $\hat{\rho} \sim 10^{-2}$ and $0.5\,\omega_{diai}\tau_A \sim 10^{-3}$, it follows that Eq. (1) is always satisfied in TCV ohmic L-modes discharges, independent of the exact contribution of $\delta\hat{W}_{KO}$.

As Eq. (1) is satisfied, the ideal kink is stabilized by FLR and diamagnetic effects, but finite resistivity enables a reconnecting mode to become unstable, namely the resistive internal kink, with a growth rate given by:

$$\gamma_\eta = s_1^{2/3}\; S^{-1/3} \;/\; \tau_A \qquad (4)$$

where S is the Lundquist number and τ_A the Alfvén time. If ρ_i is larger than δ_η, then it determines the reconnection layer width and the growth rate of the internal kink in this "ion-kinetic" regime is[6]:

$$\gamma_\rho = \left(\frac{2(1+\tau)}{\pi}\right)^{2/7} \hat{\rho}^{4/7} S^{-1/7} s_1^{6/7} / \tau_A \tag{5}$$

where $\tau = T_e/T_i$. In Ref.[2], as $\rho_i > \delta_\eta$ in ITER, only the latter growth rate was considered. However as in TCV ρ_i can be either smaller or larger then δ_η, we have to take both into account, namely use:

$$\gamma_{eff} = \max(\gamma_\rho, \gamma_\eta). \tag{6}$$

Depending on the collisionality regime of the electrons and ions, and if the electrons are adiabatic or isothermal, the stabilization of the mode γ_{eff} by diamagnetic effects enters in different ways in the relevant dispersion relations [2, 8, 9, 10]. As a general form one expects the mode to be stabilized if:

$$\left(\omega_{*e}^{\alpha_1}\, \omega_{diae}^{\alpha_2}\, \omega_{*i}^{\alpha_3}\, \omega_{diai}^{\alpha_4}\right)^{1/(\alpha_1+\alpha_2+\alpha_3+\alpha_4)} > c^* \gamma_{eff} \tag{7}$$

where $\omega_{*e;i} = T_{e;i} L_{ne;i}^{-1} / eB\rho_1$, $\omega_{diae;i} = T_{e;i} L_{pe;i}^{-1} / eB\rho_1$. The coefficient c_* also depends on collisionality. In the collisionless limit one expects $c_*=1$, while $c_* \cong (9/D)^{1/3}$ in the collisional limit[10], where $D \cong 0.3\, \beta_{e1} \sqrt{m_i T_e / m_e T_i}$ is the ratio of the resistive time to the perpendicular ion momentum diffusion time with $\beta_{e1} = 2\mu_0 n_{e1} T_{e1} / B_1^2$. As β_{e1} is typically of the order of 1% and $T_e/T_i \cong 2$, then c_* = 3-4 in the collisional limit. The exact form of Eq. (7) cannot be obtained from analytical dispersion relation as experiments are never in an asymptotic limit, however we know from these works that density and temperatures gradients of both species can play a role. Therefore, as a first step, we propose the following condition for triggering a sawtooth crash:

$$c_* \gamma_{eff} > \left(\omega_{diae}\, \omega_{diai}\right)^{1/2} \tag{8}$$

where we simply consider the electron and ion pressure gradients. A similar condition was successfully used in TFTR to discriminate between sawtoothing and sawtooth-free discharges[11]. As all the growth rates of the internal kink mode obtained in different parameter regimes are proportional to s_1, like $\gamma_\eta \sim s_1^{2/3}$ or $\gamma_\rho \sim s_1^{6/7}$, it follows that condition Eq. (8) can be rewritten for a given form of γ_{eff} as:

$$s_1 > s_{1crit} \tag{9}$$

Therefore if Eq. (1) is satisfied, the sawtooth model specifies that the crash is triggered once the shear at q=1 exceeds a critical value s_{1crit} determined by Eqs. (8) and (6).

Once the crash condition Eq. (9) is satisfied, the q profile is relaxed according to Kadomtsev complete reconnection model, as explained in Section 4.1 of Ref. [2]. In this way the profiles are modified up to the mixing radius ρ_{mix} and for simplicity the density and pressure profiles are flattened within ρ_{mix} while keeping the total particle and energy conserved. A partial relaxation model has also been implemented in PRETOR [2] but has not yet been used in this study.

III. Results

The first step before simulating the sawtooth activity is to make sure that the profiles are correctly modeled by the transport code. Indeed, as the crash criterion depends on the local values at the q = 1 surface and on some derivatives, it is important that the profiles are close to the experimental one just before the crash. This is shown in Ref. [5] where for most of the cases both the temperature and density standard deviations are within 10%-20%.

As the exact form of Eq. (8) is not well defined at this stage, we have simulated all the shots with c_* as free parameters such as to fit the experimental sawtooth period within 30%. We see in Fig.1(a) that we can simulate the sawtooth period over the wide range of parameters described above with a reasonable variation of c_*. The value of c_* vs. elongation is shown in Fig.1(b). It shows that for most of the cases we obtain the correct sawtooth period with $c_*\approx1.5$. In a few cases at low q_{edge} the predicted period is too small and a smaller $c_*\approx1$ is needed. This shows that we can simulate the experimental sawtooth period with a criterion as Eq. (9) for all the TCV ohmic L-modes discharges in the range of parameters described above. We have also changed slightly the transport coefficients in order to change the temperatures and densities profiles at the q=1 surface within the experimental error bars. We had then to change accordingly the value of c_* to obtain the same sawtooth period as before. We obtained that the value of s_{1crit} is the same as before, thus the value of s_{1crit} such as to recover the experimental sawtooth evolution is a well-defined parameter. This shows in a different way that Eq. (9) is the relevant criterion for triggering the sawtooth crash. It confirms the results of Ref. [11] but in a more detailed way as we simulated the whole sawtooth evolution.

In Fig. 2 we show the typical time evolution (a) of s_1, and the critical shear obtained with $\gamma_{eff} = \gamma_\eta$, $s_{1crit\eta}$, and $\gamma_{eff} = \gamma_\rho$, $s_{1crit\rho}$; and (b) of the q = 1 radius and T_{e0}. There are typically two phases in the evolution of the q profile. First the q = 1 radius evolves rapidly to a certain value close to the value at the crash, as seen in Fig. 2(b). This is because the q profile is relatively flat after the crash and therefore a small decrease of the q profile induces a large variation of ρ_1. Then ρ_1 is almost fixed and the shear at q=1 starts to build up until it reaches s_{1crit}. On the other hand s_{1crit} usually increases rapidly at the beginning as the profiles peak and then saturate. In this case the crash time is well determined. Note that if we had only a partial reconnection such that the q profile is flat around q=1 after the crash, but q_0 is still below one, then the first phase might be a bit shorter while the time for the shear s_1 to increase up to s_{1crit} would be similar. Therefore we would not expect much change. However for some cases the time evolution of s_1 and s_{1crit} are very close because the confinement time and the resistive time inside the q=1 surface are very similar. Then in these cases small changes can change the sawtooth behavior and the period is not as well defined as it depends on the relaxation model. This dependence needs further detailed studies.

In Fig. 2(b) we also show the time evolution of T_{e0}. Depending on the plasma parameters, its shape is either triangular with a linear increase until the next crash or more saturated-like when the increase is more rapid relatively to the sawtooth period and then saturates.

Once the sawtooth period is correctly simulated then both the sawtooth amplitude and the inversion radius are relatively well predicted as is shown in Fig. 3. Therefore the current, q, density and temperature profiles are consistent with the experimental measurements. This is true even for inversion radius varying from 0.2 to 0.6 of the minor radius. Note that there is no correlation experimentally between the inversion radius and the sawtooth period. This is also correctly simulated with the model, even if we assume complete reconnection. It is due to the fact that the sawtooth period depends on the relative time evolution of s_1 and s_{1crit}, and therefore mainly on the local plasma parameters.

This latter remark explains why the sawtooth activity is so much sensitive to electron cyclotron frequency heating (ECRH) as shown in Ref. [12]. Indeed local heating can change both $s_1(t)$, by changing the local resistive time and the current profile, and $s_{1crit}(t)$ by changing the temperature

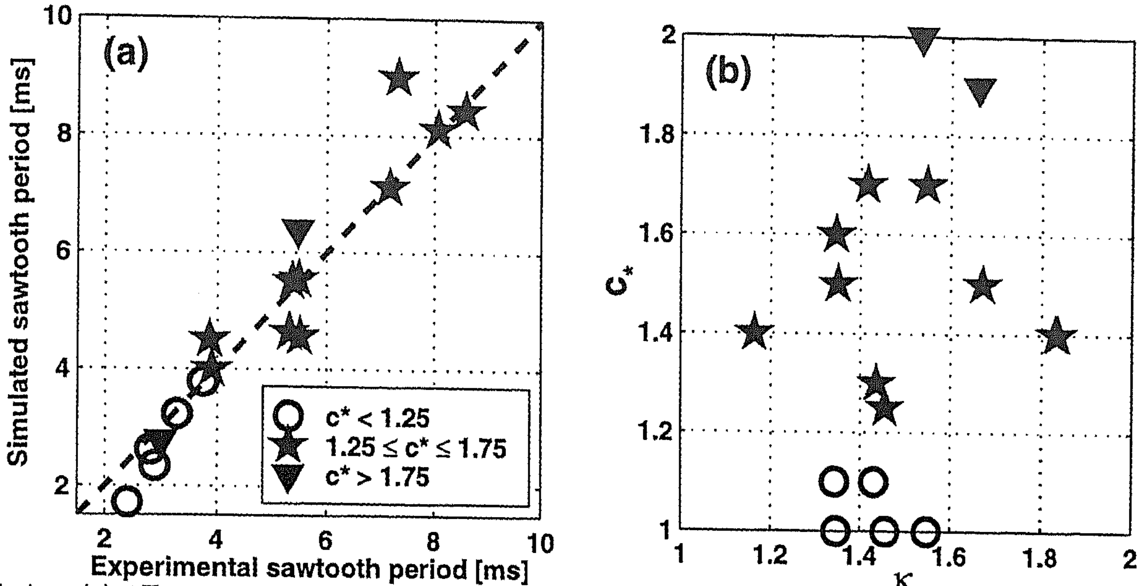

Fig.1: (a) ΔT_{PRETOR} vs ΔT_{TCV}. The value of c_* is detailed in (b) with respect to elongation.

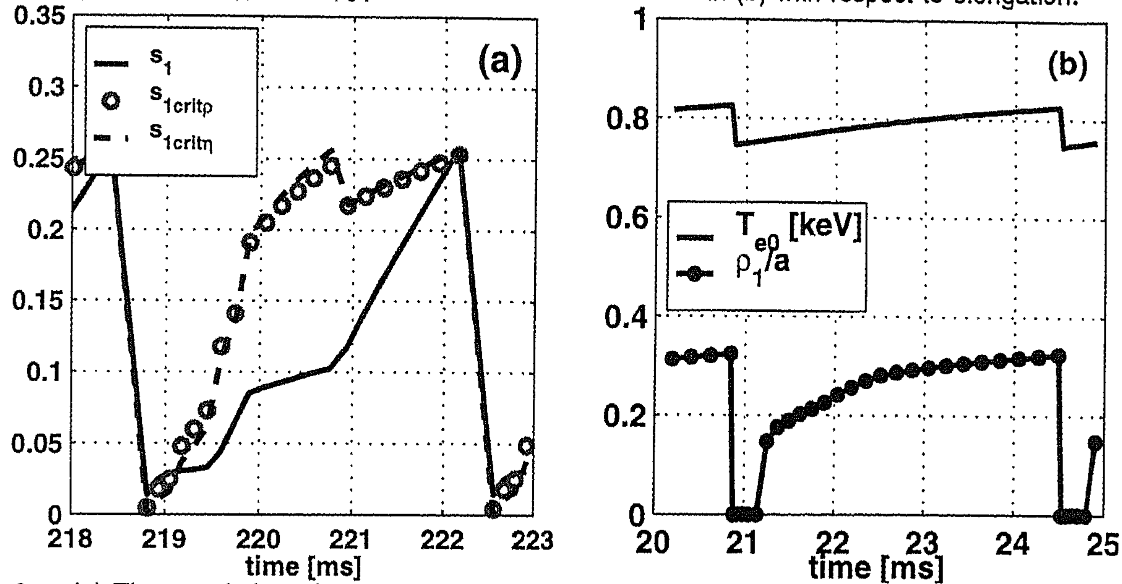

Fig.2: (a) Time evolution of s_1, $s_{crit\eta}$, and $s_{crit\rho}$ and (b) of T_{e0} and ρ_1.

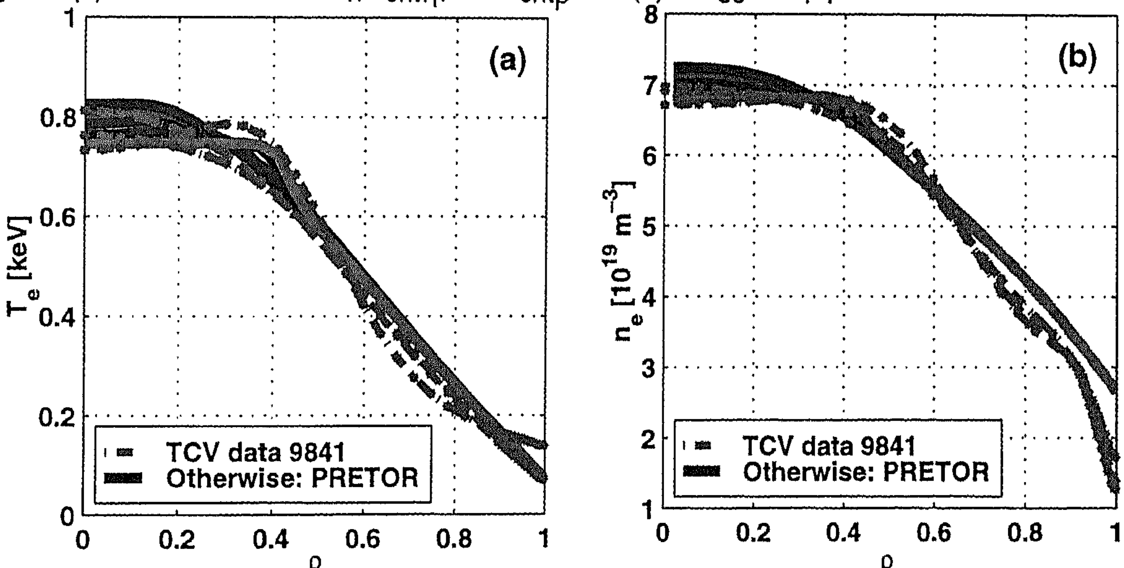

Fig.3: (a) Temperature and (b) density profiles from PRETOR (solid lines) and TCV (dashed) sample over one sawtooth cycle.

gradients. Moreover it affects the q = 1 radius. As a first check we have simulated a case with 0.5 MW of ECRH deposited over a radial width of 0.15 a. Changing the mean deposition radius from $\rho = 0$, 0.3 to 0.5 we see first that ρ_1/a = 0.44, 0.40 and 0.27 respectively. Then, in the first two cases, s_{1crit} is relatively large, 0.35, because heating inside q = 1 gives large gradients at q = 1. Therefore long sawtooth periods are obtained, while heating outside ρ_1 gives a very small s_{1crit} and short sawtooth periods. This is in qualitative agreement with the experiment as sawtooth periods of 2 ms are observed when the heating is outside q = 1 and it increases rapidly to 7-8 ms when heating near the q = 1 surface. However, heating closer to the magnetic axis decreases again the sawtooth period, which needs a more detailed study to be fully understood.

IV. Conclusion

We have shown that the crash model, which predicts that if Eq. (6) is satisfied then the crash is triggered when $\mathbf{s_1 > s_{1crit}}$, is in good agreement with all the TCV ohmic L-modes discharges with $\delta \geq 0.1$, $q_{edge} \leq 4.5$ and arbitrary κ and density. Indeed, using this criterion we are able to model correctly the inversion radius, the sawtooth period and the crash amplitude. The value of s_{1crit} depends on the local plasma parameters and their derivatives at the q = 1 surface and on the actual maximum growth rate, Eq. (7), as well as the specific diamagnetic effects. We have proposed a model, Eq. (8), which can reproduce the sawtooth period over a wide range of parameters, inversion radii and periods with a value of c_* varying only between 1 and 2. The study gives confidence in the model used in Ref. [2] to predict the ITER sawtooth period even though in this latter case another term, including the alpha particles, is the main stabilizing term.

As we are able to follow the time evolution of the sawtooth ramp and crash, using self-consistent density, temperature, current and q profile as well as toroidal MHD equilibria, we have shown that the q = 1 surface broadens relatively fast after the crash and then saturates. This is why for transport analysis of TCV-like ohmic L-modes discharges it is sufficient to use the simple criterion:

$$s_1 > 0.2$$

as trigger condition. Indeed, choosing s_1 small but not too small such that ρ_1 has time to evolve to its pre-crash value allows one to obtain correctly the inversion radius and the crash amplitude. Only the sawtooth period is then not correctly modeled if the actual s_{1crit} would be 0.3 or more for example. However it only changes slightly the profile shapes, certainly within the experimental error bars. This criterion is what has been used for the TCV transport simulations in Ref. [5].

Using this model we understand why and how the sawtooth activity is so sensitive to local ECRH[12]. We have explained the sharp increase in sawtooth period when heating outside or near the q=1 radius.

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