

Modeling Self-Organized Aggregation in a Swarm of Miniature Robots

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Abstract—We model the dynamics of self-organized robot aggregation inspired by a study on aggregation of gregarious arthropods. In swarms of German cockroaches, aggregation into clusters emerges solely from local interactions between the individuals, whereas the probabilities to join or leave a cluster are a function of the cluster size.

Rather than explicitly modeling the spatial distribution of robots in the environment, we propose a population dynamics model that keeps track of the number of robots in clusters of specific size.

The model is able to quantitatively and qualitatively predict the dynamics observed in extensive realistic simulation. In particular, we show both by modeling and simulation that the emergence of a single aggregate requires a minimal communication distance between individuals, whereas the robots remain scattered in the environment otherwise.

I. INTRODUCTION

Aggregation processes are ubiquitous in a multitude of domains ranging from physics [1] and biology [2], to swarm robotic systems [3]. The processes responsible for aggregation are likely to share similarities on different scales, from proteins, to social insects, and mammals [4], suggesting a common methodological framework for modeling their dynamics. In the case of “self-organized” [5] aggregation, which is a result from purely local interactions without central control or global information exchange, and an intrinsic amount of randomness, analysis of the resulting complex dynamics becomes cumbersome. Formal modeling of the underlying processes might lead not only for a better understanding of natural processes [6], but is also beneficial in an engineering context, for instance for designing self-organized building processes [3]. Also, aggregation can be understood as an important collective behavior in swarm-robotics, as it might be the prerequisite for more complex collective tasks that rely on local interactions.

In self-organized aggregation phenomena, structures emerge out of continuous aggregation and disaggregation of clusters. In order for sophisticated structures to emerge, the behavior of the participating agents needs to change as a function of the aggregate. Social insects for instance explicitly modulate their behavior based on their perception of the environment.

In this paper, we introduce a probabilistic macroscopic model for modeling the dynamics of self-organized robotic aggregation processes. Here, the robot behavior is inspired by

a case study concerned with aggregation of gregarious arthropods [6], which has also been successfully implemented on a swarm of real miniature Alice robots elsewhere [7].

In our model, we maintain a tight link between parameters of the individual behavior and spatio-temporal patterns observed on the collective level. This approach has the potential to allow identification of parameter values in natural collective systems, as well as optimizing individual parameters in fully engineered [8] or mixed artificial/biological systems [9].

Our probabilistic modeling approach is complementary to deterministic models for aggregation and flocking from a systems and control perspective (flocking is conceptually similar to aggregation but involves also a coordinated motion of the aggregate) as for instance [10], [11]. In these contributions, the emerging graph structure that represents the local neighborhood relations between agents, as well as the (often holonomic) agent dynamics are explicitly taken into account, allowing for proving properties such as convergence analytically. However, assumptions made on the robotic platform (perfect sensors and actuators including range and bearing), make these results rather interesting from a theoretical perspective. In our model instead, spatial aspects of aggregation are ignored and sensor and actuator noise is reflected by probabilities with which robots interact among each other and with the environment. Consequently, our model predicts the *likelihood* for convergence based on the robots’ controllers and its parameters by modeling the average number of robots in clusters of different sizes.

II. AGGREGATION IN GREGARIOUS ARTHROPODS

The aggregation of larvae of the German cockroach (*Blattella Germanica*) serves as a behavioral model in this paper. Cockroach behavior is known to be gregarious, i.e. individuals tend to associate with others of their kind, and aggregation is known to be mediated by only local interactions [6] between individuals.

Aggregation of a swarm of cockroaches in a bounded arena emerges as follows. Cockroaches move randomly through the arena [12], eventually stop, and aggregate into clusters of different sizes, in which every cockroach can sense the presence of at least one other cockroach. Clusters are not persistent, because cockroaches might resume movement and quit the cluster. Biological experiments [6] show that the average time to rest within a cluster is a function of its size, where larger clusters are preferred over smaller ones.

The behavioral parameters as a function of cluster size have been measured for *Blattella Germanica* larvae by Jeanson et al. [6]. In their experiments, Jeanson et al. used

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TABLE I

PROBABILITIES TO STOP (p_n^{join}) AND RESTART (p_n^{leave}) SEARCHING DURING ONE TIME STEP ($T = 1s$) AS A FUNCTION OF THE NUMBER OF NEIGHBORS WITHIN PERCEPTION RANGE. VALUES ARE DERIVED FROM [6].

n	p_n^{join}	p_n^{leave}
0	0.03	n.a.
1	0.42	1/49
2	0.5	1/424
3	0.51	1/700
4+	0.51	1/1306

first-instar larvae (24h old) in a circular arena and examined interaction in groups of two to four larvae, and trajectories of individual cockroaches.

Assuming that mutual perception takes place solely using their antennas, the rate at which a larva would stop when it perceived 1, 2 and 3 cockroaches within its perception radius has been measured in a suite of experiments. Similarly, the time until an individual left a cluster of 1, 2, 3 and 4 cockroaches was used to calculate the rate at which cockroaches spontaneously left a cluster. The rate of stopping when no other larvae are nearby was estimated using experiments involving a single individual.

Although cockroaches in [6] show varying behavior as a function of their location within the arena (close to the center or close to the walls), and [6] distinguishes between two different resting behaviors, these effects are not modeled in this paper.

Notice that the behavior of the cockroaches is purely probabilistic, and average macroscopic quantities (such as the average cluster size) can only be observed using systematic, large-scale experiments, which would be infeasible or difficult using biological agents.

III. ROBOT BEHAVIOR

Every robot is either moving or resting. For a finite number of robots N_0 in the arena, a robot can be part of a cluster of size 1, 2, 3, ..., N_0 . Both the transition probability for entering the rest state from the move state (p^{join}) and resuming to move (p^{leave}) are given by a non-linear function that is a function of the robots nearby (see Table I). Probabilities have been adopted from behavioral parameters of the insects. However, while the values of $p^{join}(j)$ are identical to those reported in [6], $p^{leave}(j)$ is the *average* leaving probability for short and long stops that lead to a double-exponential distribution in [6]. Note that the probability $p^{join}(0)$ is the probability to stop when no neighbors are present.

When moving, the robot senses a part of the arena, whose size is determined by its communication range, and eventually encounters other robots with which it might aggregate.

The swarm (the ensemble of robots) aggregates within the environment in clusters of different size. Due to the robots' preference to stay with larger clusters (given by the behavioral parameters extracted from cockroach experiments),

robots eventually aggregate into a single aggregate. This behavior is independent of the initial distribution of robots in the environment.

IV. A PROBABILISTIC MODEL FOR AGGREGATION DYNAMICS

The individual behavior described above can be represented by a Markov dynamical system for every individual, whereas the collective behavior can be described by averaging over the states of all individuals.

A. Individual Dynamics

We describe the dynamics of each individual v by a Markov chain with a set of states \mathcal{X} .

The state space $\mathcal{X}^{(v)}$ is discrete, finite, and reflects the size of the aggregate that a robot is part of. A static robot that does not have any neighbors (other robots in communication range) is considered an aggregate of size one. For a finite number N_0 of robots, the state space is thus given by

$$\mathcal{X}^{(v)} = j \in \{0, 1, \dots, N_0\}, \quad (1)$$

with j denoting the size of the aggregate the robot is part of, and $\mathcal{X}^{(v)} = 0$ denoting a searching robot.

The probability for a moving robot to join an aggregate of size j is given by

$$P(\mathcal{X}^{(v)}(k+1) = j | \mathcal{X}^{(v)}(k) = 0) : \mathbb{Z}_+ \rightarrow [0, 1], \quad (2)$$

and will be denoted by $p^{join}(j)$ in the remainder of this paper. Similarly the probability for a robot to leave an aggregate of size j is given by

$$P(\mathcal{X}^{(v)}(k+1) = 0 | \mathcal{X}^{(v)}(k) = j) : \mathbb{Z}_+ \rightarrow [0, 1] \quad (3)$$

and will be denoted by $p^{leave}(j)$. As the formalism above assumes that all robots in an aggregate can sense the number of robots in this aggregate, we also refer to an aggregate as a *clique*.

Notice that a robot might change its state not only due to its own action, but also when another robot is added or removed from the aggregate it is part of.

The state space and possible state transitions are summarized graphically in Figure 1.

B. Mobility

Assuming uniform distribution of objects in the arena, constant speed, and constant sensing range, we can approximate the detection of a robot within the arena by an encountering probability (see for instance [13]–[16], and references therein).

Notice that correlated random walk [12] in a bounded arena eventually leads to an uniform probability density function for the location of every cockroach in the environment [17]. Also, in our experiments we did not implement the wall-following behavior that is observed for real cockroaches [6] and promotes aggregation close to the arena border.

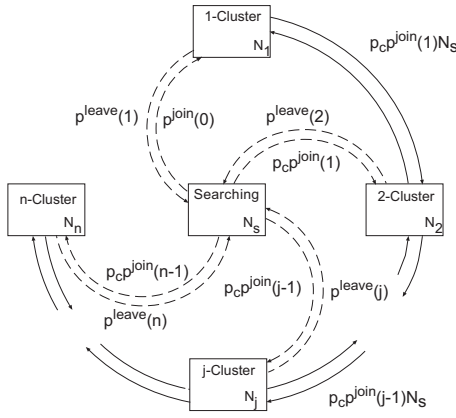


Fig. 1. State transition diagram of an individual robot.

Definition 1 (Encountering Probability): *An individual moving with constant speed and constant sensor range within a bounded arena, has a constant probability p_c to encounter another individual of constant size and appearance at every time step of length T . In environments populated with N individuals that are uniformly distributed, the probability to encounter one of them is calculated by Np_c (linear superposition of encountering probabilities).*

In [14], [15] we showed that the following relation holds for the probability p_c :

$$p_c \sim \frac{1}{A_{total}} v_r w_d T, \quad (4)$$

with A_{total} the area of the arena, v_r the average speed of an individual, w_d the individual's detection width, i.e. the width it sweeps with its sensors while moving (the sensor range is equivalent to the robot's communication range in this case), and the time discretization of the system T .

Thus, p_c is proportional to an individual's mobility as well as its communication range.

C. Collective Dynamics

Using a set of difference equations, we can summarize the *average* state transitions of each individual Markov dynamical system, and thus keep track of the number of aggregates of size 1 to N_0 .

The ensemble of individuals as well as its structural properties is now represented by a difference equation, which keeps track of the average number of individuals in each state. Inflow and outflow of each state represents the average fluctuations between states and are given by the probability for a state transition to occur and the number of robots in other states.

Using Definition 1 and following the mean-field approach described above, the average number of robots $N_j(k+1)$ in an aggregate of size j (with $1 < j < N_0$) at time $k+1$, is then given by the following difference equation

$$\begin{aligned} N_j(k+1) = & N_j(k) + p_c N_{j-1}(k) N_s(k) p^{join(j-1)} j \\ & + p^{leave(j+1)} N_{j+1}(k) j \\ & - p_c N_j(k) N_s(k) p^{join(j)} j \\ & - p^{leave(j)} N_j(k) j \end{aligned} \quad (5)$$

The term $p_c N_{j-1}(k) N_s(k) p^{join(j-1)}$ corresponds to the number of searching robots $N_s(k)$ that encounter one of the robots in a cluster of size $j-1$ (probability $p_c N_{j-1}(k)$), and decide to join this robot's cluster with probability $p^{join(j-1)}$. Then, j more robots would be in a cluster of size j . This is also the case when any robot in a cluster of size $j+1$ leaves its cluster with probability $p^{leave(j+1)}$.

The number of robots in a cluster of j diminishes by j when a searching robot joins a cluster of size j (with probability $p_c N_j(k) N_s(k) p^{join(j)}$), or when a robot leaves a cluster of j with probability $p^{leave(j)}$.

Notice that the ensemble of $N_j(k)$, $j = \{0, \dots, N_0\}$ corresponds to the degree distribution of the clusters in the environment.

For an aggregate consisting of N_0 robots, (5) simplifies to

$$\begin{aligned} N_{N_0}(k+1) = & N_{N_0}(k) \\ & + p^{join(N_0-1)} N_{N_0-1}(k) p_c N_s(k) N_0 \\ & - p^{leave(N_0)} N_{N_0}(k) N_0 \end{aligned} \quad (6)$$

as there exist no aggregates of size N_0+1 , nor any moving robots in this case. The number of aggregates of size one, at time $k+1$ is given by

$$\begin{aligned} N_1(k+1) = & N_1(k) \\ & - p^{leave(1)} N_1(k) \\ & + p^{leave(2)} N_2(k) \\ & + p^{join(0)} N_s(k) \\ & - p^{join(1)} N_1(k) p_c N_s(k) \end{aligned} \quad (7)$$

For a constant number of robots, we can calculate the number of moving robots by

$$N_s(k+1) = N_0 - \sum_{n=1}^{N_0} N_n(k+1) \quad (8)$$

As $N_j(k)$ represents the *average* number of robots in a cluster of j , and an initial random deployment of robots does not exclude cluster formations at the beginning, one can think of $\frac{N_j(k)j}{N_0}$ as the probability that an aggregate of size j exists at time k .

V. EXPERIMENTAL SETUP

We study the relevance of our model using the realistic simulator *Webots* [18] and behavioral data obtained from a systematical study with larvae of the species *Blattella Germanica* [6].

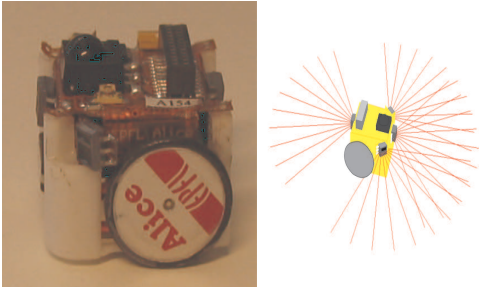


Fig. 2. The miniature robot *Alice* (left), and its simulated counterpart (right).

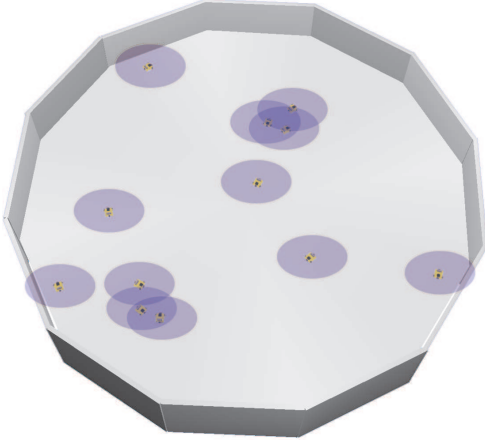


Fig. 3. The communication range of each individual is shown by superimposed discs. The scenario shows six clusters of 1 individual, and two clusters of 3 individuals.

A. Robotic Platform

The *Alice* robot [19] has a size of $2\text{cm} \times 2\text{cm} \times 2\text{cm}$, a differential wheel drive that reaches speed of up to $4 \frac{\text{cm}}{\text{s}}$, four infrared distance sensors for obstacle detection (up to 3cm), and 4Bit/s local communication up to 6cm, which can serve as well as crude low-range and bearing sensor (Figure 2). The robot as well as its sensors and actuators is faithfully implemented in *Webots* (see below).

Robots have an average speed of 4cm/s and can evaluate the number of neighbors using local communication (we assume that each robot has a unique ID or can randomly choose its ID from a sufficiently large set). Notice that behavioral probabilities in our model are independent from the robots' speed (which is solely reflected in the encountering probability).

A random communication network topology is depicted in Figure 3 where the communication range (here 10cm) of each individual is depicted by a circular disc. We refer to the ensemble of robots that can communicate with themselves directly or indirectly as a cluster.

The arena has a diameter of 1m.

B. Simulator

The experimental setup and hardware described above was implemented in *Webots* [18] a realistic, sensor-based simulator that is able to accurately model the non-linear

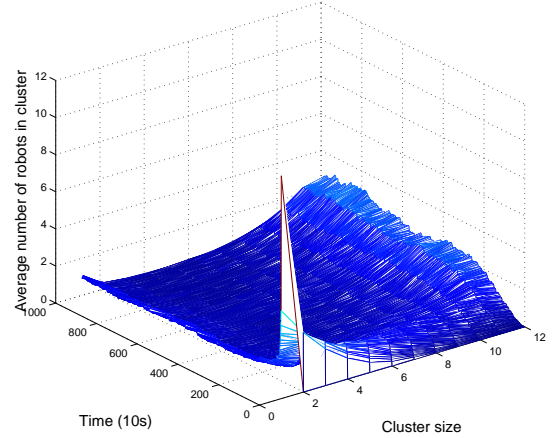


Fig. 4. Average degree distribution over 3h of simulated time for 10cm communication range (center to center). The degree distribution was recorded every 10 seconds for 1500 experiments.

sensor characteristics of the *Alice* robot, including Gaussian noise on the sensors as well as wheel-slip.

For this case study, *Webots* simulations allow us to collect results about 3 to 4 times faster than in real robot experiments. Using a computational cluster equivalent to 35 Pentium IV processors, we are able to collect a sufficient amount of data for quantitatively studying the robots' performance and its distribution for different control parameters. Using this configuration, one achieves approximately 120 simulations per hour.

VI. RESULTS

In all experiments difference equations are solved by numerical integration for 3h of simulated time (time discretization of the system $T = 1\text{s}$) with $N_0(0) = N_0 = 12$ as initial condition.

We performed 1500 runs in *Webots* for 3h of simulated time each, and the number of robots in clusters of 1 to 12 individuals was counted every 10 seconds. The communication range, i.e. the maximal distance other agents can be sensed, was set to 7cm, 10cm, and 12cm.

Notice that for experimental reasons we do not distinguish between moving robots and robots in clusters of size 1. Therefore, we add up $N_0(k)$ and $N_1(k)$ when comparing simulation and modeling results. For a communication range of 10cm simulations (Figure 4) and model (Figure 5) show good agreement for an encountering probability of $p_c = 0.03$. A close-up comparing results for the specific case of a cluster of 12 individuals is depicted in Figure 6.

The encountering probability p_c is a measure of the mobility as well as the sensor range of an agent relative to the size of the arena. It is the only parameter of our model that is not implemented in the controllers of the simulated robots. We tested the steady-state behavior of our model for different values of p_c ($0 \leq p_c \leq 0.1$), and plot the resulting degree distribution (100 simulations) in Figure 7 and 8. These results are in line with experiments with communication range of

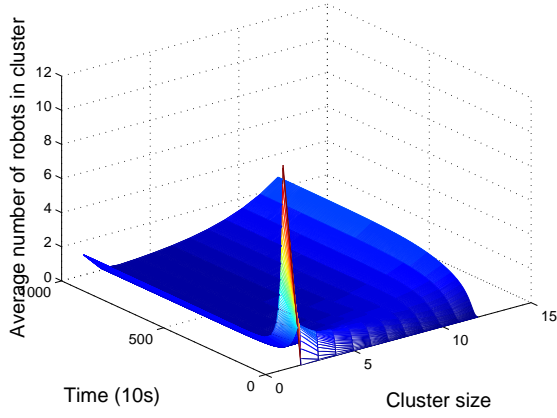


Fig. 5. Model prediction for the degree distribution over 3h simulated time (time discretization of the model 10s, $p_c = 0.03$), only the first 2.7h are plotted.

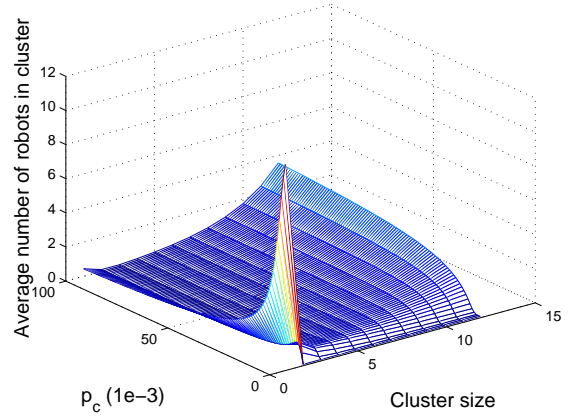


Fig. 7. Degree distribution at steady state as a function of the encountering probability ($0 \leq p_c \leq 0.1$). For a critical value of p_c , the system shows the tendency for aggregation to a single aggregate.

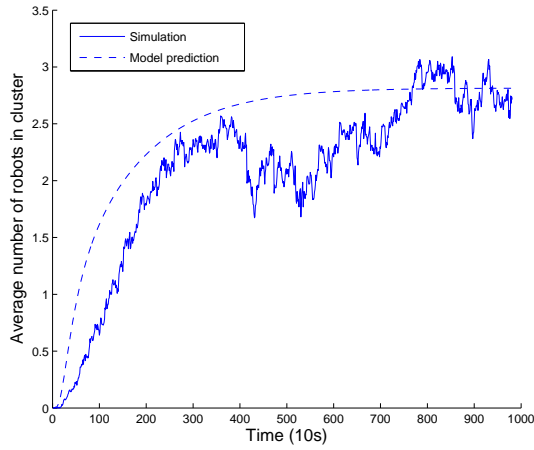


Fig. 6. Model prediction vs. simulation results (1500 experiments) for the average number of robots in a cluster of 12 for 10cm communication range (see Figure 4).

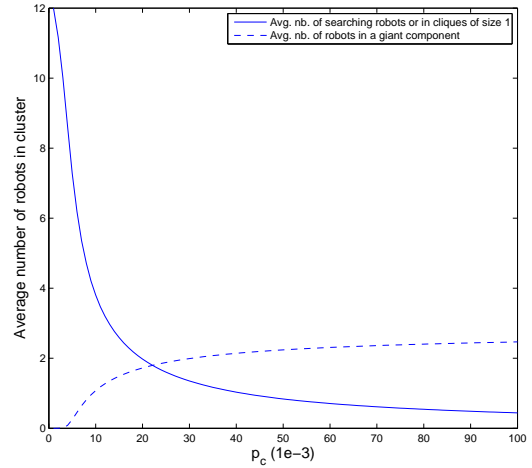


Fig. 8. Average number of individuals in a single aggregate vs. average number of individuals in a single aggregate as a function of p_c .

7cm and 12cm (Figure 9 and 10, respectively) that show similar behavior.

VII. DISCUSSION

Interestingly, the tendency of aggregating into a single aggregate comprising almost all of the robots or to remain scattered in the environment is a function of p_c , and we observe a bifurcation of the system for certain values of p_c (Figure 9 and 10). The parameter p_c is a function of the robot's speed, and its sensor range (Definition 1). Low values correspond to a low probability to encounter other robots in the arena, either due to reduced mobility or due to limited sensor range. It seems that a minimal amount of either mobility or sensor range is necessary to achieve aggregation. While we provide experimental evidence only for the influence of communication range, we conjecture that low communication ranges can be counter-effected with increased mobility, and vice versa.

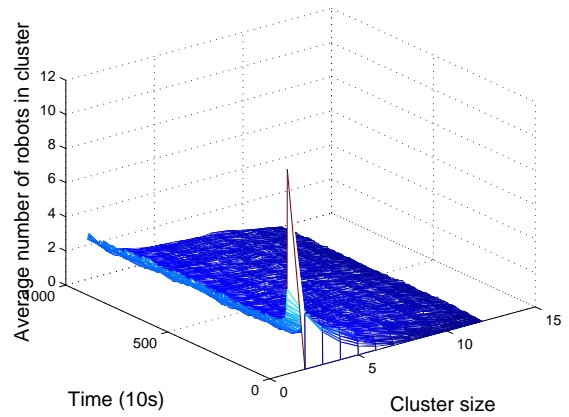


Fig. 9. Average degree distribution over 3h of simulated time for 7cm communication range (center to center). 1500 experiments, the degree distribution was recorded every 10 seconds.

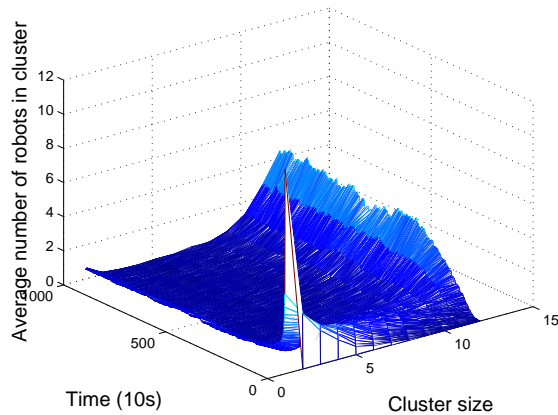


Fig. 10. Average degree distribution over 3h of simulated time for 12cm communication range (center to center). 1500 experiments, the degree distribution was recorded every 10 seconds.

We notice that the requirements of the algorithm are extremely simple although the communication range of the *Alice* seems not to be sufficient for reliably achieving aggregation into a giant component. For a real application however the achieved performance (i.e. reliability of aggregation and time to achieve a giant component) is unsatisfying, and better results could for instance be achieved by preventing robots from leaving a cluster and performing collective motions of the aggregates if cluster growth stagnates. This strategy is potentially feasible on the *Alice* platform, and would allow for coherence of aggregates while preventing deadlocks in sub-optimal solutions (multiple static clusters in the environment).

However, considering even simpler platforms than the *Alice* robot, e.g. genetically modified cells [8] or the projected *I-Swarm* robot (size below 3mm) [20], the proposed algorithm might be the only alternative for achieving aggregation.

A limitation of the proposed modeling methodology is our assumption that clusters can only grow or decay linearly, i.e. one robot at a time. For clusters that are not circular, but having a shape of a line, this assumption is not valid. Here, a leaving robot might destroy the connectivity of a cluster, and thus lead to two clusters of around half of the size. Similarly, our model does not account for robots that bridge two clusters.

VIII. CONCLUSION

The contribution of this paper is twofold: first, we introduce a probabilistic model that is capable of quantitative prediction of self-organized robotic aggregation dynamics. Second, we show how to link processes affecting the structure of the emerging cluster topology with the mobility and communication range of individual agents.

This model has the potential to not only model aggregation in robotic swarms but also self-assembly phenomena on other scales that might be governed by similar construction rules.

We are currently working on extending our formalism for modeling heterogenous swarms in which agents might

act differently as a function of their species. This is for instance the case in mixed animal-robot societies [9]. Then, modeling can be used for optimizing engineered agents to create desired patterns at the collective level in the natural society.

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REFERENCES

- [1] A. Zangwill, "Statistical physics: Advances in aggregation," *Nature*, vol. 411, pp. 651–652, 2001.
- [2] J. K. Parrish and W. M. Hammer, *Animal Groups in Three Dimensions*. Cambridge University Press, UK, 1997.
- [3] A. Martinoli, A.-J. Ijspeert, and F. Mondada, "Understanding Collective Aggregation Mechanisms: from Probabilistic Modelling to Experiments with Real Robots," *Robotics and Autonomous Systems*, vol. 29, pp. 51–63, 1999.
- [4] G. Whitesides and B. Grzybowski, "Self-assembly at all scales," *Science*, vol. 295, pp. 2418–2421, 2002.
- [5] S. Camazine, J.-L. Deneubourg, N. R. Franks, J. Sneyd, G. Theraulaz, and E. Bonabeau, *Self-Organization in Biological Systems*, ser. Princeton Studies in Complexity. Princeton University Press, 2001.
- [6] R. Jeanson, C. Rivault, J.-L. Deneubourg, S. Blanco, R. Fournier, C. Jost, and G. Theraulaz, "Self-organized aggregation in cockroaches," *Animal Behavior*, vol. 69, pp. 169–180, 2004.
- [7] S. Garnier, C. Jost, R. Jeanson, J. Gautrais, M. Asadpour, G. Caprari, J.-L. Deneubourg, and G. Theraulaz, "Collective decision-making by a group of cockroach-like robots," in *Proc. of the IEEE Swarm Intelligence Symposium (SIS)*, 2005, pp. 233–240.
- [8] S. Basu, Y. Gerchman, C. Collins, F. Arnold, and R. Weiss, "A synthetic multicellular system for programmed pattern formation," *Nature*, vol. 434, pp. 1130–1134, 2005.
- [9] G. Caprari, A. Colot, R. Siegwart, J. Halloy, and J.-L. Deneubourg, "Building mixed societies of animals and robots," *IEEE Robotics & Automation Magazine*, vol. 12, no. 2, pp. 58–65, 2005.
- [10] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, 2003.
- [11] H. Tanner, A. Jadbabaie, and G. Pappas, *Flocking in teams of nonholonomic agents*, ser. Lecture Notes in Control and Information Sciences. Springer Verlag, 2005, vol. 309, pp. 229–239.
- [12] R. Jeanson, S. Blanco, R. Fournier, J.-L. Deneubourg, V. Fourcassié, and G. Theraulaz, "A model of animal movements in a bounded space," *Journal of Theoretical Biology*, vol. 225, pp. 443–451, 2003.
- [13] W. Agassounon, A. Martinoli, and K. Easton, "Macroscopic modeling of aggregation experiments using embodied agents in teams of constant and time-varying sizes," *Autonomous Robots*, vol. 17, no. 2–3, pp. 163–191, 2004.
- [14] A. Martinoli, K. Easton, and W. Agassounon, "Modeling of swarm robotic systems: A case study in collaborative distributed manipulation," *Int. J. of Robotics Research*, vol. 23, no. 4, pp. 415–436, 2004.
- [15] N. Correll and A. Martinoli, "Modeling and optimization of a swarm-intelligent inspection system," in *Proc. of the Int. Symp. on Distributed Autonomous Robotic Systems (DARS)*, Toulouse, France, 2006, pp. 369–378, distributed Autonomous Robots 6. Springer Verlag, Tokyo.
- [16] —, "System identification of self-organized robotic swarms," in *Proc. of the Int. Symp. on Distributed Autonomous Robotic Systems (DARS)*, Minneapolis, MN, USA, 2006, pp. 31–40, distributed Autonomous Robots 7. Springer Verlag, Tokyo.
- [17] H. C. Berg, *Random Walks in Biology*. Princeton University Press, 1983.
- [18] O. Michel, "Webots: Professional mobile robot simulation," *Journal of Advanced Robotic Systems*, vol. 1, no. 1, pp. 39–42, 2004.
- [19] G. Caprari and R. Siegwart, "Mobile micro-robots ready to use: Alice," in *Proc. of the IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS)*, Edmonton, Alberta, Canada, August 2005, pp. 3295–3300.
- [20] (2006) The I-Swarm project. [Online]. Available: <http://www.i-swarm.org>