SLAM with Corner Features Based on a Relative Map

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***Abstract*— This paper presents a solution to the Simultaneous Localization and Mapping (SLAM) problem in the stochastic map framework for a mobile robot navigating in an indoor environment. The approach is based on the concept of the relative map. The idea consists in introducing a map state, which only contains quantities invariant under translation and rotation. This is done in order to have a decoupling between the robot motion and the landmark estimation and therefore not to rely the landmark estimation on the unmodeled error sources of the robot motion. The case of the corner feature is here considered. The relative state estimated through the Kalman filter contains the distances and the relative orientations among the corners observed at the same time. Therefore, this state is invariant with respect to the robot configuration (translation and rotation). Finally, an environment containing structures consisting of several corners is also investigated. Real experiments carried out with a mobile robot equipped with a 360^o laser range finder show the performance of the approach.**

I. Introduction

Simultaneous Localization and Mapping (SLAM) requires a mobile robot to autonomously explore the environment with its on-board sensors, gain knowledge about it, interpret the scene, build an appropriate map and localize itself relative to this map. Many approaches have been proposed both in the framework of the metric and the topological navigation. A very successful metric method is the stochastic map [15], where early experiments [4] [9], have shown the quality of fully metric SLAM. However, these approaches suffer from some limitations. Firstly, they rely strongly on odometry making the global consistency of the map difficult to maintain in large environments due to the odometry drift. Furthermore, they represent the robot position with a single Gaussian distribution meaning that an unmodeled event (i.e. collision) could cause a divergence between the ground truth and the estimation, which could be unrecoverable for the system (lost situation). Even though the global consistency can be better maintained by taking into account all the correlations [1], the fact that this remains suboptimal has motivated the introduction of relative reference frames [3]. In order to properly integrate information for the SLAM problem it is clearly necessary to known the statistical model characterizing the systematic and the non-systematic error of each robot's sensor as better as possible. An error on the sensor model will produce a divergence in the built map if the environment is large enough. This problem arises even if the approach is optimal respect to the dynamics of the robot and the observation and if the convergence is theoretically proven. Indeed, the divergence arises because the error model is imperfect. Dissanayake et al. [7], proved the convergence of a filter based on Kalman (absolute map filter, AMF) theoretically. However, the proof is based on an unrealistic perfect statistical knowledge of the error of each sensor and also on the strong hypothesis of a linear observation. Julier and Uhlmann proved that the AMF yields an inconsistent map, even for the special case of a stationary vehicle with no process noise [8].

In order to minimize the divergence of the built map, one has to concentrate on two important points: Adopt an optimal filter (accordingly to the dynamics and the observation); Use the best statistical model to characterize the error of the adopted sensor readings. Clearly, to deal with the second remark, it is better not to use the odometry in the estimation phase if other more precise sensors are available with a well-known error model. The AMF, using odometry, diverges when there is even a very small, undetected systematic component. This divergence is proven through simulations in [10] and through experiments on a real platform in [11]. Therefore, decoupling odometry from the estimation process becomes a main issue. For this, Csorba, Uhlmann and Durrant-Whyte [5] introduced a relative map based on quantities invariant to the robot pose (i.e. to translation and rotation). Deans and Hebert [6] adopted the same idea. Both estimate the distance between two landmarks, which is invariant to the robot pose (translation and rotation). However, their algorithms are sub optimal because they do not consider the correlation between the distances with a common landmark. Newman introduced a relative map and he used two filters in the estimation, called the relative map filter and the geometric projection filter ([13] and [14]). The second one provides a mean to produce a geometrically consistent map from the relative map, by solving a set of linear constraints. Both filters are optimal since the dynamics and the observation are linear and they are based on the Kalman Filter. However, the elements used in this approach are invariant for translation only, not for rotation. The approach adopted in [10] and [11] is to take invariant elements for both translation and rotation in order not to rely the robot motion for the estimation. Then a Kalman filter is used for estimation, contrasting to [5] and [6], who used the same invariants in combination with a non-optimal filter. The observation, as well as the dynamic, will be linear. Therefore, the only error source, which could create a divergence in the long term, is the gaussian assumption adopted in the landmark position in the robot frame evaluated through the exteroceptive sensor.

In this work, we extend the relative map introduced in [10] and [11] to the case of the corner feature. In particular, the novelty of this paper with respect to [10] and [11] consists in the introduction of the relative state among the corners and the application of the general equations based on the Kalman filter derived in [10] to this relative state (sect II and III). Furthermore, a new concept, *the structure*, was here introduced to improve the convergence (sect III). In the sections IV and V some experimental results, obtained with a mobile robot equipped with a 360^o laser range finder sensor, are shown and discussed and some conclusions and future research are also provided.

II. The Basic Equations for the Relative Map Filter

The odometry can be decoupled from the estimation process by introducing a filter whose state only contains quantities invariant under translation and rotation. This is the idea characterizing the relative filter introduced here. Once the relative map has been estimated through this filter and the absolute location of a set of landmarks is known (e.g. by using the first observation) it is possible to build the absolute map. Therefore, the entire method contains two algorithms. The former estimates the relative map, the latter builds the absolute map. In the following we provide the equations to estimate the relative state. These equations are very general and can be applied to any kind of landmarks.

Let denote with I the state containing all the relative quantities among the landmarks and with P its covariance matrix. We call the elements contained in I the *Invariants*, since they are independent of the robot pose. In the next section we define the invariants we adopt to characterize the relative information among corner features. However, in the derivation of the following equations, the explicit expression of I is not required. The only hypothesis here used for the derivation is that I contains relative quantities among the landmarks observed at the same time.

In fig. 1 the nodes represent generic landmarks (e.g. corners, segments) and the edges represent the invariants between the related landmarks (e.g. distances, relative orientations). In fig. $1a$ the vector I contains the marked invariants between the 6 landmarks. Clearly, not all of the invariants between the 6 landmarks are stored in I because not all the landmarks were observed together at the same time. At a given time step, the observation consists of a set of invariants between the landmarks observed by the robot through its external sensor (fig. $1b$). Of course, these invariants may be already observed (i.e. can be in the vector I) or may not. Let introduce the following notation:

$$I_{old} = [u, w_{old}]^T \qquad I_{obs} = [w_{obs}, v]^T \tag{1}$$

where I_{old} is the state estimated at a given time step and I_{obs} is the observation at the same time step, containing a set of invariants between the landmarks observed by the robot. u contains the invariants which are not re-observed (i.e. which do not appear in the vector I_{obs}) and w_{old} contains the invariants re-observed (denoted by w_{obs} in the vector I_{obs}). Finally, v contains the invariants observed for the first time at the considered time step. The covariance matrix of the previous vectors are:

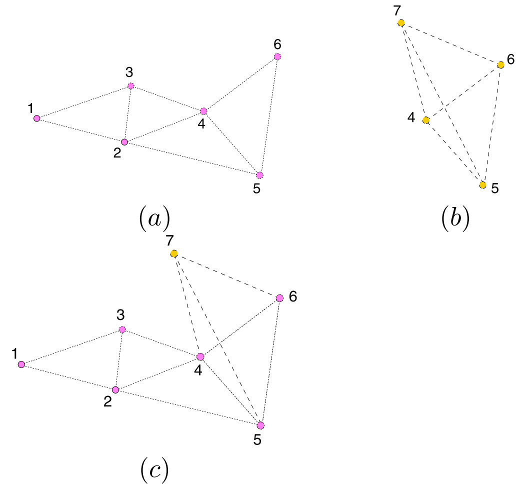

Fig. 1. Relative Map before the observation (a), the observation (b), and the relative map obtained by fusing the information coming from the old map and the observation (c). In all the three figures the map state only contains the indicated invariants between the landmarks

$$P_{old} = \begin{bmatrix} P_{uu} & P_{uw} \\ P_{uw}^T & P_{ww} \end{bmatrix} \qquad P_{obs} = \begin{bmatrix} R_{ww} & R_{wv} \\ R_{wv}^T & R_{vv} \end{bmatrix} \tag{2}$$

We adopt the following notation to denote the estimated quantities, obtained by fusing the old state with the observed one (the new estimated invariants are depicted in fig. $1c$).

$$I_{new} = [u_{new}, w_{new}, v_{new}]^T \tag{3}$$

$$P_{new} = \begin{bmatrix} Pn_{uu} & Pn_{uw} & Pn_{uv} \\ Pn_{uw}^T & Pn_{ww} & Pn_{wv} \\ Pn_{uv}^T & Pn_{wv}^T & Pn_{vv} \end{bmatrix} \tag{4}$$

We obtain the new estimation for the state and its covariance matrix by applying the equations of the Kalman filter. Observe that the observation is linear in the state (is the identity) and therefore the Kalman filter is optimal.

$$u_{new} = u + P_{uw}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{obs} - w_{old}\right) \tag{5}$$

$$w_{new} = w_{old} + P_{ww}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{obs} - w_{old}\right) \tag{6}$$

$$v_{new} = v + R_{vw}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{old} - w_{obs}\right) \tag{7}$$

$$Pn_{uu} = P_{uu} - P_{uw}\left(P_{ww} + R_{ww}\right)^{-1} P_{wu} \tag{8}$$

$$Pn_{uw} = P_{uw} - P_{uw} \left(P_{ww} + R_{ww} \right)^{-1} P_{ww} \quad (9)$$

$$Pn_{uv} = 0 \quad (10)$$

$$Pn_{ww} = P_{ww} - P_{ww} \left(P_{ww} + R_{ww} \right)^{-1} P_{ww} \quad (11)$$

$$Pn_{wv} = R_{wv} - R_{ww} \left(P_{ww} + R_{ww} \right)^{-1} R_{wv} \quad (12)$$

$$Pn_{vv} = R_{vv} - R_{vw} \left(P_{ww} + R_{ww} \right)^{-1} R_{wv} \quad (13)$$

Instead of the equations (6) and (11) it is possible to use the following equations:

$$w_{new} = w_{obs} + R_{ww} \left(P_{ww} + R_{ww} \right)^{-1} \left(w_{old} - w_{obs} \right) \quad (14)$$

$$Pn_{ww} = R_{ww} - R_{ww} \left(P_{ww} + R_{ww} \right)^{-1} R_{ww} \quad (15)$$

They are derived by observing the symmetry of the filter with respect to the change "observation" $\leftrightarrow$ "old state". Observe that the coincidence of the previous equations could be easily proven also by using the inversion lemma.

III. Invariants for the Corner Feature

The configuration of a corner in a two-dimensional environment consists of three parameters characterizing its position and orientation. In fig. 2 we display the parameters (x, y, θ) here adopted to characterize the corner configuration in the global reference frame W.

Let consider now two corners. We can attach on each one a reference frame without any ambiguity (indeed, in the SPmodel the binding matrix of a corner is the identity matrix [2]). Clearly, all the information concerning the configuration of one corner with respect to the other one is contained in the coordinates transformation between the two reference frames. Furthermore, this transformation is invariant with respect to the global reference W (i.e. to the robot pose). Therefore, a possible choice for the invariants between two corners could be the parameters defining the transformation of above. However, the previous choice has the disadvantage of being asymmetric with respect to the transformation $corner1 \leftrightarrow corner2$. For the sake of simplicity in the implementation, we define the invariants between two corners in a way that they do not depend on the order of the corners. For this reason we introduce the three invariants (d, α_1, α_2) shown in fig. 3:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (16)$$

$$\alpha_1 = \theta_1 - \arctan \frac{y_1 - y_2}{x_1 - x_2} \quad (17)$$

$$\alpha_2 = \theta_2 - \arctan \frac{y_1 - y_2}{x_1 - x_2} \quad (18)$$

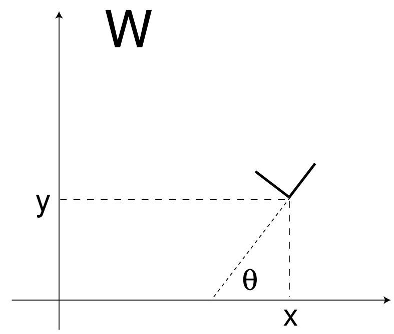

Fig. 2. The parameters defining the corner configuration in the reference W

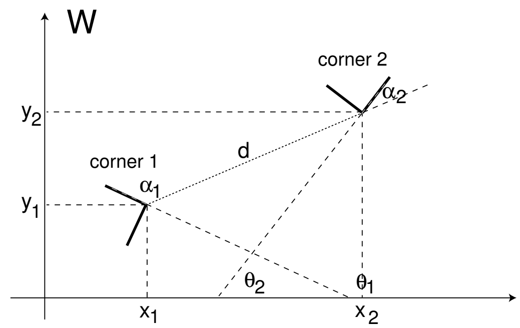

Fig. 3. The invariants between two corners

They are clearly independent of the robot configuration and symmetric with respect to the transformation $corner1 \leftrightarrow corner2$. Finally, they contain all the information concerning the configuration of one corner with respect to the other one (in other words, once the absolute configuration of one corner is known, it is possible to obtain the absolute configuration of the other one, through the previous invariants.) With this choice, the state I introduced in the previous section contains all the invariants among the corners observed at the same time. In particular, if at a given time the number of corners observed is m, the dimension of the vector I_{obs} in equation (1) is equal to $3 \times \frac{m(m-1)}{2}$. This does not mean that the dimension of the state I increases as N^2, where N is the number of the corners in the environment. Indeed, if we assume that the number m is bounded, the dimension of I increases linearly with N. Moreover, the structure of the equations in the previous section is such that the computational requirement has a complexity $O(N)$. Indeed, the covariance matrix P is block diagonal since the invariants between corners a and b are completely uncorrelated with the invariants between the corners c and d when $a \neq c, d$ and $b \neq c, d$. The equations of the previous section are used to estimate the invariants and the covariance matrix. Clearly, the elements in the state I are not independent. This means that there are some constraints on them, containing very useful information to improve the convergence of the map. A possible solution to take into account these constraints consists in considering only the independent elements in

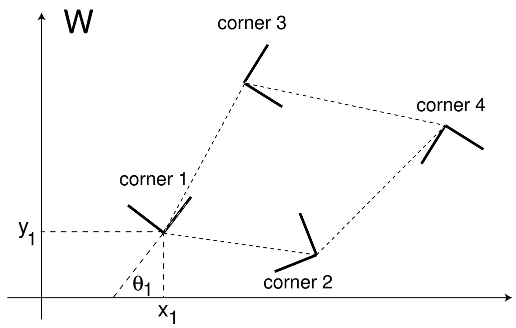

Fig. 4. To evaluate the absolute configuration of the corner 4 it is possible to follow the path $1 \rightarrow 2 \rightarrow 4$ or $1 \rightarrow 3 \rightarrow 4$

I (which corresponds exactly to applying the Projection filter suggested by Newman [13]). However, concerning all the constraints among the invariants related to a group of corners observed at least once simultaneously, they are automatically satisfied, due to the structure of the covariance matrix of the observation and the structure of the equations in the previous section [12]. Therefore, the only constraints to be imposed, are the ones involving the invariants related to a group of landmarks never observed at the same time (typically this happens when closing a loop). For these invariants a Projection filter can be applied and the covariance matrix concerning all these invariants after applying the Projection filter will have all the cross-correlations among them different from zero [13].

Once the relative map is estimated, it is possible to reconstruct the absolute map. Indeed, the absolute configuration of a given corner can be easily estimated by knowing the absolute configuration of another one and the invariants between these two corners. Since I contains invariants which are not independent, the absolute configuration of a given corner can be determined following different paths. In fig. 4, the 4^{th} corner can be located by determining firstly the configuration of the 3^{d} corner or of the 2^{nd} one. However, if all the constraints are maintained, the result does not depend on the choice. The robot configuration at a given time step is estimated by using the last observation, which provides the position and the orientation of several corners in the local frame of the robot. In particular, it is sufficient that this observation contains the configuration of one corner.

Finally, we want to consider a further hypothesis consisting in assuming the existence of structures made of two or more corners. Once this hypothesis is made two questions arise:

- how to detect a structure;
- how to use the information coming from the structure constraint in the estimation process, in order to improve the map convergence.

Clearly, to answer the first question it is necessary to better define the structure itself. In our experimental implementation we just introduce a check on the corners to verify their alignment. In the fig. 5, corner 1 and corner 2 belong to the same structure. On the other hand, corner 3

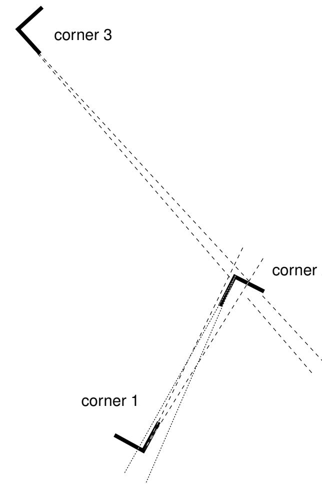

Fig. 5. Structures are found by using aperture angles around the arms of each corner. Corners belonging to the same structure must lie within their opponents aperture angle.

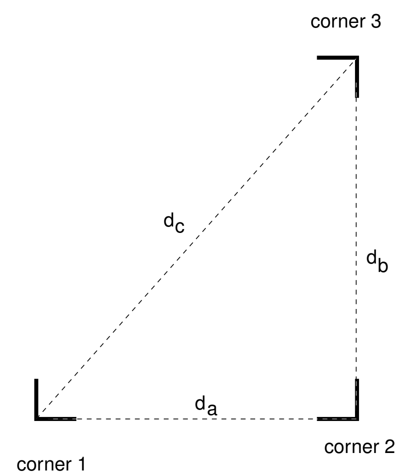

Fig. 6. Once a structure as the one here shown is detected, the number of independent invariants among the three corners reduces from 6 to 2

does not belong to the same structure since the condition to be within the aperture angle is not verified simultaneously by both corner 3 and 2.

It is important to be very conservative in deciding if some corners form one structure, since a false structure detection could degrade irreparably the quality of the map. For this reason a candidature approach is strongly recommended (i.e. it is required that the same alignment is observed several times before introducing a new structure).

Concerning the estimation process after detecting a given structure, we remark that the constraint to be imposed depends on the structure. In general, the structure constraint will remove several elements from the state I, namely the invariants that depend on the other ones through the constraint. In fig. 6, the number of independent invariants will reduce to two after imposing the constraint. Indeed, all the relative angles are determined and also the three distances satisfy the condition $d_a^2 + d_b^2 = d_c^2$

Hence, once a structure has been introduced, the number

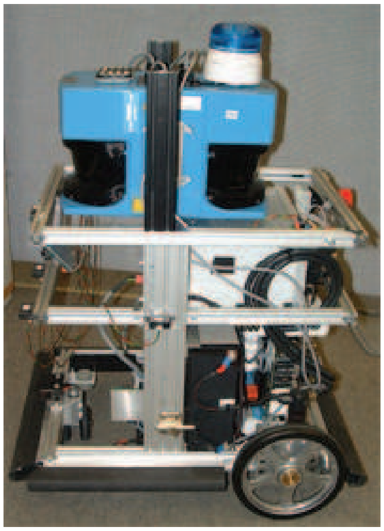

Fig. 7. The robot BIBA equipped with the laser range finders

of invariants will decrease. Moreover, some independent invariants are intrinsic to the structure, i.e. they characterize only the structure itself but do not contain any information concerning the configuration of the structure with respect to the other corners and/or structures in the environment (e.g. the two independent distances d_a and d_b in the fig. 6 are intrinsic to the structure). Therefore, to describe the structure configuration we select an anchor corner. When the structure is observed simultaneously with other corner and/or structures, the equations in section II are used to update the invariants among the anchor corner and these other corners and/or structures. This update can be carried out even if the chosen anchor corner is not directly observed but only other corners belonging to the same structure are observed.

In conclusion, the introduction of a structure will improve the convergence, since, after that, the same observations will be used to estimate a smaller number of quantities and some of the invariants will be not affected by any error. The price to pay, is the risk to detect a false structure which could cause a divergence.

IV. Results and Discussion

For the experiments, BIBA (see Fig. 7), a fully autonomous mobile robot, has been used. The robot is equipped with wheel encoders and two 180^o laser range finders. It is connected via radio ethernet only for data visualization via web and data logging for statistical purposes.

The experiment was performed in the hallway of our building department, consisting of several walls, cupboards and pillars. In the figures 8-11, the lines represent only the walls but not the cupboards and pillars. This is the reason because many times the corners are not located on the lines. The robot moved along a closed trajectory, whose length was about $50m$. We applied three different methods to solve the SLAM problem using exactly the same data (both encoder and laser). The data association problem was

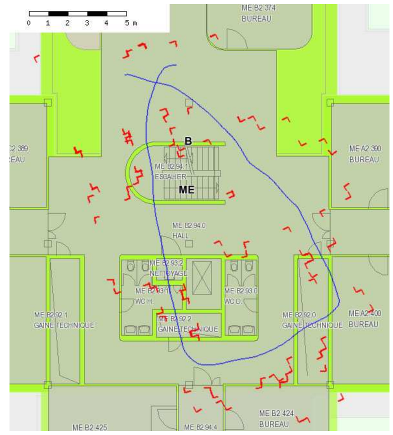

Fig. 8. SLAM with odometry

accomplished in all the three cases by using the nearest neighbor filter (NNF). Furthermore, a candidature system was employed to evaluate and admit new corners.

Method one, referred as *SLAM with odometry*, only uses odometry to determine the robot configuration. At every iteration step, once the robot configuration has been estimated through odometry, the absolute configuration of each corner is estimated in the local map. When the same corner is observed more the one time, the mean value of its position and orientation is computed. The robot trajectory and the corners are displayed in fig. 8. The map obtained with this method is completely inaccurate.

Method two, referred as *SLAM with relative map and corners*, updates the relative map in the previous sections by estimating a state containing the invariants among the corners. In fig. 9 we show the results obtained by this method. The convergence is not excellent and can be improved because we used a very simple model to characterize the error of the invariants as obtained from a single observation (the matrix P_{obs}). In particular, this error does not contain all the correlations among the invariants which are not independent. In this way, we loose a great amount of information. However, we used this error model to simplify the problems in the implementation. We want to remark that considering the correlations does not increase the computational complexity, as explained in sect. III.

Method three, referred as *SLAM with relative map and structures*, extends method two using structures. Clearly, the structure hypothesis contains very useful information as discussed in the previous section. The results shown in fig. 10 and 11, confirm this.

V. Conclusions and Future Research

This paper presented an approach to solve the SLAM problem with the corner feature in the stochastic map framework based on the concept of the relative map. The idea consists in introducing a map state which only contains quantities invariant under translations and rotations and to carry out the estimation of this relative map in an optimal way (a Kalman filter was adopted). This is an optimal way in order to have a decoupling between the

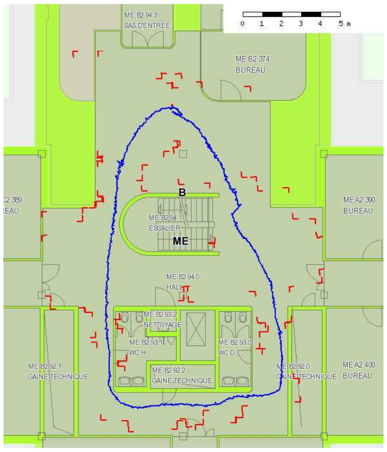

Fig. 9. SLAM with relative map and corners

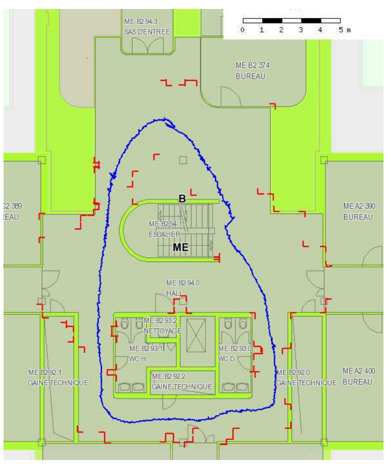

Fig. 10. SLAM with relative map and structures

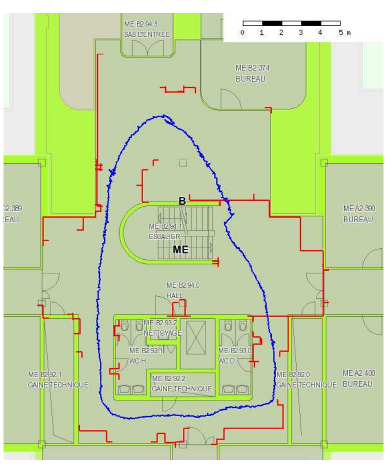

Fig. 11. SLAM with relative map and structures. With respect to the previous figure, we linked the corners belonging to the same structure through a virtual line

robot and the landmark estimation and therefore not to rely the landmark estimation on the unmodeled error sources in the robot motion. The relative state estimated through the Kalman filter contains the distances and the relative orientations among the corners observed at the same time. Therefore, this state is invariant of the robot configuration (translation and rotation). Finally, the use of structures to extend SLAM with corners was investigated.

We performed real experiments in an indoor environment validating the approach.

In a future implementation we want to introduce a more sophisticated model to characterize the error on the invariants, taking into account all the dependencies.

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