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Evaluating the Odometry Error of a Mobile Robot

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Abstract

This paper focuses on issues of odometry for the special case of synchronous drive wheeled mobile robots. In particular, the uncertainty in odometry is modeled by a four parameter statistical model already introduced in previous works together with a strategy to estimate the four parameters from actual data obtained from a given mobile robot. To obtain the data, the robot traverses a path. The error realized along that path is then used to estimate the model parameters. The accuracy on the parameter estimation through this strategy is here discussed by considering paths different for length and shape.

Key Words: Robot Navigation, Odometry, Localization

1 Introduction

Determining the odometry errors of a mobile robot is very important both in order to reduce them, and to know the accuracy of the state configuration estimated by using encoder data.

Odometry errors can be both systematic and non-systematic. In a series of papers Borenstein and collaborators [1, 2, 3, 4, 5, 6, 17] investigated on possible sources of both kind of errors. A review of all the types of these sources is given in [6]. In the work by Borenstein and Feng [5], a calibration technique called UMBmark test has been developed to calibrate for systematic errors of a two wheel robot. This method has been used by other authors [7]. Goel, Roumeliotis and Sukhatme [8] used another calibration procedure to compensate systematic errors. They referred to the differential drive mobile robot Pioneer AT. Finally, Roy and Thrun [15] suggested an algorithm that uses the robot's sensors to automatically calibrate the robot as it operates. In a series of papers Borenstein ([6] and reference therein) suggested also a method, called IPEC (Internal Position Error Correction), to improve the accuracy of the odometry data by reducing the effect of the non-systematic errors. Experimental results showed that the accuracy achieved with the IPEC method was one to two orders of magnitude better than that one of systems based on conventional dead-reckoning.

Many investigations have been carried out on the odometry error from a theoretical point of view. Wang [16] and Chong and Kleeman [7] analyzed the non-systematic errors and computed the odometry covariance matrix Q for special kind of the robot trajectory. Kelly [9] presented the general solution for linearized systematic error propagation for any trajectory and any error model. Martinelli [10], [11], [12] and [13] derived general formulas for the covariance matrix and also suggested a strategy to estimate the model parameters. This strategy is based on the evaluation of the mean values of some quantities (called *observables*) which depend on the model parameters and on the chosen robot motion.

In this paper we discuss the accuracy (Sect. 3, 4) on the parameter estimation reachable by evaluating the observables introduced in [11], [12] and [13] and computed for different robot trajectories. The odometry error model is the same as in [10] and it is here summarily discussed in Sect. 2. Finally, some conlcusions are given in Sect. 5

2 The odometry error model

We consider a mobile robot with a synchronous drive system, where each wheel is capable of being driven and steered (Fig. 1). Assuming a two-dimensional world, we can define the robot configuration with respect to a world-coordinate frame W by the vector $X = [x, y, \theta]^T$, containing its position and orientation. The robot configuration estimated by odometry measurements is different from the actual configuration X because of the odometry errors. In order to compute the global odometry error related to a given robot motion we approximated the trajectory with N small segments. We firstly model the elementary error related to a single segment. Then we compute (next sections) the cumulative error on the global path. Finally we take the limit value when $N \to \infty$.

We introduce the following assumptions about the actual motion:

- the robot moves straight along each given seg-

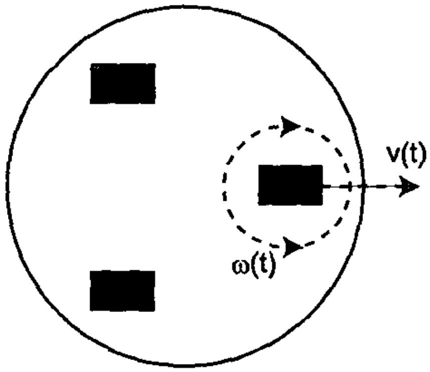


***Figure 1:** The wheels in a synchronous drive mobile robot with three wheels. All the wheels turn and drive in unison. $\omega(t)$ and $v(t)$ are respectively the rotational and translational velocity which are independent*

ment whose length, measured by the encoder sensor, is always $\overline{\delta\rho} = \frac{\rho}{N}$;

- the angle $\widehat{\delta\theta}_i$ between the actual orientations related to the $(i+1)^{th}$ and the i^{th} segment and the actual translation $\widehat{\delta\rho}_i$ covered during the same step are gaussian random variables;
- the random variable $\widehat{\delta\rho}_i$ is independent of the random variable $\widehat{\delta\theta}_i$. Moreover $\widehat{\delta\rho}_i$ is independent of $\widehat{\delta\rho}_j$ $(i \neq j)$ and $\widehat{\delta\theta}_i$ is independent of $\widehat{\delta\theta}_j$.

We therefore can write:

$$\widehat{\delta\rho}_i \sim N(\overline{\delta\rho}(1+E_\rho), \sigma^2_{\delta\rho}) \quad (1)$$

$$\widehat{\delta\theta}_i \sim N(\overline{\delta\theta_i} + E_\theta\overline{\delta\rho}, \sigma^2_{\delta\theta}) \quad (2)$$

where $\overline{\delta\theta_i}$ is the angle between the orientations related to the $(i+1)^{th}$ and the i^{th} segment measured by the encoder sensor, $E_\rho\overline{\delta\rho}$ and $E_\theta\overline{\delta\rho}$ represent the systematic components of the error and $\sigma^2_{\delta\theta}$ and $\sigma^2_{\delta\rho}$ are directly related to the rolling conditions and are assumed to increase linearly with the traveled distance, i.e.:

$$\sigma^2_{\delta\theta} = K_\theta\overline{\delta\rho} \quad (3)$$

and

$$\sigma^2_{\delta\rho} = K_\rho\overline{\delta\rho} \quad (4)$$

The odometry error model here proposed is based on 4 parameters. Two of them (E_θ, E_ρ) characterize the systematic components while the other two (K_θ, K_ρ) characterize the non-systematic components. Clearly, these parameters depend on the environment where the robot moves.

We want to remark that a statistical treatment of the non-systematic component assumes the environment homogeneous on large scale. Therefore, the expressions we are deriving in the next sections hold if the robot moves on regions larger than the scale beyond it the environment can be considered homogeneous.

The assumption that $\widehat{\delta\rho}_i$ is independent of $\widehat{\delta\theta}_i$ is clearly a simplified approximation, acceptable for a mobile robot with a synchronous drive. Let consider for example a mobile robot with a differential drive system (Fig. 2).

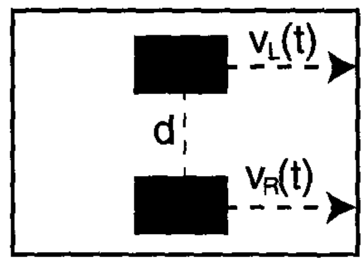


***Figure 2:** Mobile robot with a differential drive. $v_L(t)$ and $v_R(t)$ are respectively the left and right wheel velocity which are independent.*

A simple way to characterize the odometry error is obtained by modeling separately the error in the translation of each wheel [7]. The actual translation of the left/right wheel related to the i^{th} segment is assumed to be a gaussian random variable satisfying the following relation:

$$\widehat{\delta\rho}^i_{L/R} \sim N(\overline{\delta\rho}(1+E_{L/R}), K_{L/R}\overline{\delta\rho}) \quad (5)$$

The assumption that $\widehat{\delta\rho}^i_L$ is independent of $\widehat{\delta\rho}^i_R$ is similar to the assumption that $\widehat{\delta\rho}_i$ is independent of $\widehat{\delta\theta}_i$ in a synchronous drive system. However, in the case of differential drive, the robot translation $\widehat{\delta\rho}_i = \frac{\widehat{\delta\rho}^i_L + \widehat{\delta\rho}^i_R}{2}$ is not independent of the robot rotation $\widehat{\delta\theta}_i = \frac{\widehat{\delta\rho}^i_L - \widehat{\delta\rho}^i_R}{d}$.

3 The Accuracy on the Parameters Estimation

The strategy suggested in [11], [12] and [13] to evaluate the model parameters previously defined is based on the estimation of some quantities, called observables, whose mean values depend on the robot trajectory and on E_θ, E_ρ, K_θ and K_ρ. In order to make easier the experimental estimation of the observables, robot trajectories whose final configuration is very close to the initial one are chosen. The analysis carried out in [11], [12] and [13] showed that the best observables to evaluate the model parameters are Obs_θ, Obs_{θ^2}, $Obs_{\hat{y}}$ and Obs_{D^2}. They are defined as follows. Let consider a given robot motion and let suppose to repeat this motion n times. The robot motion is always the same in the world coordinate frame of the odometry system.

$$Obs_\theta = \frac{1}{n}\sum_{i=1}^{n}\Delta_i \qquad (6)$$

$$Obs_{\theta^2} = \frac{1}{n-1}\sum_{i=1}^{n}(\Delta_i - Obs_\theta)^2 \qquad (7)$$

$$Obs_{\hat{y}} = \frac{1}{n}\sum_{i=1}^{n}y_i \qquad (8)$$

$$Obs_{D^2} = \frac{1}{n}\sum_{i=1}^{n}D_i^2 \qquad (9)$$

where Δ_i is the angular difference between the initial and the final configuration, y_i is the position change along the $\hat{y}$-axis between the initial and the final configuration, D_i is the distance between the initial and the final position related to the i^{th} robot motion.

Because of the non-systematic errors, the observables are random variables whose statistics is completely defined by the hypothesis introduced in Section 2. In particular, on the basis of those hypothesis, it is possible to compute the mean value and the variance of each observable.

We want to discuss the accuracy on the parameter estimation reachable by adopting previous observables. We define the accuracy on the estimation of a given model parameter K as the relative error (in %) on its estimated value ($\frac{\Delta K}{K}100\%$). The error sources on the estimation of K are:

1. measurement errors on the difference in angle and distance between the initial and the final robot configuration;
2. resolution of the odometry system;
3. when the mean value of the adopted observable depends on parameters (previously estimated) different from K, the estimation error on these parameters propagate into the estimation error ΔK;
4. statistical variance of the observable.

When the length $\overline{\rho}$ of the chosen trajectory is large enough the effect of the first two error sources becomes negligible (the minimum $\overline{\rho}$ depends on the values of the non-systematic parameters and on the device adopted to measure the angle displacement and the distance between the initial and final configuration). Since now on we assume that this is the case. Let consider for a moment the case when also the 3^d error source disappears (i.e. the adopted observable only depends on one parameter). In this case the error on the model parameter K estimated by adopting the observable Obs_i is $\Delta K = \frac{\Delta < Obs_i >}{\left|\frac{\partial < Obs_i >}{\partial K}\right|} \simeq \frac{\sigma_{Obs_i}}{\left|\frac{\partial < Obs_i >}{\partial K}\right|}$. Therefore, the accuracy is:

$$acc\ (K, Obs_i) = \frac{1}{K}\frac{\sigma_{Obs_i}}{\left|\frac{\partial < Obs_i >}{\partial K}\right|}100\% \qquad (10)$$

To proceed we have to express the observable mean values and variances in terms of the model parameters and the robot trajectory.

Concerning Obs_θ and Obs_{θ^2} we have (see [14]):

$$< Obs_\theta >= E_\theta\overline{\rho}; \qquad \sigma_{Obs_\theta} = \sqrt{\frac{K_\theta\overline{\rho}}{n}} \qquad (11)$$

$$< Obs_{\theta^2} >= K_\theta\overline{\rho}; \qquad \sigma_{Obs_{\theta^2}} = K_\theta\overline{\rho}\sqrt{\frac{2}{n-1}} \qquad (12)$$

Since the mean values $< Obs_\theta >$ and $< Obs_{\theta^2} >$ only depend respectively on the model parameters E_θ and K_θ, for the estimation of these parameters through these observables, the accuracy definition given in (10) is correct. We obtain:

$$acc\ (E_\theta, Obs_\theta) = \frac{1}{E_\theta}\sqrt{\frac{K_\theta}{L_{Tot}}}100\% \qquad (13)$$

where $L_{Tot} = n \times \overline{\rho}$ is the total distance traveled by the robot. From equation (12) we obtain:

$$acc\ (K_\theta, Obs_{\theta^2}) = \sqrt{\frac{2}{n-1}}100\% \qquad (14)$$

Concerning $Obs_{\hat{y}}$ the computation of its mean value and variance can be found in [13] and [11] obtaining:

$$< Obs_{\hat{y}} >= (1 + E_\rho)\int_0^{\overline{\rho}} sin(\widetilde{\theta}(s))\ e^{-\frac{K_\theta s}{2}}ds \qquad (15)$$

where $\widetilde{\theta}(s) = \theta(s) + E_\theta s$ and $\theta(s)$ is the robot orientation as measured by the encoder sensor as a function of the curve length s always measured by encoder.

In a similar way the mean value and the variance of the observable Obs_{D^2} can be computed. We obtain for the mean value:

$$< Obs_{D^2} >= K_\rho \overline{\rho} + 2(1+E_\rho)^2 \times$$
$$\times \int_0^{\overline{\rho}} ds \int_0^{\overline{\rho}-s} ds' \left\{ e^{-\frac{K_\theta s'}{2}} cos[\widetilde{\theta}(s+s') - \widetilde{\theta}(s)] \right\} \equiv$$
$$\equiv K_\rho \overline{\rho} + 2(1+E_\rho)^2 F\left[\widetilde{\theta}\right] \tag{16}$$

where $F\left[\widetilde{\theta}\right]$ is just defined from the previous equation and it is a functional dependent on the robot trajectory and on the parameters E_θ and K_θ.

We obtain from equation (15)

$$(1+E_\rho) = \frac{< Obs_{\hat{y}} >}{\int_0^{\overline{\rho}} sin(\widetilde{\theta}(s))\, e^{-\frac{K_\theta s}{2}} ds} \tag{17}$$

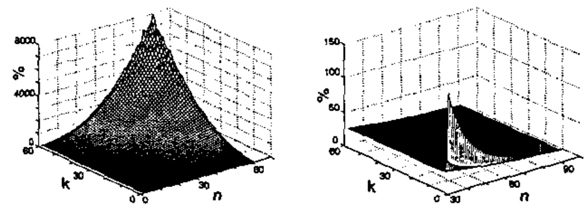
Therefore, the relative error on E_ρ is the sum of the two terms $\left|\frac{\Delta < Obs_{\hat{y}} >}{< Obs_{\hat{y}} >}\right|$ and $\left|\frac{\Delta \int_0^{\overline{\rho}} sin(\widetilde{\theta}(s))\, e^{-\frac{K_\theta s}{2}} ds}{\int_0^{\overline{\rho}} sin(\widetilde{\theta}(s))\, e^{-\frac{K_\theta s}{2}} ds}\right|$. The former corresponds to the accuracy defined in (10), the latter depends on the accuracy on E_θ and K_θ.

The only observable whose mean value depends on K_ρ is Obs_{D^2} and therefore is used for its estimation. The mean value in (16) is the sum of two terms. Therefore, the absolute error ΔK_ρ is the sum of the error on the observable evaluation ($\Delta < Obs_{D^2} > \simeq \sigma_{Obs_{D^2}}$) and the error on the quantity $2(1+E_\rho)^2 F\left[\widetilde{\theta}\right]$ due to the uncertainty on the parameters E_θ, K_θ and E_ρ. When $2(1+E_\rho)^2 F\left[\widetilde{\theta}\right] >> K_\rho \overline{\rho}$ the relative error on K_ρ becomes very large. In order to achieve high accuracy on K_ρ estimation a trajectory satisfying the relation $2(1+E_\rho)^2 F\left[\widetilde{\theta}\right] < K_\rho \overline{\rho}$ must be chosen. In particular, when $F\left[\widetilde{\theta}\right]$ is negligible the accuracy on K_ρ is given by (10) (in other words we can say that in this case $< Obs_{D^2} >$ only depends on the model parameter K_ρ).

4 Results

In this section we consider two trajectories (straight and circular) and we explicitly compute the accuracy on the model parameters through the previous observables with the two trajectories. Finally, concerning the observable Obs_{D^2}, we analytically investigate in order to find the conditions under which the requirement $2(1+E_\rho)^2 F\left[\widetilde{\theta}\right] < K_\rho \overline{\rho}$ is verified.

Clearly, the same analysis could be done for other robot trajectories by explicitly computing the integrals in Sect. 3. However, in the case of circular and straight trajectories here considered ($\theta(s)$ linear in s), the computation can be carried out analytically.



(a) (b)

Figure 3: The accuracy (in %) vs k and n on the parameter E_ρ (a) and on the parameter K_ρ (b) estimated by the observable Obs_{D^2} with the straight trajectory through equation (10). The total traveled distance is $L_{Tot} = n \times 2\ kl = 100\ m$.

4.1 Straight Trajectory

In order to estimate the model parameters in [11], [12] and [13] the following simple robot trajectory was considered. The robot moves straight forth and back k times in order to cover the distance $\overline{\rho} = 2kl$ in the odometry reference frame (l is the length of each segment). Observe that the mean values $< Obs_\theta >$ and $< Obs_\theta^2 >$ only depend on the length $\overline{\rho}$ of the robot motion while $< Obs_{\hat{y}} >$ and $< Obs_{D^2} >$ depend also on the shape. In particular for the previous trajectory we obtain (see [11] and [13])

$$< Obs_{\hat{y}} >= -(1+E_\rho) l\ Im\left\{f(z)\right\} \tag{18}$$

where $f(z) = \frac{(1-2e^{-z}+e^{-2z})(e^{-2zk}-1)}{z(e^{-2z}-1)}$ and z is a complex quantity completely defined by the rotational error model parameters:

$$z = \frac{K_\theta}{2} l + iE_\theta l \tag{19}$$

From previous equations we obtain:

$$\frac{\Delta E_\rho}{1+E_\rho} = \left|\frac{\sigma_{Obs_{\hat{y}}}}{< Obs_{\hat{y}} >}\right| + \left|\frac{\Delta Im\left\{f(z)\right\}}{Im\left\{f(z)\right\}}\right| \tag{20}$$

where we neglect $\frac{\Delta l}{l}$ since we assumed that the 2^d error source previously remarked is negligible. When z is small in absolute value we can expand the function $f(z)$ obtaining $f(z) \simeq kz$. In this case from equation (20) we have $acc\ (E_\rho, Obs_{\hat{y}}) \simeq \left(\left|\frac{\sigma_{Obs_{\hat{y}}}}{< Obs_{\hat{y}} >}\right| + \frac{\Delta E_\theta}{E_\theta}\right) \times$ 100%. (i.e., with respect to the accuracy given in equation (10) appears the effect of the uncertainty on the parameter E_θ).

In a similar way it is possible to compute the mean value $< Obs_{D^2} >$ for the considered robot trajectory, obtaining:

$$< Obs_{D^2} >= 2K_\rho kl + 2(1+E_\rho)^2 l^2 Re\{F(z)\} \quad (21)$$

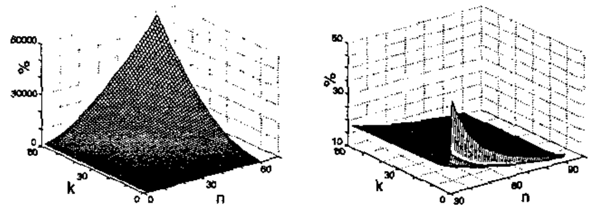
where $F(z) = \frac{(e^{-z}-1)^2(1-e^{-2kz})+4k(e^{-2z}-1)}{z^2(1+e^{-z})^2} + \frac{2k}{z}$.
In figures 3a and 3b we plot the accuracy (in %) respectively on E_ρ and on K_ρ estimated by the observable Obs_{D^2} vs k and n for the fixed $L_{Tot} = 100m$ and for the following values of the model parameters: $E_\theta = -0.2\frac{deg}{m}$, $K_\theta = 0.04\frac{deg^2}{m}$, $(1+E_\rho) = 0.98$ and $K_\rho = 4\ 10^{-6}m$ (these are about the values estimated with the mobile robot Nomad150 in an indoor environment ([11], [13])). The accuracy is computed without considering the error on the other model parameters (i.e. is computed directly from equation (10)). Observe that by changing the values of the model parameters the qualitative behaviour does not change. Concerning E_ρ the accuracy becomes very rough by increasing both k and n (i.e. for a fixed $L_{Tot} = 2knl$ by decreasing l). Regarding K_ρ we have the opposite behaviour. This opposite behaviour is crucial because it means that, when l is small, the error on the parameter E_ρ (previously estimated for example with $Obs_{\hat{y}}$ or with Obs_{D^2} with large l) does not influence the accuracy on the estimation of the parameter K_ρ. This behaviour can be explained by expanding the function $F(z)$ (indeed when l is small also z is small). We obtain $F(z) \simeq \frac{2}{3}kz - \frac{1}{2}k^2z^2$. Therefore,

$$< Obs_{D^2} > \simeq 2K_\rho kl + 2(1+E_\rho)^2 l^2 \left[\frac{K_\theta}{3}kl + \frac{E_\theta^2}{2}(kl)^2\right] \quad (22)$$

The requirement $2(1+E_\rho)^2 F\left[\tilde{\theta}\right] < K_\rho\overline{\rho}$ is then verified by considering the limit $l \to 0$, $k \to \infty$ for a fixed value of $2kl = \overline{\rho}$. In this way $< Obs_{D^2} >$ only depends on K_ρ. However, the value of $\overline{\rho} = 2kl$ must be large enough so that the quantity $K_\rho\overline{\rho}$ can be appreciated by the device adopted to measure the angle displacement and the distance between the initial and final configuration (in other words the first two error sources previously remarked can be neglected, as assumed).

4.2 Circular Trajectory

This trajectory has the advantage to be smooth without abrupt change in direction. The motion will consist of k revolution along a circumference of radius R. Actually, because of the systematic rotational error, the effective radius of the circumference will be $R_{eff} = \frac{R}{RE_\theta+1}$.



(a) (b)

Figure 4: *The accuracy (in %) vs k and n on the parameter E_ρ (a) and on the parameter K_ρ (b) estimated by the observable Obs_{D^2} with the circular trajectory through equation (10). The total traveled distance is $L_{Tot} = n \times 2\ kl = 100\ m$.*

We introduce a new complex quantity in order to investigate the observable statistics:

$$z_R = \frac{K_\theta\overline{\rho}}{2} + i\left(E_\theta + \frac{1}{R}\right)\overline{\rho} \quad (23)$$

Following computation similar to the straight trajectory we obtain:

$$< Obs_{\hat{y}} >= -(1+E_\rho)\overline{\rho}\ Im\{f_R(z_R)\} \quad (24)$$

$$< Obs_{D^2} >= K_\rho\overline{\rho} + 2(1+E_\rho)^2\overline{\rho}^2 Re\{F_R(z_R)\} \quad (25)$$

where $f_R(z_R) = \frac{1-e^{-z_R}}{z_R}$ and $F_R(z_R) = \frac{z_R - 1 + e^{-z_R}}{z_R^2}$
In figures 4a and 4b we plot the accuracy (in %) respectively on E_ρ and on K_ρ estimated by the observable Obs_{D^2} vs k and n for the fixed $L_{Tot} = 100m$ and for the values of the model parameters considered in the straight trajectory. The results are very similar to those obtained with the straight motion. Also in this case the opposite behaviour of the accuracy on the estimation of E_ρ and K_ρ can be explained by expanding the function $F_R(z_R)$. We obtain:

$$< Obs_{D^2} > \simeq K_\rho\overline{\rho} + 2(1+E_\rho)^2 K_\theta\overline{\rho}R^2 \quad (26)$$

Therefore, in this case the requirement $2(1+E_\rho)^2 F\left[\tilde{\theta}\right] < K_\rho\overline{\rho}$ is verified by considering the limit $R \to 0$, $k \to \infty$ for a fixed value of $2\pi kR_{eff} = \overline{\rho}$. As for the straight trajectory, in this limit $< Obs_{D^2} >$ only depends on K_ρ.

5 Conclusions and Future Research

In this paper the uncertainty in odometry of a mobile robot was modeled by a four parameter statistical model. The accuracy on the parameter estimation obtained by evaluating some measurable quantities (the observables, whose mean value depends on the model parameters and on the robot trajectory) was deeply analyzed. In particular two robot trajectories (straight and circular) were considered.

We are investigating in order to check the validity of the proposed model. The assumption that $\widehat{\delta\rho}_i$ is independent of $\widehat{\delta\theta}_i$ is clearly a simplified approximation. A disturbance on the robot trajectory can generate both a distance error and a dependent angle error. Moreover, more sophisticated model should also take into account the error dependence on the robot velocity and acceleration.

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