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A Possible Strategy to Evaluate the Odometry Error of a Mobile Robot

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Abstract

The odometry error of a mobile robot contains both systematic and non-systematic components. The first ones are independent of the environment while the second ones depend on the interaction of the robot with the environment where the robot moves.

This paper presents an error modeling of an odometry system for a synchronous drive system and a possible strategy in order to evaluate this error. The odometry error is modeled by introducing four parameters characterizing its systematic and non-systematic components (translational and rotational).

We introduce five experimentally measurable quantities for a given robot motion, which we call *observables*. On the basis of our odometry error model we analytically compute the average values of the observables, which depend on the previous four parameters and on the considered robot motion. We suggest a possible strategy in order to simultaneously estimate the four parameters by estimating the observables. As we will show, our strategy only requires to measure the change in the orientation and in the position between the initial and the final configuration of the robot related to suitable robot motions. In other words it is unnecessary to know the actual path followed by the robot.

The strategy has been applied to the platform Nomad150.

Key Words: Robot Navigation, Odometry, Localization

1 Introduction

Determining the odometry errors of a mobile robot is very important both in order to reduce them, and to know the accuracy of the state configuration estimated by using encoder data. Odometry, in fact, is the most widely used navigation method for mobile robot positioning. Nevertheless, this method is inaccurate since the localization error grows with the distance traveled by the robot. However, the encoder data are extensively used in the localization process by fusing these data with data coming from another (or several) sensor. Clearly, any fusion architecture needs to know the accuracy of the estimation of each sensor in order to weigh all the data in a proper manner. A widely used fusion algorithm is the Extended Kalman Filter (a good overview on sensor fusion algorithms can be found in [10]). In particular, when the fusion regards the encoder data, the accuracy is completely described by the odometry error covariance matrix, Q.

Odometry errors can be both systematic and non-systematic. While systematic errors depend only on the mobile robot independently of the environment where the robot moves, the non-systematic errors depend dramatically on the environment.

In a series of papers Borenstein and collaborators [1, 2, 3, 4, 5, 6, 17] investigated on possible sources of both kind of errors. A review of all the types of these sources is given in [6]. In the work by Borenstein and Feng [5], a calibration technique called UMBmark test has been developed to calibrate for systematic errors of a two wheel robot. This method has been used by other authors [8].

Other strategies to compensate systematic errors are suggested by Goel, Roumeliotis and Sukhatme [9] and Roy and Thrun [15]. In [15] the suggested algorithm uses the robot's sensors to automatically calibrate the robot as it operates.

In a series of papers Borenstein ([6] and reference therein) suggested also a method to improve the accuracy of the odometry data by reducing the effect of the non-systematic errors. With this method, called IPEC (Internal Position Error Correction), it was possible to detect and correct odometry errors without inertial or external-reference sensors. In particular, he implemented the IPEC method on the special designed mobile robot platforms MDOF (Multi Degree of Freedom) [2] and Omnimate [6].

Wang [16] analyzed the non-systematic errors from a theoretical point of view and computed the odometry covariance matrix Q. He referred to a differential drive mobile robot. In order to evaluate this matrix he divided the entire path in N small elementary paths. To compute the covariance matrix he had to make some assumptions about the type of the elementary path. In particular he assumed a circular path. Moreover, since the updated robot position depended non-linearly on the changes in the translation and orientation (measured by the encoders), he had to introduce another approximation. He called the non-linear term appearing in the updated position as the adjustment factor. He analyzed three different cases depending on the considered approximation for this factor. In particular he considered a Taylor approximation of the adjustment factor truncated at the zero and first orders. Finally, as third case, he considered this factor as a constant in the calculation of the covariance matrix.

The same approximation were made by Chenavier

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and Crowley [7] and by Feng and Milios [11]. They always considered a particular path and they used a Taylor approximation to compute the covariance matrix.

Chong and Kleeman [8] divided the entire path in N small segments. They found for the first time a closed form solution for the covariance matrix Q as N approaches infinity. In this way they did not require to do the Taylor approximation. However, with their method, they were able to compute this matrix only for special cases. Their expressions were applicable to circular arc motions with constant radius of curvature, included the limit cases of an infinity-radius (straight motion) and zero-radius (rotation about the center of wheel axis).

Martinelli [12] derived general formulas for the covariance matrix. In these formulas, applicable to any path, the trajectory of the robot motion appeared as a function of the curve length. In these formulas there were four parameters which depended on the robot and on the environment where the robot moved. Two parameters characterized the two non-systematic components (translational and rotational). The other two parameters characterized the translational and rotational systematic components.

In this paper we suggest a possible strategy to evaluate the same four parameters introduced in [12]. The odometry error model is the same as in [12] and it is discussed here in Sect. 2.

In Sect. 3 we introduce five experimentally measurable quantities for a given robot motion, which we call the *observables*. On the basis of our odometry error model we analytically compute the average values of the observables, which depend on the previous four parameters and on the considered robot motion. In Sect. 4 we explicitly compute the observables for a simple robot motion. Evaluating the observables for this robot motion is a possible strategy in order to simultaneously estimate both systematic and non-systematic parameters. This strategy only requires to measure the change in the orientation and in the position between the initial and the final configuration of the robot related to the considered robot motion. In other words it does not require to know the actual path followed by the robot.

The proposed strategy is illustrated in section 5 by the experimental results regarding the mobile robot Nomad150 in an indoor environment.

2 The Odometry Error Model

We consider a mobile robot with a synchronous drive system. Assuming a two-dimensional world, we can define the robot configuration with respect to a world-coordinate frame W by the vector $X = [x, y, \theta]^T$, containing its position and orientation. The robot configuration estimated by odometry measurements is different from the actual configuration X because of the odometry errors.

In order to compute the global odometry error related to a given robot motion we divided the trajectory in N small segments (see figure 1). We firstly model the elementary error related to a single segment. Then we compute (next sections) the cumulative error on the global path. Finally we take the limit value when $N \to \infty$.

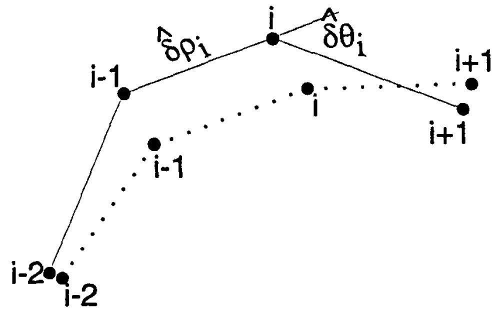

Figure 1: The robot motion is divided in N segments. The robot is assumed to move straight in each given segment. The solid line represents the real motion followed by the robot when $N \to \infty$, while the dotted line is the motion trajectory measured by the encoder

We introduce the following assumptions about the actual motion:

- the robot moves straight along each given segment whose length, measured by the encoder sensor, is always $\overline{\delta\rho} = \frac{\rho}{N}$;
- the angle $\widehat{\delta\theta}_i$ between the orientations related to the $(i+1)^{th}$ and the i^{th} segment and the translation $\widehat{\delta\rho}_i$ covered during the same step are gaussian random variables;
- the random variable $\widehat{\delta\rho}_i$ is independent of the random variable $\widehat{\delta\theta}_i$. Moreover $\widehat{\delta\rho}_i$ is independent of $\widehat{\delta\rho}_j$ $(i \neq j)$ and $\widehat{\delta\theta}_i$ is independent of $\widehat{\delta\theta}_j$.

Now we want to model the elementary error related to a single segment. A complete model should take into account the dependence of the previous random variables $\widehat{\delta\rho}_i$ and $\widehat{\delta\theta}_i$ on the length $\overline{\delta\rho}$, on the change $\overline{\delta\theta_i}$ between the robot orientation in the i^{th} and $(i+1)^{th}$ segment (measured by the encoder sensor), on the translational and rotational velocity and acceleration. We only consider here the dependence on $\overline{\delta\rho}$. We want to remark that this hypothesis is present in the models so far proposed although they mostly refer to mobile robots with a differential drive system. In this case the error of each wheel for a single segment is assumed to depend on the distance traveled by the wheel itself. In particular the error on a given wheel is assumed to depend on the distance traveled by that wheel and independent of the distance traveled by the other wheel. This hypothesis corresponds to assume the independence of $\overline{\delta\theta_i}$ in a synchronous drive system. Finally, none of these models takes into account the dependence of the elementary error on the velocity and acceleration.

Because of both systematic and non-systematic errors the encoder measurements $\overline{\delta\rho}$ and $\overline{\delta\theta}$ differ from the real values

$$\widehat{\delta\rho} = \overline{\delta\rho} + E_T\overline{\delta\rho} + \delta\rho \qquad (1)$$

$$\widehat{\delta\theta} = \overline{\delta\theta} + E_R\overline{\delta\rho} + \delta\theta \qquad (2)$$

where $E_T\overline{\delta\rho}$ and $E_R\overline{\delta\rho}$ represent the systematic components and $\delta\rho$ and $\delta\theta$ the non-systematic components.

Let consider the i^{th} segment. From the previous assumptions we can write:

$$\widehat{\delta\rho}_i \sim N(\overline{\delta\rho}(1+E_T), \sigma^2_{\delta\rho}) \qquad (3)$$

$$\widehat{\delta\theta}_i \sim N(\overline{\delta\theta_i} + E_R\overline{\delta\rho}, \sigma^2_{\delta\theta}) \qquad (4)$$

where $\overline{\delta\theta_i}$ is the angle between the orientations related to the $(i+1)^{th}$ and the i^{th} segment measured by the encoder sensor and $\sigma^2_{\delta\theta}$ and $\sigma^2_{\delta\rho}$ are directly related to the rolling conditions.

The actual orientation after the i^{th} step is:

$$\widehat{\theta}_i = \theta_0 + \overline{\Delta\theta_i} + iE_R\,\overline{\delta\rho} + \sum_{j=1}^{i}\delta\theta_j \qquad (5)$$

where θ_0 is the initial orientation of the robot and $\overline{\Delta\theta_i}$ is the global change in orientation measured by the encoder sensor.

By defining

$$\widetilde{\theta}_i = \theta_0 + \overline{\Delta\theta_i} + iE_R\,\overline{\delta\rho} \qquad (6)$$

and

$$\Delta\theta_i = \sum_{j=1}^{i}\delta\theta_j \qquad (7)$$

we obtain

$$\widehat{\theta}_i = \widetilde{\theta}_i + \Delta\theta_i \qquad (8)$$

where $\Delta\theta_i$ is still a random variable satisfying the following relation

$$\Delta\theta_i \sim N(0, i\sigma^2_{\delta\theta}) \qquad (9)$$

So far we have introduced the parameters E_R and E_T characterizing the systematic components of the odometry errors. We introduce now two new parameters (K_θ and K_ρ) in order to characterize the non-systematic components, namely the two variances $\sigma^2_{\delta\theta}$ and $\sigma^2_{\delta\rho}$.

From the definition of $\overline{\delta\rho} = \frac{\overline{\rho}}{N}$ we can write:

$$\sigma^2_\theta = N\sigma^2_{\delta\theta} = \overline{\rho}\frac{\sigma^2_{\delta\theta}}{\overline{\delta\rho}} \qquad (10)$$

namely there is a linear dependence of the variance σ^2_θ on the distance $\overline{\rho}$ measured by the encoder sensor. We therefore introduce the parameter K_θ, whose estimation strategy is discussed in the section 4, defined by the following relation

$$K_\theta = \lim_{N\to\infty}\frac{\sigma^2_{\delta\theta}}{\overline{\delta\rho}} \qquad (11)$$

We therefore have

$$\sigma^2_\theta = K_\theta\overline{\rho} \qquad (12)$$

In the same way we compute the variance σ^2_ρ obtaining:

$$\sigma^2_\rho = K_\rho\overline{\rho} \qquad (13)$$

where

$$K_\rho = \lim_{N\to\infty}\frac{\sigma^2_{\delta\rho}}{\overline{\delta\rho}} \qquad (14)$$

The odometry error model here proposed is based on 4 parameters. Two of them (E_R, E_T) characterize the systematic components while the other two (K_θ, $K\rho$) characterize the non-systematic components.

3 The Observables

In this section we introduce the observables which are measurable quantities related to a given robot motion. These observables depend on the path followed by the robot and on the four error parameters (E_R, E_T, K_θ and K_ρ). As we will show in the next section, in order to easier experimentally estimate the observables, we consider robot motions whose initial configuration coincides with the final configuration in the world coordinate frame of the odometry system.

Let consider a given robot motion and let suppose to repeat this motions n times. The robot motion is always the same in the world coordinate frame of the odometry system. The observables are:

$$\wp_1 = \frac{1}{n}\sum_{i=1}^{n}\widehat{\Delta}_i \qquad (15)$$

$$\wp_2 = \frac{1}{n-1}\sum_{i=1}^{n}(\widehat{\Delta}_i - \wp_1)^2 \qquad (16)$$

$$\wp_3 = \frac{1}{n}\sum_{i=1}^{n}x_i \qquad (17)$$

$$\wp_4 = \frac{1}{n}\sum_{i=1}^{n}y_i \qquad (18)$$

$$\wp_5 = \frac{1}{n}\sum_{i=1}^{n}D_i^2 \qquad (19)$$

where $\widehat{\Delta}_i$ is the angular difference between the initial and the final configuration, x_i and y_i are the position change respectively along the x-axis and y-axis between the initial and the final configuration, D_i is the euclidean distance between the initial and the final position related to the i^{th} robot motion.

Because of the non-systematic errors, the observables are random variables whose statistics is completely defined by the hypothesis introduced in Section 2. In particular we have from equation (5)

$$\widehat{\Delta}_i \sim N(\overline{\Delta\theta} + E_R\overline{\rho}, K_\theta\overline{\rho}) \quad (20)$$

By defining $\Delta_i = \widehat{\Delta}_i - (\overline{\Delta\theta} + E_R\overline{\rho})$ we have

$$\Delta_i \sim N(0, K_\theta\overline{\rho}) \quad (21)$$

In the following we compute the average values of the observables. The computation is quite troublesome and we address the interested reader to [13] for details.

3.1 $\wp_1$

We obtain for the average value of $\wp_1$ (from equation (20))

$$< \wp_1 >= \overline{\Delta\theta} + E_R\overline{\rho} \quad (22)$$

3.2 $\wp_2$

From equation (15) and (16) we obtain

$$\wp_2 = \frac{1}{(n-1)n^2}\sum_{ijk}(\widehat{\Delta}_i - \widehat{\Delta}_j)(\widehat{\Delta}_i - \widehat{\Delta}_k) =$$

$$= \frac{1}{(n-1)n^2}\sum_{ijk}(\Delta_i - \Delta_j)(\Delta_i - \Delta_k) \quad (23)$$

Since $< \Delta_i\Delta_j >= 0$ $(i \neq j)$ we obtain

$$< \wp_2 >= \sigma_\theta^2 = K_\theta\overline{\rho} \quad (24)$$

3.3 $\wp_3$

The average value is

$$< \wp_3 >=< x > \quad (25)$$

i.e. the average position change along the x-axis between the initial and the final configuration of the robot. We suppose $x = 0$ at the initial configuration.

In order to compute $< x >$ we need to express the final position coordinate x in terms of $\widehat{\delta\rho}_i$ and $\widehat{\theta}_i$. We have:

$$x = \lim_{N\to\infty} x_N = \lim_{N\to\infty}\sum_{i=1}^{N}\widehat{\delta\rho}_i cos(\widehat{\theta}_i) \quad (26)$$

We firstly compute $< x_N >$ and then we take the limit value when $N \to \infty$. We have:

$$< x_N >= \int d\widehat{\delta\rho}_1 ... d\widehat{\delta\rho}_N d\widehat{\delta\theta}_1 ... d\widehat{\delta\theta}_N f_G(\widehat{\delta\rho}_1, \sigma_{\delta\rho}) ...$$

$$... f_G(\widehat{\delta\rho}_N, \sigma_{\delta\rho}) f_G(\widehat{\delta\theta}_1, \sigma_{\delta\theta}) ... f_G(\widehat{\delta\theta}_N, \sigma_{\delta\theta})\{$$

$$\sum_{i=1}^{N}\widehat{\delta\rho}_i cos(\widehat{\theta}_i)\} \quad (27)$$

where we denoted with $f_G(w, \sigma_w)$ the gaussian distribution function of the random variable w whose variance is σ_w.

By a direct calculation we obtain:

$$< x_N >= \overline{\delta\rho}(1 + E_T)\sum_{i=1}^{N} cos(\widetilde{\theta}_i)\, e^{-\frac{i\sigma^2_{\delta\theta}}{2}} \quad (28)$$

When $N \to \infty$ the sum in equation (28) becomes an integral. We obtain

$$< x >= \lim_{N\to\infty} < x_N >= (1+E_T)\int_0^{\overline{\rho}} cos(\widetilde{\theta}(s))\, e^{-\frac{K_\theta s}{2}} ds \quad (29)$$

where $\widetilde{\theta}(s)$ is the robot orientation as measured by encoder (included the systematic component) as a function of the curve length s always measured by encoder which does not include the systematic component.

3.4 $\wp_4$

Analogously to the previous case we obtain

$$< \wp_4 >=< y >= (1+E_T)\int_0^{\overline{\rho}} sin(\widetilde{\theta}(s))\, e^{-\frac{K_\theta s}{2}} ds \quad (30)$$

3.5 $\wp_5$

We have

$$< \wp_5 >= \frac{1}{n}\sum_{i=1}^{n} < D_i^2 >=$$

$$= \frac{1}{n}\sum_{i=1}^{n} < x_i^2 + y_i^2 >=< x^2 > + < y^2 > \quad (31)$$

The computation of the second order product averages $< x^2 >$ and $< y^2 >$ is a little bit more troublesome than the computation of $< x >$ and $< y >$. We do not give here their expression (given in [12]). For details about their computation we address the reader to [13]. Their sum simplifies giving:

$$< \wp_5 >= K_\rho\overline{\rho} + 2(1 + E_T)^2 \times$$

$$\times \int_0^{\overline{\rho}} ds \int_0^{\overline{\rho}-s} ds' \left\{ e^{-\frac{K_\theta s'}{2}} cos[\widetilde{\theta}(s + s') - \widetilde{\theta}(s)] \right\} \quad (32)$$

Clearly $< \wp_5 >$ does not depend on the initial orientation. Moreover this observable is very important since it is the only one whose average value depends on the parameter K_ρ.

In the next section we suggest the strategy in order to estimate the error parameters (E_R, E_T, K_θ and K_ρ) through experimentation.

4 The Strategy to Estimate the Errors Parameters

Let consider the following simple robot motion. The robot moves straight forth and back k times in order to cover a fixed distance $\overline{\rho} = 2kl$ (measured by encoder). We obtain

$$< \wp_1 >= 2E_R kl \tag{33}$$

$$< \wp_2 >= 2K_\theta kl \tag{34}$$

To compute the other observables for this robot motion is very useful to introduce the following complex quantity:

$$z = \frac{K_\theta l}{2} + iE_R l \tag{35}$$

This complex quantity characterizes the rotational components of the odometry error. In particular its real part contains the non-systematic component while the imaginary part the systematic one.

By a direct calculation we obtain from equations (29) and (30):

$$< x > -i < y >= (1 + E_T) l f(z) \tag{36}$$

where

$$f(z) = \frac{(1 - 2e^{-z} + e^{-2z})(e^{-2kz} - 1)}{z(e^{-2z} - 1)} \tag{37}$$

From equation (36) we see that the real part of $(1 + E_T)lf(z)$ gives the average value of the position change along the $x-$axis and the opposite of the imaginary part the average value of the position change along the $y-$axis. Therefore we have:

$$< \wp_3 >=< x >= (1 + E_T) l Re\{f(z)\} \tag{38}$$

and

$$< \wp_4 >=< y >= -(1 + E_T) l Im\{f(z)\} \tag{39}$$

The computation of $< \wp_5 >$ is a little bit more troublesome. We obtain

$$< \wp_5 >= 2K_\rho kl + 2(1 + E_T)^2 l^2 \times$$

$$\times Re\left\{\frac{(e^{-z} - 1)^2(1 - e^{2kz}) + 4k(e^{-2z} - 1)}{z^2(1 + e^{-z})^2} + 2\frac{k}{z}\right\} \tag{40}$$

Our strategy consists of the estimation of the average values of the observables for the considered robot motion. In order to obtain the error on the estimation of these average values we need to compute, on the basis of the odometry model introduced in section 2, the variances of the observables. The computation of them is troublesome and we do not give it here. However, the explicit computation can be found in [13], where the accuracy on the parameter estimation through the proposed strategy is theoretically discussed. Clearly, the error on the parameters estimation depends on the accuracy on the estimation of the average values of the observables.

The advantage of the proposed strategy is that we only have to consider the initial and the final configuration of the real robot motion. In the next section we apply the described strategy to the mobile robot Nomad150 in order to estimate its error parameters in an indoor environment.

5 Results and Conclusions

5.1 Experimental Context

Our experiments consist of the estimation of the parameters E_R, K_θ, E_T and K_ρ related to our mobile robot Nomad150 in the hallway of our department. Of course it is possible to apply the same method to other mobile robots and the results will depend not only on the robot but also on the environment where the robot moves.

The Nomad150 is a three-wheeled, cylindrical, zero-gyro radius robot. Its diameter is $0.457m$ and its height $0.406m$. It is equipped with 16 sonar sensors placed at 22.5^0 increments, which we did not use. Odometry sensors, located at the synchronous drive system, provide an estimation of the robot's configuration. The sensitivity errors of this configuration estimation are $0.13cm$ in the translation and $0.05deg$ in the orientation. The robot configuration is defined by the vector $X = [x, y, \theta]^T$ introduced in section 2 and its motion, in absence of the odometry errors, is described by the well known equations of the unicycle model.

In order to evaluate the actual configuration change between the initial and the final configuration in the robot motion, we fixed on the basis of our platform three screws. When the robot was in the initial and in the final configuration we marked the floor in correspondence of the three screws. We only measure the three distances between the initial and final position of the three screws. The error associated to these measurements was taken equal to $0.1cm$. In this way it was possible to estimate the change in the orientation with an accuracy equal to $0.3deg$.

5.2 Results

Using the observable $\wp_1$ we found the results showed in Fig.2 where we plot the average change in orientation related to four values of $\overline{\rho}$. From the figure it is possible to see a linear behaviour in accordance with our theory.

Concerning the estimation of the parameters E_R, K_θ and E_T we considered the robot motion suggested in section 4 setting $k = 1$, $l = 6\ m$ and $n = 30$. We set $k = 1$ because the error on the previous parameters estimation increases with k for a fixed $\overline{\rho}(= 2kl)$, as shown in [13]. Moreover this error decreases with n very strongly only for $n \leq 30$ ([13]). The error on the parameter K_ρ estimation decreases with k, as shown in [13]. Therefore in this case we set $k = 20$, $l = 0.50\ m$ and $n = 30$. We did not consider higher values of k because of the sensitivity error in the translation of the odometry system of the mobile robot Nomad150. In fact, for a fixed $\overline{\rho} = 2kl$, increasing k means decrease l.

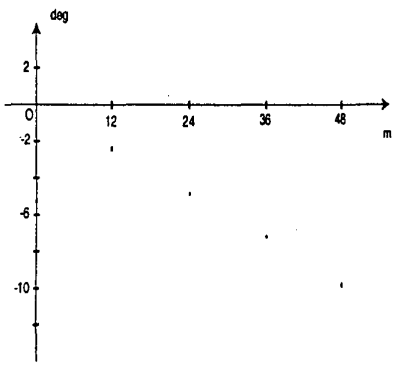

Figure 2: The systematic error in orientation.

Since $\wp_3$, $\wp_4$ and $\wp_5$ depend on more than one parameter we firstly estimated E_R and K_θ, using respectively $\wp_1$ and $\wp_2$. Then we used $\wp_3$, $\wp_4$ and $\wp_5$ in order to estimate the other parameters.

We found the following results: $E_R = (-0.202 \pm 0.004)\frac{deg}{m}$, $K_\theta = (0.012 \pm 0.003)\frac{deg^2}{m}$, $(1 + E_T) = 0.98 \pm 0.04$ and $K_\rho = (3.6 \pm 1.9)10^{-5}m$.

5.3 Conclusions and Future Work

In this paper we presented a theory about the odometry error for a mobile robot with a synchronous drive system and a strategy in order to evaluate the error model parameters. This strategy enables us to estimate the error parameters by only requiring to measure the change in the orientation and in the position between the initial and the final configuration of the robot related to suitable robot motions. In other words it is unnecessary to know the actual path followed by the robot.

We showed the results obtained in the most difficult case of an indoor environment, i.e. where the non-systematic components of the odometry error are very small and very difficult to be evaluated.

We are theoretically investigating on the possibility to find other possible strategies (i.e. other robot motions or also other observables) to estimate the non-systematic parameters with higher accuracy.

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