Using Symmetries and Equivalent Moments in Improving the Efficiency of the Subdomain Multilevel Approach

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***Abstract*—In this paper, we present further improvements in the subdomain multilevel approach (SMA), one of the techniques used for fast full-wave analysis of large printed antennas and circuits. The time for filling the method of moments (MoM) matrix is improved in two ways: by taking advantage of translational or rotational symmetries and by using the equivalent moments. The first way obviates the recomputation of the subdomains that represent translated or rotated replicas of an already computed subdomain. The second one speeds up the computation of the interaction between two different subdomains. To show the advantages of the implemented improvements in simulating a printed structure with repetitive subdomains, an 8 $\times$ 8 corporate-fed array of patches is chosen.**

***Index Terms*—Fast algorithms, macro-basis functions, method of moments, multilevel approaches, printed antennas.**

I. Introduction

Solving electromagnetically large structures with an integral equation/method of moments (IE-MoM) technique using subsectional basis functions is a very demanding procedure in terms of both computer memory and time. The memory needed for solving a problem of N unknowns increases with $\mathcal{O}(N^2)$ and the complexity with $\mathcal{O}(N^3)$ if the resulting MoM linear system is solved with a direct method (standard Gaussian elimination or equivalent). Computationally efficient techniques are thus needed to accelerate the IE-MoM procedures and allow modeling of large circuits and antennas on standard desktop PCs.

There are a number of techniques used to accelerate the MoM calculations and improve the $\mathcal{O}(N^2)$ and $\mathcal{O}(N^3)$ factors [1]. The fast multiple method (FMM) [2], the multilevel fast multiple algorithm (MLFMA) [3], the impedance matrix localization (IML) [4], the adaptive integral method (AIM) [5], and the multilevel matrix decomposition algorithm (MLMDA) [6] are all iterative techniques keeping the same number of unknowns but using very efficient matrix-vector product schemes. Another large group of approaches is based on the size-reduction of the matrix. A nonexhaustive list includes the diakoptics-based multilevel moments method (MMM) [7], the synthetic basis function (SBF) [8], the characteristic basis function (CBF) [9], and the sub-entire-domain (SED) basis function methods [10]. In this paper, we present further improvements in the subdomain multilevel approach (SMA) with macro-basis functions (MBF) [11], a technique that belongs to the latter group and that has proven to be very efficient in modeling large printed antenna arrays.

Using the SMA with MBFs, the size of the MoM matrix is significantly reduced. This leads to a sizeable drop in the CPU time needed to solve the problem, which becomes comparable to or smaller than the MoM fill-in time. We present two strategies that improve the MoM matrix filling. The first way takes advantage of the subdomains that are translated or rotated replicas of an already computed subdomain. Using the principles of translational or rotational symmetries of the subdomains in which the structure is divided, we do not need to recompute MoM submatrices for every subdomain but rather copy the corresponding submatrix. All this presumes, however, that the subdomains are not only geometrically equal, but that they have exactly the same mesh and exactly the same basis function numbering scheme. The strategy for improving the subdomain mutual-interaction filling time is based on reducing the MBFs defined over subdomains to their equivalent moments [12]. Instead of computing the mutual interactions between every pair of subsectional basis functions belonging to two different subdomains and then summing them up, the mutual interaction between two subdomains is computed as the sum of a significantly lower number of equivalent-moment interactions. In this paper, we describe these improvements and demonstrate their capabilities on a practical example of an 8 $\times$ 8 corporate-fed array of patches.

II. SMA Algorithm

The standard SMA algorithm can be summarized in three steps [11].

A. Cut and Estimate

Cut the whole structure into a number of subdomains S_p ($p = 0, 1, \ldots, N_\mathrm{s}$). S_0 is a special subdomain, called the root domain, that contains at least all the excitations (subsectional basis functions where circuit ports are defined). The root domain may include subsectional basis functions that have been considered irrelevant for the SMA procedure and where no MBFs will be defined. The remaining subdomains S_p ($p = 1, \ldots, N_\mathrm{s}$) will be the support for the MBFs in the SMA. Depending on the geometry, subdomains may or may not be connected between them by subsectional basis functions $\mathbf{f}_k^\mathrm{b}$, called "bridge rooftops". Each subdomain S_p ($p = 1, \ldots, N_\mathrm{s}$) is solved now

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independently, using some artificial excitations that translate the effect of the outside world on the subdomain. Typically, bridge rooftops with unit current are used as excitations. Thus, a set of values for the currents $[\hat{\alpha}_k^p]$ in the isolated subdomain under a given excitation is obtained and stored.

B. Compress

The p^{th} MBF defined over the subdomain S_p is expanded using the stored coefficients $[\hat{\alpha}_k^p]$

$$\mathbf{m}_p = \sum_k \hat{\alpha}_k^p \mathbf{f}_k^p \tag{1}$$

where $\mathbf{f}_k^p$ denotes all the subsectional basis functions defined over surfaces $\sigma_k^p \subset S_p$. Now, the global current is expanded using the subsectional basis functions $[\alpha_k^0]$ on the root domain S_0, the MBFs $[\beta_p]$ defined over the subdomains S_p, and the bridge basis functions $[\alpha_k^{\text{b}}]$ that connect them

$$\mathbf{J} = \sum_k \alpha_k^0 \mathbf{f}_k^0 + \sum_k \alpha_k^{\text{b}} \mathbf{f}_k^{\text{b}} + \sum_p \beta_p \mathbf{m}_p$$

resulting in a compressed MoM matrix. The number of unknowns in the MoM matrix equation is reduced, as all the subsectional basis functions belonging to a subdomain are merged into an MBF. The compression of the MoM submatrices that contain all the interaction integrals between two subdomains S_p and S_q can be done using vector-matrix-vector multiplication

$$\begin{aligned}\langle \mathbf{m}_p, \mathcal{L}\mathbf{m}_q \rangle &= \int_{S_p}\int_{S_q} \mathbf{m}_p(\mathbf{r}) \cdot \overleftrightarrow{\mathbf{G}}(\mathbf{r} \mid \mathbf{r}') \cdot \mathbf{m}_q(\mathbf{r}')\, \mathrm{d}S\, \mathrm{d}S' \\ &= \sum_k \hat{\alpha}_k^p \sum_l \hat{\alpha}_l^q \int_{\sigma_k^p}\int_{\sigma_l^q} \mathbf{f}_k^p \cdot \overleftrightarrow{\mathbf{G}} \cdot \mathbf{f}_l^q \, \mathrm{d}S\, \mathrm{d}S' \\ &= [\hat{\alpha}_k^p]^T [Z_{k,l}^{pq}][\hat{\alpha}_l^q]\end{aligned} \tag{2}$$

where

$$Z_{k,l}^{pq} = \int_{\sigma_k^p}\int_{\sigma_l^q} \mathbf{f}_k^p \cdot \overleftrightarrow{\mathbf{G}} \cdot \mathbf{f}_l^q \, \mathrm{d}S\, \mathrm{d}S'$$

designates the elements of the MoM submatrix that correspond to the interactions between subsectional basis functions belonging to the subdomains S_p and S_q. It should be noted here that the mutual coupling between different subdomains is accounted for through these MoM elements and that none of the MoM elements is set to zero. The final MoM matrix is reduced in size, but still fully populated.

The solution of the compressed system gives the unknowns for subsectional basis functions over the root domain, for bridge basis functions and for MBF's $\beta_p, p = 1, \ldots, N_{\text{s}}$.

C. Expand

The solution over the compressed subdomains is finally recovered through a superposition of the MBFs and their global solutions

$$\alpha_k^p = \beta_p \hat{\alpha}_k^p.$$

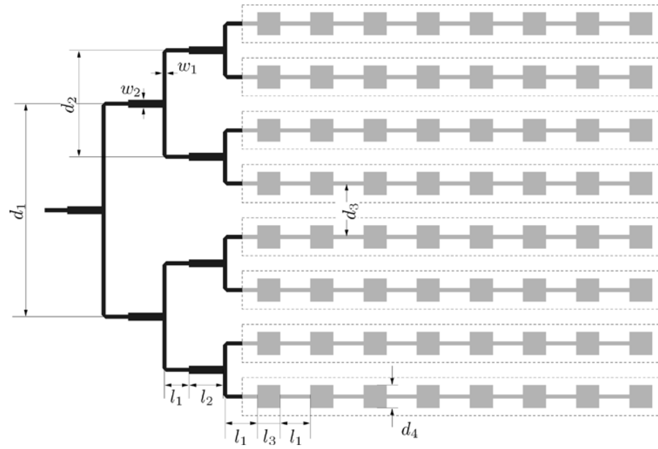


Fig. 1. The layout of the antenna array. $w_1 = 1.3$, $w_2 = 3.93$, $d_1 = 94.32$, $d_2 = 47.16$, $d_3 = 23.58$, $d_4 = 11.79$, $l_1 = 12.32$, $l_2 = 18.48$, $l_3 = 10.08$. All dimensions given in mm. Printed on Duroid-5870 with $\varepsilon_r = 2.35$, $\tan\delta = 0.0012$ and $h = 1.57$ mm. The dashed lines define the eight subdomains (fingers) in which the whole structure is subdivided.

This way the problem is solved for every single unknown. MBFs can be considered as "entire-domain" basis functions numerically defined on every subdomain S_p for a set of specific excitations. The different MBFs can be obtained by changing the position of the excitation (orthogonal space harmonics) or by using different frequencies. Another very simple possibility, which performs very well for structures near resonance, is to consider as separate MBFs, the real and imaginary parts of the original complex function. The success of this approach is due to the different physical behavior of in-phase and quadrature parts of the current, related respectively to the radiation and induced fields [13]. The real part is close to the eigencurrent and rather independent of excitations, while the imaginary part is strongly connected to the specific nature and position of the excitation. A linear combination of both should fit better the actual current. The proposed expansion shows now an additional degree of freedom

$$\mathbf{J} = \sum_k \alpha_k^0 \mathbf{f}_k^0 + \sum_k \alpha_k^{\text{b}} \mathbf{f}_k^{\text{b}} + \sum_p \beta_p^{\text{r}} \mathrm{Re}\{\mathbf{m}_p\} + \mathrm{j}\beta_p^{\text{i}} \mathrm{Im}\{\mathbf{m}_p\}.$$

In order to speed up the interaction between two MBFs, each subdomain p is further subdivided into K equal square regions s_p^k $(k = 1, \ldots, K)$ and the equivalent moment computed at the center $\mathbf{r}_{c_k}^p$ of each square, approximating (2) as

$$\sum_{k=1}^{K}\sum_{l=1}^{L} \left(\int_{\mathrm{s}_p^k} \mathbf{m}_p(\mathbf{r})\, \mathrm{d}S \right) \overleftrightarrow{\mathbf{G}}(\mathbf{r}_{c_k}^p \mid \mathbf{r}_{c_l}^q) \left(\int_{\mathrm{s}_q^l} \mathbf{m}_q(\mathbf{r}')\, \mathrm{d}S' \right). \tag{3}$$

Although the application of the SMA does not depend either on the type of integral equations or on the Green's functions, we used here the mixed potential integral equations (MPIE) with multilayered media Green's functions computed as explained in [13].

III. 8 × 8 Corporate-Fed Patch Array

As a benchmark we consider the 8×8 corporate-fed array of patches [14] shown in Fig. 1. It consists of a single layered microstrip design with a combination of lines entering straight into the patches.

The feeding lines and patches are densely meshed, which leads to a large number of $N = 9947$ triangular and rectangular basis functions for the whole antenna array and to the memory occupation of 1.6 GB. An AMD Athlon 1.4 GHz personal computer with 512 MB of RAM has been used in the simulations. Solving the problem using the conventional MoM could not be done on this PC because of the lack of the available memory (1.6 GB needed against the available 512 MB). The antenna structure with mesh density that yields 4000 unknowns is solved in 190 min per frequency point on this PC. Taking into account that the time dependency on the number of unknowns in the direct solution is $\mathcal{O}(N^3)$, the time needed for solving the problem with the mesh density fine enough to lead the accurate results (9947) would take about 54 h per frequency point. This value is extrapolated and it does not take into account the memory resources. Actual time would be even longer due to the inevitable swapping to the hard disk.

Using the SMA, the structure is split into nine parts: the corporate feed network on the left-hand side and eight "fingers," each including eight patches (Fig. 1). Although nothing prevents theoretically the definition of MBFs on the corporate beamforming network, this option has not been retained here. The beamforming network is a nonresonant structure that appears only once in the problem. Approximating the currents on it would certainly require more than one complex MBF and the lack of geometric redundancy minimizes the eventual benefits. The SMA computation starts with the evaluation of the currents on the isolated fingers, each with $N = 1131$ BFs being merged into a complex MBF. The global system of equations is then compressed to 923 unknowns, what corresponds to a memory drop from 1.6 GB to 13 MB. The direct solution is used for solving the compressed MoM system. The time needed per frequency point is 24.35 min. This is equivalent to a reduction of computer time by a factor greater than 100.

By inspecting our 8×8 array, we can see that eight subdomains (fingers) are identical. A block of the MoM matrix for one subdomain could be filled, compressed, and repeated in the MoM matrix 7 more times. In addition, the solution for the subdomain could be computed only once and then reused in compressing the MoM submatrices that correspond to the interactions between different subdomains. However, the eight subdomains need to have the same mesh and basis function numbering scheme. By reconstructing the mesh and including the concept of repeated subdomains in our solver, further improvements in the computational time in addition to these obtained using the unrefined SMA (that does not take into account the repetitive subdomains) are achieved. The same structure is solved in only 12 min per frequency point, giving exactly the same input impedance and the radiation pattern as before. The computational time as compared to that one of the unrefined SMA has been reduced to 49%.

As a final step, we use the concept of equivalent moments to represent the MBFs defined over subdomains when computing their mutual interactions. It has been shown [12] that using 20 moments per wavelength ($\mathrm{mp}\lambda$) in computing the MBF mutual interactions yields satisfactory results with the error in computed input impedance up to 1% as compared to the one obtained using the unrefined SMA. When we make use of the equivalent moments with the density of 20 $\mathrm{mp}\lambda$, the antenna is solved in 5.7 min, which is only 23.5% (four times faster) of the time needed for the unrefined SMA.

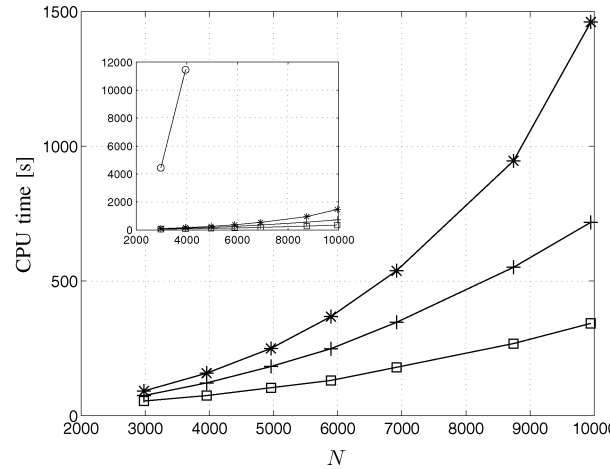


Fig. 2. CPU time versus number of unknowns using unrefined SMA ($*$), SMA with repeated subdomains ($+$) and SMA with repeated subdomains and fast mutual interactions with 20 $\mathrm{mp}\lambda$ ($\square$). In the upper left corner, the time needed for direct solution of the conventional MoM system of equations is shown.

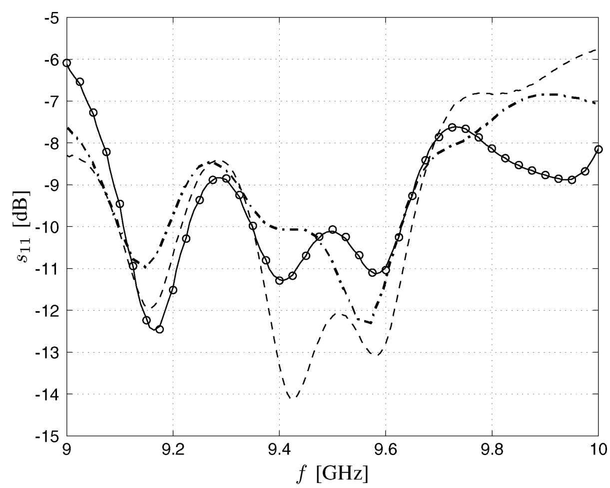
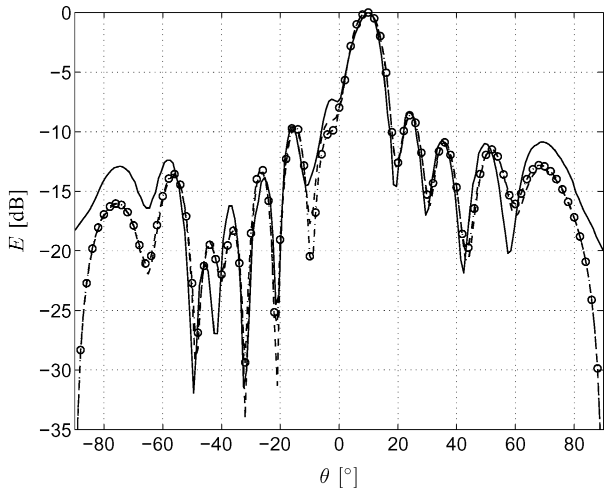


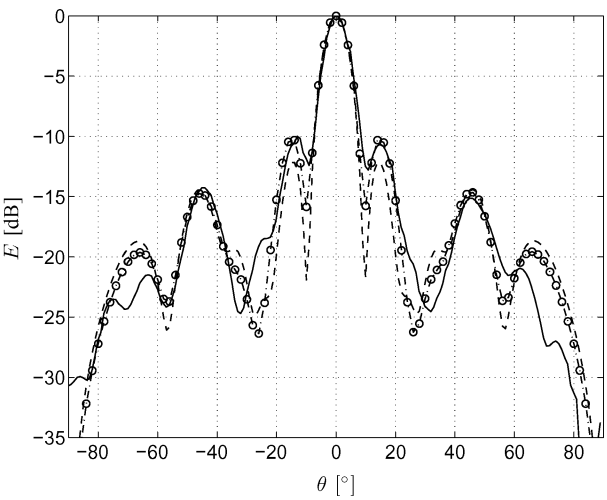
Fig. 3. The reflection coefficient of the 8×8 corporate-fed patch array. Measurements (dashed line), MLMDA [6] (dash-dotted line), the unrefined SMA (solid line), and the SMA with repeated subdomains and 20 $\mathrm{mp}\lambda$ ($\circ$).

Fig. 2 shows the CPU time as a function of the number of unknowns of the studied problem. The CPU time needed to solve the problem using a conventional MoM with direct solution grows with the number of unknowns as $\mathcal{O}(N^3)$. The time for solving the problem decreases as we use the unrefined SMA ($*$), the SMA with repeated subdomains ($+$) and it becomes minimal when the SMA with repeated subdomains and 20 $\mathrm{mp}\lambda$ in MBF mutual interactions are used ($\square$).

The reflection coefficient of the antenna is shown in Fig. 3. In this figure, three different results can be seen. The unrefined SMA (solid line) and the SMA with repeated subdomains and 20 $\mathrm{mp}\lambda$ ($\circ$) give practically the same results, the only difference being in more advantageous computational time. The results taken from [6] and obtained using another approximate technique, the multilevel matrix decomposition algorithm (MLMDA), are presented using a dash-dotted line. By inspecting the third curve (dashed line), which represents



(a) E–plane.



(b) H–plane.

Fig. 4. Copolar radiation patterns of the 8×8 corporate-fed patch array at $f = 9.43$ GHz. Measurements (solid lines), conventional MoM solved using [15] (o), and SMA with repeated subdomains and 20 $\mathrm{mp}\lambda$ (dashed lines).

the measured values, one can conclude that the differences between the compared numerical methods are of the order of the differences from the measured values. Therefore, our method provides an accuracy comparable to the MLMDA.

Fig. 4 shows the copolar radiation patterns of the 8×8 corporate-fed patch array. Solid lines represent measured values, dashed lines represent the results obtained using the SMA with repeated subdomains, and 20 $\mathrm{mp}\lambda$ in MBF mutual interactions, and circles – the results obtained using the conventional MoM. In the conventional MoM, the solution of the problem that takes 1.6 GB of memory on a PC with 512 MB of RAM was made possible using the block ILU preconditioner scheme [15], where matrix blocks are stored on the hard disk and swapped to memory. One can observe a very good agreement of the SMA results and the ones obtained using the conventional MoM with the measurements in both E- [Fig. 4(a)] and H-planes [Fig. 4(b)].

IV. Conclusion

The SMA technique is one of the most efficient algorithms for reducing the computer memory occupation and time needed in analyzing complex printed antennas. Therefore, it brings to the reach of standard PCs the study of geometries that would otherwise require extraordinary computer resources. When applied properly to many practical designs, it provides reduction factors greater than 100 in both memory size and CPU time. Moreover, the use of symmetries and equivalent moments, as shown in this paper, allow a further reduction in CPU time by a factor of four or better.

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