INTERPOLATED IDENTIFIED REFLECTION COEFFICIENTS FOR ACOUSTIC RAY METHOD

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Abstract

Due to calculation time in the medium frequency range, the acoustic ray method may be more appropriate than finite element methods, despite the fact that the latter are more rigorous than the former in the presence of damping materials. Indeed, the use of rays rests on specular reflection, with its errors, particularly from grazing rays [1]. For more precise information, another type of reflection is sought, as far as possible local to keep the computation speed typical of specular reflection. To this end, in the elementary situation of half-space, a reflection coefficient has been identified through the integral method which leads here to an exact solution. Such a coefficient shows a significantly different value from the specular one, especially for efficient damping material on the boundary and non-normal incidences, all the more so for grazing rays. Unfortunately, excessive computation time with integral equations does not allow the coefficient of each ray in a cavity to be identified rigorously. Were this possible, the reflection would not be described exactly since the walls of a cavity are of finite dimensions. However, between these two calculation methods – with specular coefficient, and with coefficient rigorously identified for each ray – there may be a compromise: interpolating the identified coefficient from precalculated sampled values, which is presented in the present paper.

INTRODUCTION

In optimization procedures for absorbing materials [2], the rapidity of sound field prediction is an important issue, forbidding the use of a complex method such as finite element or boundary element methods that require great computation time in the medium frequency range. The best compromise could be the one offered by the acoustic ray-tracing method. However, this method rests on specular reflection, the localization of which (point of impact of the ray on the reflecting wall) leads to erroneous results in situations where the geometry is not trivial (near corners of a cavity, for example) or for rays with a grazing incidence.

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Nevertheless, thanks to a substitute for these rays, the spectra obtained when compared with those resulting from the finite element method in configurations where both methods can be implemented, shows useful information in thirds or twelfths of an octave [1].

PROPOSED METHOD

For more precise information, another type of reflection is sought, as far as possible local to keep the computation speed typical of specular reflection. To achieve this, one possibility is to identify a local reflection coefficient out of an exact expression of the acoustic field. In order to describe the reflection on one surface only, the exact expression is sought in a half-infinite space (that is, an open space bounded by an infinite absorbing surface). For the time being, only the 2D case is considered.

Integral solution inside a half-infinite space

The wave equation inside a two-dimensional, half-infinite space $\Omega_{1/2}$ bounded by an absorbing surface Γ and filled with an infinite volume of air – seen as isotropic and compressible – is

$$(\Delta + k^2)p(\mathbf{x}) = -f_s\delta(\mathbf{x} - \mathbf{x_S}^1) \tag{1}$$

with $(\Delta + k^2)$ the Helmholtz operator. In case of a perfectly reflecting surface, the solution of equation 1 is the 2D Green kernel

$$g(\mathbf{x}, \mathbf{x}') = -\frac{i}{4}H_0^-(k \cdot |\mathbf{x} - \mathbf{x}'|) \tag{2}$$

with H_0^- is the Hankel function of the second type at order zero. It can be shown that this kernel is equivalent to the superposition of the two infinite-space kernels corresponding to two point sources in $\mathbf{x_S}$ and $\mathbf{x_{S_I}}$

$$g_{1/2}(\mathbf{x}, \mathbf{x_S}) = g(\mathbf{x}, \mathbf{x_S}) + g(\mathbf{x}, \mathbf{x_{S_I}}) \tag{3}$$

To extend this solution to the case of an absorbing surface Γ, the Green theorem is used on equation 1. This leads to a contour integral form for the acoustic pressure $p(\mathbf{x})$ in any point $X \in \Omega$:

$$p(\mathbf{x}) = f_s g_{1/2}(\mathbf{x}, \mathbf{x_S}) + \frac{ik}{Z_r}\int_\Gamma g_{1/2}(\mathbf{x}, \mathbf{y})p(\mathbf{y})d\mathbf{y} \quad \text{and thus} \tag{4}$$

$$\mathbf{x} \in \Omega \qquad p(\mathbf{x}) = f_s g(\mathbf{x}, \mathbf{x_S}) + f_s g(\mathbf{x}, \mathbf{x_{S'}}) + 2\frac{ik}{Z_r}\int_\Gamma g(\mathbf{x}, \mathbf{y})p(\mathbf{y})d\mathbf{y} \tag{5}$$

So, the problem must first be solved on the absorbing surface. Equation 5 being satisfied in the whole half-space $\Omega_{1/2}$ and the Green function leading to what is called a single layer, equation 5 is also verified on the surface Γ, resulting in

$$\mathbf{x} \in \Gamma \qquad p(\mathbf{x}) = 2f_s g(\mathbf{x}, \mathbf{x_S}) + 2\frac{ik}{Z_r}\int_\Gamma g(\mathbf{x}, \mathbf{y})p(\mathbf{y})d\mathbf{y} \tag{6}$$

[1] Thorough this paper, two-dimensional points are noted in bold letters, that is $\mathbf{x_S} \equiv \{x_S, y_S\}$.

Discretizing the surface Γ in n elements Γ_{f_i} of equal length leads to the following matrix form

$$\{p(\mathbf{x_{f_i}})\} = 2f_s\{g(\mathbf{x_{f_i}},\mathbf{x_S})\} + 2\frac{ik}{Z_r}[S_{ij}]\{p(\mathbf{x_{f_i}})\} \tag{7}$$

with S_{ij} the $(n \times n)$-matrix containing the coefficients coming from the single layer integral:

$$S_{ij} = \int_{\Gamma_{f_i}} g(\mathbf{x_{f_i}},\mathbf{y})d\mathbf{y} \tag{8}$$

The acoustic field on the surface Γ is obtained by matrix inversion:

$$\{p(\mathbf{x_{f_i}})\} = 2f_s\left[[Id] - 2\frac{ik}{Z_r}[S_{ij}]\right]^{-1}\{g(\mathbf{x_{f_i}},\mathbf{x_S})\} \tag{9}$$

Identification of the reflection coefficient

By analogy with the specular case where

$$p^{spec}_{refl}(\mathbf{x}) = R_{spec}(\mathbf{x},\mathbf{x_{S'}})f_s g(\mathbf{x},\mathbf{x_{S'}}) \tag{10}$$

(for a given situation with a receiver point in $\mathbf{x}$ and a source in $\mathbf{x_S}$, $\mathbf{x_{S'}}$ being the position of the image source of $\mathbf{x_S}$), one can identify a reflection coefficient in the expression of the acoustic field obtained by the integral method (equation 5), leading to an identified coefficient [1]

$$R_{id}(\mathbf{x},\mathbf{x_{S'}}) = 1 + 2\frac{ik}{Z_r}\frac{\int_\Gamma g(\mathbf{x},\mathbf{y})p(\mathbf{y})d\mathbf{y}}{f_s g(\mathbf{x},\mathbf{x_{S'}})} \tag{11}$$

Storage and interpolation of the identified coefficient

The point in having a pseudo-local reflection coefficient is to be able to use it in a ray-tracing method and therefore profit of the time efficiency of such a method. The coefficient must also be quickly available. To achieve this, it is computed for a given set of source-receiver configurations and then stored in a multidimensional matrix, the dimension of which corresponds to the number of variables. For now, the matrix is computed for a given frequency (500 Hz) and a given impedance ($Z_r = 9$) and the following variables are chosen: source height h_S, receiver height h_R and the incidence angle θ, the raster being linear.

$$\left.\begin{array}{rcl} h_{S_i} & = & h_{S_0} + i\Delta h_S \\ h_{R_j} & = & h_{R_0} + j\Delta h_R \\ \theta_k & = & \theta_0 + k\Delta\theta \end{array}\right\} \longrightarrow R_{id_{ijk}}(h_{S_i}, h_{R_j}, \theta_k) \tag{12}$$

The numerical values used in the presented example are $\Delta h_S = \Delta h_R = 0.2 \cdot \lambda$, $h_{S_0} = h_{R_0} = 0$ and $h_{S_{max}} = h_{R_{max}} = 2 \cdot \lambda$, where λ is the acoustic wavelength.

For every other configuration, the identified coefficient is interpolated out of the nearest values in the matrix. The method used for this is a linear interpolation out of the nearest values in the coefficient matrix:

$$\left.\begin{array}{ccccc} h_{S_i} & < & h_S & < & h_{S_{i+1}} \\ h_{R_j} & < & h_R & < & h_{R_{j+1}} \\ \theta_k & < & \theta & < & \theta_{k+1} \end{array}\right\} \longrightarrow R_{id}(h_S, h_R, \theta) = \mathrm{mean}(R_{id_{i,j,k}}, \ldots, R_{id_{i+1,j+1,k+1}}) \tag{13}$$

The value for R_{id} is the mean value of all neighboring coefficients.

Control results

Figure 1 compares an exact solution (the acoustic pressure prediction obtained by the integral solution presented above) with two approximated solutions obtained by the ray-tracing method: one using a specular reflection coefficient, the other the interpolated identified coefficient.

The situation in Figure 1 has been chosen so that each value of the identified coefficient has to be interpolated (no exact value is used). It can be seen that the obtained result is much nearer to the exact result than the specular reflection result. When the incidence angle aperture grows (right side of the figure), greater "steps" in the interpolation appear, due to the non-linearity of the angle function. This can easily be improved.

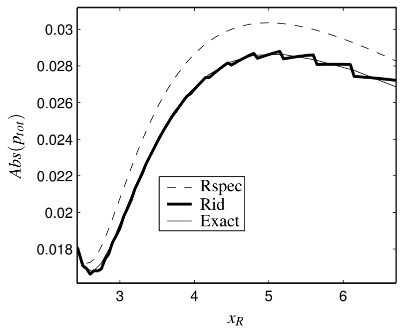


Figure 1: *Acoustic pressure (in Pa) versus distance between source and receiver in a test situation.* $Z_r = 9$, $f = 500Hz$

Now that the interpolation method is shown to be trustworthy, the next step is to use it in a more complex environment. First, the method is tested in the 1/4-infinite space (that is, a second absorbing surface is added, perpendicularly to the first one), then it will be tested in a more complex closed 2D cavity using a ray-tracing program.

1/4-INFINITE SPACE

By adding a second absorbing surface, perpendicular to the first one, two important modifications are brought: a) the finiteness of the surface at the corner point and b) the geometrical effects of the corner itself. So, this configuration can be used to test the relevance of the proposed method while remaining simple enough, so that an exact integral solution can still be computed for comparison.

Integral method in the 1/4-infinite space

The numerical experiences show that the following relation is valid, although it has not been formerly demonstrated:

$$g_{1/4}(\mathbf{x}, \mathbf{x_S}) = g(\mathbf{x}, \mathbf{x_S}) + g(\mathbf{x}, \mathbf{x_{S'}}) + g(\mathbf{x}, \mathbf{x_{S''}}) + g(\mathbf{x}, \mathbf{x_{S'''}}) \tag{14}$$

Following the same steps than in the half-infinite case, the Galerkine form of the Helmholtz equation in the 1/4-infinite space $\Omega 1/4$ leads to

$$p(\mathbf{x}) = f_S g_{1/4}(\mathbf{x}, \mathbf{x_S}) + \frac{ik}{Z_r} \int_{\Gamma_1 \cup \Gamma_2} g_{1/4}(\mathbf{x}, \mathbf{y}) p(\mathbf{y}) d\mathbf{y} \tag{15}$$

By setting the coordinate system so that the axes x and y are on the walls, with the origin at the corner, the acoustic pressure in each point of the space $\Omega_{1/4}$ can be written as the sum of the contributions of the source and the image sources and correction terms corresponding to the absorption of the surfaces Γ_1 and Γ_2:

$$\begin{aligned} p(\{x_R, y_R\}) &= f_S[g(\{x_R, y_R\}, \{x_S, y_S\}) + g(\{x_R, y_R\}, \{-x_S, y_S\}) \\ &\quad + g(\{x_R, y_R\}, \{x_S, -y_S\}) + g(\{x_R, y_R\}, \{-x_S, -y_S\})] \\ &\quad + \mathrm{corr}(\Gamma_1) + \mathrm{corr}(\Gamma_2) \end{aligned} \tag{16}$$

with

$$\begin{aligned} &\mathrm{corr}(\Gamma_1) = \\ &\tfrac{2ik}{Z_r} \left\langle \int_{\Gamma_{1f_i}} [g(\{x_R, y_R\}, \{x', 0\}) + g(\{x_R, y_R\}, \{-x', 0\})] dx' \right\rangle \cdot \{p(x_{f_j})\} \end{aligned} \tag{17}$$

$$\begin{aligned} &\mathrm{corr}(\Gamma_2) = \\ &\tfrac{2ik}{Z_r} \left\langle \int_{\Gamma_{2f_i}} [g(\{x_R, y_R\}, \{0, y'\}) + g(\{x_R, y_R\}, \{0, -y'\})] dy' \right\rangle \cdot \{p(y_{f_j})\} \end{aligned} \tag{18}$$

The acoustic pressure on the surfaces is obtained via the matrix equations

$$\begin{aligned} &\{p(x_{f_j})\} = \\ &2f_S \left[[\mathrm{Id}] - \tfrac{4ik}{Z_r} [S_{ij,\Gamma_1}] \right]^{-1} \cdot \{g(\{x_{f_i}, 0\}, \{x_S, y_S\}) + g(\{x_{f_i}, 0\}, \{-x_S, y_S\})\} \end{aligned} \tag{19}$$

$$\begin{aligned} &\{p(y_{f_j})\} = \\ &2f_S \left[[\mathrm{Id}] - \tfrac{4ik}{Z_r} [S_{ij,\Gamma_2}] \right]^{-1} \cdot \{g(\{0, y_{f_i}\}, \{x_S, y_S\}) + g(\{0, y_{f_i}\}, \{x_S, -y_S\})\} \end{aligned} \tag{20}$$

with

$$S_{ij,\Gamma_1} = \int_{\Gamma_{1f_j}} \left[g(\{x_{f_i}, 0\}, \{x', 0\}) + g(\{x_{f_i}, 0\}, \{-x', 0\}) \right] dx' \tag{21}$$

$$S_{ij,\Gamma_2} = \int_{\Gamma_{2f_j}} \left[g(\{0, y_{f_i}\}, \{0, y'\}) + g(\{0, y_{f_i}\}, \{0, -y'\}) \right] dy' \tag{22}$$

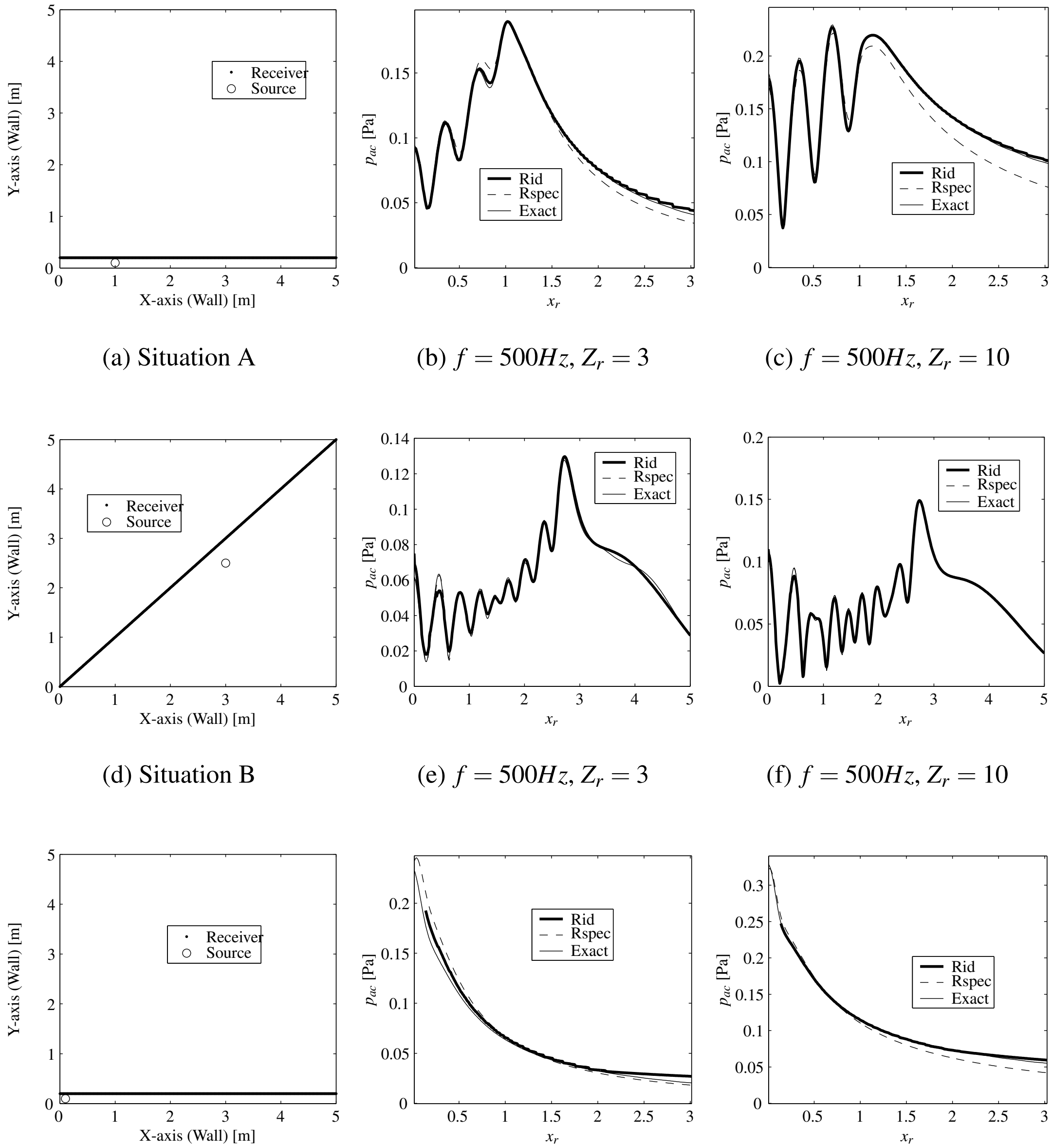
Use of the identified coefficient

Again, the results obtained via the integral method are compared with ray-tracing results, using a specular coefficient or the identified interpolated coefficient discussed herein, identified out of the half-infinite case.

Of course, since neither the specular nor the identified coefficient take the finiteness of the surfaces into account, one expects erroneous results for reflections near the corner.

DISCUSSION OF THE RESULTS

Figure 2 shows the comparison between the integral solution of three particular problems and the values obtained by ray-tracing. In interpreting these results, one must take into account the fact that the integral solution (dubbed "exact") considers the walls with a finite size of 5 meters, also the boundaries of the depicted graphs. Outside this, the walls are considered perfectly reflecting ($Zr = \infty$, the integral term vanishes). This leads to a certain deviation of the "exact" solution near the end of the walls ($x_r > 3m$



(a) Situation A (b) $f = 500Hz$, $Z_r = 3$ (c) $f = 500Hz$, $Z_r = 10$

(d) Situation B (e) $f = 500Hz$, $Z_r = 3$ (f) $f = 500Hz$, $Z_r = 10$

(g) Situation C (h) $f = 500Hz$, $Z_r = 3$ (i) $f = 500Hz$, $Z_r = 10$

Figure 2: *Comparison between the acoustic pressure obtained by an exact solution and ray-tracing using both a common specular reflection coefficient and the interpolated identified coefficient. The abscissa of the pressure curves corresponds to the x-coordinate of the receiver location.*

in situations A and C). When the absorption is important ($Zr = 3$), this effect is even greater, since the change of boundary conditions at the end of the considered walls is more important. In situation B, this effect is less visible, since the receiver points are away from both walls. For this reason, the "exact" solution should not be taken as an absolute reference, and the curves are only plotted for the first 3 meters. Simulations run with complex impedances show a similar trend.

This being stated, the following observations are of interest:

- In situations where the greater contribution comes from rays that are reflected under a grazing angle (situations A and C), the identified coefficients show better results.

- In situations near walls, but where there still exists a grazing contribution and where the ray lengths are small, the identified coefficient still performs very well (situation A for $x_r < 1$).

- On the contrary, when the ray length are of greater importance and when most rays are reflected under closed angles ($\theta < \pi/4$), the specular reflection leads to similar results (situation B).

From an overall point of view, this is already an improvement compared to the results yielded by ray-tracing using purely specular reflection. To verify this, the interpolated identified coefficient has to be implemented inside an existing 2D ray-tracing program.

INTEGRATION INSIDE A RAY-TRACING CODE

This step of the project is still in its early stages of development, however the goal is clear enough to be stated. The ray-tracing program that is going to be extended was developed by one of the authors (T. Courtois). In it, the specular reflection coefficient $R_{spec}(\theta)$, obtained by the cosine form, will be replaced by the interpolated identified coefficient $R_{id}(h_S, h_R, \theta)$. The reflection on each wall will be treated as a separate case. For h_S, the height of the previous image source will be used and h_R will be computed geometrically out of the total ray length. The incidence angle θ will be obtained geometrically, as in the specular case. Experience seems to show that the first rays are the most relevant for the acoustic pressure at the receiver point. The interpolated identified coefficient will also substituted to the specular one only in the first 10 rays.

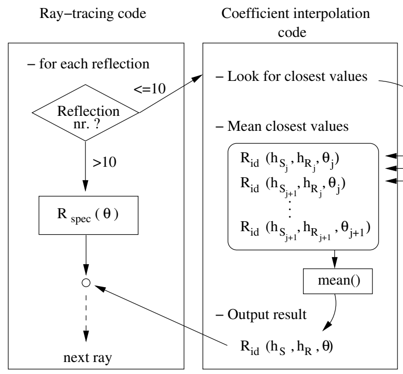


Figure 3: *Flow-chart of the software integration*

For situations falling out of the range of the precalculated coefficients, the specular coefficient will be used. Limiting the use of the interpolated coefficient to the first 10 reflections should enable the use of a coefficient matrix of reasonable size.

CONCLUSION

In the research phase related to a better use of the ray-tracing method, it is shown how to improve the specular reflection, which is the source of errors in the description of sound fields in cavities.

First, a sober interpolation of the identified reflection coefficient R_{id} is used in the half-infinite space $\Omega_{1/2}$ in order to confirm that the exact solution is better approximated than with a specular reflection coefficient. Then, the 1/4-infinite space $\Omega_{1/4}$ is treated, where only the specular reflection is the source of errors. When both the source and the receiver are away from the borders, the specular reflection is sufficient. On the contrary, for situations with grazing rays, the interpolated identified coefficient R_{id} leads to better results. This new information being acquired, the influence of the finiteness of the walls remains to be treated. Until a generic problem that can show the influence of this factor has been found, one has to limit the scope of investigation to the case of a rectangular cavity where the source and receiver are near the walls and use the interpolation presented herein. Depending on the importance of the influence of the finiteness of the walls, an interpolation of an identified coefficient taking the dimensions of the walls into account will have to be developed.

At this stage of research, the next step is to concentrate on situations where the solution brought by the ray-tracing method does not match the exact solution inside a cavity with rigid walls, also before addressing the problem of absorption by the walls. Since the proposed improvement cannot be demonstrated, it will have to be proved by numerical results. If this improvement is confirmed, the errors in a situation with absorbing walls will be a mix of errors due to the specular reflection and due to the image sources approximation. The proportions taken by these two kind of errors remains yet to be discovered. Since the problems discussed herein are essential to problems of acoustic prediction in cavities or for virtual acoustics, any progress made in this field is welcome.

Also it is to be hoped that the error due to the specular reflection is predominant and – in the description of the reflection – that the interpolation out of identified coefficients R_{id} outgrows a) the finiteness of the walls and b) the geometry of the cavity. In other terms, that a) the non-locality of the reflection is confined to the neighborhood of the point of impact of the ray on the wall and b) that the solution approximated with the ray-tracing method inside a cavity with rigid walls is near the exact solution. In such a situation, a matrix of precomputed R_{ids} out of $\Omega_{1/2}$ only would be enough to deal with any geometry.

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