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Technical Note

Viscoelastic constitutive law in large deformations: application to human knee ligaments and tendons

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Abstract

Traction tests on soft tissues show that the shape of the stress–strain curves depends on the strain rate at which the tests are performed. Many of the constitutive models that have been proposed fail to properly consider the effect of the strain rate when large deformations are encountered. In the present study, a framework based on elastic and viscous potentials is developed. The resulting constitutive law is valid for large deformations and satisfies the principles of thermodynamics. Three parameters — two for the elasticity and one for the viscosity — were enough to precisely fit the non-linear stress–strain curves obtained at different strain rates with human cruciate ligaments and patellar tendons. The identification results then in a realistic, three-dimensional viscoelastic constitutive law. The developed constitutive law can be used regardless of the strain or rotation values. It can be incorporated into a finite element program to model the viscoelastic behavior of ligaments and tendons under dynamic situations. © 1998 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The mechanical behavior of soft tissues is known to be viscoelastic (Fung, 1973). The shape of the stress–strain curves depends on the strain rate at which the traction test is performed. Few experimental studies have been conducted in this field. Nevertheless, the effect of the strain rate on the stress–strain curves has been demonstrated with anterior cruciate ligament (Kennedy et al., 1976), tendons (Haut, 1983; Sanjeevi, 1982), intrinsic and extrinsic wrist ligament (Nowalk and Logan, 1991), incisor periodontal ligament (Chiba and Komatsu, 1993), and inferior glenohumeral ligament (Ticker et al., 1996).

From a theoretical point of view, many of the constitutive models that have been proposed to date fail to properly consider the effect of the strain rate. Extending elastic models (Chiba and Komatsu, 1993; Danto and Woo, 1993) or adding spring and dashpot elements (Jamison et al., 1968; Sanjeevi, 1982) are two popular approaches. With the extension of elastic models, the interpretation of the strain rate effect depends on the elastic model used, while the addition of spring and dashpot elements is difficult to generalize in large deformations. The development of the quasi-linear viscoelastic theory (Fung, 1993) to incorporate the strain rate effect gives good results at low strain rates (0.06–0.75% s^{-1}) (Haut and Little, 1972) but imprecise results for higher strain rates (up to 10% s^{-1}) (Woo et al., 1981). Recently, a new viscoelastic model has been developed for large deformations (Johnson et al., 1996), however, no strain rate effect has been considered.

The purpose of the present study is to develop a realistic three-dimensional viscoelastic constitutive law which takes into account the strain rate effect in soft tissue. The proposed constitutive law will verify four points. First, it will describe the non-linearity of the stress–strain curves. Second, the strain rate will be considered as an explicit

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variable. Third, the constitutive law will be valid for large deformations. Finally, the principles of thermodynamics will be satisfied. The parameters of the constitutive law will then be identified with traction tests performed at different strain rates on human cruciate ligaments and patellar tendons.

2. Material and methods

A general constitutive law, valid for large deformations and with the strain rate as explicit variable, is expressed by (Noll, 1958)

$$\mathbf{S} = \mathbf{S}(\mathbf{C}, \dot{\mathbf{C}}), \tag{1}$$

where $\mathbf{S}$ is the second Piola–Kirchhoff stress tensor, $\mathbf{C}$ is the right Cauchy–Green strain tensor and $\dot{\mathbf{C}}$ its first time derivative. In an isothermal process (corresponding to the present experimental situation), the thermodynamical principles reduce to the satisfaction of the Clausius–Duhem inequality (Truesdell and Noll, 1992)

$$\left(\mathbf{S} - 2\rho_0 \frac{\partial W_e}{\partial \mathbf{C}}\right) : \frac{\dot{\mathbf{C}}}{2} \geqslant 0 \quad \forall \mathbf{C}, \dot{\mathbf{C}}, \tag{2}$$

where $W_e = W_e\,(\mathbf{C})$ is the Helmholtz free energy (also called elastic potential) per unit of mass, and ρ_0 the density expressed relative to the reference configuration. The notation ':' represents the scalar product between second-order tensors. If there is no dissipation, Eq. (2) is trivial

$$\mathbf{S} = 2\rho_0 \frac{\partial W_e}{\partial \mathbf{C}}. \tag{3}$$

In the presence of viscosity, the existence of a viscous potential $W_v\,(\mathbf{C}, \dot{\mathbf{C}})$ accounting for energy dissipation is assumed (Germain, 1973)

$$\mathbf{S} - 2\rho_0 \frac{\partial W_e}{\partial \mathbf{C}} = 2 \frac{\partial W_v}{\partial \dot{\mathbf{C}}}. \tag{4}$$

The verification of the thermodynamical principles (2) then becomes:

$$\frac{\partial W_v}{\partial \dot{\mathbf{C}}} : \dot{\mathbf{C}} \geqslant 0 \quad \forall \dot{\mathbf{C}}. \tag{5}$$

Eq. (5) holds true when the potential W_v is continuous, positive (or zero), and convex. Moreover, the value of W_v must be zero when the strain rate is equal to zero (Coussy, 1995).

In order to get a tractable identification process, the specimens are considered as homogeneous, incompressible and mechanically isotropic. The elastic potential W_e and viscous potential W_v can then be expressed as a function of strain and strain rate invariants (Boehler, 1987)

$$\begin{aligned}\mathbf{S} = &-p\mathbf{C}^{-1} + 2\rho_0\left(\frac{\partial W_e}{\partial I_1} + I_1 \frac{\partial W_e}{\partial I_2}\right)\mathbf{I} - 2\rho_0 \frac{\partial W_e}{\partial I_2}\mathbf{C} \\ &+ 2\frac{\partial W_v}{\partial J_1}\mathbf{I} + 4\frac{\partial W_v}{\partial J_2}\dot{\mathbf{C}} + 6\frac{\partial W_v}{\partial J_3}\dot{\mathbf{C}}^2 \\ &+ 2\frac{\partial W_v}{\partial J_4}\mathbf{C} + 2\frac{\partial W_v}{\partial J_5}\mathbf{C}^2 \\ &+ 2\frac{\partial W_v}{\partial J_6}(\mathbf{C}\dot{\mathbf{C}} + \dot{\mathbf{C}}\mathbf{C}) + 2\frac{\partial W_v}{\partial J_7}(\mathbf{C}^2\dot{\mathbf{C}} + \dot{\mathbf{C}}\mathbf{C}^2),\end{aligned} \tag{6}$$

where I_1–I_3 and J_1–J_7 are the strain and strain rate invariants, respectively. p is the hydrostatic pressure, and represents the indeterminate part of the stress arising due to the constraint of incompressibility. $\mathbf{I}$ is the identity tensor. For details, see Appendix. The constitutive law (6), with verification of inequality (5), is a rather general isotropic, incompressible, viscoelastic law which is fully compatible with the principles of thermodynamics and is valid for large deformations. The originality of the present formulation resides in the fact that different elastic and viscous potentials can be tested in one framework given by Eq. (6).

Results of tensile tests made on isolated cruciate ligaments (ACL and PCL) and patellar tendons (PT) (four males, mean age: 74.8 ± 2.1 yr) with their bony insertions were used for the identification of the constitutive law. The tests were performed under controlled temperature (37°C) and humidity (100%) at four different rates of elongation (0.3, 6, 9 and 12 mm s^{-1}). The specimens are preconditioned with the 0.3 mm s^{-1} elongation rate. The first test was started 30 min after preconditioning. In order that the specimens were in an identical mechanical situation for each test, a 30 min interval separated each test (Hubbard and Chun, 1988). At the end of the tests, the specimens were reloaded with the 0.3 mm s^{-1} rates of elongation to assess the reproducibility of the tests. More details of the experimental procedures can be found elsewhere (Pioletti, 1997; Pioletti et al., 1996).

Only the deformation gradient $\mathbf{F}$ and the nominal stress $\mathbf{P}$ are accessible through experimental measurements. Direct relations link the stress and strain variables used in the experimental and theoretical parts: $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ and $\mathbf{P} = \mathbf{F}\mathbf{S}$ (T for transposition). Use of the strain $\mathbf{C}$ and the stress $\mathbf{S}$ in the theoretical description is a convenient method to obtain a frame invariant constitutive law necessary for large deformations.

The elastic stress–strain curve is defined by the result of the test made at 0.3 mm s^{-1}. The elastic potential W_e proposed by Veronda (Veronda and Westmann, 1970) closely fits the non-linear elastic stress–strain curve

$$W_e = \alpha \exp[\beta(I_1 - 3)] - \frac{\alpha\beta}{2}(I_2 - 3), \tag{7}$$

where α and β are two elastic parameters. According to Eq. (4), the viscous potential W_v is obtained by the difference between the elastic stress–strain curve (here obtained from the $0.3\ \mathrm{mm\,s^{-1}}$ test) and the stress–strain curves obtained at higher rates of elongation (6, 9 and $12\ \mathrm{mm\,s^{-1}}$). The viscous potential has to be convex in $\dot{\mathbf{C}}$ and equal to zero when $\dot{\mathbf{C}} = 0$ to be thermodynamically acceptable (Eq. (5)). Hence, we propose the following original viscous potential to satisfy these two requirements

$$W_v = \frac{\eta}{4} J_2(I_1 - 3), \tag{8}$$

where η is a viscous parameter. Incorporation of the elastic and viscous potentials in Eq. (6) then furnishes the particular viscoelastic law applicable to the ligaments and tendons

$$\mathbf{S} = p\mathbf{C}^{-1} + \alpha\beta(2\exp[\beta(I_1 - 3)] - I_1)\mathbf{I} + \alpha\beta\mathbf{C} + \eta(I_1 - 3)\dot{\mathbf{C}}. \tag{9}$$

A least-squares fit of the experimentally obtained stress–strain curves furnishes the values of the parameters α, β and η.

3. Results

The strain rate effect is visible in the experimental stress–strain curves obtained from the human ACL, PCL and PT (Fig. 1a–c). Using the proposed constitutive law, the experimental stress–strain curves obtained at different strain rates are well fitted (Fig. 1a–c). Fig. 1 then illustrates the ability of the developed constitutive law to model the strain rate effect in ligaments and tendons. The mean elastic and viscous parameters are reported in Table 1.

4. Discussion

The development of a constitutive law that takes into account the strain rate effect in soft tissues is presented. Using a new viscous potential, the strain rate is considered as an explicit variable. The resulting constitutive law, valid for large deformations and thermodynamically correct, closely fits the stress–strain curves obtained at different strain rates.

Mechanical isotropy is assumed in the constitutive law. However, for a more comprehensive model, a transversely isotropic hypothesis would be more appropriate for ligaments and tendons tissue. In view of the specimens' dimensions, transverse tractions are difficult to apply without severe artifacts which will result in an imprecise parameters' identification. This experimental limitation led to use of the isotropic hypothesis.

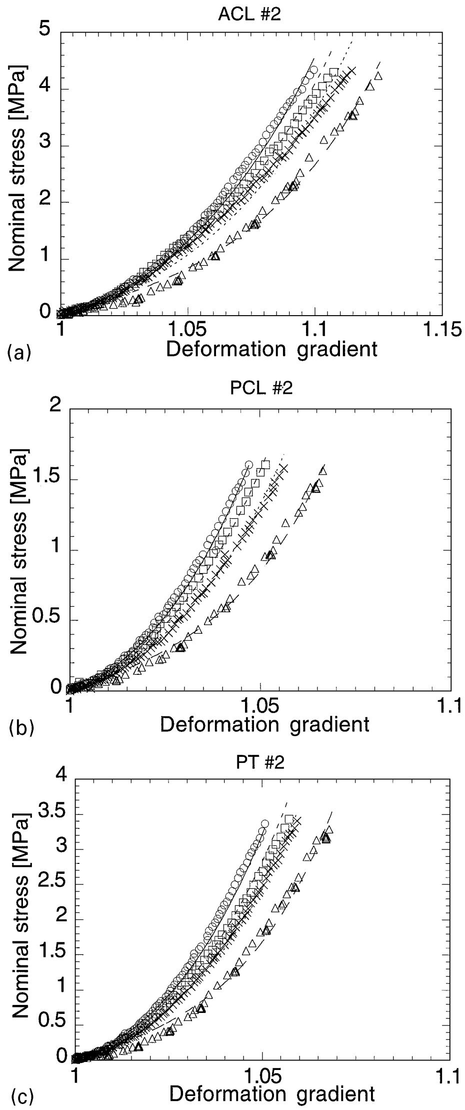

Fig. 1. Experimental and theoretical stress–strain curves obtained at four different rates of elongation for one human ACL (a), PCL (b), and PT (c) specimen. Good correlation between experimental and theoretical curves are found. The proposed constitutive law with only three parameters is then able to fit non-linear stress–strain curves obtained at different strain rates: (○) exp. $12\ \mathrm{mm\,s^{-1}}$; (—) theor. $12\ \mathrm{mm\,s^{-1}}$; (□) exp. $9\ \mathrm{mm\,s^{-1}}$; (– – – –) theor. $9\ \mathrm{mm\,s^{-1}}$; (×) exp. $6\ \mathrm{mm\,s^{-1}}$; (- - - - -) theor. $6\ \mathrm{mm\,s^{-1}}$; (△) exp. $0.3\ \mathrm{mm\,s^{-1}}$; (– –) theor. $0.3\ \mathrm{mm\,s^{-1}}$.

Table 1
Mean viscoelastic parameters ± SE

Human	α (MPa)	β	η (MPa s)
ACL	0.30 ± 0.08	12.20 ± 2.18	39.29 ± 10.98
PCL	0.18 ± 0.05	17.35 ± 8.82	48.67 ± 35.20
PT	0.09 ± 0.02	66.96 ± 12.99	438.13 ± 232.20

The physical motivation behind the assumption of incompressibility is that ligaments and tendons are primarily composed of water which is nearly incompressible. The incompressible hypothesis is a convenient way to simplify the identification process and has been used in several works (Demiray, 1972; Veronda and Westmann, 1970; Weiss, 1994). A recent study has shown that the water contained in the ligament can flow out of the specimen during tensile stretch (e.g. Chen et al., 1993). The incompressibility assumption can then give an imprecise description of the strain rate effect if a large amount of water flows out of the specimens.

In soft tissue biomechanics, a precise description of the strain rate effect with a constitutive law valid for large deformation and thermodynamically correct was missing. The extension of the quasi-linear viscoelastic theory of Fung (e.g. Fung, 1993) gives an imprecise description of the strain rate effect (Woo et al., 1981). More recent theories like the single integral finite strain viscoelastic model (Johnson et al., 1996) do not incorporate the strain rate effect. In contrast, the present constitutive law is developed with the strain rate as an explicit variable and then furnishes a more precise description of its effect. The present model is not based on structural considerations as some other studies (e.g. Lanir, 1979). An extension of the model described here could therefore be to determine a correlation between the geometrical structure of the specimens and the three parameters of the constitutive law.

Formulation of constitutive laws with invariant tensors is a convenient and powerful method of describing biological tissue properties in the presence of such irreversible processes as viscosity or plasticity (e.g. Rakotomanana et al., 1992). The resulting general constitutive law (Eq. (6)) has the ability to describe the strain rate effect for other soft or hard tissues. The strain rate effect could be modeled by identifying different mathematical forms of the viscous potential (Eq. (8)) or by using supplementary strain rate invariants.

The developed constitutive law, with only three parameters, has the advantage of being as simple as rheological spring and dashpot models. However, the present constitutive law is thermodynamically valid and meaningful for large deformations which is not generally the case for rheological models. The proposed constitutive law can then be used for soft tissue biomechanics regardless of the strain and rotation values. This law can be incorporated into a finite element program to realistically model the mechanical behavior of ligaments and tendons.

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Appendix A

In case of isotropy, the elastic potential, which depends on one symmetric second-order tensor ($\mathbf{C}$), can be expressed as a function of three strain invariants (Boehler, 1987)

$$W_e = W_e(I_1, I_2, I_3). \tag{A.1}$$

The three strain invariants used in the present study are:

$$I_1 = \operatorname{tr}\mathbf{C}, \quad I_2 = \tfrac{1}{2}((\operatorname{tr}\mathbf{C})^2 - \operatorname{tr}\mathbf{C}^2), \quad I_3 = \det\mathbf{C} = 1 \tag{A.2}$$

tr and det are the trace and the determinant, respectively. $I_3 = 1$ is due to the incompressibility assumption. The viscous potential depends on two symmetric second-order tensors ($\mathbf{C}$ and $\dot{\mathbf{C}}$). The general isotropic representation of the viscous potential involves ten invariants (Boehler, 1987)

$$W_v = W_v(I_1, I_2, I_3, J_1, J_2, J_3, J_4, J_5, J_6, J_7) \tag{A.3}$$

with

$$\begin{aligned} &J_1 = \operatorname{tr}\dot{\mathbf{C}}, \quad J_2 = \operatorname{tr}\dot{\mathbf{C}}^2, \quad J_3 = \operatorname{tr}\dot{\mathbf{C}}^3, \quad J_4 = \operatorname{tr}(\mathbf{C}\dot{\mathbf{C}}) \\ &J_5 = \operatorname{tr}(\mathbf{C}^2\dot{\mathbf{C}}), \quad J_6 = \operatorname{tr}(\mathbf{C}\dot{\mathbf{C}}^2), \quad J_7 = \operatorname{tr}(\mathbf{C}^2\dot{\mathbf{C}}^2). \end{aligned} \tag{A.4}$$

The isotropic stress is obtained by derivation of W_e with respect to $\mathbf{C}$ and W_v with respect to $\dot{\mathbf{C}}$ with W_e and W_v given by (A.1) and (A.3), respectively:

$$\begin{aligned} \mathbf{S} = &-p\mathbf{C}^{-1} + 2\rho_0\left(\frac{\partial W_e}{\partial I_1} + I_1\frac{\partial W_e}{\partial I_2}\right)\mathbf{I} - 2\rho_0\frac{\partial W_e}{\partial I_2}\mathbf{C} \\ &+ 2\frac{\partial W_v}{\partial J_1}\mathbf{I} + 4\frac{\partial W_v}{\partial J_2}\dot{\mathbf{C}} + 6\frac{\partial W_v}{\partial J_3}\dot{\mathbf{C}}^2 + 2\frac{\partial W_v}{\partial J_4}\mathbf{C} \\ &+ 2\frac{\partial W_v}{\partial J_5}\mathbf{C}^2 + 2\frac{\partial W_v}{\partial J_6}(\mathbf{C}\dot{\mathbf{C}} + \dot{\mathbf{C}}\mathbf{C}) \\ &+ 2\frac{\partial W_v}{\partial J_7}(\mathbf{C}^2\dot{\mathbf{C}} + \dot{\mathbf{C}}\mathbf{C}^2), \end{aligned}$$

where p is the hydrostatic pressure and $\mathbf{I}$ the identity tensor. Of course, a representation with less invariants can be obtained from the identification process.

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