Primordial constraint on the spatial dependence of the Newton constant

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A Nordtvedt effect at cosmological scales affects the acoustic oscillations imprinted in the cosmic microwave background. The *gravitational baryonic mass density* of the universe is inferred at the first peak scale from WMAP data. The independent determination of the *inertial baryonic mass density* through the measurement of the deuterium abundance in the framework of standard big bang nucleosynthesis leads to a new constraint on a possible violation of the strong equivalence principle at the recombination time.

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The cosmic microwave background (CMB) anisotropies provide a unique laboratory for achieving precision cosmology. The recent analyses of the corresponding temperature (and polarization) power spectrum, combined with other cosmological tests, lead to a coherent picture of the structure, energy content, and evolution of our universe. The corresponding cosmological parameters are already determined with a rather high precision by the one-year WMAP data [1, 2]. However, in this context full credit may not be given to the concordance cosmological model before the theoretical hypotheses on which it is based are tested, notably through a thorough analysis of the CMB. The inflation scenario [3, 4] and the cosmological principle must be questioned [4–6], as well as, perhaps most fundamentally, the theory of gravitation itself on which cosmology is developed, namely general relativity. In this letter we investigate the effect of a strong equivalence principle violation induced by the spatial variation of the newtonian gravitational coupling at cosmological scales. The corresponding cosmological Nordtvedt effect on the CMB provides a new test of general relativity, at the recombination time.

The equivalence principle, postulating the universality of free fall, is an important fundament of any theory of gravitation. It is however implemented at different levels in different theories, thus distinguishing them from one another in their most fundamental structure. This distinction may be structured in terms of the already deeply discussed question of the spacetime variation of fundamental coupling constants [7], such as the fine structure constant α for electromagnetic interactions (but also the speed of light c, the weak and strong interaction couplings, etc.) or the newtonian gravitational constant G. This spacetime variability of coupling constants is natural in the framework of the present unified theories for the fundamental interactions, such as string theories. In this theoretical framework, auxiliary fields of gravitation indeed appear beyond the tensor metric field, upon which the fundamental coupling constants naturally depend. The variation of α in gravitational fields violates the universality of non-gravitational experiments in free fall, the so-called Einstein equivalence principle. Many constraints have been established on the variation of α at low redshifts ($z \leq 4$), at the big bang nucleosynthesis (BBN) epoch, and lately at recombination time through the analysis of its influence on the CMB anisotropy spectra [8]. Recent evidence for a time variability of the fine structure constant has been found through the analysis of quasar absorption line spectra, while the same data indicate no spatial variation [9]. Any spacetime variation of G violates the universality of gravitational experiments in free fall, known as the strong equivalence principle (SEP). This principle actually distinguishes general relativity from any other theory of gravitation of interest, such as scalar-tensor or vector-tensor alternatives [10, 11]. The time variation of G originally proposed by Dirac has been extensively analyzed, leading to constraints at present times, at the BBN epoch, and lately as well at recombination time through the analysis of the CMB [12–17]. The possible scale dependence of G has been envisaged [18–21]. Recent studies also contemplate a dependence of the strength of the gravitational coupling on the nature of interacting particles [22, 23]. Here, we consider the explicit spatial dependence of the Newton constant, never yet studied at cosmological scales.

If the newtonian gravitational coupling is a function of the position x in spacetime, $G \rightarrow G(x)$, the mass m of a compact body also depends on the position through its internal gravitational binding energy. An effective action for the geodesic motion of compact bodies may therefore be defined as $S_{mat} = -c \int m(x) ds$. Energy-momentum conservation is therefore broken through the introduction of a source term in the general covariant conservation equations. We adopt the corresponding covariant expression as our mathematical implementation of a possible SEP violation:

$$T^{\mu\nu}{}_{;\nu} = G^{,\mu} \frac{\partial T}{\partial G} \quad , \tag{1}$$

where T is the trace of the energy-momentum tensor $T^{\mu\nu}$. This relation defines a simple modification of the theory

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of gravitation, which singles out the spacetime dependence of the Newton constant as the unique perturbation to the gravitational interaction defined in general relativity. In the following, we restrict ourselves to a pure spatial dependence of the newtonian coupling.

The dependence of the gravitational coupling on the spatial position $\vec{x}$ is parametrized through the relation $G(\vec{x}) = G_0(1 + \eta_g V(\vec{x})/c^2)$, where $V(\vec{x})$ stands for the gravitational potential at the point considered, G_0 is the background value of the gravitational constant in the absence of this potential, and η_g is the amplitude of the SEP violation. Let us define the compactness s of a body as the sensitivity of its mass relative to G. It is equivalently given by the ratio of its internal gravitational binding energy E_g to its total mass energy: $s = -d\ln m/d\ln G = |E_g|/mc^2$. From the definition (1), one may easily show that the newtonian acceleration of a body in a gravitational field now explicitly depends on its proper sensitivity s. In other words, the SEP violation induces a departure of the gravitational mass m_g of a body relative to its inertial mass m, proportionally to its own compactness: $m_g = m(1 - \eta_g s)$. The SEP violation defined in (1) therefore reduces to the well-known Nordtvedt effect [12, 24], once we neglect the possible time variation of G.

We now have to understand this effect on cosmological grounds and analyze its particular implication for the CMB physics. In the primordial universe the photon gas may be considered to be tightly coupled to baryons through the interplay of Compton scattering and Coulomb interaction. We may therefore consider a photon-baryon plasma in the gravitational potentials produced by the dominant cold dark matter component of the expanding universe. The cosmic microwave background radiation observed today corresponds to a snapshot of the photon gas decoupled from the rest of the universe at the time of the last scattering. The structure of the anisotropy distribution on the sky today is defined by the multiple physical phenomena which governed the evolution of the plasma before recombination. The well known acoustic peaks in the corresponding temperature power spectrum originate from electromagnetic acoustic oscillations of the photon gas. Odd and even peaks respectively correspond to scales which had reached maximum compression and rarefaction at the time of last scattering in potential wells (conversely in potential hills). However, the action of gravity is also introduced through a purely newtonian coupling of the baryonic content of the plasma to the dark matter potentials. The effect of this coupling is to shift the equilibrium point of the oscillations toward more compressed states in potential wells (rarefied states in potential hills). Consequently, the height of odd peaks relative to even peaks is enhanced proportionally to the total baryon weight in the dark matter potentials [25–28]. The temperature power spectrum peaks height therefore bears the imprint of a possible SEP violation through the Nordtvedt effect as it essentially originates from a gravitational interaction and thus depends on a gravitational, rather than inertial, baryonic mass density:

$$R_g\left(s_b, \eta_g\right) \;=\; R\left(1 - \eta_g s_b\right) \quad . \tag{2}$$

The compactness s_b is now associated with a baryon-region seen as a homogeneous (under the hypothesis of the cosmological principle) compact body at the relevant cosmological scale. The canonical variable $R = 3\rho_b/4\rho_\gamma$ stands for the baryonic density ρ_b normalized by the photon density ρ_γ, as it still appears in the continuity and Euler equations derived from (1) for the evolution of the photon-baryon plasma [29]. For simplicity, the SEP violation parameter is assumed to be constant throughout the cosmological evolution before recombination: $\eta_g = \eta_g^*$, where the superscript * evaluates quantities at the recombination time. This approximation is natural in the framework of string-inspired theories.

The compactness of a homogeneous spherical baryon-region of radius L and total mass M_b, calculated as the ratio of the internal gravitational binding energy over the total mass energy reads: $s_b = 3GM_b/5Lc^2 = 4\pi G\rho_b L^2/5c^2$. At each instant in the course of the universe expansion, the maximal size of the radius L is set by the event horizon. This hypothesis is natural as the event horizon defines at each moment the maximal distance through which particles may have interacted gravitationally since the primordial ages of the universe (after inflation), and therefore the maximal size of a cosmological body. For the sake of the analogy with the Nordtvedt effect on compact bodies in a gravitational field, we consider in the following a constant compactness over the course of the universe evolution until recombination: $s_b = s_b^*$. Assuming that the time dependent Friedmann-Lemaître equations remain essentially unchanged, one may justify this hypothesis, knowing that recombination takes place inside the matter era. In terms of physical quantities (the Hubble constant, the age of the universe and the relative baryon density), we then get for the maximal radius

$$s_b^{1*} \;=\; \frac{27}{10}\left(H^0 t^0\right)^2 \Omega_b \simeq 0.1 \quad , \tag{3}$$

where the superscript ^{0} evaluates quantities at the present time. The low baryon density is indeed largely compensated by the cosmological scales involved to give a non-negligible compactness. This compactness is the sensitivity to be considered at the scale of the wavelength λ_1 associated with the first acoustic peak. The sensitivity of the baryonic body relevant for the subsequent acoustic peaks (λ_n) scales like n^{-2}: $s_b^{n*} \simeq 0.1 n^{-2}$.

The independent measurements of both the gravitational baryonic mass density of the universe and its inertial counterpart lead to a constraint on the SEP in terms of the cosmological Nordtvedt effect defined in (2). On the one hand, the value for the parameter $\Omega_b h^2$ obtained from CMB data, is understood in first approximation as a measurement of the relative height of the temperature power spectrum odd and even peaks. The small contributions of the inertial baryonic content to the power spectrum, notably through the sound speed in the primordial plasma itself affecting the peaks position, are neglected in this approximation. The one-year WMAPext results (i.e. WMAP extended to the CBI and ACBAR experiments) give $\Omega_b h^2 = (22 \pm 1) \times 10^{-3}$, essentially measuring the relative height of the first two peaks [1, 2]. The corresponding value for the *gravitational baryonic mass density* $R_g^*(s_b^{1*}, \eta_g^*)$ of the universe at last scattering, and at a scale corresponding to the maximum oscillation wavelength therefore reads: $R_g^* = 0.613 \pm 0.028$. On the other hand, from the determination of light element (D, ^3He, ^4He, ^7Li) abundances, the standard BBN theory may infer the *inertial baryonic mass density* of the universe, essentially counting nuclei on astrophysical scales and through non-gravitational interactions. The deuterium abundance is extremely sensitive to the primordial baryon content. Moreover it may only have been produced in significant quantities during BBN. Its measurement in quasar absorption line systems is therefore an extremely good probe of the baryon content of our universe. The most recent estimate of the primordial deuterium-to-hydrogen abundance ratio reads: $D/H = 2.78^{+0.44}_{-0.38} \times 10^{-5}$ [30]. The corresponding baryon content is given through standard BBN by $\Omega_b h^2 = (21.4 \pm 2) \times 10^{-3}$, or $R^* = 0.596 \pm 0.056$. Combined with the one-year WMAPext value, this measure gives the first constraint on a possible violation of the SEP through a cosmological Nordtvedt effect:

$$\eta_g^* = -0.3 \pm 1 \quad . \tag{4}$$

Large systematic uncertainties still affect the other light element abundance estimation, therefore leading to less reliable assessments [31]. These measurements, taken at face value only affected by statistical errors, would indicate a sizeable SEP violation.

To be more accurate, the constraint (4) should be determined through a best fit of our modified theory (1) and present experimental data, taking into account the substitution (2) in the plasma evolution equations before recombination. Also notice that, in the more complete approach of a specific scalar-tensor or vector-tensor alternative to general relativity, our cosmological Nordtvedt effect would no longer remain the only new effect. The introduction of auxiliary gravitational fields indeed affects the fundamental nature of gravitation and notably leaves complex signatures in the CMB [14–16] as well as in the BBN [14, 32, 33]. This would inevitably modify the proposed constraint. In such a framework, the bound on η_g^* could also be run backward or forward over cosmological timescales in terms of the evolution of the auxiliary fields themselves. This would allow its comparison, either with theoretical predictions on initial conditions (string theories suggest a violation parameter of order unity at the outset of the radiation era), or with present experimental constraints ($\eta_g^0 \leq 1 \times 10^{-3}$ [12]). Finally, other implications of a cosmological Nordtvedt effect should also be studied beyond its impact on the CMB, at different epochs of the universe evolution.

The present considerations are further developed on the ground of theory and data analysis in [29].

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