A comparison of different quantization strategies for subband coding of medical images

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ABSTRACT

In this paper different methods for the quantization of wavelet transform coefficients are compared in view of medical imaging applications. The goal is to provide users with a comprehensive and application–oriented review of these techniques.

Coding methods based on subband transforms are commonly used for image compression in a wide range of applications. In recent years, wavelet based methods have also been applied to medical images, and the first commercial implementations of such techniques have appeared.[1] The issues related to clinical reliability of processed images, together with additional features which will be offered by future coding schemes render the conventional evaluation criteria (e.g. PSNR) insufficient. In this paper, the performance of four quantization methods (namely standard Scalar Quantization , Embedded Zerotree, Variable Dimension Vector Quantization and Pyramid Vector Quantization) are compared with regard to their application in the field of medical imaging. In addition to the standard rate–distortion criterion, we took into account the possibility of bitrate control, the feasibility of real-time implementation, the genericity (for use in non-dedicated multimedia environments) of each approach. In addition, the diagnostical reliability of the decompressed images has been assessed during a viewing session and the help of a specialist.

Classical scalar quantization methods are briefly reviewed. As a result, it is shown that despite the relatively simple design of the optimum quantizers, their performance in terms of rate–distortion tradeoff are quite poor. For high quality subband coding, it is of major importance to exploit the existing zero–correlation across subbands as proposed with the embedded zerotree wavelet (EZW) algorithm. This approach is based on four main blocks: 1) a hierarchical subband decomposition, 2) prediction of the absence of significant information across scales using zerotrees, 3) entropy–coded successive–approximation quantization, and 4) lossless source coding via adaptive arithmetic coding. In this paper an improved EZW–algorithm is used which is termed Embedded Zerotree Lossless (EZL) algorithm –due to the importance of lossless compression in medical imaging applications– having the additional possibility of producing an embedded lossless bitstream.

VQ based methods take advantage of statistical properties of a block or a vector of data values, yielding good quality results of reconstructed images at the same bitrates. In this paper, we take in account two classes of VQ methods, random quantizers (VQ) and geometric quantizers (PVQ). Algorithms belonging to the first group (the most widely known being that developed by Linde-Buzo-Gray) suffer from the common drawback of requiring a computationally demanding training procedure in order to produce a codebook. The second group represents an interesting alternative, based on the multidimensional properties of the distribution of the source to code. In particular a Pyramid Vector Quantization has been taken into account. Despite being based on the implicit geometry of independent and identically distributed (i.i.d.) Laplacian sources, this method proved to achieve good results with other distributions.

Tests show that zerotree yields the most promising results in the rate–distortion sense. Moreover, this approach allows an exact rate control and has the possibility of a progressive bitstream which can be used either for data browsing or up to a lossless representation of the input image.

Keywords: subband coding, quantization, medical imaging, image compression.

1 INTRODUCTION

Image compression is one of the most typical applications of digital signal processing. The great number of medical imaging techniques, together with the increasing importance of digital imaging in radiology departments makes the compression of medical images, both for transmission and for storage purposes, more and more important. The evaluation criteria which are normally used for the validation of compression techniques -such as the PSNR- prove insufficient when applied to medical imaging. It goes without saying that the artifacts which can be introduced by the coding-decoding loop might be unacceptable for the diagnostic reliability of an image, and the same applies for the loss of visual information. The subjective rating of physicians and extensive diagnostic accuracy tests are needed for a thorough evaluation of compression methods.[1]

In addition to that, the development of multimedia environments (and a Picture Archiving and Communicatyion System, PACS, can be considered as a special example of it) makes it necessary to take into account other aspects of the techniques under evaluation, such as the possibility of accurate bitrate control, the gerericity of the method, its flexibility in view of its implementation in non–dedicated environments (such -for example- as a PC in a remote location for teleradiology applications).

The above mentioned considerations show that the validation of newly developed techniques should extend to different aspects of the compression scheme, and also a review of existing methods according to more variate criteria is desirable.

As well known, coding methods based on subband transforms are commonly used for image compression in a wide range of applications. In recent years, wavelet based methods have also been applied to medical images, and the first commercial implementations of such techniques have appeared.[1] In this paper different methods for the quantization of wavelet transform coefficients are compared in view of medical imaging applications. In particular, we take into consideration four methods, namely standard Scalar Quantization (SQ), Variable Dimension Vector

Quantization (VQ), Embedded Zerotree (ZTR) and Pyramidal Vector Quantization (PVQ). SQ and VQ are taken as a reference, while ZTR and PVQ are more extensively described and evaluated.

The paper is organized as follows. In Sec. 2 the techniques under evaluation are described; the reference ones are briefly sketched, while the latter two are explained in deeper detail. In Sec. 3 the results of the simulations are presented, for three of the techniques graphs of the PSNR are shown. The simulation results are then compared and discussed in Sec. 4, with regard to the different criteria presented in the introduction.

2 DESCRIPTION OF TECHNIQUES

2.1 Wavelet transform

A general subband decomposition divides the input signal into different subbands. The objective of this decomposition is to achieve a high energy compaction, that is compact most of the information in few subbands. For this purpose, the choice of the filters is an important issue. It is shown in[2,3] that the filters represent an important factor in the performance of the decomposition for compression purposes.

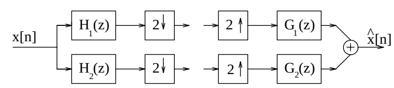

Figure 1: *Two–band analysis/synthesis filter bank system.*

Fig. 1 shows a two–band filter bank. The input/output relationship is given by

$$\hat{X}(z) = X(z)T(z) + X(-z)A(z) \tag{1}$$

where

$$T(z) = \frac{1}{2}\left[H_1(z)G_1(z) + H_2(z)G_2(z)\right] \tag{2}$$

and

$$A(z) = \frac{1}{2}\left[H_1(-z)G_1(z) + H_2(-z)G_2(z)\right]. \tag{3}$$

Perfect reconstruction can be achieved by removing the aliasing distortion, $A(z)$, and imposing the transfer function, $T(z)$, to be a pure delay of the form $T(z) = z^{-\delta}$, where δ is the delay of the system. By choosing the synthesis filters as $G_1(z) = H_2(-z)$ and $G_2(z) = -H_1(-z)$ the aliasing component is removed. Under these constraints the system transfer function becomes

$$T(z) = F(z) + F(-z) \tag{4}$$

where the product filter is $F(z) = H_1(z)H_2(-z)$. Perfect reconstruction is obtained when the product filter, $f(k) = \mathcal{Z}^{-1}(F(z))$, is a power–complementary half–band filter. This means that every odd sample, except one sample $f(\delta)$, is equal to zero, that is

$$f(2n+1) = \begin{cases} 1/2 & 2n+1 = \delta \\ 0 & otherwise \end{cases} \qquad n = 0, 1, 2, \ldots, \frac{L-1}{2} \tag{5}$$

where L is the length of the product filter $F(z)$. All filters satisfying these equations enjoy the perfect reconstruction property.

The wavelet decomposition is a special case of a subband decomposition with two–band filter banks for which at each step only the lowest (DC) band is re-decomposed. The main property that the wavelet decomposition exploits is that in most pictures the energy is concentrated in the low frequency bands and hence a finer resolution in these frequency ranges is needed to augment the energy compaction.

The design of the subband filters is an important issue. Linear phase and quasi–perfect-reconstruction filters have been proposed by Johnston[4] and are called Quadrature Mirror Filters (QMFs). It has been shown that from this family of filters the filter bank "16B" is the most appropriate for image coding. Other filters known as AFB, Asymmetrical Filter Banks, have been proposed which introduce less ringing effect.[2] They are especially optimized for the wavelet decomposition. Based on intensive simulations we decided that the AFB filters were best suited for the applications of this paper.

2.2 Scalar Quantization

Scalar quantization is the most straightforward quantization method, and is presented in this work as a reference model. The approach followed for the tests presented in this paper is depicted in Fig. 2a, and was proposed in as a coding scheme for intra frames in a digital video codec for medium bitrate transmission.[5]

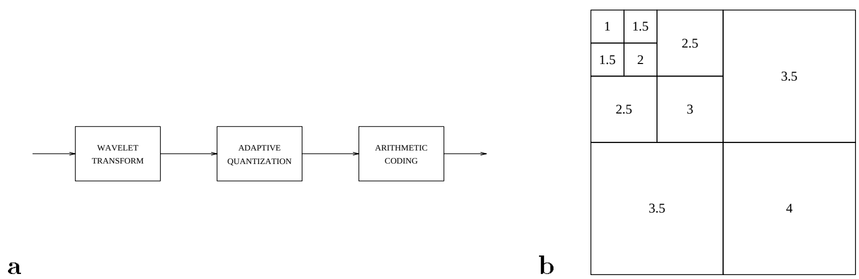

Figure 2:**a)** SQ coding scheme. **b)** SQ weighting matrix.

The input images are wavelet transformed by means of the filters described in section 2.1. The wavelet transformed coefficient are then scalar quantized with a different step according to the subband to which they belong. The ratios between the quantization step that was used for the lowest frequency band and the coarser ones used in the other subbands for a 3 level wavelet transform are shown in table 2b.

After the quantization, coefficients are eventually coded by means of an arithmetic coder. The scalar quantization scheme, when analyzed according to the criteria sketched in Sec. 1, proves relatively simple, compared to the other methods, as far as the implementation is concerned. Unfortunately, the performance of this coding scheme is quite poor, and suffers from two major drawbacks:

- The performance in compression of this scheme is mainly related to the characteristics of the entropy coder; the high degree of correlation among the transform coefficients both within each band, both across different subbands is not taken into account and exploited.
- The control of the bitrate is not straightforward: the influence of the quantization steps on the compression rate is biased by the action of the arithmetic coder, and the adaptation of the weights for each subband (which can add some freedom in the determination of the compression rate) is not easy.

2.3 Embedded Zerotree Wavelet Algorithm

The Embedded Zerotree Wavelet Algorithm (EZW) has been proposed by Shapiro.[6] It is based on a wavelet decomposition followed by a successive–approximation quantization procedure which predicts the insignificant information across scales with zerotrees of wavelet coefficients. This simple procedure can be summarized as follows. After a standard wavelet transform an initial threshold is determined. Then, a scan through all the wavelet coefficients produces a symbol stream consisting of the following four symbols:

1. POS: If the coefficient is significant with respect to the threshold and positive.
2. NEG: If the coefficient is significant with respect to the threshold and negative.
3. ZTR: The symbol zerotree root is used if the wavelet coefficient is insignificant with respect to the threshold and all his children are insignificant, too. The parent–children relationship is shown in Fig. 3.
4. IZ: The symbol isolated zero is used for all other wavelet coefficients.

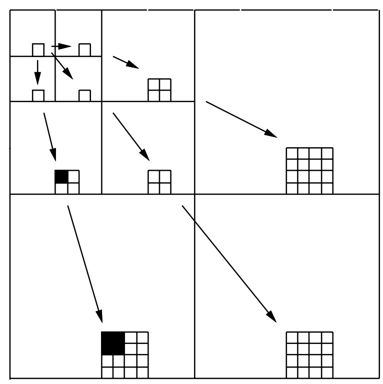

Figure 3: *Parent–Children relationship.*

After each iteration the threshold is divided by two and a new iteration will refine the quantized coefficients. The symbol stream is then encoded using an arithmetic coder to produce a completely embedded bitstream.

The EZW algorithm has shown excellent performances in compressing natural images at low data rates. In fact, it outperforms JPEG for most of the test images.

2.4 Vector Quantization

2.4.1 Introduction

Vector Quantization (VQ) takes advantage of the statistical correlation among data samples. Image coding using vector quantization has emerged as a powerful alternative for image compression.[7] It is well known how VQ can improve performance over Scalar Quantization (SQ)[8,9] at high compression rates. VQ performs quantization on several samples simultaneously and achieves a performance closer to the rate distorsion bound, according to the result of Shannon's rate distortion theory: a better performance can be achieved when the data source has memory, and even if it is memoryless. The most important characteristics of VQ is the exploitation of the statistical dependencies present inside the vectors, allowing to take into account linear and non-linear correlation between the scalars. For this reason, increasing the vector dimension can generally increase the coding efficiency. On the other hand, the computational and storage complexity is an exponential function of the dimension and they both represent the major drawbacks in implementation of vector quantizers. Furthermore, the performance of VQ depends on the procedure that is followed for the creation of the codebook, in particular on the images used in the training. This represents a major drawback in terms of loss of genericity.

2.4.2 Variable Dimension VQ

In vector quantization, the signal samples are grouped into fixed size vectors. The bitrate resulting from the VQ of the signal samples depends then on the size of the codebook, i.e. on the bits needed to address each vector in the codebook, and on the performance of the entropy coder. It is therefore difficult to fix *a priori* the compression rate, also because the address length does not vary continuously, and it is thus possible to select only a limited number of values. This is indeed one of the major limitations of VQ when compared to the other quantization methods presented in this paper.

Variable dimension VQ represents an improvement of VQ, allowing us to enjoy a higher degree of freedom in the choice of the compression rate, and at the same time to better adapt the codebooks to the characteristics of the images (or of the wavelet subbands in our case).

The input data are mapped into a sequence of vectors of variable dimension, each represented by a pair $Cdbk_i, Add_i$, in which $Cdbk_i$ indicates the codebook, Add_i the address of the chosen vector in that particular codebook. As an example, when VQ is used to code grey level images, a simple criterion for the creation of the codebooks could be that of using smaller blocks and richer codebooks for highly detailed areas, while coding larger uniform areas with fewer and longer vectors.[10]

In our case, a different codebook has been created for each level of decomposed subbands, according to the scheme of Fig. 4.

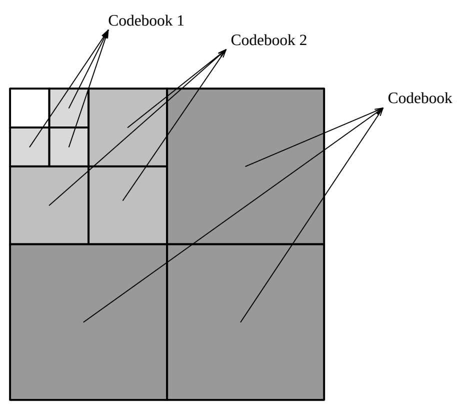

Figure 4: Creation of codebooks

As it is the case for SQ, VQ does not allow us to fix *a priori* the exact bitrate for each image, so that we decided to select the sizes of the blocks and the codebooks in order to achieve the desired average compression rate over the set of images corresponding to the two exams.

2.5 Pyramidal Vector Quantization

2.5.1 General on Pyramid Vector Quantization (PVQ)

The PVQ is based on the implicit geometry of independent and identically distributed (i.i.d.) *Laplacian sources*. It is based on the cubic lattice points that lie on the surface of a N-dimensional pyramid and it has a simple encoding and decoding algorithm. Its performance in terms of reconstructed image quality has been proved to be equal or better than the classical vector quantization methods. In particular PVQ outperforms Linde Buzo Gray (LBG) most of all in terms of computational time. The former requires no off-line codebook design and its encoding time grows linearly with rate or dimension, while the LBG algorithm has a design complexity that grows exponentially with the rate and dimension and a full-search encoding complexity that also depends exponentially on the rate distortion product. The possibility to have an on-line codebook design offers two further advantages. The first is a higher genericity: in fact the change of the input data does not require a new codebook design. The second is the possibility of working at different bit-rates, which is impossible by using LBG, since in this latter case the dimension of the codebook determins the bit-rate. Furthermore, at higher compression rates, PVQ outperforms SQ in the rate distortion sense and achieves for *Laplacian* sources the performances of a scalar quantizer with entropy coding without the disadvantage of a Variable Length Coding, which can cause synchronization problems with noisy channel.

2.5.2 Laplacian source

Let $X = (x_1, ..., x_N)$ be a vector of N independent and identically distributed Laplacian random variables with zero mean and variance $2/\lambda^2$ with probability density function:

$$p_x(x_i) = \frac{\lambda}{2} e^{-\lambda |x_i|}. \tag{6}$$

Contours of constant joint density $f_x(x)$ are specified by the condition:

$$r = \sum_{i=1}^{N} |x| = constant \tag{7}$$

which is satisfied by all vectors belonging to the surface $S(N, r)$ of a hyper-pyramid in N-dimensional space, whose dimensions depend on r (radial parameter). The scalar random variable r indexes a particular contour of constant density $f_x(X)$ or, equivalently, the pyramid $S(N, r)$. It can be shown that, for large N, vectors tend to be highly localized on the surface of the *hyper-pyramid* of maximum probability characterized by $r = N/\lambda$. This means that the distance per dimension between a generic multidimensional source realization X and an appropriate point X_r on the pyramid $S(N, N/\lambda)$ tends to zero for arbitrarily large N; the appropriate point being the one giving the minimum norm-difference. More generally if the distortion measure is based on the norm: $||X||_v = \left(\sum_{i=1}^{N} |Xi|^v\right)^{1/v}$, for $v \geq 1$, then the dimension distance (norm) between X and its closest point X_r on pyramid $S(N, N/\lambda)$ tends to zero as long as dimension N increases.[11]

2.5.3 A Pyramid Vector Quantization

A Pyramid Vector Quantization (PVQ) can be constructed based on a subset of the points in the cubic lattice (that is, the set of all the vectors with integer components). As such, the PVQ is a type of lattice quantizer, but significantly, a lattice quantizer that is not based on a uniform source probability distribution function. The construction of the PVQ will proceed in three distinct steps.

- First the number of the cubic lattice points on the pyramid is evaluated.
- Second, an encoding/decoding algorithm is developed.
- Then the resulting source code is denoted the pyramid vector quantizer.

The PVQ codewords are scaled versions of the lattice points that lie on a specific pyramid. Enumeration encoding and decoding algorithms are provided for representing the pyramid codewords as binary codewords for transmission or storage. Although asymptotic in dimension, it is sufficient to base an optimum VQ design on a single ($S(N, N/\lambda)$) pyramid, for moderate sizes of dimension distortion may be introduced in approximating X by X_r. As a final modification to the PVQ design, concentric pyramids are used to generate a hybrid scalar-vector quantizer in product code[12] form. In this design a fraction of the total bit rate is allocated to the scalar quantization of the radial random variable r with the remaining bit rate allocated to the PVQ for a normalized pyramid. The VQ construction is carried out for the *mse* distortion measure but may be readily extended to any norm-based distortion criteria.
In the cases of different statistical distributions, the procedures of distributing the codewords on the surface and coding the vectors are more difficult than in the Laplacian case (where the surface is a pyramid). Simulations show[13] that PVQ still works well also in these cases. Infact, by using PVQ (designed for Laplacian variables) on Gaussian or Generalized Gaussian ($\nu = 0.5$) is possible to obtain the same reconstructed image quality while saving computational effort.
Hereafter a simple application of PVQ, in a case where the vector has two elements, is given. Referring to the fig. 5 a, $S(N, I)$ is the surface with integer components , $S(N, r)$ is the *average* pyramid and c_i are the codewords. Following the quantization procedure, first X is projected on the pyramid $S(N, r)$ on X_1 (that is X_r). Then X_1 is further projected on the pyramid $S(N, I)$ in X_2 and represented by the nearest codeword with integer components, X_3. The components of this latter vector will be transmitted. At receiver, the inverse operations will be done obtaining the output X_4. The difference between X and X_4 represents the quantization error.

a

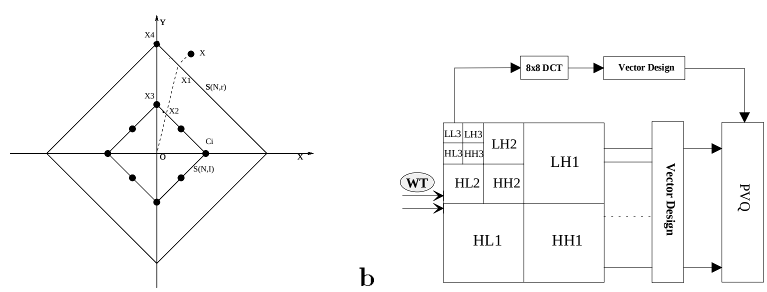

Figure 5: **a)** Codeword selection procedure. **b)** Proposed coding scheme.

2.5.4 Proposed coding scheme

As previously mentioned, a three level analysis structure has been implemented. The $LL3$ sub-image (see fig. 5b) can be seen as a low pass version of the input image, and it is coded in a different way with respect to the others that contain only details. In order to remove the correlation still present, a DCT transformation is applied on the $LL3$ band by using a 8x8 block dimension.[14] Then the elements belonging to each sub-image are collected in vectors to be quantized by PVQ. For $LL3$ sub-image the VQ input vectors are obtained by homologous DCT coefficients,[14] while for the others they are obtained considering wavelet coefficients belonging to the same row (column) depending on the direction of their main still present correlation. A two level bit-allocation strategy has been used. First the available bit-rate is sub-divided on the sub-images by taking into account both the variance of the coefficients and their correlation.[15] Then for each sub-image an allocation strategy is used to subdivide the available bits through the vectors to quantize (the dimension of the vectors is always 64).

3 SIMULATIONS RESULTS

The test images consisted of two series of slices, one corresponding to a CT scan of the liver, the second to a MR scan of the brain. Two sample slices corresponding to the two exams are shown in Fig. 6a and Fig. 6b respectively.

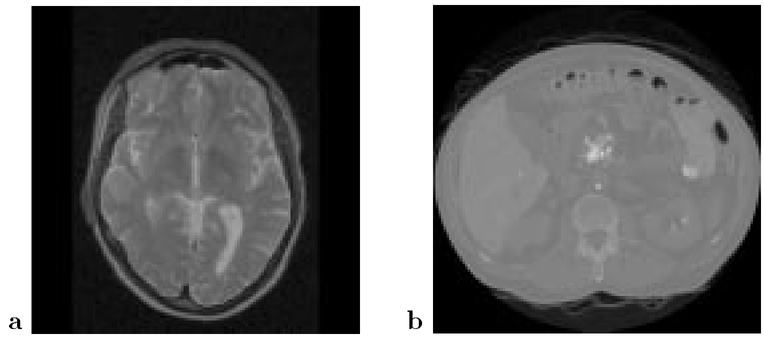

Figure 6: Examples of the test images used in the tests: **a)** MR scan of the brain; **b)** CT scan of the liver.

The two series of images were compressed at 0.5 bit/pixel and 1 bit/pixel; uit should be pointed out that for SQ and VQ these values could be achieved on ly in average, since an exact bitrate control is not allowed by these schemes.

The results of the simulations in terms of PSNR are summarized in the graphs of Fig. 7 and Fig. 8. It should be remarked that, for SQ and VQ tests, it was impossible to fix exactly the bitrate, and the parameter have been chosen so as to achieve an average value as near as possible to the one fixed for PVQ and ZTR.

It was particularly difficult to tune the coder in the case of VQ. The adopted reference method seems to be not suitable for these low compression rates: the quality of the obtained images, both in terms of PSNR and visual quality was well below that obtained by the other methods, and showed a strong dependence on the used codebooks. Due to these considerations, we do not include the results obtained by VQ in the following graphs.

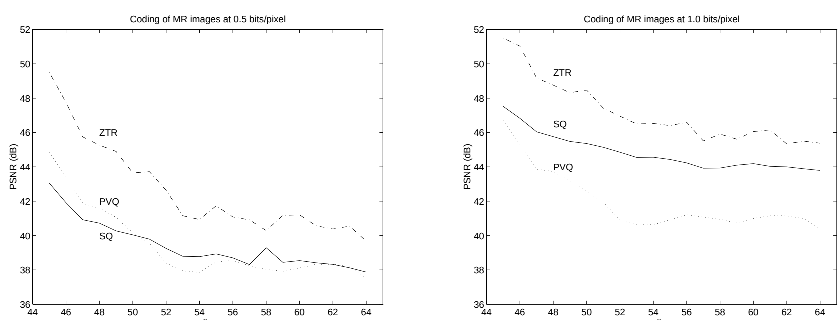

Figure 7: PSNR results for the coding of MR images at 0.5 and 1.0 bit per pixel.

4 DISCUSSION

The four quantization approaches which are described in this paper have their own merits and drawbacks. There are several factors which are important for any compression scheme. The most important ones are rate-distortion performance, rate control and computational complexity. The rate-distortion curves for three of the

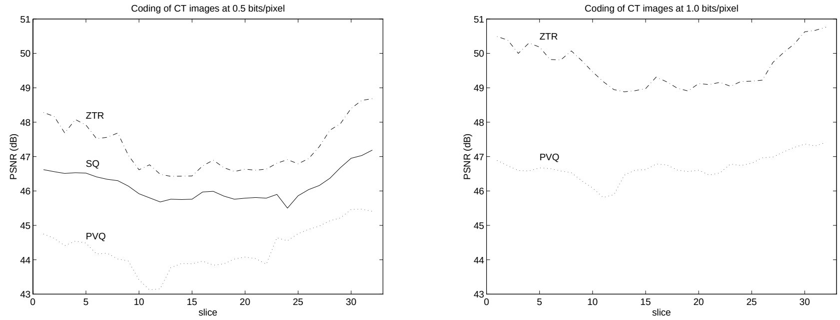

Figure 8: PSNR results for the coding of CT images at 0.5 and 1.0 bit per pixel.

methods applied on the two series of images are shown in Sec. 3. It can be concluded that the zerotree approach outperformed all other approaches in the sense of this objective measure.

As far as the visual quality is concerned, a physician was asked to rate the processed images during a viewing session. It should be stated that the viewing conditions were not controlled and the evaluation tests were not extensive. The goal of the viewing sessions was that of giving an overall judgment on the performance of the methods, as well as to identify in the test images elements which could at the same time be difficult to code and important to be correctly retrieved.

For both the series of images, methods based on vector quantization confirmed the bad results obtained in terms of PSNR: it can be said that such methods do not confirm at high bitrate the good performances they show for higher compression rates. In particular for the images coded at 1 bit/pixel, the apparent quality of the images seemed comparable to that of the images obtained by ZTR. However, by employing appropriate luminance and contrast changes, it appeared clear that, at both the tested compression rates, clinically important details were deleted or altered by VQ based methods, in particular when tiny details or texturing occurred.

Scalar quantization, although apparently more performant than PVQ in terms of PSNR, gave non acceptable results as far as the diagnostic reliability was concerned. In this case the major drawback is a global smoothing, associated with the introduction of artifacts.

Embedded Zerotree showed the best results, compared to the other methods, also in term of diagnostic reliability. Images coded at 1 bit/pixel did not show the introduction misleading artifacts, and the details that were identified as clinically important were preserved.

As an example, in the following images we show a detail of the MR from the original image (Fig. 9a), and coded with ZTR (Fig. 9b), SQ (Fig. 10a) and PVQ (Fig. 10b) at 1 bit/pixel.

Another important factor is the rate control functionality. Indeed, it is not possible to get an exact rate control with the SQ and the VQ methods. Since there are outside parameters (quantization matrices and codebook size) which affect the bitrate, these parameters have to be changed until the correct bitrate is found. On the other hand, the PVQ and the zerotree approach can fix the bitrate without an iteration. Moreover, the embedded zerotree quantization produces a progressive bitstream which could be of strong use for data browsing.

It is very difficult to measure exactly the computational complexity of a method. But it is clear that the simplest approach is the scalar quantization. The other three methods are about approximately equivalent in these terms.

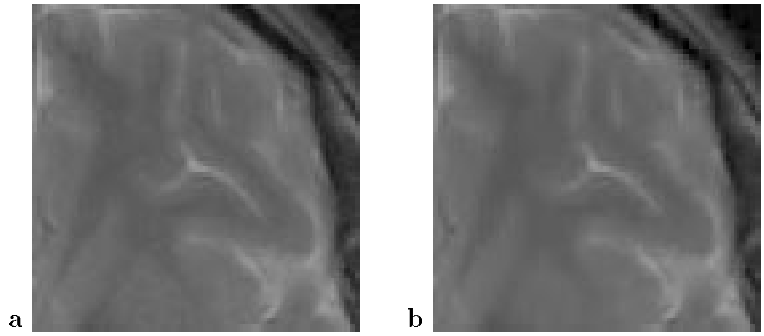

Figure 9: A detail of a MR scan slice. **a)** original; **b)** coded by ZTR at 1 bit/pixel

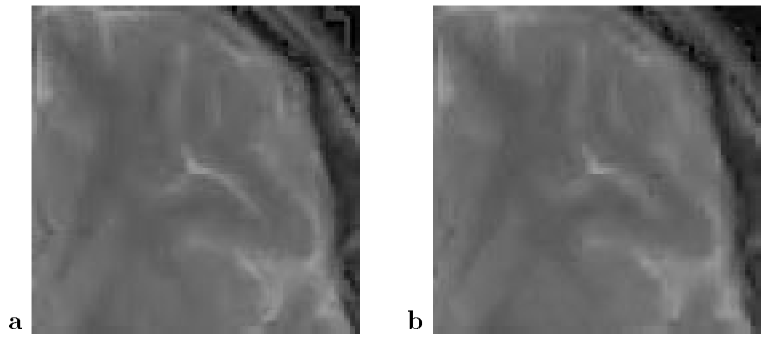

Figure 10: A detail of a MR scan slice. **a)** coded by SQ at 1 bit/pixel; **b)** coded by PVQ at 1 bit/pixel

5 CONCLUSIONS

In this paper a comparison of different quantization methods for subband coding have been compared for medical image compression. The approaches taken into account are scalar quantization, embedded zerotree successive approximation quantization, classical vector quantization and pyramidal vector quantization. For all the test images and test bitrates it can be verified that the embedded zerotree approach performed best from an objective and subjective point of view.

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