Collaborating Hearing Aids

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***Abstract*—Hearing aids are electronic, battery-operated sensing devices which aim at compensating various kinds of hearing impairments by means of appropriate signal processing. Most of today's hearing aid systems consist of two appliances working independently of each other. However, collaboration using a wireless communication link would allow to improve the overall beamforming capability of the system, hence providing better rejection of interfering signals. In this paper, the problem is considered from an information-theoretic viewpoint. We provide the necessary theoretical background to precisely quantify the gain achieved by collaboration as a function of the communication bit-rate. The beamforming capability is then discussed for the setup considered in this work.**

I. Introduction

A hearing aid system consists of two audio capture devices which acquire and process incoming signals in order to overcome some of the user's hearing deficiencies. One such task amounts to combine coherently the acquired signals in order to extract a sound source coming from a particular direction. It thus allows to mitigate the effect of interfering signals. In the array processing literature, this is commonly referred to as *beamforming* [1]. The quality of this operation depends notably on the number of microphones and the spatial extent provided by the array. In this context, the availability of a wireless communication link would make use of the natural spatial extent offered by the head in order to achieve better speech intelligibility in noisy environments [2].

In this paper, we investigate the beamforming gain provided by collaborative hearing aids as a function of the available communication bit-rate. From the perspective of one hearing device, our setup is identified as a remote source coding problem with side information at the decoder. The latter is referred to as remote, indirect or noisy Wyner-Ziv coding in the literature and has been addressed by various researchers [3], [4], [5] in the scalar case. Extension to vector sources was investigated in [6] in the context of high-rate transform coding and the corresponding rate-distortion formula can be found in [7]. To assess the gain achieved by Wyner-Ziv coding schemes, we compute the rate-distortion tradeoff obtained when the encoder disregards the side information available at the decoder. We then apply these results to our hearing aid problem. For various scenarios of interest, we provide numerical evidences of the gain achieved by this collaboration.

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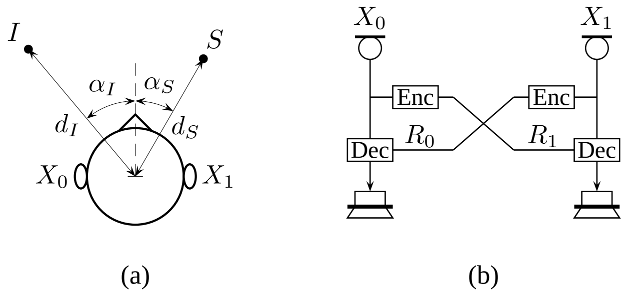

Fig. 1. Our hearing aids setup. (a) Typical head-related configuration. (b) Collaboration using a wireless communication link.

The paper is organized as follows: in Section II, we describe our hearing aids setup and identify the problem from an information-theoretic standpoint. Section III provides the necessary theoretical results. Our rate-constrained beamforming gain analysis is presented in Section IV. We finally offer some conclusions in Section V.

II. Problem Statement

The problem that we consider consists of two hearing aids, each equipped with an omnidirectional microphone, a processing unit and wireless communication capabilities. As illustrated in Figure 1 (a), the signal received at microphone k ($k = 0, 1$) can be expressed as

$$X_k(t) = S_k(t) + I_k(t) + N_k(t) \quad (1)$$

$$= h_k(t) * S(t) + \tilde{h}_k(t) * I(t) + N_k(t) \quad (2)$$

where S is the point source of interest, I an interfering signal and N_k some ambient noise. The quantity h_k (resp. $\tilde{h}_k$) corresponds to the head-related impulse response (HRIR) from the source (resp. the interferer) to the kth microphone. Their Fourier transform is referred to as head-related transfer function (HRTF). This allows us to consider the shadowing effect introduced by the head [8]. The involved sources are assumed to be independent stationary jointly Gaussian random processes with mean zero and (real) bandlimited power spectral densities (PSD) Φ_S, Φ_I and Φ_{N_k}. The position of the source, the interferer and microphone k is given in polar coordinates with respect to the center of the head by (α_S, d_S), (α_I, d_I) and (α_k, d_k), respectively. Note that the above parameters are assumed to be known at both hearing aids.

In this context, the goal of hearing aid k is to reconstruct S_k with minimum mean-squared error (MSE)[1]. As depicted in Figure 1 (b), the reconstruction is based on the observed signal X_k and a compressed version of its neighbor's observation. In the sequel, we look at this problem from the perspective of hearing aid 0 and wish to characterize the best achievable gain offered by a wireless communication link at rate $R_0 = R$. Under these assumptions, our setup simply corresponds to a remote Wyner-Ziv problem [4]: hearing aid 1 encodes X_1 such that hearing aid 0 reconstructs S_0 with minimum MSE, provided X_0 as side information. The corresponding distortion-rate tradeoff is denoted $D(R)$ and the gain achieved by this collaboration is computed as

$$G(R) = \frac{D(0)}{D(R)} \,. \tag{3}$$

The results needed to evaluate the above gain-rate function are presented in the next section.

III. Distributed Source Coding

As pointed out previously, our hearing aids setup corresponds to a remote Wyner-Ziv problem where S_0 is the remote source and X_k is the observation made at hearing aid k. In this context, the optimal distortion-rate tradeoff is given by the following parametric formulas [7]

$$R(\theta) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \max\left\{0, \log_2 \frac{\Phi_e(\Omega)}{\theta}\right\} d\Omega \tag{4}$$

$$D(\theta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{S_0|X_0,X_1}(\Omega)\, d\Omega + \frac{1}{2\pi}\int_{-\infty}^{\infty} \min\left\{\theta, \Phi_e(\Omega)\right\} d\Omega \tag{5}$$

where $\Phi_e = \Phi_{S_0|X_0} - \Phi_{S_0|X_0,X_1}$ and $\theta \in (0, \operatorname{ess\,sup}_\Omega \Phi_e(\Omega)]$. $R(\theta)$ is expressed in units of bits per second and $D(\theta)$ in MSE per second. In the above notation, $\Phi_{X|Y}$ denotes the PSD of the error process $X - \mathrm{E}[X|Y]$. In practice, coding schemes that allow to approach the rate-distortion tradeoff predicted by Equations (4) and (5) usually require a significant computational load at the encoder and/or at the decoder. It is therefore useful to quantify the loss incurred by an encoding technique that disregards the presence of X_0 at the receiver, i.e. that encodes X_1 in a non-Wyner-Ziv fashion. Let us denote by U the compressed signal received at the decoder in this case. Using [7, Lemma 1], the reconstruction error can be split as

$$\begin{aligned} &\mathrm{E}\left[\|S_0 - \hat{S}_0\|^2\right] \\ &= \mathrm{E}\left[\|S_0 - \mathrm{E}[S_0|X_0,U]\|^2\right] &(6)\\ &= \mathrm{E}\left[\|S_0 - \mathrm{E}[S_0|U]\|^2\right] - \mathrm{E}\left[\|\,\mathrm{E}[S_0|V]\|^2\right] &(7)\end{aligned}$$

where $V = X_0 - \mathrm{E}[X_0|U]$. The first term corresponds to the error made in a remote setup where no side information

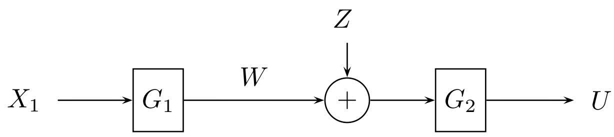

Fig. 2. An optimal forward test channel for the remote rate-distortion problem.

is available. The second term is the gain provided by the availability of X_0 for the reconstruction. The distortion-rate tradeoff in this case can be computed as

$$\tilde{R}(\theta) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \max\left\{0, \log_2 \frac{\tilde{\Phi}_e(\Omega)}{\theta}\right\} d\Omega \tag{8}$$

$$\tilde{D}(\theta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{S_0|X_1}(\Omega)\, d\Omega + \frac{1}{2\pi}\int_{-\infty}^{\infty} \min\left\{\theta, \tilde{\Phi}_e(\Omega)\right\} d\Omega - \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{S_0}(\Omega) - \Phi_{S_0|V}(\Omega)\, d\Omega \tag{9}$$

where $\tilde{\Phi}_e = \Phi_{S_0} - \Phi_{S_0|X_1}$ and $\theta \in (0, \operatorname{ess\,sup}_\Omega \tilde{\Phi}_e(\Omega)]$. Note that by construction, the above function is decreasing but is not necessarily convex. The process U can be described by an optimal forward test channel [9]. For a given "reverse water-filling" parameter θ, it first amounts to apply the (Wiener) filter G_1 with transfer function

$$G_1(\Omega) = \Phi_{S_0X_1}(\Omega)\Phi_{X_1}^{-1}(\Omega) \tag{10}$$

to obtain the process W whose PSD is given by

$$\Phi_W = \Phi_{S_0X_1}\Phi_{X_1}^{-1}\Phi_{S_0X_1}^{*} \,. \tag{11}$$

We then add an independent Gaussian noise Z with mean zero and PSD

$$\Phi_Z = \max\left\{0, \frac{\theta\,\Phi_W}{\Phi_W - \theta}\right\} \,. \tag{12}$$

Finally, the resulting spectrum is bandlimited by the filter G_2 whose frequency response is

$$G_2(\Omega) = 1_{\{\Phi_W \geq \theta\}}(\Omega) \,. \tag{13}$$

The overall process is depicted in Figure 2.

IV. Rate-Constrained Beamforming Gain

We are now in the position to apply the results derived in the previous section to our hearing aid problem. For the rest of the discussion, we will assume that S, I and N_k have flat PSDs over the frequency band $[-\Omega_0, \Omega_0]$, i.e.

$$\Phi_S(\Omega) = \sigma_S^2\, 1_{[-\Omega_0,\Omega_0]}(\Omega) \tag{14}$$

$$\Phi_I(\Omega) = \sigma_I^2\, 1_{[-\Omega_0,\Omega_0]}(\Omega) \tag{15}$$

$$\Phi_{N_k}(\Omega) = \sigma_N^2\, 1_{[-\Omega_0,\Omega_0]}(\Omega) \tag{16}$$

for $k = 0, 1$. The shadowing effect introduced by the head is taken into account by means of the model exposed in [8]. In that paper, the head is considered to be a sphere of radius

[1]The signal S_k corresponds to what would be received at hearing aid k in the absence of interfering signals.

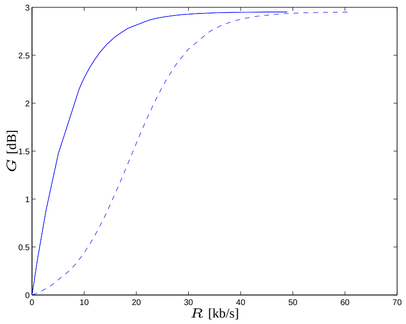

Fig. 3. Gain provided by the wireless communication link as a function of the communication rate R with (solid) and without (dashed) Wyner-Ziv coding.

d_R. The HRTF between a point (α_R, d_R) on the sphere and a point (α_T, d_T) outside the sphere ($d_T > d_R$) can be computed as

$$H_{R,T}(\Omega) = -\frac{c}{4\pi d_R^2 \Omega} \Psi^*(\Omega) \tag{17}$$

with

$$\Psi(\Omega) = \sum_{m=0}^{\infty} (2m+1) L_m \left(\cos(\alpha_R - \alpha_T)\right) \frac{p_m(\Omega d_T/c)}{p'_m(\Omega d_R/c)} \tag{18}$$

where L_m denotes the Legendre polynomial of degree m, p_m the mth-order spherical Hankel function, p'_m it's derivative and c the speed of sound. Under these assumptions, we can easily compute the PSDs involved in the computation of the gain-rate function. Details of this derivation are however omitted here for lack of space.

We first evaluate the gain-rate function obtained in the absence of interferer. The desired source is assumed to be in front of the observer and both Wyner-Ziv and non-Wyner-Ziv coding techniques are considered. The relevant parameters are chosen as follows: $\alpha_0 = -\alpha_1 = 90$ [deg], $d_0 = d_1 = d_R = 0.09$ [m] (typical head radius), $\alpha_S = 0$ [deg], $d_S = 1.5$ [m], $\sigma_S^2 = 100$, $\sigma_N^2 = 0.01$, $c = 340$ [m/s], $f_0 = \Omega_0/(2\pi) = 4000$ [Hz]. The results are plotted in Figure 3. We observe that the loss incurred by neglecting the side information in the encoding process can be quite significant.

Keeping the above parameters, we now consider the impact of an interferer on the gain provided by our hearing aid system. For a given rate R and frequency Ω, we compute the reconstruction error as a function of the direction of arrival α_I. We then normalize the results to have a maximal distortion of 1. The corresponding polar plot is shown in Figure 4 for $\sigma_I^2 = 100$, $d_I = 1.5$ [m], $f = \Omega/2\pi = 3000$ [Hz] and $R = 0, 0.1, 1$ [b/s]. The lobes correspond to directions for which the source and the interferer cannot be properly disambiguated. We clearly see the impact of a limited communication bit-rate on the ability of the system to reject interfering signals. When $R = 0$ (no collaboration), the observed pattern reflects the natural rejection provided by the head. As $R \to \infty$ (full collaboration), it becomes symmetric since both signals X_0 and X_1 are available to hearing aid 0.

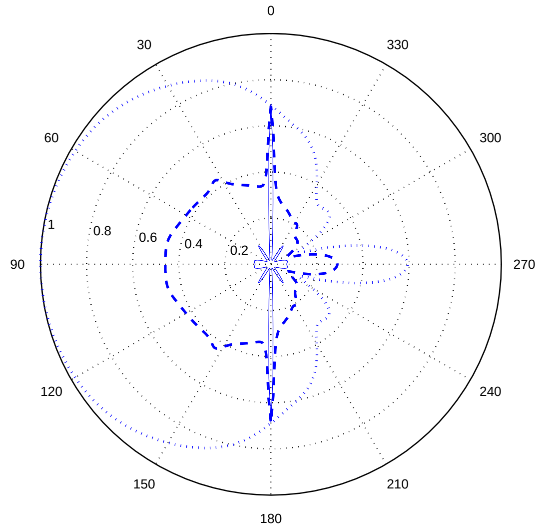

Fig. 4. System's response to an interferer at $f = 3000$ [Hz] and $R = 0$ [b/s/Hz] (dotted), $R = 0.1$ [b/s/Hz] (dashed) and $R = 1$ [b/s/Hz] (solid).

V. Conclusions

In this work, we have studied the beamforming gain provided by hearing aids that are allowed to collaborate using a rate-constrained wireless link. The problem has been identified and solved from an information-theoretic standpoint. We have carried numerical simulations to assess the performance of the system in a realistic scenario.

Acknowledgment

This research was supported by the National Competence Center in Research on Mobile Information and Communication Systems (NCCR-MICS), http://www.mics.org), a center supported by the Swiss National Science Foundation under grant number 5005-67322.

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