Sensible and latent heat flux predictions using conditional sampling methods

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Abstract. The conditional sampling formulation typically used in eddy accumulation flux measurements was tested at two sites using velocity, temperature, and specific humidity time series measurements. The first site was at Owen’s Lake in southern California, and the second site was an irrigated bare soil field at the Campbell Tract facility in Davis, California. The constant $\beta$ relating the turbulent flux to the accumulated concentration difference between updrafts and downdrafts was related to the statistics of the vertical velocity. It was found that this formulation for $\beta$ reproduces the extreme fluxes (both sensible and latent heat) better than previous $\beta$ formulations. The overall comparison between eddy correlation measured fluxes and conditional sampling predicted fluxes for a wide range of atmospheric stability conditions was excellent ($R^2 = 0.97$). On average, the mean $\beta$ obtained from our experiments for temperature and water vapor is in good agreement ($\pm 4\%$) with other reported $\beta$ values for carbon dioxide, water vapor, ozone, and temperature. These results suggest that the conditional sampling formulation used in eddy accumulation methods is useful for routine flux measurements of passive greenhouse gases.

1. Introduction

Interest in atmospheric transport of greenhouse gases and other passive agricultural chemicals is motivating development of simple and accurate methods for measuring trace gas fluxes [Baker et al., 1992; Lenschow, 1982; Dabberdt et al., 1993; Wofsy et al., 1993; Desjardins et al., 1992; Gao et al., 1992]. One recently developed method, known as “eddy accumulation,” has received wide interest from a variety of disciplines, including ecology [Baldocchi et al., 1988], meteorology [Hicks and McMillen, 1984; Baker et al., 1992; Desjardins, 1977; Speer et al., 1985], hydrology, and climatology [Businger and Oncley, 1989]. The method, originally developed by Desjardins [1977] for sensible heat flux estimation, requires that air be sampled and placed in two independent containers. One container collects the air only during updrafts (i.e., positive vertical velocity), and the other collects air only during downdrafts (i.e., negative vertical velocity). At the end of the sampling period, the accumulated concentration in each container is measured. The flux is considered to be proportional to the concentration difference between the two containers. Hence in contrast to the eddy correlation technique, this conditional sampling strategy does not require rapid sensor response time. This is due to the fact that mixing ratios are first accumulated and then measured. However, other requirements on pumping rates, switching response lag, air equilibration, temperature effects on concentration measurements, laminar-flow-induced longitudinal mixing in the sampling line, and bias in the mean vertical velocity, among others, can play a critical role in the design of appropriate instrumentations for the eddy accumulation method [see Hicks and McMillen, 1984; Speer et al., 1985].

Using the eddy accumulation sampling method, Businger and Oncley [1989] proposed that the turbulent vertical flux $J$ of some admixture may be determined from

$$J = \beta \sigma_w \left( \bar{C}_+ - \bar{C}_- \right) \quad (1)$$

where $\bar{C}_+$ and $\bar{C}_-$ are the mean concentrations in the upward and downward moving eddies, respectively; $\sigma_w$ is the standard deviation of the vertical velocity $w$; and $\beta$ is an empirical constant. Using velocity, humidity, and temperature measurements, Businger and Oncley [1989] found that $\beta = 0.6 (\pm 10\%)$ for both temperature and humidity with little dependence on stability. Measurements by Baker et al. [1992] for carbon dioxide ($CO_2$) and water vapor $q$ over a soybean canopy as well as over bare soil suggest that $\beta \approx 0.56$. The experimental results by Baker et al. [1992] reinforce Businger and Oncley’s [1989] conclusion that $\beta$ does not depend on atmospheric stability. In their analysis, Baker et al. [1992] showed that $\beta$ depends on the probability that updrafts or downdrafts occur. They showed that $\beta$ approaches 0.62 if $w$ is normally distributed. This value is well within the range obtained by Businger and Oncley [1989]. The prospect of $\beta$ being determined from the vertical velocity statistics has motivated this study.

The objectives of this study are (1) to compare the predictions of sensible ($H$) and latent ($LE$) heat fluxes from the conditional sampling formulation employed by eddy accumulation methods with the standard eddy correlation measurements and (2) to test a proposed formulation for $\beta$ that depends on the statistics of $w$ only. Vertical velocity,
temperature, and humidity time series were measured at two sites. The first site is a desert-like, smooth, sandy, dry lake bed, while the second site is a Yolo clay loam field, periodically raked, with small soil clods scattered throughout the area. The site is equipped with an irrigation system so that the performance of the conditional sampling formulation employed in eddy accumulation methods can be investigated for a wide variety of atmospheric stability conditions. We note that in this study we do not use an eddy accumulation device to actually sample the air volume; however, our time series measurements permit a thorough examination of the conditional sampling technique and formulation employed in typical eddy accumulation experiments.

2. Theory

The mean vertical turbulent flux $J$ of a passive admixture over a horizontally homogeneous surface under steady state conditions can be expressed as

$$J = \langle w'c' \rangle$$

(2)

where $c = \langle c \rangle + c'$ is the mixing ratio of the passive admixture, $c'$ is the fluctuation about the time average, $w = \langle w \rangle + w'$, $\langle w \rangle$ and $w'$ are the mean and time-fluctuating vertical velocity, and the angle brackets denote the time-averaging operator [e.g., Brutsaert, 1982, pp. 52–54; Monin and Yaglom, 1971, chapters 3–4]. Here we assume that the time average and the ensemble average are identical [see Lumley and Panofsky, 1964; Lumley, 1970].

The covariance in (2) can be written as

$$\langle w'c' \rangle = r \sigma_w \sigma_c$$

(3)

where $r$ is the correlation coefficient between $c$ and $w$, and $\sigma_w(= \langle w'^2 \rangle^{1/2})$ and $\sigma_c(= \langle c'^2 \rangle^{1/2})$ are the standard deviations of the vertical velocity and mixing ratio, respectively. We consider next the product of $r$ and $\sigma_c$ and its possible relation to the statistics of $w$.

Let $f(x, y)$ denote the joint probability density function (pdf) of the two variables $w$ and $c$, and let $f_{w}(x)$ denote the marginal pdf of $w$. By definition, the pdf for $y = c$, given that $x = w$, is

$$f(y = c | x = w) = f(x, y) / f_{w}(x).$$

(4)

From (4), the conditional mean of $y = c$, given that $x = w$ $[E(y = c | x = w)]$, is

$$E(c | x = w) = \int_{-\infty}^{+\infty} y f(y | x) dy = \int_{-\infty}^{+\infty} y f(x, y) dy / f_{w}(x).$$

(5)

Notice that $E(y = c | x = w)$ is a function of $x$ only because we integrate over all possible values of $y$ in (5). For the special case $E(y = c | x = w) = a + bx$, it can be shown that [see Hogg and Craig, 1978, pp. 65–78]

$$r = b \sigma_w / \sigma_c.$$

(6)

The linearity assumption of $E(y = c | x = w)$ is reasonable if (1) significant correlation between $w$ and $c$ exists and (2) $\sigma_c$ and $\sigma_w$ are nonzero. Condition (2) is generally satisfied for any turbulent flow, and condition (1) may be reasonable if condition (2) is satisfied and $J$ (described by the covariance in (2)) is significant. We note that while condition (2) is almost always satisfied in atmospheric surface layer flows, condition (1) may be violated. Replacing (6) in (3), we get

$$\langle w'c' \rangle = \sigma_w (b \sigma_c).$$

(7)

Since the conditional sampling formulation in the eddy accumulation method relies on sampling the air in two separate containers depending on whether $w$ is positive ($w_+$) or negative ($w_-$), it is convenient to sort the $c$ data into two categories: $c_+$ and $c_-$. For $w > 0$ (updrafts), we label $c$ as $c_+$ and place it in container 1, and for $w < 0$ (downdrafts), we label $c$ as $c_-$ and place it in container 2.

The result of this sampling scheme is that our $w$ and $c$ data are clustered in two quadrants ($w_+, c_+$, and $w_-, c_-$). From (6) it can be shown that for a large number of samples, $b$ can be approximated by [see Hogg and Craig, 1978, pp. 73–77, 296–303; Baker et al., 1992]

$$b = (\langle c_+ \rangle - \langle c_- \rangle) / (\langle w_+ \rangle - \langle w_- \rangle).$$

(8)

An interpretation of $b$ is also shown in Figure 1 for temperature. In Figure 1 the slope of the best fit line through the 50,400 points is simply approximated by (8). Notice also in Figure 1 that the $E(y = c | x = w)$ appears to be nonlinear, which agrees with an earlier finding by Baker et al. [1992].

The consequence of this nonlinearity is discussed in the following section.

By combining (8) and (7), we get

$$J = (\sigma_w / (\langle w_+ \rangle - \langle w_- \rangle)) \sigma_c (\langle c_+ \rangle - \langle c_- \rangle).$$

(9)

The above equation is the same as (1) with

$$\beta = \sigma_w / (\langle w_+ \rangle - \langle w_- \rangle).$$

Hence $\beta$ may be estimated from the statistics of $w$ (see also Baker et al. [1992]).

3. Experiments

The experiments were carried out at two sites. The first site is in Owen's Lake, while the second site is at the University of California Campbell Tract Facility, Davis.

3.1. Owen's Lake

Turbulence measurements were carried out from June 20 to July 2, 1993, over a dry lake bed (Owen's Lake) in Owen's Valley, California. The lake bed is part of a basin bounded from the west by the Sierra Nevada and from the east by the White and Inyo Mountains. The measurements were performed on the northeast end of the lake bed (elevation = 1100 m). The portion of the lake bed surface surrounding the site is a uniform heaved sandy soil extending 11 km in the north-south direction and 4 km in the east-west direction.

The vertical velocity component was measured using a triaxial ultrasonic anemometer (Gill Instruments model 1012R2) to an accuracy of ±1%. The sonic anemometer path distance $d_{st}$ is 0.149 m, and the maximum sampling frequency is 56 Hz. The maximum sampling frequency was used in this study. To ensure that the steady state assumption is valid, a sampling period of 15 min was chosen. This yielded 50,400 measurements per $w$ component.

The absolute temperature $T$ was computed directly from the measured fluctuations in the speed of sound ($c_v$) using
$T = \frac{c_s^2}{\alpha R_d}$

where $\alpha = C_p/C_v$, $C_p$ and $C_v$ are the specific heat capacities of dry air under constant pressure and volume, respectively, and $R_d$ is the gas constant for dry air [see Suomi and Businger, 1959; Wyngaard, 1981; Friehe, 1986; Katul et al., 1994a, b]. The influence of humidity variation on the speed of sound was neglected at this site since the maximum measured relative humidity (RH) was 13%. A validation of how well the sonic anemometer captures temperature fluctuations is discussed by Katul [1994]. At this site, only mean relative humidity was measured.

3.2. Campbell Tract

The Campbell Tract facility is located at the University of California, Davis (elevation = 16 m). The site is a uniform 500 m by 500 m bare soil field. The facility is equipped with an irrigation system that is capable of irrigating 120 m x 110 m area with a typical uniformity coefficient of 85% [Katul and Parlange, 1992]. The tower was installed at the northeast end of the field to maximize the fetch. Measurements were conducted during August 21–23, 1993, following an irrigation on the night of August 21. Only south wind directions were analyzed when the field was irrigated. Two Campbell Scientific one-dimensional sonic anemometers and Krypton hygrometers, as well as the Gill ultrasonic triaxial sonic anemometer, were used at this site. A one-dimensional sonic anemometer and Krypton hygrometer were placed 1.12 m above the ground surface, while the other one-dimensional sonic anemometer and Krypton hygrometer were placed 2.9 m above the ground surface. The Krypton hygrometers were 15 cm away from the one-dimensional sonic anemometers at both heights. The triaxial sonic anemometer was placed 1.96 m above the ground surface (see Figure 2). The sampling frequency was 21 Hz and the sampling period was 26 min, resulting in 32,768 data points from each sensor. The conditional sampling formulation was tested at two heights for $LE$ and at the triaxial sonic anemometer height for $H$. Further details regarding the experimental setup for the Campbell Tract facility are given by Katul and Parlange [1994].

4. Results and Discussion

We compare how well the conditional sampling formulation used in eddy accumulation methods reproduces the eddy correlation flux described in (2). The following procedure is used to simulate this conditional sampling scheme:

1. At each discrete time step $dt$, determine the sign of $w$.
2. If $w > 0$, then store the corresponding $c$ value in a vector $c_+$ (i.e., compartment 1) and $w$ data in vector $w_+$; otherwise, store the $c$ data in a vector $c_-$ (i.e., compartment 2) and the $w$ data in a vector $w_-$. The end result is identical to that of conditional sampling used in eddy accumulation methods.
3. Compute $\langle c_+ \rangle$ and $\langle c_- \rangle$ by accumulating the mixing ratio in each compartment.
4. Compute $\langle w_+ \rangle$ and $\langle w_- \rangle$ similarly.
5. Compute the overall variance of $w$.
6. Estimate the flux using (9).
7. Compare this estimated flux with the flux "measured" by the eddy correlation system (see, for example, (2)).

In this study, temperature $T$ and specific humidity $q$ are used to represent the admixture $c$ used in section 3. It should be noted that if concentration (instead of mixing ratio) in the two compartments is measured, then Webb et al.'s [1980] correction for the dry air density should be used [e.g., Baldocchi et al., 1988] when computing the flux. Density fluctuations can cause errors as large as 40% in the flux estimation [see Leuning et al., 1982]. For temperature and water vapor, Webb et al.'s [1980] correction is not very significant [Baldocchi et al., 1988].

4.1. Sensible Heat Flux Comparison

In Figure 3 a comparison between predicted $H$ and measured $H_{ec}$ is shown. A linear regression model of the form
H = A H_{ec} + B was used to assess the performance of the eddy accumulation method, and the results are shown in Table 1. We also determined the constant B by regressing $H_{ec}$ versus $\sigma_u((C_+ - C_-))$ and forcing the intercept of the regression model to be zero. Our mean estimate of $\beta = 0.57$ agrees with the values reported by Businger and Oncley [1989] ($\beta = 0.6 \pm 0.10$), Baker et al. [1992] ($\beta = 0.56$), and Pattey et al. [1993] ($\beta = 0.57$). We used $\beta = 0.57$ and estimated $H$ for both sites (see Figure 3). It appears that the choice of $\beta = 0.57$ slightly underestimates the maximum sensible heat flux, while the proposed $\beta = \sigma_u((\langle w \rangle - \langle w \rangle))$ appears to better reproduce the maximum sensible heat flux (see Figure 3). In all cases the correlation coefficient $R^2$ was in excess of 0.96, indicating that 96% of the variation in the eddy correlation measurements can be explained by the conditional sampling formulation. The standard error of estimate (SEE) (assuming that the eddy correlation measurements are error-free) was relatively small (= 15 W m$^{-2}$). Note that the conditional sampling formulation appears to be fairly robust to the nonlinearities in $E(y = T|x = w)$. Baker et al. [1992] suggested that departure from linearity of $E(y = T|x = w)$ may result in an overestimation of the true sensible heat flux by about 11%. It appears from our data that this overestimation is not very significant. It is interesting to note that the $\beta = 0.56$ employed by Baker et al. [1992] consistently underestimated the peak sensible and latent heat fluxes in agreement with the results of Figure 3.

### 4.2. Latent Heat Flux Comparison

The predicted ($LE$) and measured latent heat flux ($LE_{ec}$) at the Campbell Tract facility are compared in Figure 4 for both heights ($z = 1.12$ m and $z = 2.9$ m). No LE comparison was made for Owen's Lake. Using $\beta = 0.57$ and (1), we estimated LE and compared it with $LE_{ec}$ in Figure 4. The predicted LE is in good agreement with $LE_{ec}$. A linear regression model of the form $LE = A LE_{ec} + B$ was also used to evaluate the performance of the conditional sampling method. The regression statistics are shown in Table 1. The agreement between latent heat flux predictions and measurements are much less scattered (maximum SEE = 15 W m$^{-2}$) when compared with those reported by Baker et al. [1992] for the soybean canopy and bare soil surface (SEE = 57 W m$^{-2}$). The higher SEE in the Baker et al. [1992] study is probably due to errors in the energy balance closure. Nevertheless, both studies clearly indicate that the conditional

![Figure 3](image-url)
Table 1. Comparison Between Latent (LE) and Sensible (H) Heat Fluxes Predicted by Eddy Accumulation and Measured by Eddy Correlation for Owen's Lake and the Campbell Tract Sites

<table>
<thead>
<tr>
<th>Site</th>
<th>β Model</th>
<th>Flux</th>
<th>Intercept B</th>
<th>R²</th>
<th>SEE, W m⁻²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Owen's Lake</td>
<td>1</td>
<td>H</td>
<td>0.97</td>
<td>11.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>H</td>
<td>0.97</td>
<td>11.0</td>
<td></td>
</tr>
<tr>
<td>Campbell Tract 1</td>
<td>H</td>
<td>1.02</td>
<td>0.395</td>
<td>3.15</td>
<td></td>
</tr>
<tr>
<td>Campbell Tract 2</td>
<td>H</td>
<td>1.10</td>
<td>0.83</td>
<td>3.49</td>
<td></td>
</tr>
<tr>
<td>(z = 1.96 m)</td>
<td>0.95</td>
<td>0.95</td>
<td>0.88</td>
<td>2.63</td>
<td></td>
</tr>
<tr>
<td>Campbell Tract 2</td>
<td>LE</td>
<td>1.10</td>
<td>-2.02</td>
<td>6.07</td>
<td></td>
</tr>
<tr>
<td>(z = 2.9 m)</td>
<td>0.96</td>
<td>0.96</td>
<td>-1.40</td>
<td>3.30</td>
<td></td>
</tr>
<tr>
<td>Campbell Tract</td>
<td>LE</td>
<td>0.99</td>
<td>7.24</td>
<td>14.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.90</td>
<td>0.90</td>
<td>2.47</td>
<td>7.53</td>
<td></td>
</tr>
</tbody>
</table>

Linear regression models of the form $LE = A LE_0 + \beta$ and $H = A H_0 + \beta$ are used in evaluating the eddy accumulation predictions. Two β models are considered: model 1 estimates β using $\alpha_\beta (w) = \omega (w)$; while model 2 assumes that β is a constant equal to 0.57. The standard error of estimate (SEE) and the coefficient of determination ($R^2$) are also shown for each β model.

4.3. Further Comments on β

Baker et al. [1997] showed that for bireflectively correlated variables, such as w and c, with w being approximately Gaussian, β approaches $p_\beta^2/N_d$. Here $p_\beta$ is the probability that an updraft occurs, $p_d = 1 - p_\beta$ (i.e., probability that a downdraft occurs), and $N_d$ is the ordinate of the unit Gaussian curve at abscissa $p_d$. Baker et al. [1992] found that $p_d$ did not fall below 0.46, at least for the range of measurements they carried out. They estimated β to be 0.627 if $p_d = 0.5$ and found that this estimate of β systematically overestimates the sensible heat flux measurements. They attributed this overestimation error in β to the nonlinearity in the correlation between c and w and the non-Gaussian pdf of the vertical velocity.

In Figure 5 the pdf of w for a variety of unstable atmospheric conditions (i.e., $z/L < 0$) is shown. Here z is the height above the ground surface, and L is the Obukhov length [e.g., Bruun, 1982, p. 65]. The pdf in Figure 5 is computed as follows:

1. Normalize the vertical velocity (for each stability condition) so that each vertical velocity time series has a zero mean and unit variance.
2. Compute the frequency distribution of each time series using 100 bins. This constitutes the measured pdf (which is discrete).
3. Compare the measured pdf with a zero-mean and unit variance Gaussian pdf. A semilog axis is used in Figure 5 to emphasize the behavior of the pdf tails.

Notice how the tails of the updraft events in Figure 5 depart from Gaussian behavior faster than the downdraft events. Hence even though $p_\beta$ and $p_d$ are approximately equal, the extreme updraft events are more likely to occur than the extreme downdraft events (and hence transport more concentration). This departure from Gaussian distribution of the updraft tails can be an important factor responsible for the overestimation of β as proposed by Baker et al. [1992].

It is evident from this study and previous studies that the variation in β (0.56–0.63) for many tracers (e.g., q, T, CO₂, and O₂) is not very large (11%). Hence the maximum error in the computed flux due to an arbitrary choice of β between 0.56 and 0.63 will not exceed 11% and is probably not very large in relation to the unavoidable instrumentation errors typically associated with the eddy accumulation methods. These errors are due to switching response lag, air equilibration, temperature effects on concentration measurements, laminar-flow-induced longitudinal mixing in the sampling line, and some bias in the mean vertical velocity. Nevertheless, the errors in β can result in systematic bias in the flux measurements.

5. Conclusions

A comparison between the conditional sampling formulation employed in eddy accumulation methods and the eddy correlation method was carried out at two sites for latent and sensible heat fluxes. The first site was a dry lake bed with large amounts of sensible heat transported to the atmosphere. In contrast, the second site was an irrigated, bare soil field with large amounts of latent heat flux. Fast response measurements of temperature, vertical velocity, and specific humidity were carried out at both sites under a variety of atmospheric stability conditions.

Following the work of Baker et al. [1992], a formulation for the empirical factor β was derived from the statistics of the vertical velocity. A key assumption was the linearity of

Figure 4. Similar to Figure 3 but for latent heat flux.
the expected value of the probability distribution of admixture concentration, conditioned on the vertical velocity. Some arguments regarding the validity of this assumption were also discussed. It was found that such a formulation reproduces the extreme sensible and latent heat flux events better than a simple constant $\beta$ model. However, over a wide range of sensible and latent heat flux measurements, it was found that a constant value for $\beta$ also performs reasonably well (but underestimates extreme heat fluxes). We estimated a mean constant $\beta$ to be 0.57, which appears to agree well with other reported $\beta$ values (0.56–0.63) for carbon dioxide, water vapor, temperature, and ozone. Also, the value of $\beta$ appears to be insensitive to atmospheric stability or momentum roughness variation as evidenced in this study and other studies. Hence the conditional sampling formulation employed in eddy accumulation methods is reliable and may prove to be a potential alternative to eddy correlation methods when rapid sampling of concentrations is not possible.

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References


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