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SIMILARITY RULES OF CAVITATION TESTS: THE CASE OF THE FRANCIS TURBINE.

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SUMMARY

On the basis of experimental studies on the influence of the test head and the water nuclei content on the performance of a Francis turbine, we have shown that outlet cavitation is influenced by the active nuclei content of the test loop water.

Numerical simulations of the cavitation nuclei dynamics in a Francis runner allowed us to associate the leading role played by the water nuclei content in the development of outlet cavitation, with the particular pressure distribution over the blades. It was found that the saturation of the nuclei content influence was driven by a limitation of the volume available to the bubble growth in the region where the flow pressure was lower than the vapour pressure. If this fundamental result is applied to the problem of transposing model cavitation tests to the prototype case, it can be shown that, if the saturation effect is reached during the model test, this effect will also be present in the prototype. In order to guarantee the same cavitation development and characteristics using the model or the prototype, it is necessary to perform the cavitation tests with Froude similarity. The saturation characteristics for the model will then be the characteristic cavitation behaviour of the prototype.

RESUME

Les études expérimentales que nous avons menées sur l'influence de la chute d'essai et la teneur en germes de l'eau sur les performances d'une turbine Francis, ont montré que la cavitation de sortie est fortement influencée par la teneur en germes actifs de l'eau d'essai. Les résultats de la modélisation numérique de l'évolution dynamique d'un germe unique de cavitation, permet d'associer le rôle prépondérant joué par la teneur en germes de l'eau d'essai sur le développement de la cavitation, au distribution de pression sur les aubes de la machine. Ainsi nous voyons que la saturation de l'influence de la teneur en germes est directement liée à une limitation de place dans la région d'explosion des germes, soit dans la zone de l'écoulement où la pression est inférieure à la pression de vapeur. En appliquant directement ce résultat fondamental au problème de la transposition des essais de cavitation sur modèle au prototype, nous montrons que si l'effet de saturation est atteint sur le modèle, alors il l'est également sur le prototype. D'autre part, si nous voulons garantir les mêmes figures de cavitation ainsi que les mêmes caractéristiques en cavitation sur le modèle et sur le prototype il est nécessaire de mener ces essais en respectant la similitude de Froude. Ainsi, les caractéristiques mesurées en saturation de l'influence de la teneur en germes de l'eau d'essai sur le modèle, seront celles du prototype.

1. INTRODUCTION

During the testing of hydraulic machine models the test head and the nuclei content of the water used play a very important role. The influence of these elements has been written about in numerous publications. In 1978, at Fort Collins, our Institute presented the importance of these 2 parameters to the development of cavitation [1]. In order to respect similarity with the prototype, it was suggested to keep the same quantity of nuclei in the prototype as in the model [2]. An attempt to quantify the importance of the nuclei content and the test head was made in 1980 [3].

On the basis of the resolution of Rayleigh-Plesset equation, which modelizes the dynamic behaviour of a single nucleus, the proponderant effects were shown, which occurred only with outlet cavitation (travelling cavitation) [4]. In Montréal [5] systematic cavitation tests on Francis turbines models confirmed that the nuclei content greatly influences outlet cavitation. As an important result, these investigations have revealed that there is a threshold for the test head, or for the concentration of cavitation nuclei, beyond which the model performance is no longer affected by these parameters

Making a synthesis of all our investigations, we now try to explain the threshold phenomenon. By studying the behaviour of a single bubble we can explain the saturation effect, and show that if saturation is reached on the model, it leads to the saturation on the prototype. These investigations, which were made using a high specific speed Francis turbine, show the important Froude effect on development cavitation and, of course, on the performance of the model. These results show the importance of testing the model with the Froude head and injecting nuclei. It can be concluded that the σ-η saturation curve obtained for the Froude head on the model will be the cavitation characteristic of the prototype with similar cavitation patterns.

2. SIMILARITY RULES FOR CAVITATION TESTS

As far as predicting the cavitation behaviour of a prototype is concerned, at least the usual requirements for performance tests on a model must be fulfilled. A rigorous geometrical homology between model and prototype should be verified and, of course, the operating conditions must be homologous (same φ and ψ) in order to obtain the same flow conditions. The effect of the Reynolds number, which obviously has an influence according to the test head efficiency, can be overcome by referring the results to the value obtained using a regime where there is no cavitation. This simple practice remains valid as long as cavitation tests are carried out to determine the setting level of the machine.

In order to ensure the same relative location of the vapour pressure level in the model and in the full-size runner, the Thomas number σ and the Froude number have to be the same on model and prototype respectively. Though the reference setting level can be modified to compensate the Froude condition not being respected, this simple method is not practicable because it produces a change in the pressure gradient. Thus, if for a fixed-cavity-type cavitation these conditions are sufficient and well confirmed such as in the case of inlet cavitation in pumps and turbines, or for whirl in turbines, in the case of bubble cavitation, the nucleation effect prevents us from obtaining reliable cavitation test results. However, the dynamics of a single cavitation nucleus is described well by the Rayleigh-Plesset equation which explains these nucleation effects.

2.1. Dynamics of a Single Cavitation Nucleus

Using a numerical model of bubble dynamics based on Rayleigh-Plesset equation (Equation.1) we can confirm the influence of the water nuclei content and the test head on the performance of a turbine [4].

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 + \frac{4\nu}{R}\dot{R} = \frac{1}{\rho}\left[P_v - P(t) - \frac{2\gamma}{R} + \left(\frac{2\gamma}{Ro} - (P_v - P_o)\right)\left(\frac{R_o}{R}\right)^{3\Gamma}\right]$$

Equation 1 : Rayleigh-Plesset Equation

where

- R : bubble radius of initial value Ro
- Pv : vapour pressure
- P(t) : bubble far field pressure, given by the flow, of initial value Po
- γ : gas-liquid surface tension
- ν : liquid kinematic viscosity
- ρ : liquid density
- Γ : ratio of the specific heats of the gas.

The study of the dynamics effect on the behavior of cavitation nuclei leads to an improved relation of the previous quasi-static Rayleigh analysis. As the main result of the parametric study of the above equation, the minimum active bubble radius is found to be an inverse function of the test head and the maximum size reached by the bubbles at the end of the blade is a function of 1/H .

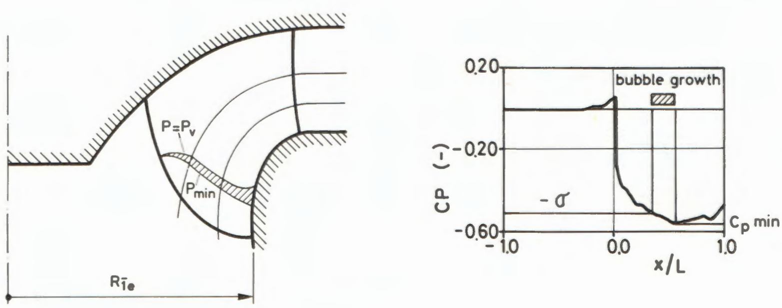
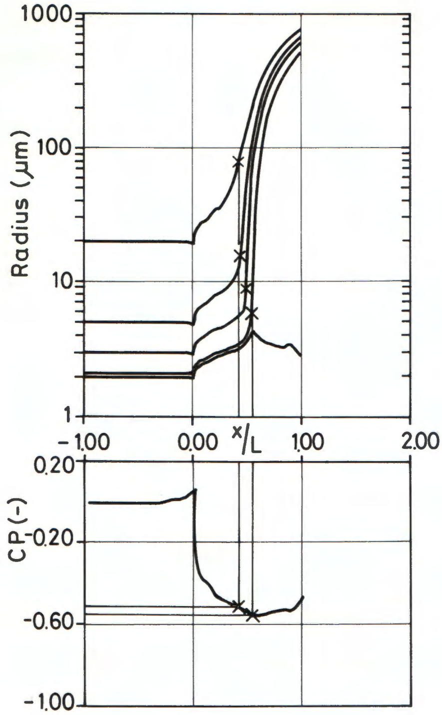


Figure 1 Pressure distribution.

If we consider a given runner design, we can obtain the pressure distribution along a streamline corresponding to a given flow condition (Figure 1). Furthermore for a given setting level and a given test head, which lead to a given cavitation number and Froude number, the flow region of a pressure level less than the vapour pressure Pv is limited by the surface indicated in Figure 1. Since any bubble will be subjected to a pressure lower than the vapour pressure, it is suspected of exploding, provided its radius is greater than Ri, the radius limit of the active nuclei, corresponding to the actual streamline.



The region of the streamline where the bubbles explode corresponds to the flow region between the vapour pressure (Cp = - σ) and the minimum pressure Cp_{min}, see Figure 1. A nucleus with a radius equal to the minimum active one will explode at the Cp_{min} point, and a nucleus with a large radius, near the Cp = - σ point (Figure 2). Moreover a compression of the dynamic effect during the bubble growth is observed. Indeed, for larger bubble radii the bubble growth at the end of the blade takes place in the same way as for the smallest radius. This important observation shows us that in developed outlet cavitation, nuclei with different initial radii will reach about the same sizes, and then occupy the same volume. So it means that whether the active nuclei are big or small the effect on the cavitation volume is the same. The only difference is that larger nuclei will explode just a little sooner (closer to the p = pv isobar).

Figure 2 Growth of active cavitation nuclei.

So, if we observe a cavitation development, its total volume depends on the number of active nuclei, and not on their initial size. For a given pressure distribution, corresponding to a streamline, the exploding bubble radius has been found to be scaled by the head H and the reference length L according to (Equation 2):

$$R(M) = \frac{\gamma L}{\mu\sqrt{2gH}} f(M) + R_0 \qquad \text{Equation 2}$$

where M : represents any point along the streamline

μ : dynamic viscosity

g : gravity acceleration

f is a function which depends only on the pressure coefficient Cp (M) and the cavitation number σ related to the corresponding streamline.

This means that, for a given setting of the machine, this function f remains unchanged along a streamline; it does not depend on the initial size of the nuclei. This function f characterizes the cavitation behaviour of the machine, along this streamline (Figure 3).

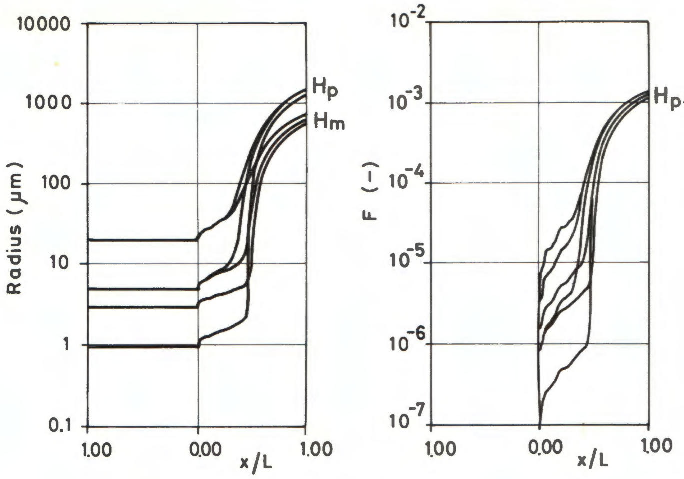


Figure 3 Comparison between the bubble radius and the f function evolutions for the prototype and the model of a Francis turbine.

2.2. Nuclei Content

The existence of this function, which is in direct relation to the asymptotic behaviour of the nuclei growth, allows us to impose the same relative volume of cavitation bubbles in the model and in the prototype. This leads to modifying the law for the nuclei content in order to take into account the above-mentioned dynamic effects.

If the water nuclei content is measured, it can be represented as a density n (Ro) related to the number N (Ri) per unit volume of nuclei with a radius greater than Ri, by the relation:

$$N(R_i) = \int_{R_i}^{\infty} n(R_o)\, dR_o$$

The volume occupied by the exploding bubbles in the unit volume can then be expressed as follows:

$$\delta_v = \int_{R_i}^{\infty} n(R_o)\, \frac{4\pi}{3} R^3(R_o, M)\, dR_o$$

By substituting the expression for R (Eqn 2), and integrating it in the unit volume, we obtain the total volume of cavitating bubbles per unit volume which should be the same in the prototype (subscript p).

$$\delta_v = \frac{4\pi}{3}\left(\frac{\gamma L_p}{\mu\sqrt{2gH_p}}\right)^3 N(R_i)\, f^3 + \frac{4\pi}{3}\int_{R_i}^{\infty} n(R_o) R^3\, dR_o$$

Moreover, owing to the interaction of the explosive nuclei with each other, which is not taken in account in the previous analysis, the saturation effect should occur. This effect, of which experimental evidence has been given, is due to the volume where the cavitation takes place being limited upstream by the Pv pressure level surface in the flow. As soon as the volume of cavitating bubbles equals a proportion K of the unit volume, saturation occurs. We can write the following saturation condition (Equ. 3) :

$$\frac{4\pi}{3}\left(\frac{\gamma L}{\mu\sqrt{2gH}}\right)^3 N(R_i)\, f^3 + \frac{4\pi}{3}\int_{R_i}^{\infty} n(R_o) R^3\, dR_o \geq K \quad \text{and} \quad K \leq 1 \qquad \text{Equation 3}$$

Since the radius R of an exploding bubble is much larger than its initial value Ro, we can neglect the 2nd volume integral against the value of the 1st. Owing this approximation, the water nuclei content should satisfy the following relation in order to obtain the saturation effect (Equation 4).

$$N(R_i)\, f^3 \geq K \frac{3}{4\pi} \left(\frac{\mu}{\gamma}\right)^3 \left(\frac{\sqrt{2gH}}{L}\right)^3 \qquad \text{Equation 4}$$

This very important result should be pointed out since it explain the influence of the head and the nuclei content on outlet cavitation. Then the respect of the equality of the cavitation total volume on the model and the prototype can be written as follows (Equation 5) :

$$\frac{N_m}{N_p} = \left(\frac{L_p}{L_m}\right)^3 \left(\frac{H_m}{H_p}\right)^{3/2} \qquad \text{Equation 5}$$

anf if the test head is corresponding to the Froude condition,

$$\frac{N_m}{N_p} = \left(\frac{L_p}{L_m}\right)^{3/2} = \lambda^{3/2}$$

The active nuclei content depends on the total nuclei content of the water used for the test and on the test head (which, for a given setting of the machine defines the Ri minimum active radius along a streamline). By increasing the test head the active nuclei content will be greater, which means that the cavitation volume will be greater, as long as saturation condition is not fulfilled.

By assuming that the nuclei content follows an inverse n - power law, we have :

$$N(R_i) = N_o \left(\frac{a}{R_i}\right)^n$$ where a is a characteristic nuclei size corresponding to a content N_o

Moreover, the limit active radius R_i has been found to be scaled by the head H as :

$$R_i = \phi(\sigma + c_p)\, \frac{\gamma}{\rho}\, \frac{1}{2gH}$$

where ϕ is a function which depends only on the cavitation number σ and the pressure coefficient distribution Cp (M) of the corresponding streamline. If we substitute the last two relations in the saturation condition (Equation 5) , the following equation must be verified in order to obtain the saturation effect on the prototype if this saturation effect is obtained on the model :

$$N_o\, a^n \frac{L_m^3}{H_m^{3/2}} \left(\frac{2gH_m}{\phi} \frac{\rho}{\gamma}\right)^n \leq N_o\, a^n \frac{L_p^3}{H_p^{3/2}} \left(\frac{2gH_p}{\phi} \frac{\rho}{\gamma}\right)^n$$

Then $1 \leq \lambda^3 \left(\frac{H_p}{H_m}\right)^{n - 3/2}$ or $1 \leq \lambda^3 (\alpha)^{n - 3/2}$

where	$L_{prototype}$	$= \lambda L_{model}$	with $\lambda \approx 10$ to 20
	$H_{prototype}$	$= \alpha H_{model}$	with $\alpha \approx 1$ to 5
			and $n \approx 15$ to 40

which obviously proves the result that saturation in the model leads to saturation in the prototype.

3. REQUIRED EXPERIMENTAL EQUIPMENT

To define the $\eta - \sigma$ saturation curve, it is usually necessary to inject nuclei of a size range greater than the minimum active radius Ri. A range of injected nuclei radius of 1 - 10 µm is required. Based on rapid saturated water expansion through a series of 1 mm diameter diaphragms (Figure 4), the nuclei generator is built on the following principle. A gaseous cavitation is produced at the diaphragm outlets and, by adjusting the downstream pressure recovery time in the generators, it is then possible to provide nuclei of different diameters. The pressure recovery time, which governs the diffusive process of nuclei growth, depends on the downstream diffusing part length and the flow velocity into the generators. The saturation of the water with air is obtained by producing a bubbly flow in a separate vessel with an air pressure of up to 16 bar.

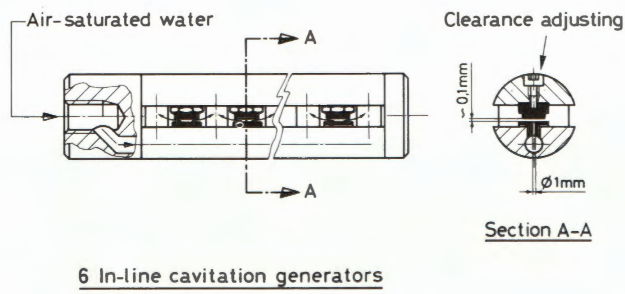


Figure 4 Cavitation Nuclei Generator.

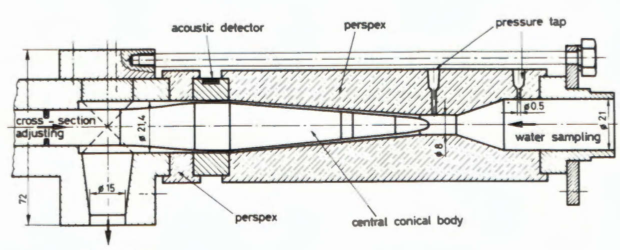


Fig 5 New Nuclei Counter.

The nuclei content measurements were done with a venturi system in the same way as during hydraulic machine mode testing [4]. A new cavitation nuclei counter has been designed to overcome the restricted possibilities for counting nuclei during low head tests (Figure 5). In this new nuclei counter the flow is accelerated, promoting explosive growth of the nuclei, by using a restricted section bounded by a central conical body and a cone diffuser. By adjusting the central body, the restricted section can be modified to match different pressure and flow values during water sampling. To calibrate this new nuclei counter, real time information was analyzed, by measuring the signal from the piezo-ceramic, compared with the number of detected explosions and simultaneous high-speed photographs of the exploding bubbles (Figure 6). All the measurements have shown a good reliability of this counting technics.

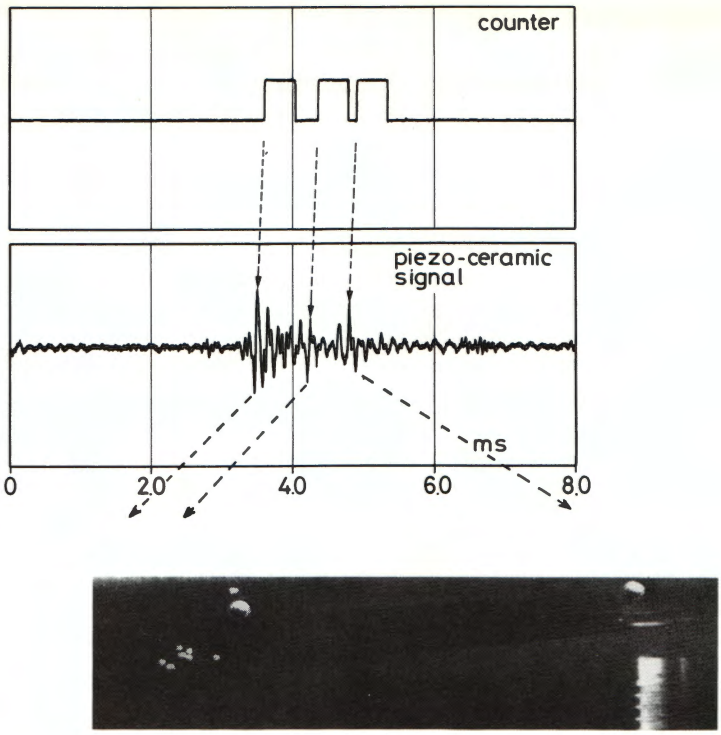


Figure 6 Piezo-ceramic and counter signals compared with the photograph of the corresponding exploding nuclei.

This counter is also useful since it can provide a real time true degassing level of a test installation. This can be seen in the following nuclei distributions measured with our new nuclei counter (Figure 7). After one hour of degassing, the distribution was "A", after 2 hours it was "B", and finally after 5 hours "C". The advantage of measuring the degassing of the test water using this nuclei counter is that we know exactly the amount of active cavitation nuclei in the water which will govern the development of the bubble cavitation.

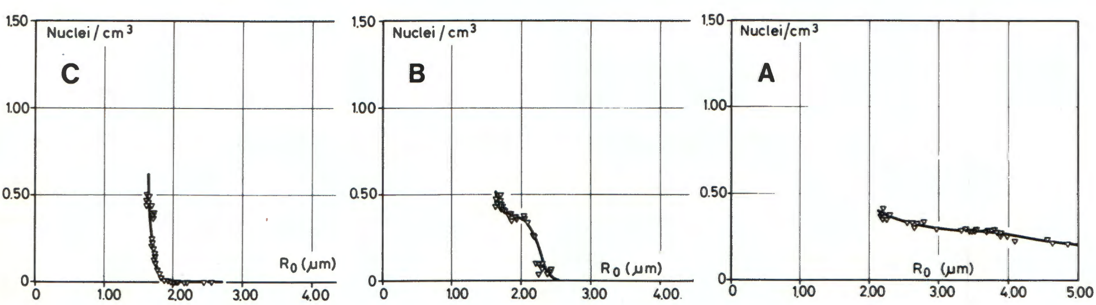


Figure 7 Nuclei distribution during the degassing operation.

4. CAVITATION TESTS ON FRANCIS TURBINES

By carrying out systematic cavitation tests on several Francis turbine models [5], we can show that the only type of cavitation influenced by the nuclei content and the head is outlet bubble cavitation . To investigate the combined influences of the test head and the nuclei content of the water on the development of the flow and efficiency characteristics, as a function of the cavitation coefficient σ, all the model tests were done with 2 and even 3 heads, both with a different nuclei content in the water. All the tests were done in a closed circuit; the nuclei content characteristic of the "without injection" condition is not completely uniform, since it is related to a more or less thorough degassing of the water, which nevertheless corresponds to the usual values for obtaining good visualizations. For the "with injection" condition, very similar distributions were obtained from one test to another. Tests have shown the effect of saturation very well. In fact, above a certain nuclei injection rate the performance of the machine is no longer affected. This saturation phenomenon also occurs when the head is increased, since the number of active nuclei increases.

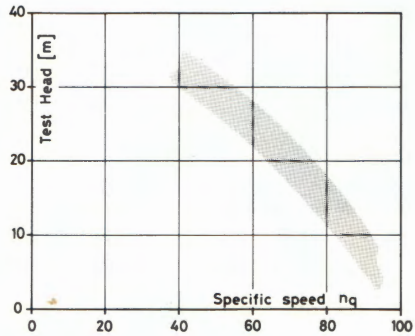


Figure 8 Saturation test head function.

A curve (Figure 8) is also obtained which determines, in principle, the minimum test head, corresponding to a saturation, as a function of the specific speed. It should be noted that the high specific speed turbines tested had inlet cavitation, which was therefore not sensitive to the nuclei content of the water. In order to investigate the trend of this curve for the high specific velocities, we paid special attention to this kind of Francis turbine. The tests presented here concern a high specific speed Francis of ν = 0.6, nq = 94.

This machine is of an improved design which completely avoids inlet cavitation. At rated head, and low σ the bubble cavitation is located only near the outlet edge. The tests were performed with heads of 6 m and 15 m respectively. The maximum test head was limited by the mechanical design of the model (deformation limits), and the minimum head by the low σ values. Figure 9 shows the performance characteristics and the nuclei contents in each case.

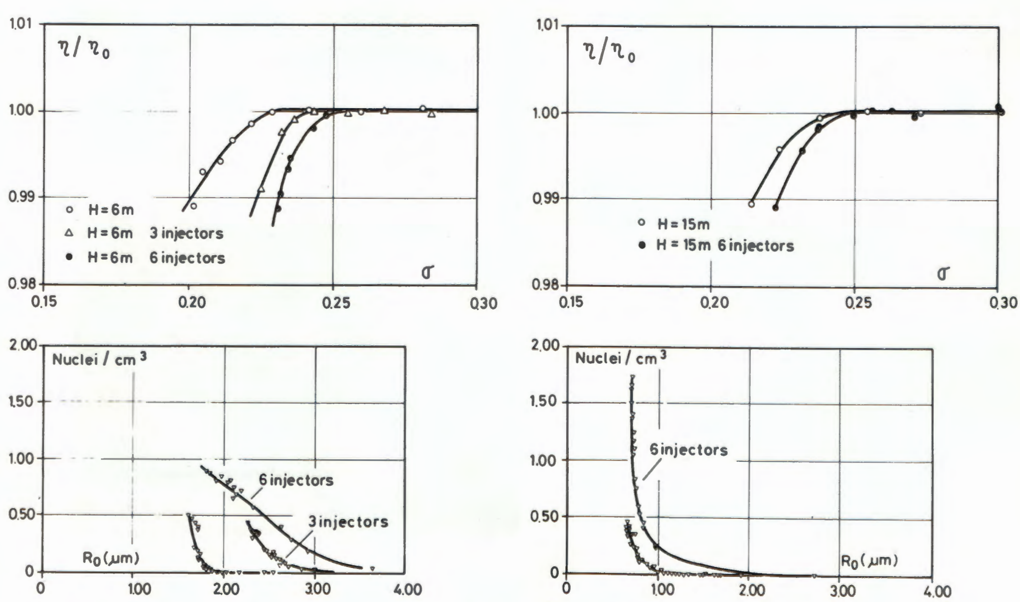


Figure 9 Performance characteristics in cavitation of a ν = 0.60 Francis turbine.

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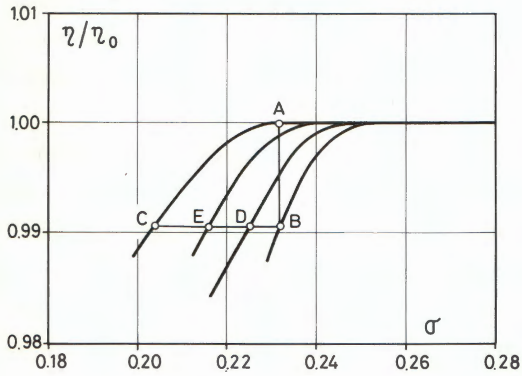
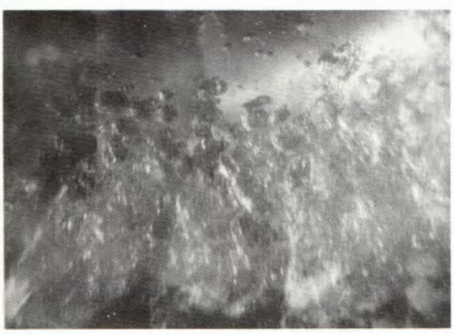
The diagrams in Figure 9 show that the (η–σ) curve is influenced by nuclei injection. This means that the test head is still too low to avoid scale effect without nuclei injection. The test head given in Figure 8 is therefore far too low for this specific speed.

$\sigma - \eta$ curves



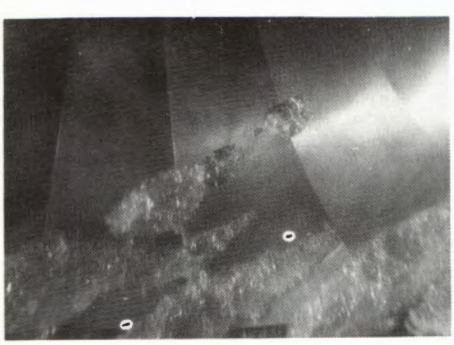
A : H = 6 m, no nuclei injection , σ_A

B : H = 6 m , nuclei injection, $\sigma_B = \sigma_A$,$\Delta\eta$



C : H = 6 m, no nuclei injection , $\Delta\eta$

D : H = 15 m, nuclei injection, $\Delta\eta$



E : H = 15 m, no nuclei injection, $\Delta\eta$

Figure 10 Cavitation observations.

The photographs in Figure 10 show a large difference in cavitation development between heads of 6 and 15 m. For this machine, the Froude head is 4.5 m. The photographs show clearly the importance of Froude similarity to keep the real vertical extension of the cavitation development. The largest cavitation development observed at 6 m head also gives a lower efficiency at constant σ. This last result should be pointed out because by testing Francis turbine in a nuclei saturation condition, we can overcome the nuclei effect, which could prevent us from following the true cavitation development. For instance for this high specific speed Francis turbine Froude effect was revealed clearly by injecting cavitation nuclei.

6. CONCLUSIONS

It is very difficult to respect the similarity law concerning the number of nuclei per unit volume [3]. However, if there are enough active nuclei to reach saturation, similarity is respected. The required concentration of active nuclei can be obtained either by a rather high test head, or by nuclei injection. The contribution presented in Montréal [5] arrived at the conclusion that similitude could be respected with a test head above a certain limit, without nuclei injection. However, the results presented above show clearly that this test technique is not possible, for two reasons:

- the test head required for obtaining saturation is of the order of 20 m, which involves very high flows and outputs for high specific speed turbines;
- the test head is often very different from the Froude head, and this causes unacceptable distorsions in the cavitation development.

Nevertheless, owing to the easy use of the nuclei generators, there is no over-duty in injecting the proper active nuclei content. For instance, in the case of a high specific speed Francis turbine ($\nu = 0.6$), we were able to perform reliable cavitation test for a test head (6 m) close to that given by the Froude condition by injecting 1 nucleus/cm^3 with a radius greater than 1.7 µm.

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NOTATIONS

R	: bubble radius of initial value Ro	m
Pv	: vapour pressure	Pa
nq	: specific speed	t/min
H	: net head	m

Cp	: pressure coefficient of minimum value Cp_{min}	-
σ	: cavitation number	-
N	: number of nuclei	$1/cm^3$
n(R)	: number of nuclei with an R radius	$1/cm^3$
R_i	: active limit radius	m

SUBSCRIPTS

m	: model
p	: prototype

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