Understanding Collective Aggregation Mechanisms: From Probabilistic Modelling to Experiments with Real Robots

A. Martinoli [a], A. J. Ijspeert [b,a] and F. Mondada [a]

[a] *Microprocessor and Interface Lab, Swiss Institute of Technology, IN-F Ecublens, CH-1015 Lausanne, Switzerland*

[b] *Istituto Dalle Molle di Studi sull'Intelligenza Artificiale, C.so Elvezia 36, CH-6900 Lugano, Switzerland*

Abstract

This paper presents an experiment of clustering implemented at three different levels: in a hardware implementation, in a sensor-based simulation and in a probabilistic model. The experiment consists of small reactive autonomous robots gathering and clustering randomly distributed objects. It is shown that, while the behaviour of the real robots can be faithfully reproduced in a sensor-based simulation, the evolution of the cluster sizes is perfectly described, both qualitatively and quantitatively, by a simple probabilistic model. Rather than simulating robots moving within an environment, the probabilistic model represents the clustering activity as a sequence of probabilistic events during which cluster sizes can be modified depending on simple geometrical considerations.

Key words: Collective Autonomous Robotics, Modelling, Robot Simulation, Real Robots, Clustering

1 Introduction

Autonomous collective robotics favours decentralised solutions, i.e. solutions where coordination is *not* taken over by a special unit using private information sources, or concentrating and redistributing most of the information gathered by the individual robots. Inspired by the so-called collective intelligence demonstrated by social insects [2], autonomous collective robotics studies robot-robot and robot-environment interactions leading to robust, goal-oriented, and perhaps emergent group behaviours. Such a decentralized approach in collective robotics seems to be a promising way to solve problems which are hard to tackle using classical control methods.

Often, fully decentralized control is combined with minimal robotic skills: robots are not able to communicate to each other, to plan their activity or to adapt their behaviour continuously. With such simple controllers, a gathering task becomes essentially a geometrical problem with a probabilistic nature which is well adapted to be described by simple probabilistic models. In this paper, we present such a model based on the probabilities of the robots to interact with clusters and other robots, and on the probabilities of clusters to be incremented or decremented.

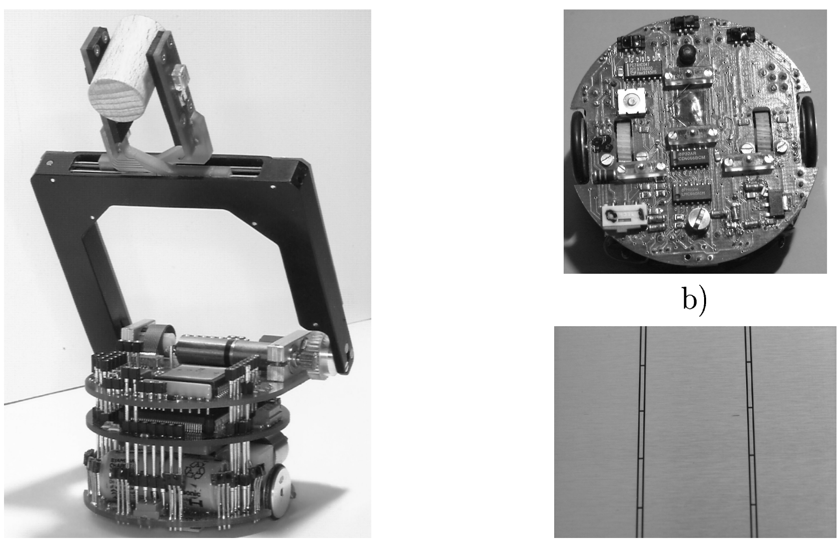
The motivations for such a modelling are two-fold. *Firstly*, because of its minimalist essence, it enables the investigation and the determination of which characteristics of the experiment are most influential on the clustering process. It allows, for instance, the evaluation of the influence on the collective performance of the number of involved robots, of parameters of the programmed behaviours, or of the interaction geometry. *Secondly*, working with probabilistic simulations means time saving. If modelling is reliable enough, the results of several runs are available in a few minutes, instead of a few or several hours required if the experiments are performed with sensor-based simulators (i.e. simulations which reproduce, as closely as possible, the sensory and motor capacities of a physical robot) or real robots, respectively. Probabilistic simulations can then be used as a prediction tool, and, in particular, it would be interesting to dispose of a tool which allows evaluations of critical characteristics of an experiment before the robot final design is accomplished or before a much more complicated sensor-based simulator is developed.

We have chosen experiments concerned with gathering and clustering of randomly distributed objects as a benchmark for our modelling study. This choice was motivated by the following considerations: *first*, this kind of experiments are well-suited to autonomous collective robotics and several experimental results are available; *second*, due to the robots capability of modifying the environment with the help of their gripper, these experiments present a double dynamics, the one of the environment (aggregation process) and the one caused by the robots movements and interactions with each other. However, we believe that the modelling presented in this paper is simple and general enough to be applied to other experiments in collective autonomous robotics.

Collective decentralized clustering of spread objects is inspired from studies of aggregation processes with social insects. Deneubourg [3] showed that a simple mechanism involving the modulation of the probability of dropping corpses as a function of the local density was sufficient to generate the observed sequence of clustering of corpses. In [1,5], similar experiments were carried out with real robots architectures based basically on reactive behaviour. In these two papers, a precise statistical analysis was carried out but neither a modelling of the experiment nor a comparison with simulation results were presented. In [8] we reported preliminary results on a similar experiment. However, while it was possible to quantitatively analyse the data, difficulties for the recognition algorithm to distinguish between the small objects to be clustered and other robots led to a high rate of destructive interferences and experimenter interventions which prevented the creation of an adequate probabilistic model which could have generated similar results. In [9] the experiments were repeated with a more reliable algorithm which allowed for a first attempt to develop a probabilistic model. This paper presents an improved version of that model and compares its predictions with new data delivered by the 3D-version of the Webots simulator [11] over long experiments. The improved model has also shown to be able to reproduce data presented in [1]. A detailed comparison at the model level of both experiments will be published elsewhere [7].

2 Materials and Methods

This section presents the three different implementations of the clustering experiment. The experimental set-up with the real robots and their control program are first presented. We then explain how the same experiment is carried out in Webots, a sensor-based simulator. Finally, we present how the dynamics of the clustering experiment can be represented in a probabilistic model.



a) b) c)

Fig. 1. a) Khepera equipped with gripper. b) View from the bottom of the redesigned Khepera base module. 4 electrical contacts (rotation symmetric to the center), a castor wheel and a battery charger have been added to the standard base module. c) A zoom of the special electrical floor shows main bands (alternatively connected to the plus and minus poles of a standard power supply unit) separated by thin unconnected bands.

Fig. 2. a) Seed scattering at beginning of the experiment and b) after about 2 hours, at the end of the longest experiment.

2.1 Experiments with Real Robots

2.1.1 Experimental Set-Up

Khepera is a miniature mobile robot developed to perform "desktop" experiments [12]. Its distinguishing characteristic is a small diameter of 55 mm. Each robot can be extended with a gripper module, which can grasp and carry objects with a maximum diameter of 50 mm (see fig. 1a). The energetic autonomy of Khepera in this configuration is about between 18 and 20 minutes. In order to extend the autonomy of Khepera for performing longer clustering experiments without battery recharging breaks, we have developped an original device to supply the robot from the floor. Fig. 1a and b show the two main components of this device. A more detailed description of this tool has been presented in [6].

The experiments are carried out with a group of 1 to 10 Kheperas and 10 to 40 *seeds* to be clustered (see fig. 2 as an example using the special extended autonomy tool mentioned above). The seeds have a cylindrical form, with a diameter of 15 mm and a height of 25 mm. We use two square arenas with different sizes, the largest having double the surface of the smallest ($113 \times 113\,\mathrm{cm}^2$ and $80 \times 80\,\mathrm{cm}^2$). The initial scattering of the seeds and the starting position of the robots are arbitrarily predefined and differ from replication to replication. Several experiments which differ in the number of scattered seeds, the number of robots, and the working surface are performed and the team performances are measured on the basis of the aggregation evolution (see section 3).

It is worth emphasising that in all the experiments the robots operate completely autonomously and independently; all sensors, motors and controls are on-board, and there is no explicit communication (IR or radio link) with other robots or with the experimenters. The only possible interactions among robots are the reciprocal avoidance of collisions and an indirect form of communication through modifications of the environment (stigmergic communication).

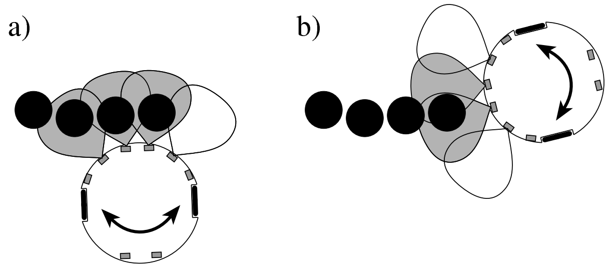


Fig. 3. The robot discriminating behaviour is based on a “wobble” movement, sampling continuously its proximity sensors in front of the found object. The picture shows this behaviour in front of a cluster of seeds: in case a) the robots discriminates the cluster as an obstacle, in case b) as a single seed.

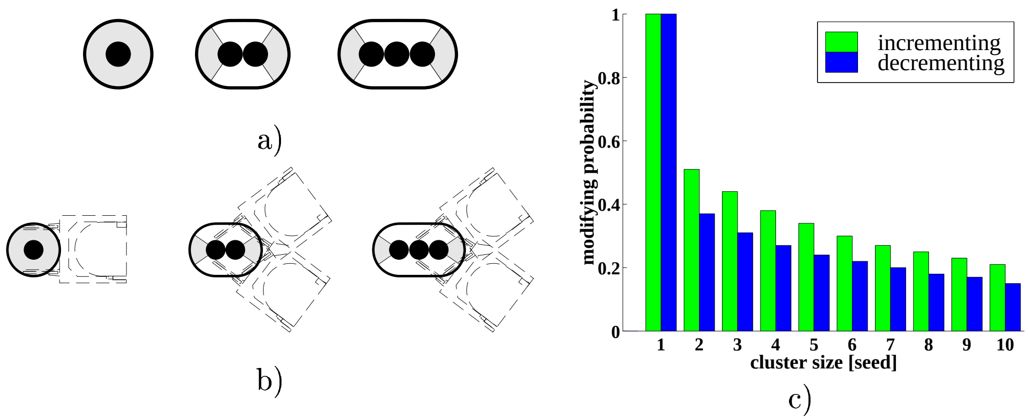
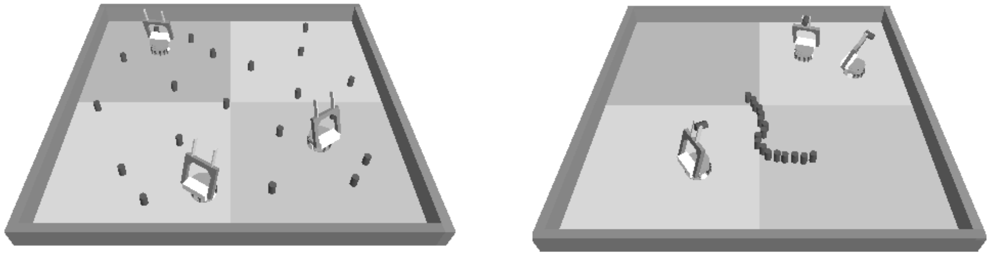


Fig. 4. a) Geometrical representation of the cluster incrementing probability. The ratio between the identification perimeter (arc delimiting the grey zone) and the total detection perimeter of the cluster represents the probability to increment the cluster size by one seed. b) Geometrical representation of the cluster decrementing probability. The robot, in order to decrement the size of the cluster by 1 seed first has to detect the cluster as in figure 4a and then grasp a seed (the angle from which a seed can successfully be grasped from a cluster is slightly smaller than its detection angle, see arc delimited by the grey zone in fig. 4b). c) The numerical values of both modifying probabilities represented separately on the same plot.

2.1.2 Control Algorithm

We can summarize the robot behaviour with the following simple rules: the robot moves on the arena looking for seeds. When its sensors are activated by an object, the robot starts a discriminating procedure. Two cases can occur: if the robot is in front of a large obstacle (a wall, another robot or an array of seeds), the object is considered as an obstacle and the robot avoids it. In the second case, the small obstacle is considered as a seed. If the robot is not carrying a seed, it grasps the seed with the gripper; if the robot is already carrying a seed, it drops the seed it is carrying close to the one it has found; then, in both cases, it resumes looking for seeds. Note that, because only the 2 extreme seeds of a cluster can be identified as seeds (in opposition to obstacles) by the robots, clusters are build in lines.



a) b)

Fig. 5. a) Example of aggregation process with 3 robots in Webots. Seed scattering at beginning of the experiment and b) after about 4:15 h of simulated time.

The discriminating behaviour is the same to that reported in [9]. In order to distinguish objects with the help of its proximity sensors only, the robot takes an increased number of spatial and temporal samples (see fig. 3). A test of reliability has shown that this algorithm correctly discriminates objects with a probability of 0.89 [13].

Figure 4 illustrates the geometrical and numerical results considered in the calculations of the size incrementing and decrementing probabilities of a cluster once the robot has found it. These probabilities are used in the probabilistic model.

2.2 Sensor-Based Simulations

As basis for comparison and as tool for performing systematic long experiments with 1 to 10 robots, we reproduced the different clustering experiments using the Webots simulator [11], which is at the moment only Khepera oriented. Webots is based on the as realistic as possible reproduction of the sensor capabilities as well as of the robot-robot and robot-environment interaction kinematics. Noise is added to the data delivered by virtual sensors and actuators. Algorithms implemented and tested on Webots can directly be transfered on real robots. The mean acceleration ratio for this experiment between Webots and real time is about 15 [1] on a workstation Ultra Sun 1 with 5 robots, if the display output is disabled.

[1] The acceleration factor in Webots depends strongly on the number of robots simulated simultaneously as well as from their configuration (vision turrets, gripper turrets, and so on).

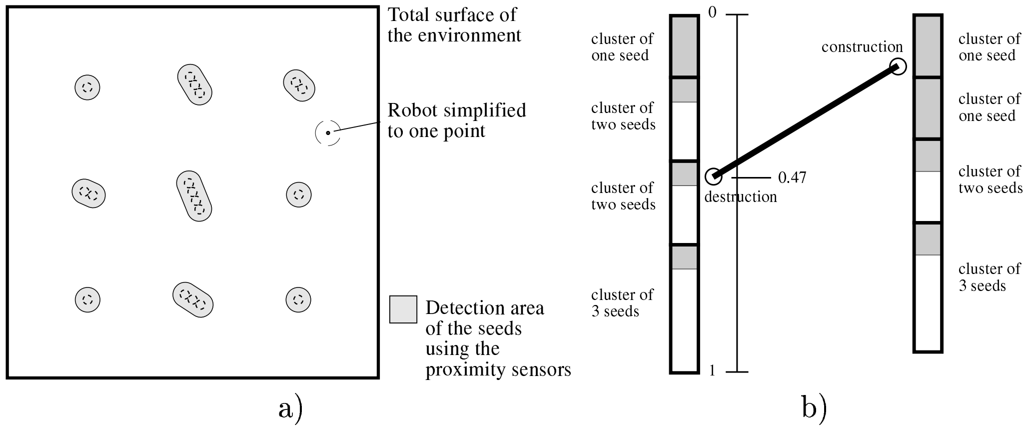


Fig. 6. a) The first random process: the clusters are represented with their detection area scaled with the total surface of the environment; the probability that a robot encounters a cluster is proportional to the detection area of the cluster. b) The second random process: the robot can modify the size of the cluster, incrementing or decrementing it by 1 seed, only if the robot runs into the cluster from the positions described in Fig. 4. The two bars represent the whole set of clusters scattered at a given moment on the arena. Depending on the charge status of the robot (uncharged or charged), a random pointer is drawn on the decrementing respectively incrementing bar. If the random pointer falls on a grey zone, the selected cluster will be decremented (left bar) or incremented (right bar) by 1 seed.

2.3 Probabilistic Modelling

The central idea of the probabilistic model is that instead of simulating robots moving within an environment, the clustering process is represented as a sequence of probabilistic events during which cluster sizes can be increased or decreased depending on geometrical aspects as illustrated in Figure 4. Robots could therefore be seen as dice being thrown into the arena at each iteration, with their random location as well as their current state (e.g. carrying or not a seed), determining their next state and the next state of the environment (i.e. the state of the clusters). The model takes into account the robot-robot and robot-environment geometry, the time needed to manage them, the sensor range for detecting seeds, walls or teammates,[2] and the reliability of the discriminating algorithm.

There are three fundamental approximations in our simulation:

- Robots are not moving in the environment (i.e. no trajectories are calculated): the simulation calculates the global probability of finding a cluster or another teammate based on their detection area and the arena surface.
- Boundary effects are only taken in account in a limited way: arena bound-

[2] Because all robots are equiped with an infrared reflecting band, teammates are detected at a distance of 6.5 cm while seeds and walls at 1.7 cm only.

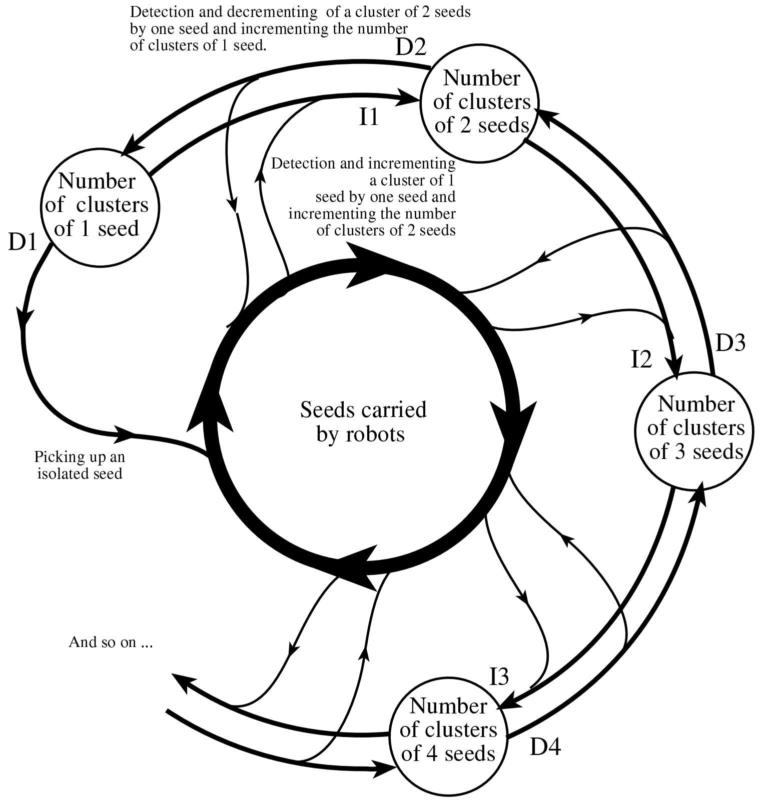


Fig. 7. The dynamics of the clustering experiment represented as a Markov chain. Notice that the transition probabilities from state to state are time-dependent in this representation. See text for further explanations.

aries are considered as obstacles which project their detection zone inside the arena. However, due to the fact that no position is attributed to clusters, no further interaction effect between clusters and boundaries is taken in account (for instance a possible reduced accessibility of the clusters tips, which are the "sensitive" points for modifying a cluster, as depicted in fig. 4).

- In order to convert the number of iterations into time, we calculate a fixed conversion factor as follows. We assume that all the seeds are scattered on the arena (worst case hypothesis) and that one robot alone has to find all them with probability equal to one. Furthermore, we assume that the robot is exploring the surface in a systematic and exhaustively way so that all the arena area has been visited once and only once till the end of exploration. Dividing the arena area by the *detection* surface of a single seed, we can obtain a discrete partitioning of the probability space. This problem is therefore equivalent to establish how many trials (which means iterations in the simulation) do we need for drawing k red balls (k = number of seeds scattered on the arena) from an urn which contains n balls (n = arena surface/detection surface of 1 seed). Notice that, to be coherent with the above mentioned systematic exploration of the robot, we do *not* put the balls already drawn back in the urn. It can be shown (it is also intuitively clear) that the answer to this problem is that we need n trials (or iterations) to achieve with probability of one that all the red balls have been drawn (left side of eq. 1). On the other hand, considering the robot physical size and its mean forward velocity (right side of eq. 1), we can establish the following equivalence:

$$F_{\text{it2t}} \frac{A_{\text{arena}}}{A_{\text{detseed}}} = \frac{A_{\text{arena}}}{\bar{v}_{\text{robot}} D_{\text{robot}}} \tag{1}$$

And therefore:

$$F_{\text{it2t}} = \frac{A_{\text{detseed}}}{\bar{v}_{\text{robot}} D_{\text{robot}}} [\text{s/iterations}] \tag{2}$$

With the used numerical values ($A_{\text{detseed}} = 20.4$ cm, $\bar{v}_{\text{robot}} = 8.0$ cm/s, $D_{\text{robot}} = 5.5$ cm) the resulting conversion factor is $F_{\text{it2t}} = 0.46$ [s/iterations]). The conversion factor is also used for taking in account the duration of the actions of the robots. For instance, with the implemented discriminating behaviour, it takes 2 s for avoiding obstacles and 10 s for modifying the size of a cluster. The algorithm translates these time lapses in number of iterations during which the searching behaviour is frozen.

Every robot can increment or decrement the size of a cluster by one seed at a time. The cluster modifying probabilities are conditioned by four stochastic processes. The first two of them are explained in Fig. 6. First, a random position in the environment is assigned to the robot. If this position is inside the detection area of a cluster, the second random process is started. According to the state of the robot (carrying or not carrying a seed) the size of the found cluster is incremented or decremented by one seed if the number delivered by the second random process is within the construction or destruction region (calculated with the values of Fig. 4c). The third stochastic process, related to the 0.89 efficiency of the discriminating algorithm, is always taken in account before each pick and drop operation. The fourth stochastic process, the interference with other teammates, is always overlapped to the first two processes: interference can occur during the search as well as during seed pick up or drop activity. Each random process is repeated for each robot independently before the next iteration of the program is started.

The aggregation process can also be represented as a Markov chain (see Fig. 7). The chain has as many states as different cluster sizes, which actually corresponds to the number of seeds scattered on the arena. The transition probabilities from state to state are calculated as a function of the product of the probabilities linked with the 4 stochastic processes mentioned above. Notice that a state of the Markov chain is represented by all the clusters with the same size. As a consequence, the rules to calculate the transition probabilities are pre-established by the geometrical constraints of the set-up but their values are updated every time that the number of clusters of a given size changes. Therefore, if we wanted to obtain a Markov chain with fixed transition probabilities, we would had to expand every state for all possible number of cluster of a given size. Finally, for a cluster of size n, if there is no cluster of size n-1, its construction probability is zero. As term of reference, the mean acceleration ratio between the probabilistic simulation and real time is about 4000 on a workstation Ultra Sun 1 with 5 robots.

3 Results and Discussion

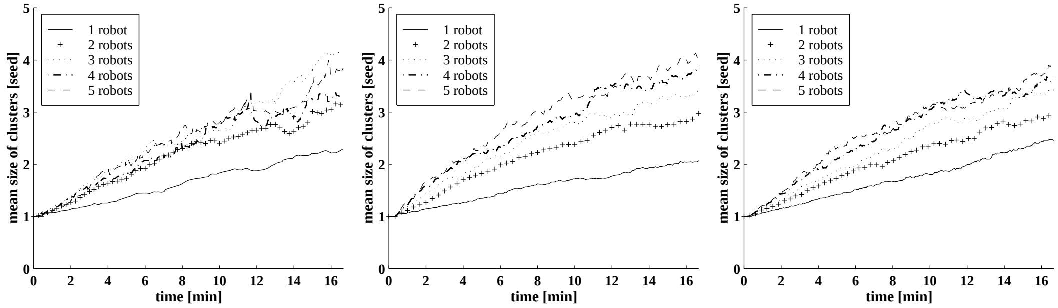
We have carried out several sets of experiments with different number of robots, different number of seeds and in two different sizes of arena (see Table 1). All experiments are carried in the three different implementations (real robots, Webots simulator and probabilistic model) except for the longest experiments (20 hours) which have not been realized with the real robots.

In order to quantify the evolution of the aggregation process, we have chosen 3 kinds of measurements: the mean size of the clusters, the size of the biggest cluster and the number of clusters. The first measurement is mainly used for the short experiments (up to 16 minutes). Due to the discontinuities of the mean size of cluster around the end of the longer experiments (up 1200 minutes), when for instance the number of clusters jumps from 2 to 1, we prefer to plot the two latter measurements. In all figures, the average value from several repetitions is plotted (see Table 1 for the number of repetitions).

In order to check whether or not there is a significant difference between data collected from the simulations and the real experiment, we performed a Mann-Whitney test [4] on the distributions of mean cluster size at the end of the shorter experiments and on the time needed to gather all seeds in the longer experiments. With the help of this non-parametric test, we compared the distributions of pairs of data sets (real robots vs. prob. model, real robots vs. Webots, Webots vs. prob. model). The results show that there is no statistically significant difference ($p<0.05$) between all the data sets except for long experiments with a single robot (Webots vs. probabilistic model).

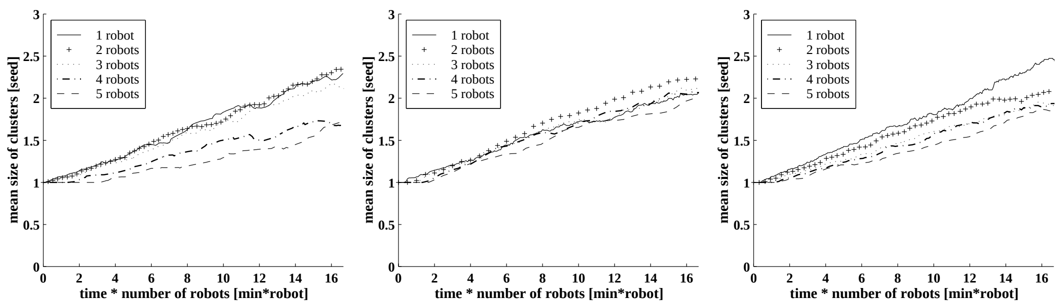
Table 1
Characteristics of the experiments carried out.

Arena	Seeds	Robots	Duration	Nb of repetitions			Figures
$[cm^2]$				real	Webots	prob. model	
80x80	20	1–5	16 min	5	10	10	8, 9, 10
113x113	40	2,6,10	16 min	5	10	10	10
80x80	20	2	120 min	2	5	10	11
80x80	20	1–10	20 hours	–	5	10	12, 13



a) b) c)

Fig. 8. Aggregation evolution with increasing number of robots (1 to 5) on the arena of 80x80 cm^2 and 20 seeds to be gathered. (a) Results of the experiments with real robots. (b) Results of the Webots simulator. (c) Results of the probabilistic modelling.



a) b) c)

Fig. 9. Mean contribution to the aggregation process of single teammate within a group composed of an increasing number of robots (1 to 5) on the arena of 80x80 cm^2 and 20 seeds to gather. (a) Results of the experiments with real robots. (b) Results of the Webots simulator. (c) Results of the probabilistic modelling.

Figure 8 shows the clustering evolution for a group of 1 to 5 robots. Although the results of both simulations, Webots and probabilistic model, are slightly smoother than those of the real ones (they are namely the average of twice the number of the experimental replications), the three plots present a good agreement. The main difference is that the performances of the group of 3 real robots is less rapidly saturated than the one in the simulations. Also, the performances of the group of 4 and 5 real robots saturate in a stronger way after 10 minutes (when the number of clusters is already reduced) than in both simulations.

The main observation which can be made from these results is that the increasing rate of the mean cluster size does not increase significantly with the number of robots when there are more than three robots in the arena. This is even more evident in Figure 9 which shows that in both the probabilistic modelling and the experiments with real robots there is no superlinearity in the team performances. On the contrary, in the experimental and probabilistic results with 4 and 5 robots there is a substantial sublinearity because of the destructive interferences, i.e. robots in a group contribute significantly less to

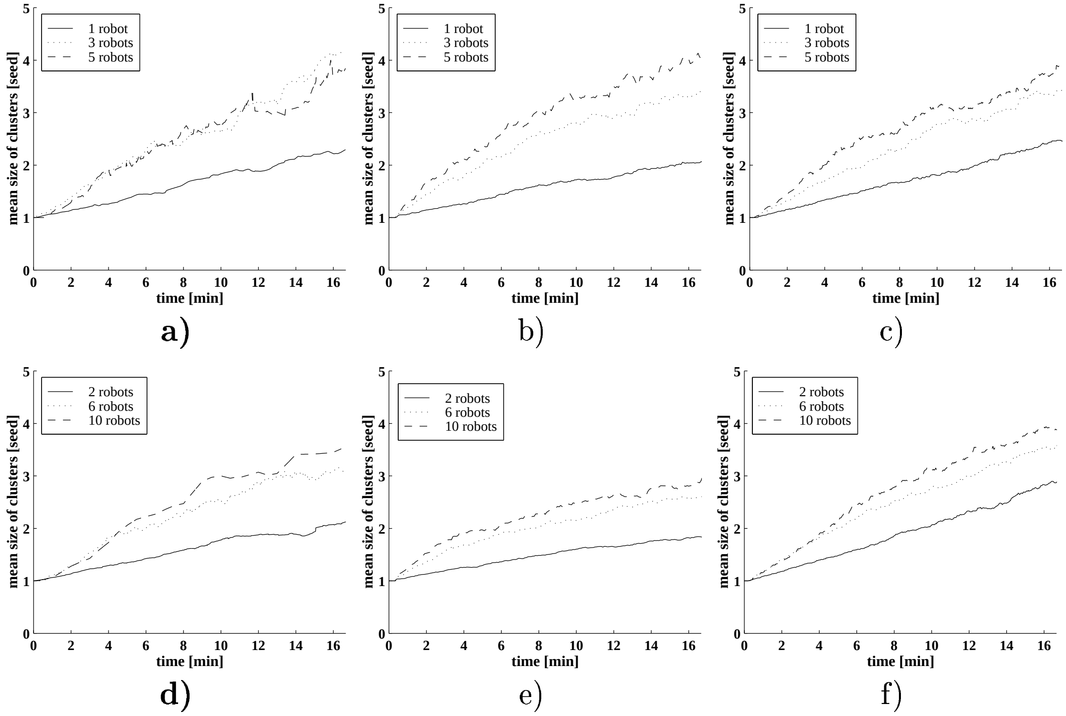


Fig. 10. Aggregation evolution with an increasing number of robots (1,3 and 5) on the arena of 80x80 cm^2 with 20 seeds to gather in a) and, similarly, with the double of teammates (2,6 and 10), the double of seeds (40) and an arena twice bigger (113x113 cm^2) in d). The corresponding Webots results are depicted in Figures b) and e) and the ones of the probabilistic modelling in c) and f).

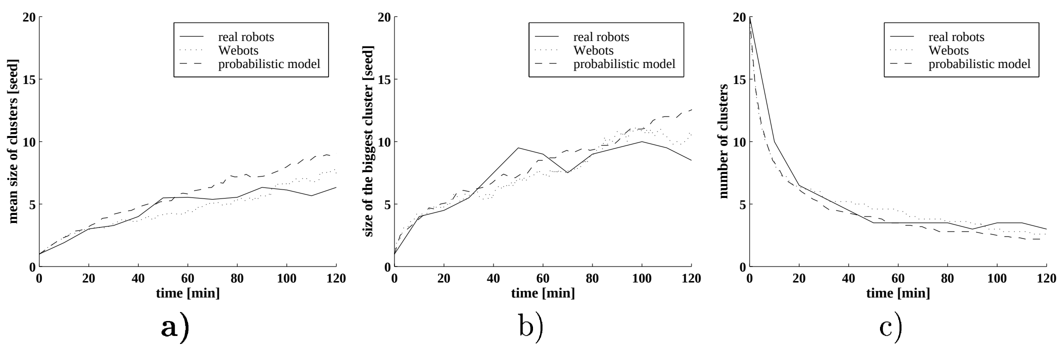


Fig. 11. Evolution of aggregation process during 2 hours. The results of real robots, Webots simulator and probabilistic modelling are overlapped in the same plot. Notice that the resulting plots are the average of different sets of runs with different sample times for each kind of experiment (2 runs and 10 minutes for real robots, 5 runs and 10 s for Webots and 10 runs and 30 s for the probabilistic modelling).

the increase of the mean cluster size than if they had worked alone. The Webots simulator shows a slightly better performance of the group of 2 robots. However, we guess that this is just an artefact due to the high variance [3] of this kind of measurements and the relative small set of runs.

[3] The coefficient of variation (standard deviation/mean) of these short experiments lies between 0.15 and 0.25 for simulations and real robots.

Fig. 10 compares the team performances where the number of robots, the surface, and the number of seeds to gather is doubled. The purpose here is to demonstrate that robot- and seed-density (meant as amount of work to do) are two key parameters of the experiments and that we can obtain the same results in the team fitness with the same density of robots and seeds (this is very important because of the rare availability of a greater number of robots). This plots confirm the good agreement between simulations and real robots shown in fig. 8.

As illustrated by fig. 11 which shows experimental and simulated results of 2 hours experiments with a group of 2 robots (i.e. the longest experiment realized with the real robots), both the sensor-based simulation and the probabilistic model provide a good prediction for the evolution of different variables used to characterise the clustering process.

Fig. 12 shows a comparison between the Webots simulator and the probabilistic modelling for 20 hours-long experiments. Three interesting points can be outlined. *First*, there is no substantial acceleration of the aggregation process with groups of 5 and 10 robots compared to a group of 3. *Second*, due to the fact that there are only 20 seeds on the arena and that, on average, half the number of robots are charged and the other half uncharged, the biggest cluster which can be built is about 20 decreased by half the number of robots (e.g. for 10 robots $20\text{-}0.5{*}10 = 15$ seeds). *Third*, since clusters of isolated seeds are in an irreversible way eliminated during the aggregation process and since aggregation is enhanced by a positive building gradient (the incrementing probability is consistently greater than the decrementing probability, see fig. 4), the seeds will always be gathered in a single cluster if enough time is available.

Fig. 13 shows a comparison of Webots simulator and probabilistic model based on mean and variance of the time needed by the robots to gather all the seeds in a single cluster. Fig. 13a shows good agreement between both simulations results. Fig. 13b shows that also in this experiment there is no superlinearity with an increasing number of robots working together. With more than 3 robots, the obtained team performance is sublinear. An interesting difference between the probabilistic model and the Webots simulator is outlined by the performances of the groups of 7 and 10 robots. For both group sizes, in one of the Webots runs (only 5 runs have been performed for this experiment with Webots), one of two biggest clusters has grown in a special position, close to an arena corner. As a consequence, it has taken much more time, due to the access difficulty of this cluster, to destroy it and increment a second bigger cluster which still had sufficient space to grow. This explains the larger mean values and standard variations for these 2 sets in the Webots simulation. This is typically a boundary effect not taken in account by the probabilistic model because, as mentioned before, the cluster position is not considered.

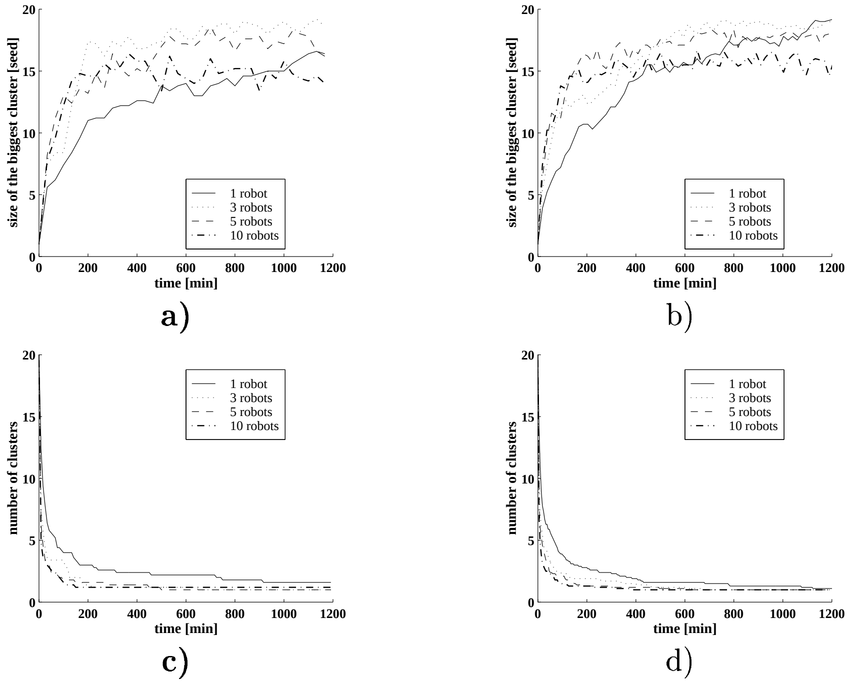


Fig. 12. Evolution of the size of the biggest cluster and the number of clusters on the 80x80 cm^2 arena with 20 seeds during 20 hours with 1 to 10 robots. (a,c) and (b,d) correspond to the results with Webots and the probabilistic model, respectively.

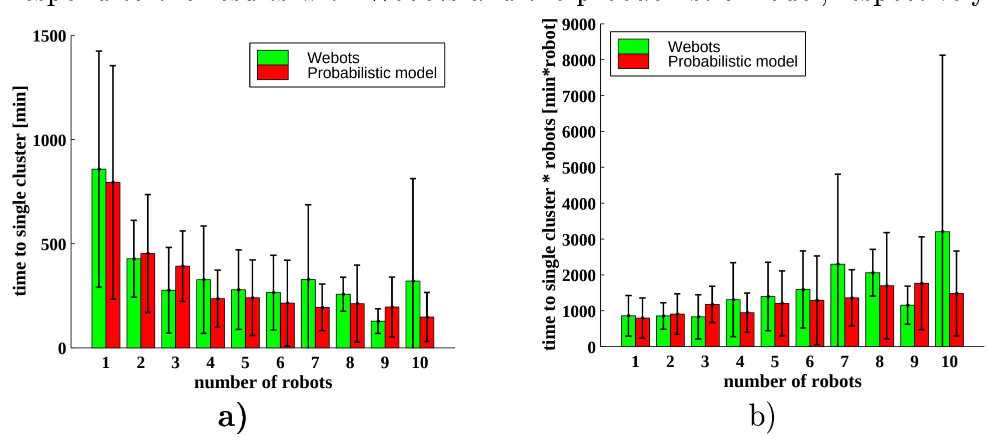


Fig. 13. Comparison between the results of Webots and the probabilistic model on the time needed to gather all the seeds in a single cluster. The arena size is 80x80 cm^2 with 20 seeds, the number of robots varies between 1 and 10. Each bar represent an average value over 5 runs for the Webots simulator and over 10 runs for the probabilistic model. The standard deviation over the 5, respectively, 10 runs is depicted by the black lines. In a) is depicted the comparison between absolute values and in b) between the single teammate contribution to the aggregation process.

4 Conclusion

This paper has presented a comparison between a clustering experiment implemented at three different levels: in real robots, in a sensor-based simulation and in a probabilistic model.

The good agreement of the clustering dynamics described by the probabilistic model with data collected with the two other implementations shows that that minimalist model incorporates the essential characteristics of the clustering problem. These characteristics have been identified to be probabilities of modifying the size of clusters and probabilities of having interferences with other robots. These probabilities are essentially based on geometrical considerations and can be derived from the sensory capacity of single robots. Once these probabilities are established and interaction time lapses measured, the probabilistic model has the interesting feature of being a prediction tool of the same quality as a detailed sensor-based simulation, while being significantly simpler and faster. Another interesting feature of the model is that the identification of the primary characteristics of this particular clustering problem is a step forward towards the understanding of collective mechanisms underlying clustering in general. In [7], we will show, for instance, that the model quantitatively predicts the data found in [1] once the probabilities are adapted to that experimental setup.

The results show that in this kind of experiments, where the coordination between robots is essentially probabilistic, the data obtained present a high variance (see also for instance [10] as another example for segregation mechanisms with robots). One possible solution to increase coordination capabilities of robots, while keeping the team control fully decentralised, would be to introduce a form of explicit local communication (signalling or symbolic communication).

Acknowledgements
We would like to thank André Guignard for the important contribution in the redesign of Khepera basis module, Aude Billard for the reviewing of this paper, Luca Gambardella for helpful discussions on autonomous mobile robotics, and Masakazu Yamamoto for the improvement of the distinguishing algorithm of Khepera. Alcherio Martinoli, Auke Jan Ijspeert and Francesco Mondada have been partially supported by the Swiss National Research Foundation.

References

[1] R. Beckers, O.E. Holland, and J.L. Deneubourg. From local actions to global tasks: Stigmergy and collective robotics. In R. Brooks and P. Maes, editors, *Proceedings of the Fourth Workshop on Artificial Life*, pages 181–189, Boston, MA, 1994. MIT Press.

[2] E. G. Bonabeau and G. Theraulaz. *Intelligence Collective.* Hermès, Paris, France, 1994.

[3] J. C. Deneubourg, S. Goss, N. Franks, A. Sendova, A. Franks, C. Detrin, and L. Chatier. The dynamics of collective sorting: Robot-like ant and ant-like robot.

In J. A. Mayer and S. W. Wilson, editors, *Simulation of Adaptive Behavior: From Animals to Animats*, pages 356–365. MIT Press, 1991.

[4] W. L. Hays. *Statistics*. Harcourt Brace College, Orlando, US, 1991.

[5] M. Maris and R. te Boekhorst. Exploiting physical constraints: Heap formation through behavioral error in a group of robots. In *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems IROS-96*, volume 3, pages 1655–1660, Osaka, Japan, November 1996.

[6] A. Martinoli, E. Franzi, and O. Matthey. Towards a reliable set-up for bio-inspired collective experiments with real robots. In A. Casals and A. T. de Almeida, editors, *Proc. of the Fifth International Symposium on Experimental Robotics ISER-97*, pages 597–608, Barcelona, Spain, June 1997. Springer Verlag.

[7] A. Martinoli, A. I. Ijspeert, and L. M. Gambardella. In preparation.

[8] A. Martinoli and F. Mondada. Collective and cooperative group behaviours: Biologically inspired experiments in robotics. In O. Khatib and J. K. Salisbury, editors, *Proceedings of the Fourth International Symposium on Experimental Robotics*, pages 3–10, Stanford, U.S.A., June 1995. Springer Verlag.

[9] A. Martinoli and F. Mondada. Probabilistic modelling of bio-inspired collective experiments with real robots. In T. Lüth, R. Dillman, P. Dario, and H. Wörn, editors, *Proceedings of the Third International Symposium on Distributed Autonomous Robotic Systems*, pages 289–308, Karlsruhe, Germany, May 1998. Springer Verlag.

[10] C. Melhuish, O. Holland, and S. Hoddell. Collective sorting and segregation in robots with minimal sensing. In R. Pfeifer, B. Blumberg, J.-A. Meyer, and S. W. Wilson, editors, *From Animals to Animats V: Proceedings of the Fifth International Conference on Simulation of Adaptive Behavior*, Zürich, Switzerland, August 1998.

[11] O. Michel. Webots: Symbiosis between virtual and real mobile robots. In *Proceedings of the First International Conference on Virtual Worlds, VW'98*, pages 254–263, Paris, France, July 1998. Springer Verlag.

[12] F. Mondada, E. Franzi, and P. Ienne. Mobile robot miniaturization: A tool for investigation in control algorithms. In T. Yoshikawa and F. Miyazaki, editors, *Proceedings of the Third International Symposium on Experimental Robotics*, pages 501–513, Kyoto, Japan, 1993.

[13] M. Yamamoto. Software Development of Basic Behaviors for Autonomous and Collective Robotics. Technical report, Microcomputing Laboratory, Swiss Federal Institute of Technology, October 1996.