Optimal Estimator-Detector Receivers for Space-Time Block Coding

Olivier Roy
Dept. of Communication Systems
Swiss Federal Institute of Technology
1015 Lausanne, Switzerland
e-mail: olivier.roy@epfl.ch

Sylvie Perreau
Inst. for Telecommunications Research
University of South Australia
Mawson Lakes 5095, Australia
e-mail: sylvie.perreau@unisa.edu.au

Alex J. Grant[1]
Inst. for Telecommunications Research
University of South Australia
Mawson Lakes 5095, Australia
e-mail: alex.grant@unisa.edu.au

Abstract — **Most space-time coding schemes can be classified either as non-coherent or coherent. In this paper we prove that optimal non-coherent decoding can always be decomposed into a channel estimation step followed by coherent decoding step. Surprisingly the required estimators do not in general minimise the mean squared error between the estimated and actual channel.**

Summary

Since the appearance of seminal works on space-time information theory and coding, two main philosophies have emerged for the design of codes and associated decoding algorithms. The first strategy takes the view that fundamentally, the parameters of the space-time channel are unknown and information theoretic principles would direct us to design codes directly for the channel with unknown parameters. This is the *non-coherent* approach taken in [1] and related works. The second strategy is to design the system such that the receiver can easily form some kind of estimate of the fading channel parameters, which is subsequently used within a coherent metric as if it were in fact the actual channel realisation [2]. We shall refer to this second class of strategies as *coherent*. In this paper we wish to compare these two approaches. We prove that optimal non-coherent decoding can always be decomposed in a coherent way using estimator-detector receivers and give the required channel estimators.

Consider a t transmit, r receive space-time channel operating in Rayleigh flat fading environment with $l \geq t$ consecutive channel uses,

$$Y = XH + N \tag{1}$$

where $Y \in \mathbb{C}^{l\times r}$ is the received matrix, X is the $l \times t$ transmitted codeword chosen equiprobably from a codebook, $X \in \{X_0, X_1\}$. The matrix $H \in \mathbb{C}^{t\times r}$ contains the channel gains and $N \in \mathbb{C}^{l\times r}$ is an additive noise matrix. The elements of H are i.i.d. circularly symmetric Gaussian with unit variance, those of N are i.i.d. circularly symmetric Gaussian with variance σ^2. Maximum Likelihood (ML) detection is done according to $\omega(Y) \underset{1}{\overset{0}{\gtrless}} |\Lambda_0|^r|\Lambda_1|^{-r}$, where $\Lambda_i = \left(X_iX_i^* + \sigma^2 I_l\right)$.

Consider decoding according to the following (possibly suboptimal) two step process. First, compute two linear channel estimates $\hat{H}_i = K_iY$ $(i = 0, 1)$ where K_i is a $t \times l$ matrix. Now, using these channel estimates, we can define a coherent detection approach. Let $\mu_i(Y) = \exp\|Y - X_i\hat{H}_i\|^2$ be the coherent metric for X_i, using $\hat{H}_i$ as if it were the true channel realisation. Within this framework, the decision statistic is

$$\mu(Y) = \exp\left(\|Y - X_1\hat{H}_1\|^2 - \|Y - X_0\hat{H}_0\|^2\right) \tag{2}$$

with decision rule

$$\mu(Y) \underset{1}{\overset{0}{\gtrless}} \gamma \tag{3}$$

for some decision threshold γ. The channel estimators K_i that, along with a carefully chosen γ, allow the estimation-detection approach given by (2) and (3) to be equivalent to the non-coherent ML decision rule will be referred to as *minimum codeword error probability* (MCEP) estimators.

The main results of [3] are summarised in the following theorems.

Theorem 1 *Let X_0 and X_1 be unitary codewords. Both zero-forcing (ZF) and minimum mean square error (MMSE) estimators are MCEP estimators. The corresponding decision threshold γ must be set to 1.*

Theorem 2 *Let $X_i = U_i\Sigma_iV_i^*$ be the singular value decomposition (SVD) of X_i with U_i (resp. V_i) an $l \times l$ (resp. $t \times t$) unitary matrix and Σ_i an $l \times t$ matrix of the form $\Sigma_i = \left[\mathrm{diag}(\sigma_{i,1}, \sigma_{i,2}, \ldots, \sigma_{i,t})\ O_{t\times(l-t)}\right]^T$. The MCEP channel estimators are given by $K_i = V_iD_iU_i^*$ where D_i is a $t \times l$ matrix of the form $D_i = \left[\mathrm{diag}(d_{i,1}, d_{i,2}, \ldots, d_{i,t})\ O_{t\times(l-t)}\right]$ with*

$$d_{i,j} = \begin{cases} \frac{\sigma_{i,j}^*}{|\sigma_{i,j}|^2}\left(1 - \sqrt{\frac{\sigma^2}{|\sigma_{i,j}|^2+\sigma^2}}\right) & \text{if } \sigma_{i,j} \neq 0 \\ 0 & \text{if } \sigma_{i,j} = 0 \end{cases}$$

for $i = 0, 1$ and $j = 1, \ldots, t$. The corresponding decision threshold γ must be set to $|\Lambda_0|^{r\sigma^2}|\Lambda_1|^{-r\sigma^2}$.

Optimal non-coherent detection can always be decomposed in a coherent way. The required estimators do not in general correspond to an MMSE estimate of the channel. Thus, trying to find a channel estimate as close as possible to the actual channel realisation is not necessarily the best strategy to adopt in order to minimise the decoding error probability. Extensions of Theorem 2 to spatially correlated fading with $H \sim N(0, P_r \otimes P_t)$ have also been achieved and can be found in [3].

[1] Alex J. Grant is supported by the Australian Government under ARC Grants DP0209658 and DP0344856. This work was performed while O. Roy was visiting the University of South Australia.

References

[1] T. L. Marzetta and B. M. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. Inform. Theory*, vol. 45, no. 1, pp. 139–157, Jan. 1999.

[2] V. Tarokh, N. Seshadri and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744–765, March 1998.

[3] O. Roy, *Optimal estimator-detector receivers for space-time block coding.* Master's thesis, Dept. of Communication Systems, Swiss Federal Institute of Technology, Lausanne, 2004.

0-7803-8280-3/04/$20.00 ©2004 IEEE