WATER RESOURCES RESEARCH, VOL. 28, NO. 8, PAGES 2111–2122, AUGUST 1992

A Mathematical Model of Hillslope and Watershed Discharge

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A mathematical water balance model describing major hydrological processes operating within wet forested watersheds is proposed. The model is capable of predicting hillslope and watershed discharge, evapotranspiration demands, hillslope moisture status, and surface and subsurface flow rates. It is based on soil physical principles and requires the following input variables: average hillslope angle and width, average soil depth, precipitation, average daily evaporation rates, effective saturated hydraulic conductivity, soil moisture holding capacity and initial moisture content. These variables are often easily measured from field studies. However, in some cases, the absence of field data may require that some of the variables in the model, e.g., saturated hydraulic conductivity, be estimated or calibrated from hillslope hydrograph records. The watershed model is composed of two submodels: a storage model and a hillslope model. The storage model describes the dynamic variation in water table elevation in recharge zones and the hillslope model is used to predict runoff and seepage through flow from surrounding hillsides. Application of the model is illustrated on a small watershed located in North Madison, Connecticut.

INTRODUCTION

The development of mathematical models provides a cost-effective and time-effective method of studying hydrological processes. Many models rely on *Horton*'s [1933] explanation of streamflow generation as a result of infiltration excess overland flow. However, persistent failure to observe this phenomenon in a wide variety of forested watersheds has cast doubt on its validity [*Ward*, 1984]. The infiltration excess overland flow concept is more appropriately applied to watersheds with thin vegetation or disturbed land use located in semiarid and arid regions [*Dunne*, 1983]. A conceptual model of streamflow generation and watershed discharge was proposed by *Dunne* [1983]. In Dunne's model, infiltration excess overland flow forms only one extreme in a continuum of hydrological response. The other extremes are saturation excess overland flow and rapid subsurface storm flow. *Whipkey* [1965], *Weyman* [1970], *Mosley* [1979], *Sloan et al.* [1983] and *Moore et al.* [1986] attribute the rapid subsurface storm flow response in forested watersheds to preferential flow paths in the soil layer during both saturated and unsaturated conditions. Even though these flow paths represent only a small percentage of the total pore volume, they may account for the bulk of water movement [*Jones*, 1971, 1978; *Ward*, 1984; *Watson and Luxmore*, 1986].

Undoubtedly, Dunne's conceptual model should be used not only to classify hydrological-mathematical models, but more importantly, to examine the appropriateness of modeling assumptions. If the physical assumptions pertaining to the model are incorrect in relation to the nature of the watershed, then any modeling results, even if they fit the observations, must be viewed with caution when they are used for predictive purposes. Many of the hydrological-mathematical models that rely solely on Horton's explanation of streamflow generation cannot be applied with confidence to forested regions. The central aim of this paper is to present a hydrological-mathematical model based on soil physical principles that is suitable for predicting soil moisture status, hillslope discharge and watershed yield for forested regions. The model is applied to data collected from an experimental watershed described by *Stagnitti et al.* [1989].

THEORY

Changes in the overall moisture storage in small watersheds is dependent on changes in the volume of water stored in recharge zones such as ponds, swamps and lakes and changes in moisture stored in the surrounding hillslopes. The water balance for water stored in recharge zones or storage areas may be written as

$$\frac{d[A_s h(t)]}{dt} = A_s[p(t) - E_a(t)] + H(t) - D(t) \qquad t \geq t_a \tag{1}$$

where $h(t)$ is the elevation of water in the recharge areas as measured as a height above some datum level, e.g., sea level, and A_s is the surface area, t is time and t_a is an arbitrary starting time, $p(t)$ is the precipitation rate per unit area, $E_a(t)$ is the evaporation rate per unit area, $H(t)$ is the contribution from the hillslopes surrounding the recharge area and $D(t)$ is the discharge. In principle, the discharge $D(t)$ is related to changes in the water elevation and therefore may be represented as function of $h(t)$. For example, in the simplest case, $D(t)$ may be represented as a piecewise linear function given by

$$D(t) = \Omega_k h(t) + \beta_k \tag{2}$$

where the coefficients are the gradients and intercepts of lines corresponding to the various flow zones and are measured experimentally. Note that is difficult to obtain a straightforward solution to (1) if $D(t)$ is a nonlinear function

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Paper number 92WR00705.
0043-1397/92/92WR-00705$05.00

of $h(t)$, and with the exception of a very few specific cases the solution of (1) would have to be evaluated by numerical methods. Substituting (2) into (1) and assuming that A_s is constant results in the following general analytical solution for (1):

$$h(t) = e^{-\nu_k t} \int_{t_a}^{t} [p(\tau) - E_a(\tau) + A_s^{-1}H(\tau)]e^{\nu_k \tau}\, d\tau - \left(\frac{\beta_k}{\Omega_k}\right) + \left[h(t_a) + \left(\frac{\beta_k}{\Omega_k}\right)\right]e^{-\nu_k(t - t_a)} \quad (3)$$

where $\nu_k = A_s^{-1}\Omega_k$. Equation (3) is valid for each piecewise segment. In practice the solution must be applied separately for each flow zone or linear portion of $D(t)$. Therefore application of (3) requires that whenever $h(t)$ crosses from one flow zone to another, then the solution process is restarted at that time. For example, suppose at a time $t = (t^* - \delta t)$, where δt is a small time interval, $h(t^* - \delta t)$ is on one line segment or flow zone and then, if at time $t = t^*$, $h(t^*)$ crosses into another flow zone, then the solution process is restarted with $t_a = t^*$ and $h(t_a) = h(t^*)$ and with the appropriate values for the coefficients defined by (2). This method can be very easily implemented into a computer algorithm as we have done. Furthermore, it avoids the complications that may arise in a numerical solution of (1) if a nonlinear $D(t)$ was adopted.

If the instantaneous behavior of the functions $p(t)$, $E_a(t)$ and $H(t)$ is not known, which indeed is often the case, then some further simplifications may be required. As these variables are often measured experimentally in a cumulative or averaged sense, the integrand of (3) may be simplified by the following approximations. First, the precipitation term is simplified by assuming that within suitably short periods of time, a storm event may be represented by a single impulse and consequently a sequence of storms may be approximated by a series of impulses summed over a given time period. Thus

$$p(t) = \sum_{m=0}^{M} \delta^*(t - t_m)P_m \quad (4)$$

where t_m denotes the time at which a storm event occurs, P_m is the amount of precipitation that fell during t_m and $\delta^*(t - t_m)$ is a delta function with the following properties:

$$\begin{aligned} &\delta^*(t - t_m) = 0 \qquad t \neq t_m \\ &\int_{-\infty}^{\infty} \delta^*(t - t_m)f(t)\, dt = f(t_m) \end{aligned} \quad (5)$$

where $f(t)$ is any finite function of t (see, for example, *Boyce and DiPrima* [1986, p. 311]). Defining a suitably small time interval, δt, where

$$j\delta t = t \qquad j = 0, 1, 2, 3, \cdots, (t - t_a) \quad (6)$$

then $H(t)$ and $E_a(t)$ may be replaced by their average rates defined as

$$\overline{X_j} = A_s^{-1}\overline{H_j} - \overline{E_j} \quad (7)$$

Substituting (4)–(7) into (3) results in the following simple scheme for predicting the elevation:

$$h(t) = \sum_{m=0}^{M} P_m e^{-\nu_k(t - t_a - t_m)} + \left[h(t_a) + \left(\frac{\beta_k}{\Omega_k}\right)\right]e^{-\nu_k(t - t_a)} + \nu_k^{-1}\left[\sum_{j=1}^{t - t_a} (\overline{X_{j-1}} - \overline{X_j})e^{-\nu_k t_{j-1}}\right] - \left(\frac{\beta_k}{\Omega_k}\right) \quad (8)$$

The discharge $D(t)$ from the recharge area can also be obtained from this scheme by substituting (8) back into (2). At this point, the only parameter that has yet to be determined is the hillslope contribution rate, $H(t)$. The average rate of hillslope discharge is required in (7) and (8).

In principle a watershed may be treated as a series of hillslope segments contributing discharge to the recharge area (e.g., streamflow). Each hillslope segment is selected on criteria based on uniform topography and soil properties. Dividing watersheds into smaller units for analysis is a common practice in hydrology [e.g., *Engman and Rogowski*, 1974; *Beven et al.*, 1984]. The number and size of units or hillslopes depend on the limitations of computer data storage and manipulation. Consider the two rates of flow: seepage through flow $S(t)$ and saturation excess runoff $R(t)$. Seepage through flow is the contribution to the recharge area of soil moisture traveling through the soil matrix under normal hydraulic gradients. Saturation excess runoff is defined here as the combined contribution of overland flow and/or rapid subsurface storm flow. In both cases, saturation excess runoff is generated whenever the soil layer is saturated and unable to absorb or transmit additional moisture.

The average rate of discharge from all hillslopes in a time interval δt is therefore given by

$$\overline{H_j} = \sum_{c=1}^{N} (R_j^{(c)} + S_j^{(c)}) \qquad j = \left[\frac{t - t_a}{\delta t}\right] \quad (9)$$

where c denotes a particular hillslope segment and j is a time counter. For any hillslope segment c, let

$$i\delta y = y \qquad i = 0, 1, 2, 3, \cdots, (L/\delta y) \quad (10)$$

measure the downslope distance along a typical transect of hillslope segment c, where $i = 0$ corresponds to the hilltop and $i = L/\delta y$ corresponds to the interface between the hillside and recharge zone. Also define W_i as the cross-section area at a point i, where W_i is calculated by multiplying the length of the contour at i by its soil depth. The runoff for any hillslope segment c is given by the sum of saturated excess moisture generated at each point i in the hillslope. That is,

$$R_j^{(c)} = \sum_{i=1}^{L/dy} \left[\delta_{(i)}(\theta_{(i,j)} - \theta_s)\, \frac{\delta y}{\delta t}\, \frac{(W_i + W_{i-1})}{2}\right] \quad (11)$$

where

$$\begin{aligned} &\delta_{(i)} = 1 \qquad \theta_{(i,j)} > \theta_s \\ &\delta_{(i)} = 0 \qquad \theta_{(i,j)} \leq \theta_s \end{aligned} \quad (12)$$

and where

$$\theta_{(i,j)} = \theta(i\delta y,\, t_a + j\delta t) \qquad 0 \leq y \leq L,\ t \geq t_a \quad (13)$$

is the local moisture content of the soil. An implicit assumption of (11) is that runoff is delivered to the recharge area within the time interval δt. This is a reasonable approximation for many small watersheds. However, if the length of the hillslope is much greater than the distance typically traveled by water through macropores or overland flow in a time δt, then (11) would have to be replaced with a more accurate model. In such cases an overland flow model like the one proposed by *Campbell et al.* [1984] might be employed.

The seepage through-flow component is defined by

$$S_j^{(c)} = K[\theta_{(i,j)}] \sin \phi W_i \tag{14}$$

when $\sin \phi$ is the average slope along the transect and $K(\theta)$ is the unsaturated hydraulic conductivity function. The seepage through-flow component at the interface between the hillslope and the recharge area is given by (14) for $i = L/\delta y$. Note that $K(\theta_s)$ is the saturated hydraulic conductivity. The final task is to develop an appropriate hillslope model to predict the local moisture content $\theta_{(i,j)}$. *Stagnitti and Parlange* [1987] propose the following equation as a model for water transport in hillslopes:

$$\frac{\partial[\theta(y, t)W(y)]}{\partial t} + \frac{\partial[K(\theta) \sin \phi W(y)]}{\partial y} = \frac{W(y)[p(t) - E_a(t)]}{B(y)} \qquad 0 \le y \le L, \ t_a \le t \le \infty \tag{15}$$

where $B(y)$ is soil depth. Equation (15) assumes that diffusion tends to maintain a uniform moisture content in the soil layer and that the variation in the moisture content results primarily from gravitational forces which act to move moisture in a downslope direction. As a consequence, (15) is only valid for thin soil layers, that is, when the typical soil depth is much smaller than the typical hillslope length.

A similar but simpler version of (15) was applied by *Stagnitti et al.* [1986] to study drainage from a uniform hillslope segment. Equation (15) reduces to the equation studied by *Stagnitti et al.* [1986] when $W(y)$ and $B(y)$ are simple constants and when uniform rates of $p(t)$ and $E_a(t)$ are assumed. Under these assumptions, (15) has a straightforward analytical solution. This solution was compared to experimental data collected by *Hewlett and Hibbert* [1963, 1967] and to numerical solutions of other hillslope models developed by *Nieber and Walter* [1981], *Nieber* [1982], *Beven* [1981, 1982] and two models developed by *Sloan et al.* [1983]. Predictions of the hillslope discharge and cumulative discharge based on the simplified version of (15) were in very good agreement with predictions based on the more complex numerical models and with the experimental data. Equation (15) is an extension of this earlier work. The model can now be applied to complex hillslope geometries and variable rates of evapotranspiration and precipitation. In this paper we assume that the capillary forces are negligible in comparison with gravitational forces. However, if capillary forces are important then an additional approximation which corrects the downslope flux as described by *Stagnitti et al.* [1986] could be incorporated into (15).

A piecewise linear model for the hydraulic conductivity is assumed and defined by

$$K[\theta_{(i,j)}] = K_s\left[\frac{\theta_{(i,j)} - \theta_h}{\theta_s - \theta_h}\right] \qquad \theta_{(i,j)} > \theta_h \tag{16}$$

$$K[\theta_{(i,j)}] = 0 \qquad \theta_{(i,j)} \le \theta_h \tag{17}$$

where θ_h is a soil moisture content which we call the holding capacity. At the holding capacity the downslope drainage ceases. This variable can be related to soil physical properties by the following equation:

$$\theta_h \approx \theta_s(1 - n) \tag{18}$$

where n is the typical soil parameter in the unsaturated hydraulic conductivity function. Values for n typically vary from $n = 1/2$ (coarse sand) to $n = 1/14$ (fine clay). Whenever the local soil moisture content falls below the holding capacity, the water becomes stagnant and immobile. Therefore, moisture ceases to move downslope and is only affected by precipitation and evapotranspiration. This condition is defined by (17). On the other hand, if the local moisture content is above the holding capacity, then this water will be transported downslope with an average hydraulic gradient defined by (16). If after extensive precipitation the soil can no longer absorb moisture, i.e., the local moisture content reaches saturation, then any additional moisture is treated as saturation excess runoff and is routed to the recharge area according to (11).

Substituting (16) and (17) into (15) and assuming that the slope is constant results in the following first-order partial differential equation:

$$\frac{\partial \chi}{\partial t} + \left[\frac{K_s \sin \phi}{\theta_s - \theta_h}\right]\frac{\partial \chi}{\partial y} = \frac{W(y)[p(t) - E_a(t)]}{B(y)} \qquad \chi \ge 0 \tag{19}$$

where

$$\chi = [\theta(y, t) - \theta_h]W(y), \qquad \theta(y, t) \ge \theta_h, \qquad W(y) \ge 0 \tag{20}$$

Equations similar to (20) appear in many other branches of physics, e.g., kinematic surface waves [*Lighthill and Whitham*, 1955*a*] and traffic flow theory [*Lighthill and Whitham*, 1955*b*]. One-dimensional models based on gravity-dominated flow like (19) have been successfully applied to study unsaturated, vertical drainage [*Sisson et al.*, 1980; *Smith*, 1983], saturated subsurface storm flow [*Beven*, 1981, 1982], coupled unsaturated-saturated flow [*Smith and Hebbert*, 1983; *Hurley and Pantelis*, 1985], preferential drainage [*Steenhuis et al.*, 1988] and preferential solute transport [*Steenhuis et al.*, 1990, 1991; *Stagnitti et al.*, 1991]. These examples clearly indicate the usefulness of this approach to studying watershed hydrology. These models may be distinguished by their application (e.g., downslope or vertical drainage) and their initial and boundary conditions. The differences between these models have been previously discussed by *Stagnitti et al.* [1986], *Stagnitti et al.* [1987] and *Hurley and Pantelis* [1987]. A feature of (19) which further distinguishes it from other models is that even though the model is one-dimensional in nature, it simulates approximate three-dimensional behavior by including variable terms for soil depth and hillslope width.

The solution to (19) is defined on a set of trajectories or characteristics which propagate from the initial condition $\chi(y, t = 0)$. A general analytical solution of (19) was presented by *Stagnitti and Parlange* [1987]. However, this solution is difficult to apply in practice. Therefore, a simple numerical scheme based on the method of characteristics is proposed. A similar scheme was used by *Stagnitti and Parlange* [1987] to simulate streamflow generation from a variety of hypothetical hillslope types (e.g., uniform or parallel side boundaries, convergent and divergent side boundaries) for a wide range of input parameters (e.g., initial soil moisture profiles, hillslope angles, soil depths, saturated hydraulic conductivities, etc.). This study, however, is the first attempt to apply the hillslope model on a watershed scale. Along the characteristics, (19) reduces to a system of ordinary differential equations given by

$$dt = \left[\frac{\theta_s - \theta_h}{K_s \sin \phi}\right] dy = \left[\frac{B(y)}{[p(t) - E_a(t)]W(y)}\right] d\chi \tag{21}$$

The characteristics of (19) are straight lines which are obtained by integrating the first two terms of (21).

$$y - \left(\frac{K_s \sin \phi}{\theta_s - \theta_h}\right)t = y_0 \qquad \text{for } y > \left(\frac{K_s \sin \phi}{\theta_s - \theta_h}\right)t \tag{22}$$

Letting

$$d\chi \approx [\theta_{(i,j)} - \theta_h]W_i - [\theta_{(i-1,j-1)} - \theta_h]W_{i-1} \tag{23}$$

and substituting (6) and (23) into the first and last terms of (21) results in the following solutions for the local moisture content at any point and for any time:

$$\theta_{(i,j)} = \theta_{(i,j-1)} + \theta_a \qquad \text{for } \theta_p \le \theta_{(i-1,j-1)} < \theta_h, \quad \theta_p \le \theta_{(i,j-1)} < \theta_h \tag{24}$$

$$\theta_{(i,j)} = \theta_h + \theta_a \qquad \text{for } \theta_p \le \theta_{(i-1,j-1)} < \theta_h, \quad \theta_h \le \theta_{(i,j-1)} < \theta_s \tag{25}$$

$$\theta_{(i,j)} = \theta_h + \frac{W_{i-1}}{W_i}[\theta_{(i-1,j-1)} - \theta_h + \theta_a] \qquad \text{for } \theta_h \le \theta_{(i-1,j-1)} < \theta_s \tag{26}$$

where θ_p is the moisture content at the permanent wilting point. Note that θ_p represents the minimum water content that can be extracted from the soil layer due to transpiration. The value for θ_a may be positive or negative and is defined by

$$\theta_a = \frac{1}{B(i\delta y)} \int_{t-\delta t}^{t} [p(\tau) - E_a(\tau)]\, d\tau \tag{27}$$

An implicit assumption of (27) is that any excess precipitation is readily absorbed into the soil layer during δt until soil moisture saturation is attained. If the local soil moisture is saturated, then the excess precipitation is distributed as saturation excess runoff.

A zero-flux boundary condition applies on the hilltop. Therefore,

$$\theta_{(0,j)} = \theta_h + \theta_a \qquad \text{for } \theta_{(0,j-1)} \ge \theta_h \tag{28}$$

$$\theta_{(0,j)} = \theta_{(0,j-1)} + \theta_a \qquad \text{for } \theta_{(0,j-1)} < \theta_h \tag{29}$$

It is necessary to smooth irregularities that propagate in the solution as a result of subsequent periods of wetting and drying which result from the irregular pattern of precipitation. This can be achieved readily by averaging the moisture contents between two adjacent cells after (27) has been applied and before the next time step in the following manner: For values of i starting at $i = L/\delta y$ and decreasing to $i = 2$, let

$$\theta_{(i,j)} = \frac{[\theta^*_{(i,j)} + \theta^*_{(i-1,j)}]}{2} \tag{30}$$

where the asterisk represents the moisture content just prior to the averaging. Finally, from (21), the method of characteristics requires that the value for the minimum distance interval δy be related to the minimum time interval δt in the following manner:

$$\delta y = \frac{K_s \sin \phi}{(\theta_s - \theta_h)} \delta t \qquad \text{for } \theta_h \le \theta_{(i,j)} \le \theta_s \tag{31}$$

Equation (31) also shows that moisture in the mobile pore group will travel downslope at a velocity given by $[K_s \sin \phi/(\theta_s - \theta_r)]$. Clearly, the values of the saturated hydraulic conductivity, saturated moisture content and soil moisture holding capacity determine the magnitude, duration and timing of both saturation excess runoff and seepage through flow. Therefore, the interaction among these variables uniquely determines the shape of the hillslope hydrograph and watershed yield.

The solution strategy for finding the hillslope moisture content given by (24)–(26) and consequently for calculating the hillslope contribution to the recharge area given by (11) and (14) is algorithmically simple and computationally efficient. The model can be easily implemented on a basic microcomputer to study long-term response. In fact, the model can even be implemented in a spread sheet. This is an important consideration as the adoption of other hillslope models [e.g., *Freeze*, 1974] cannot be applied to study long-term prediction of discharge and other characteristics on the time scale we propose (e.g., months and years). The next section describes an experimental watershed that is used to evaluate the model.

APPLICATION OF THE THEORY TO AN EXPERIMENTAL WATERSHED

Description of the Watershed and Experimental Methods

The study area is a small, wet, forested region located in North Madison, Connecticut. The watershed has an area of about 2.3 ha and is situated in a larger mixed deciduous forest. The soil is composed of a spongy and well-developed humus layer which lies on a shallow layer of fine sandy loam to a depth of 0.3–1 m. The soil layer is supported by an impervious granite ledge which extends to the watershed's boundaries. Located approximately in the center of the watershed is a small swamp (the recharge area) with an area of about 0.4 ha and an elevation of about 47 m above sea level. This swamp feeds an underground stream which exits the watershed at site A in Figure 1. The swamp's bed consists of mud and leaf debris which has sedimented in

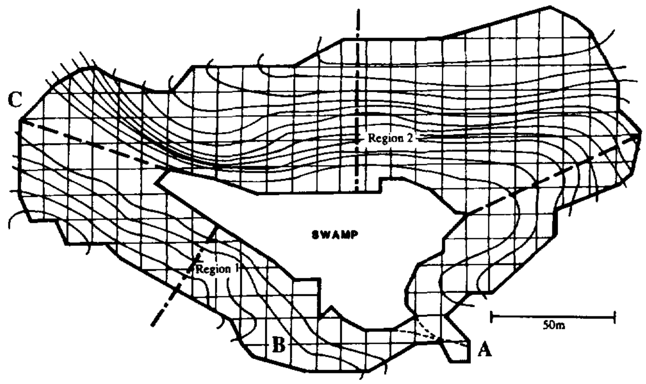

Fig. 1. Sketch of the 2.3-ha experimental catchment. The grid lattice points are at 10 m intervals. Soil measurements were taken at each vertex. The transect for each hillslope segment is represented by the heavy dot-dash lines (Modified from *Stagnitti et al.* [1989].)

geological time. Precipitation, solar radiation, humidity, air temperature, soil moisture, swamp water level and watershed yield were collected for about 6 months in 1973 and 1 month in 1975. The hydrology of this watershed was studied by *Stagnitti et al.* [1989].

The Relationship Between the Elevation of Water in the Swamp (Recharge Area) and Its Discharge

Figure 2 exhibits the relationship between the discharge D and the elevation of water in the swamp, h. As the discharge from the swamp exits the watershed at site A in Figure 1, it also represents the watershed discharge. The figure comprises two sets of data points: squares and circles. The squares are observed measurements of swamp level and discharge at site A. These data were recorded on site by field technicians during calibration of the weir. The other measurements (circles) were constructed later by comparing data read from the discharge hydrograph and swamp level charts [*Stagnitti et al.*, 1989]. Four straight lines defined by (2) were fit to the data in Figure 2. The values for the intercept and slope of each line are given by *Stagnitti et al.* [1989]. These data are used to classify the flow characteristics of the watershed into four zones defined by (1) high flow ($h > 0.467$ m), (2) medium flow ($0.444 \text{ m} < h < 0.467$ m), (3) low flow ($0.410 \text{ m} < h < 0.444$ m), and (4) no flow ($h < 0.410$).

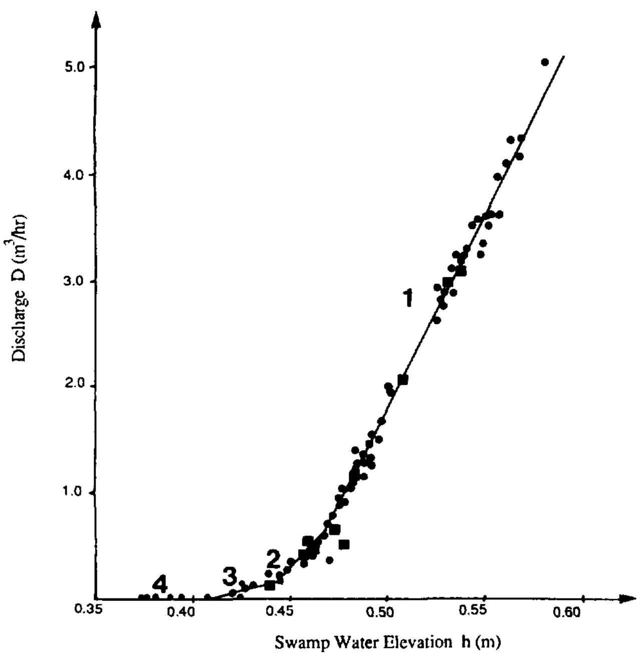

Fig. 2. Relationship between the discharge $D(t)$ and the swamp water elevation h. Lines 1 to 4 indicate high flow, medium flow, low flow and no flow zones, respectively. (Reproduced from *Stagnitti et al.* [1989].)

Equations (3) and (24)–(26) are subject to the initial conditions $h(t = t_a)$ and $\theta(y, t = t_a)$ given. Also, the solution process is dependent on other parameters such as precipitation, hillslope width, saturated hydraulic conductivity, soil moisture holding capacity, permanent wilting point, length of hillslopes, etc. The following sections describe how values of the initial and boundary conditions and other input parameters were determined.

Determining Values for the Initial Conditions

Experimental data expressing the average moisture content as a function of downslope distance y at any time were not available in this study. Therefore, the initial moisture profile is estimated from some other data. *Stagnitti et al.* [1989] found that the watershed's overall moisture storage is approximately correlated to the swamp's water elevation and hence watershed yield. This assumption makes intuitive sense since during low flows the surrounding hillslopes are relatively dry and during very high flows the surrounding hillslopes are relatively wet. During very high discharges following abundant rain, the surrounding hillslopes are very near or at saturation and the lower portions of the hillslope certainly are. Therefore, when these extreme events occur the following will be a reasonable approximation for the initial condition:

$$\theta(y, t_a) = \theta_{(i,0)} = \theta_s \tag{32}$$

For the North Madison watershed, *Stagnitti et al.* [1989] found that $\theta_s = 0.47$. In the absence of any better experimental data, the only reliable starting point is to choose conditions which are favorable to the application of (32). The influence of the initial conditions on the model's predictions was checked by simulating watershed discharge with different values for the initial condition ranging from the moisture holding capacity to soil saturation. As would be expected the initial condition has a considerable influence on the characteristics of the outflow hydrograph for the first couple of weeks but after that all the simulated hydrographs stabilized to the one curve regardless of the values adopted for the initial condition and thus these values have little influence on long-term predictions. Therefore some error in the initial moisture profile can be tolerated for long-term predictions.

Predicting Evapotranspiration

An average watershed evapotranspiration rate is predicted using the *Priestley and Taylor* [1972] equation given by

$$\overline{E_j} = \left[\frac{\alpha\Delta}{\lambda(\Delta + \gamma)}\right]\frac{1}{(t_j - t_{j-1})}\int_{t_{j-1}}^{t_j} R_n(t)\, dt \tag{33}$$

where α was found by *Stagnitti et al.* [1989] to equal 1.22. No distinction was made between evaporation from the swamp and evapotranspiration from the surrounding hillslopes even though the watershed model can handle both rates independently.

Determination of the Optimal Time Interval δt

The theory has been derived for any arbitrary value for the time interval δt. However, δt should be sufficiently small so that the numerical solution is accurate. The optimal magnitude of δt which minimizes the numerical error but retains computational efficiency is not known a priori but needs to be determined by simulation. The results of simulation runs for this experiment indicate that when δt is approximately 1 hour, accurate numerical solutions are obtained for a realistic range of values for the saturated hydraulic conductivity and holding capacity.

Hillslope Parameters

The watershed consists of a swamp and several hillslope segments. For purposes of this simulation the watershed was discretized into two hillslope segments as illustrated in Figure 1. The soil depth for each region was determined by averaging all the grid point measurements for each hillslope. For region 1 the average soil depth was 0.493 m and for region 2 it was 0.566 m. Two transects, one for each hillslope, were chosen to represent averaged hillslope characteristics. The length of the transect for the regions is 44 m and 63 m for regions 1 and 2, respectively. The hillslope function $W(y)$ is determined by multiplying the length of the contour at every 1-m rise above sea level by the average hillslope depth. A quadratic least squares regression gave the best statistical fit to these data points. The results of the regression are

$$W(y) = 0.796 + 9.476y - 0.1980y^2 \qquad R^2 = 0.972 \qquad (34)$$

$$W(y) = 85.81 + 3.598y - 0.0701y^2 \qquad R^2 = 0.991 \qquad (35)$$

The coefficient of determination R^2 was determined by comparing the predicted $W(y)$ from (34) and (35) to the actual values at each 1-m rise above sea level. In each case the correlation is significant. Differentiating these equations with respect to y gives an indication of hillslope convergence or divergence. Both hillslopes exhibit regions of divergent and convergent flow. Region 1 exhibits convergent flow for $y < 24$ m and divergent flow for $y > 24$ m and region 2 is convergent for $y < 26$ m and divergent for $y > 26$ m.

Runoff Travel Times

Overland flow was seldom observed in this watershed. Also, field studies with water and dye injected continuously at the boundary of the watershed and directly at the granite ledge showed that water never took longer than 10 min to reach the swamp. This observation is consistent with other studies of wet, forested watersheds [*Ward*, 1984]. Therefore the saturation excess runoff $R(t)$ is largely composed of rapid subsurface flow traveling along the granite ledge.

Saturated Hydraulic Conductivity and Soil Moisture Holding Capacity

Point source measurements of the saturated hydraulic conductivity may change from location to location even by orders of magnitude. However, it can be replaced by an "effective" value which represents an overall watershed-scale average [*Talsma and Hallan*, 1980]. We have no field measurements for these variables and hence cannot determine such effective values without some form of calibration. The effective values for both the saturated hydraulic conductivity and the holding capacity may be determined by calibrating the predicted watershed discharge with a small portion of the original record. The rest of the record can then be used to validate the calibrated values. This is a common procedure (see, for example, *Smith and Hebbert* [1983]).

Since we only had four weeks of data in 1975 they were chosen as the calibration period for these parameters. Values for the saturated hydraulic conductivity were allowed to vary from 0.005 m h^{-1} to 0.5 m h^{-1}. These values correspond to a wide range of soil types anywhere from a clay to a fine sand [*Brooks and Corey*, 1964]. The holding capacity was allowed to vary between 0.25 to 0.44 which corresponds to values for $1/n$ ranging from approximately 2 to 14. The best results were obtained when the saturated hydraulic conductivity equalled 0.05 m h^{-1} and the holding capacity equalled 0.36. These values correspond to soils comprising sandy loam mixtures and are therefore consistent with the observed soil type in the watershed. We can now use these values to predict the hillslope discharge and hence watershed yield for all of the 1973 data without further curve fitting.

Moisture Content at the Permanent Wilting Point

The permanent wilting point is attained when the soil matrix dries to a point where the forces retaining moisture within the matrix are greater than forces due to evapotranspiration. The moisture content at the wilting point varies considerably depending on location, soil and vegetation type. The value of θ_p for the North Madison watershed is unknown. Typically, a value of 0.1 represents regions like the North Madison watershed (see, for example, *Steenhuis et al.* [1984]) and is the value adopted in this study. However, as shown shortly, the value of the wilting point has no influence in this study as the hillslope moisture content is never found to drop anywhere near 0.1.

A summary of all the model input parameters and other hillslope and swamp statistics for the North Madison watershed is given in Table 1.

RESULTS AND DISCUSSION

Figures 3–9 illustrate application of the watershed model to the 1973 record. The initial conditions for the model were specified only once on May 21. Figure 3 illustrates the precipitation and predicted evapotranspiration rate. Figure 4 shows the predicted average hillslope moisture contents for each of the two hillslope regions. Hillslope region 1 consistently dries faster than region 2. This is because region 1 has a wider seepage face and a shallower soil depth. The average soil moisture content for both regions seldom drops below 0.3, indicating that vegetation always had ample moisture for transpiration. Figure 5 illustrates the predicted saturated

TABLE 1. Summary of the Model's Input Parameters and Other Statistics

Variable or Parameter Description	Region 1	Region 2	Both
Hillslope and catchment areas, ha	0.727	1.197	1.924
Swamp area, ha			0.363
Average soil depth, m	0.49	0.57	
Number of measurements taken	66	127	
Standard deviation	0.23	0.26	
Average hillslope angle	0.15	0.25	
Number of measurements	8	15	
Coefficient of determination, %	97.2	99.1	
Swamp perimeter, m	70	60	
Slope length measured along transect, m	44	63	
Saturated moisture content			0.47
Soil moisture holding capacity			0.36
Moisture content at permanent wilting			0.10
Initial soil moisture profile	0.47	0.47	
Saturated hydraulic conductivity, m/h			0.05
Model time increment, h			1
Distance increment, m	0.068	0.113	
Number of elements per hillslope	647	558	

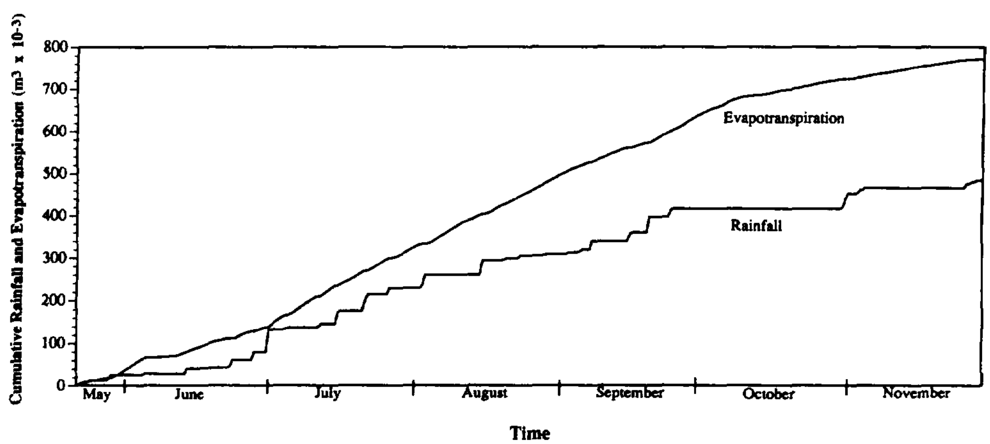

Fig. 3. Cumulative precipitation and cumulative predicted evapotranspiration for 27 weeks in 1973.

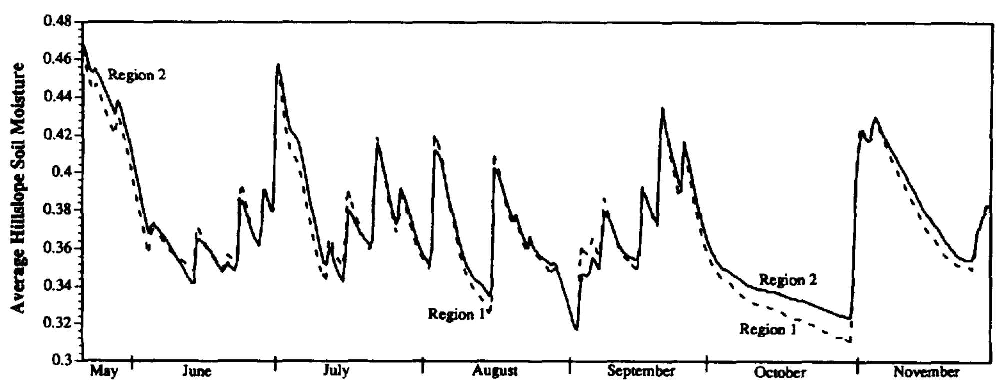

Fig. 4. Comparison of predicted average hillslope moisture contents for the two hillslope regions in the North Madison watershed.

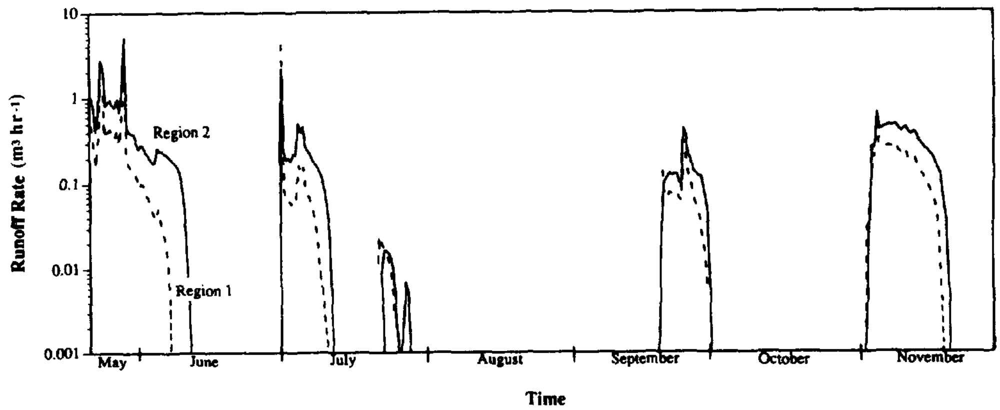

Fig. 5. Predicted saturation excess runoff for the two hillslope regions in the North Madison watershed.

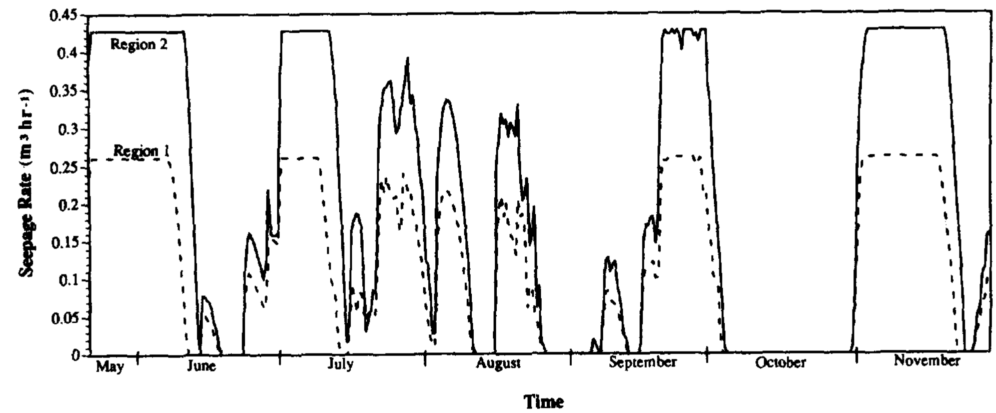

Fig. 6. Predicted seepage rates for each hillslope.

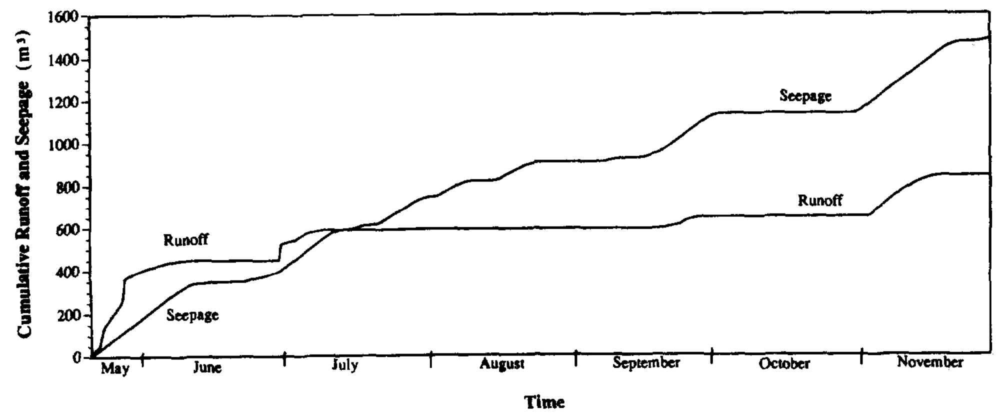

Fig. 7. A comparison of cumulative runoff and cumulative seepage from both hillslopes for the 1973 record.

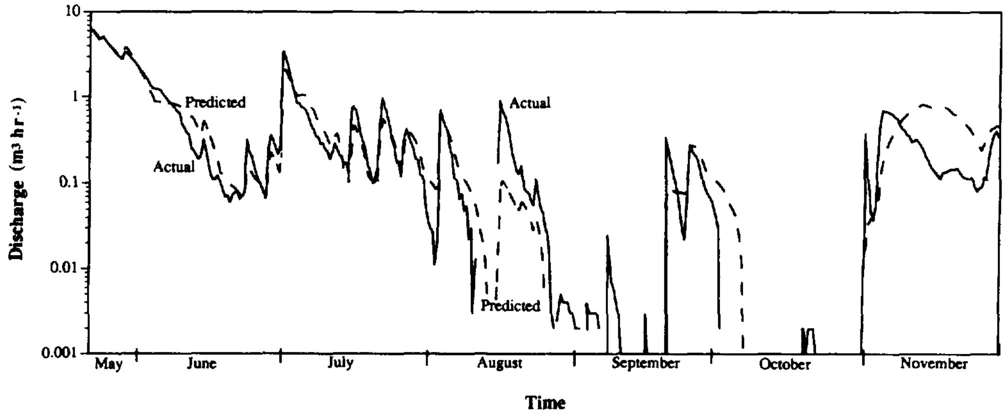

Fig. 8. Predicted versus actual watershed yield for the 1973 record.

excess runoff generated from each hillslope. The predicted saturation excess runoff from region 1 is consistently less than that for region 2. The seepage through-flow rates are shown in Figure 6. The maximum possible seepage rate is given by

$$S_{\max} = K_s \sin \phi W(L) \tag{36}$$

The maximum seepage rates for regions 1 and 2 are 0.26 m^3 h^{-1} and 0.43 m^3 h^{-1} respectively. Hillslope seepage occurs even during extended periods of no watershed discharge. Figure 7 compares the cumulative volume of seepage to runoff.

Figure 8 compares the predicted hourly watershed discharge with the actual discharge. Predicted discharge compares very favorably with actual discharge. However, the model tends to underpredict the storm peak discharge. Some of the contribution to the actual storm peak is attributed to precipitation falling in front of the swamp [*Stagnitti et al.*, 1989]. No adjustment for this amount was made in the model. The underprediction of the storm peak is usually followed by an overprediction of the hydrograph recession limb. The predicted yield is less accurate during periods of very low flow, e.g., when the average daily discharge is around 3 m/d or less (see Table 2). During these periods evapotranspiration plays a more significant role in determining watershed yield than does hillslope discharge. In this study we have adopted the value for the Priestley-Taylor coefficient α of 1.22. However, *Stagnitti et al.* [1989] found that α varied between 1.17 and 1.24 and in fact the lower values for α fell during periods of low flow. The seemingly poor agreement in predicted and actual yield during low flow periods is caused by inaccuracies in predicting evapotranspiration rather than inaccuracies in the hillslope model. In terms of the overall long-term water budget these discharges are inconsequential and hence the inaccuracies in prediction are largely unimportant (see, for instance, Figure 9).

Figure 9 compares the predicted cumulative yield with actual yield. The agreement is excellent except for a small portion at the end of the year in which the model predicts more outflow than is observed. The explanation for this is that toward the end of the year the water in the swamp and the channel froze with the effect of temporarily increasing the storage in the watershed and decreasing the flow [*Stagnitti et al.*, 1989]. The model in its current form does not account for this additional moisture storage, although it could easily be incorporated and it would be useful to do so if we had a longer record to examine.

The totals for precipitation, evapotranspiration, actual and predicted yield and hillslope runoff and seepage along with their average daily rates are described in Table 2. Table 2 also presents statistics of hillslope seepage, hillslope runoff and their combined total as a percentage of the total watershed yield. When the percentage is greater than 100% the hillslope contributions are replenishing the swamp storage

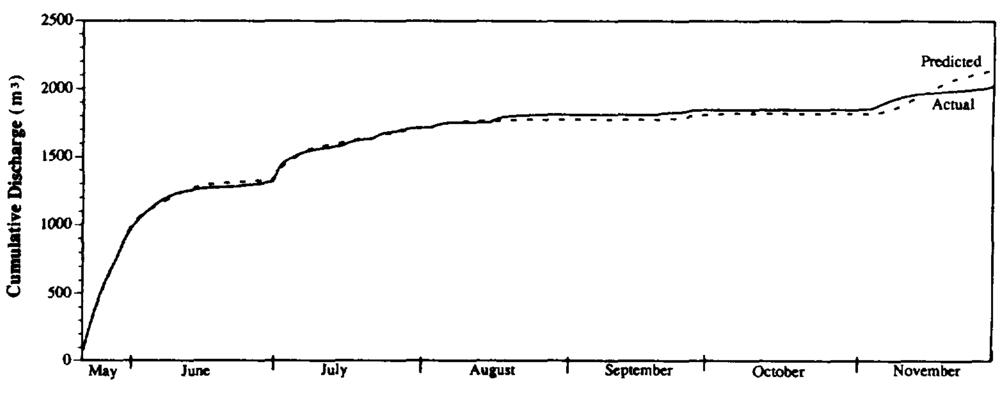

Fig. 9. Cumulative predicted and actual yield for the 1973 record.

TABLE 2*a*. Summary of the Model's Output Statistics: Monthly Totals

Date	Rainfall, $m^3 \times 10^{-3}$	Evaporation, $m^3 \times 10^{-3}$	Actual Basin Yield, m^3	Predicted Basin Yield, m^3	Hillslope Runoff, m^3	Hillslope Seepage, m^3	Hillslope Runoff as a Percentage of Predicted Basin Yield	Hillslope Seepage as a Percentage of Predicted Basin Yield	Hillslope Yield as a Percentage of Predicted Basin Yield
May 21–31, 1973	25.3	36.5	979.7	995.4	398.5	173.6	40.0	17.4	57.5
June 1–30, 1973	107.2	102.9	382.4	367.4	129.5	232.1	35.2	63.2	98.4
July 1–31, 1973	95.8	187.8	350.3	356.0	67.1	339.0	18.8	95.2	114.1
Aug. 1–31, 1973	79.2	174.1	95.5	60.1	0.0	162.2	0.0	269.9	269.9
Sept. 1–30, 1973	108.0	146.7	36.1	39.7	54.7	225.6	137.8	568.3	706.0
Oct. 1–31, 1973	37.0	78.1	7.5	5.9	0.5	33.7	8.5	571.2	579.7
Nov. 1–26, 1973	31.7	45.3	166.8	323.8	191.0	314.7	59.0	97.2	156.2
Nov. 13 to Dec. 12, 1975	56.1	13.5	1817.6	1830.0	1146.1	325.1	62.6	17.8	80.4

but when they are lower than 100% the swamp storage is diminishing at a rate faster than the surrounding hillslope contributions. The swamp regulates a large proportion of the watershed's discharge with the exception of that resulting from a small amount of precipitation falling in front of the swamp [*Stagnitti et al.*, 1989]. Following rain the swamp gradually depletes at a rate dependent on the evaporation rate and the available water in the swamp. The swamp often supports a noticeable recession flow long after storm events. This is a characteristic of many wetlands [*Bay*, 1967; *Woo and Valverde*, 1981]. The rate of attenuation depends on the water level and the size of the surrounding zone of soil saturation. For example, during dry periods the surrounding hillslopes are readily able to absorb a large proportion of precipitation from a storm event without causing any noticeable rise in watershed discharge. However, during wet periods, only a small amount of precipitation is needed to cause a noticeable rise in the water level and consequently watershed yield. Hillslope contributions to recharge or storage areas and hence watershed yield are important during large storm events or in periods of high flow. The seepage through-flow component dominates the recession limb of the hydrograph and saturated excess runoff determines the characteristics of the peak flow or storm flow hydrograph.

CONCLUSION

A simple water balance model consisting of a first-order ordinary differential equation describing the dynamics of recharge or storage areas and a first-order partial differential equation describing the dynamics of hillslope discharge was presented. The contribution from surrounding hillsides into the recharge area is easily calculated by the model and only requires knowledge of a few fundamental parameters which can be measured experimentally, or in the absence of field data, these parameters may be estimated or calibrated from a portion of the discharge hydrograph. The model was successfully used to predict the discharge from the North Madison watershed. The predicted discharge is in good agreement with actual flows for nonwinter months.

Some management-oriented conclusions for the North Madison watershed drawn from this study include: (1) Overland flow is only a minor contributor to watershed discharge. (2) Subsurface flow along preferential flow paths is a significant contributor to the storm period hydrograph. (3) Subsurface flow through the soil matrix primarily acts to sustain watershed discharge, particularly during nonstorm periods. (4) There is always ample moisture for plant transpiration. (5) Hillslope topography, soil water hydraulic conductivity, soil depth and hillslope length dynamically interact to produce unique signatures of the hillslope hydrographs and hence watershed yield. As data were not collected during the winter months a more comprehensive year-round water balance incorporating the effects of snowmelt runoff could not be performed. It may be possible to improve the prediction of watershed discharge by adjusting the parameters in the $D(t)$ relationship and hillslope parameters and/or by including small corrections for the storm pulse discharge. However, the emphasis of this work was to develop a

TABLE 2*b*. Summary of the Model's Output Statistics: Average Daily Rates

Date	Number of Days	Rainfall, $m^3 \times 10^{-3}$	Evaporation, $m^3 \times 10^{-3}$	Actual Basin Yield, m^3/d	Predicted Basin Yield, m^3/d	Hillslope Runoff, m^3/d	Hillslope Seepage, m^3/d
May 5–21, 1973	10.5	2.4	3.5	93.3	94.8	38.0	16.5
June 1–30, 1973	30.0	3.6	3.4	12.7	12.2	4.3	7.7
July 1–31, 1973	31.0	3.1	6.1	11.3	11.5	2.2	10.9
Aug. 1–31, 1973	31.0	2.6	5.6	3.1	1.9	0.0	5.2
Sept. 1–30, 1973	30.0	3.6	4.9	1.2	1.3	1.8	7.5
Oct. 1–31, 1973	31.0	1.2	2.5	0.2	0.2	0.0	1.1
Nov. 1–26, 1973	26.0	1.2	1.7	6.4	12.5	7.3	12.1
Nov. 13 to Dec. 12, 1975	29.7	1.9	0.5	61.3	61.7	38.6	11.0

physically based model avoiding unnecessary calibration. Indeed, it is remarkable that with only two parameters obtained by curve fitting a small portion of the discharge hydrograph in 1975, we were able to predict the outflow so accurately for the 1973 record.

NOTATION

A_c watershed area (m^2).
A_s storage or recharge area (m^2).
$B(y)$ soil depth as a function of downslope distance y (m).
c hillslope counter.
$D(t)$ watershed discharge rate (m^3 h^{-1}).
$\overline{E_j}$ average rate of actual evapotranspiration (m^3 m^{-2} h^{-1}).
$E_a(t)$ actual evapotranspiration rate (m^3 m^{-2} h^{-1}).
$h(t)$ water elevation in storage or recharge area (m).
$H(t)$ hillslope discharge (m^3 h^{-1}).
$\overline{H_j}$ average rate of $H(t)$ (m^3 h^{-1}).
$K(\theta)$ hydraulic conductivity function (m^3 h^{-1}).
K_s saturated hydraulic conductivity (m^3 h^{-1}).
L hillslope length for hillslope segment c (m).
m storm counter.
n soil porosity parameter in unsaturated hydraulic conductivity function.
N number of hillslope segments.
$p(t)$ precipitation rate (m^3 m^{-2} h^{-1}).
$P, P_0, P_1, \cdots$ cumulative precipitation (m^3).
$R_j^{(c)}$ runoff from cth hillslope segment (m^3 h^{-1}).
R_n net solar radiation (W m^{-2}).
$S_j^{(c)}$ seepage from cth hillslope segment (m^3 h^{-1}).
t time (hours).
t_a initial time (hours).
t_m time at which storm m commences (hours).
T_a air temperature (°C).
$W(y)$, W_i length of contour at y multiplied by soil depth (m^2).
$\overline{X_j}$ average rage of $[A_s^{-1}H(t) - E_a(t)]$ (m^3 h^{-1}).
y spatial coordinate directed downslope along transect (m).
α Priestley and Taylor coefficient.
β_k coefficient in $D(t)$ relationship.
δt, δy small intervals in time and space, respectively (hours, m).
γ psychrometric constant (mbar).
Δ gradient of saturation vapor pressure–temperature curve evaluated at the air temperature T_a.
$\theta(y, t)$, $\theta_{(i,j)}$ volumetric soil moisture content (m^3 m^{-3}).
θ_a additional moisture to hillslope (m^3 m^{-3}).
θ_h volumetric holding capacity (m^3 m^{-3}).
θ_p moisture content at permanent wilting (m^3 m^{-3}).
θ_r residual moisture content (m^3 m^{-3}).
θ_s saturated moisture content (m^3 m^{-3}).
λ latent heat of vaporization (J m^{-3}).
$\nu_k = A_s^{-1}\Omega_k$.
τ time (hours).
ϕ hillslope angle.
Ω_k coefficient in $D(t)$ relationship (m^3 h^{-1}).

Acknowledgments. Experimental data were obtained while the second author was a research scientist at the Connecticut Agricultural Experimental Station in New Haven. Part of this research was conducted while the first author was on sabbatical leave at Cornell University in 1991. Remarks by two anonymous referees significantly improved the final manuscript and are gratefully appreciated.

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(Received April 5, 1991;
revised March 11, 1992;
accepted March 20, 1992.)