When Power Adaptation is Useless or Harmful

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***Abstract*— We consider the design of optimal strategies for joint power adaptation, rate adaptation and scheduling in a multi-hop wireless network. Most existing strategies for ad-hoc networks control either power and scheduling, or rates and scheduling, but not the three together as we do. We assume the underlying physical layer allows fine-grained rate adaptation (like in 802.11a/g, HDR/CDMA, UWB). Our goal is to find properties of the power control in an optimal joint design. In the linear regime (i.e when the rate of a link can be approximated by a linear function of signal-to-noise ratio, SNR), we prove analytically that it is always optimal to use the simple $0 - P^{MAX}$ power control (when a node is sending it uses the maximum transmitting power allowed). This holds in both important networking scenarios: high rate networks where the goal is to maximize rates under power constraints, and low power networks where the goal is to minimize average consumed power while meeting minimum rate constraints. Moreover, we prove that, when maximizing rates, $0-P^{MAX}$ is the only possible optimal power control strategy. Outside the linear regime, we do not know what the optimal power control is. We show that in the power minimization scenario, in some cases, rate adaptation and $0 - P^{MAX}$ power control performs much worse than power adaptation. Nevertheless, we conjecture, and we demonstrate numerically that when maximizing rates, even outside the linear regime, $0 - P^{MAX}$ is very close to the optimal power control, and the rate adaptation with $0 - P^{MAX}$ outperforms power adaptation with fixed link rates.**

***Index Terms*— System design, Mathematical programming/optimization**

I. Introduction

A. Power Control and Optimal Wireless MAC Design

The first wireless MAC protocols for multi-hop networks were designed to control only medium-access. A typical example is the original 802.11 MAC. It always uses maximum power for transmitting a packet, and aims to establish communication on a fixed, predefined link rate. Then several improvements to the initial approach have been proposed. According to the type of improvement, the MAC protocols can be divided globally in two groups. One group of protocols [1], [2], [3] is focused on rate adaptation: the transmission power is still kept fixed, but the rate is adapted to the actual channel conditions and the amount of interference. The other group of protocols [4], [5], [6], [7] considers power adaptation while keeping the rates fixed. However, there are no MAC protocols that adapt both rate and power at the same time, and the fundamental issues in this joint adaptation problem are not well understood.

We consider a wireless network with arbitrary constraints on scheduling, rate adaptation and routing, and we are interested in characterizing properties of the optimal power allocation strategy in this setting.

B. Physical Layer and Rate Adaptation

The physical layer of a wireless link defines communication parameters such as bandwidth, modulation and coding that can be used to establish communication with some level of bit or packet errors. One of the most important parameters of the physical layer is signal-to-noise ratio (SNR) at the receiver. The higher the SNR is, the higher communication rates can be attained, and one of the goals of networking design is to efficiently tracks and adapts SNRs and/or rates on links.

Some of the existing wireless systems use fix communication rates. A typical example is a cellular voice network, where one voice channel has a fixed rate. There, a goal of the system is to maintain the SNR of each user above a threshold, such that there are no outages. Initially, the first version of 802.11 used the same approach. Although it has been defined for best-effort data traffic, it did not support variable rates due to simplicity in design. In contrast, most of the recently proposed wireless physical layers allow rates to vary with SNR. A typical examples are 802.11a/g [8], CDMA/HDR [9], TH-UWB [10]. Those physical layers use adaptive modulation [11], [8] and/or adaptive coding [10] to adjust the rate to the SNR at the receiver while maintaining a constant, guaranteed bit-error rate. The function that gives the maximum achievable rate for a given SNR is called *rate function*. Examples of protocols that use rate adaptation can be found in [1], [2], [3].

C. Linear Regime

The rate function of an efficiently design system is a concave function of SNR. Furthermore, in many cases, it is a linear function. Some examples of physical layers where rate function is linear are low or moderate-gain CDMA [12] or TH-UWB [10]. Also, physical layers with non-linear rate functions may operate in the linear regime if the transmitted power is low. We show in Section IV-A that the part of the rate functions of 802.11a/g physical layer for low SNR is approximately linear. Our main findings are for the linear regime, where we obtain exact, analytically proven results; for the non linear regime we have only some numerical results.

D. Rate Maximization and Power Minimization

There are two typical deployment scenarios for wireless networks: high bit-rate networks and low power consumption networks. The first one considers real-time video and audio communication, web surfing, data transfer, and alike. The primary design focus is to maximize available rates. Typical examples of this type of networks are 802.11 wireless LANs and HDR cellular systems. Rate maximization is performed under given power constraints which are typically due to regulations and battery constructions. In the rate maximization scenario, we are interested in the set of feasible rates.

The second scenario is focused on low power networks like sensor networks or networks of computer peripherals. The main goals in this type of networks is to maximize network lifetime, or equivalently, to minimize average consumed power. At the same time, end-to-end flow rates are lower bounded by application requests, and each sender typically has a minimum amount of information to send to a destination in a given time. Here we are interested in the set of feasible long-term average power consumptions on links in networks, subject to minimum long term rate constraints. Long-term average power consumption is defined in Section II-D, and different performance objectives for comparing the sets are presented in detail in Section II-E.

E. Power Control

The goal of power control is to determine which power a transmitter should use when transmitting a packet. The optimal transmitted power of a packet depends on a large number of parameters, such as distance from the destination, background noise, amount of interference incurred by concurrent transmissions, etc. Since power control is tightly coupled with scheduling, it is typically implemented within the MAC protocol.

Perhaps the simplest way to choose the transmitted power is to do no power control. In other words, whenever a packet is sent, it is sent with maximum allowed power. We call this $0 - P^{MAX}$ power control. The $0 - P^{MAX}$ power control was widely used in the design of the first wireless MAC protocols, such as 802.11, due to its simplicity, and due to the fact that the optimal power control was not well understood.

Most of the research on power control is focused on voice cellular systems. Those systems typically use orthogonal channels for different users (e.g. CDMA spreading) in order to decrease the inter-channel interference. However, if the signal of an interfering user is very strong, the interference cannot be completely filtered out, and the transmission might fail. This problem is known as the *near-far problem*, and it is solved by power control. Some pioneering work in this area can be found in [13], [14], [15]. These papers propose iterative algorithms that converge to a power allocation where all nodes' SNRs are above thresholds, should such allocation be possible. Those ideas have been extended to multi-hop wireless networks in [16].

An attempt to design an optimal power control protocol for 802.11 networks has been done in [4], [5], [6], [7]. They consider the 802.11b physical layer with fixed rate, and the common conclusion is that the power should be adjusted to the minimal value required to be successfully decoded at the destination. One of the most recent protocols defined along these lines, and which we will use for numerical performance comparison, is CA/CDMA [7]. There, every transmitter transmits with the minimum necessary power, increased by some margin. This margin allows it to resist some amount of interference caused by concurrent transmission. The MAC protocol guarantees that the interference from concurrent transmissions is not going to exceed the margin.

The above power control protocols are optimal only when the physical layer offers a fixed rate, regardless of the signal-to-noise level at the receiver. Not too much work is done on power control for networks with variable link rates. An adaptive power control mechanism for cellular networks with variable link rates has been presented in [11]. However, this mechanism is adapted to voice traffic. It does not consider scheduling and thus leaves out an important design parameter of data wireless networks. Protocols that consider rate adaptation, power adaptation and scheduling, and that maximize the sum of rates for this scenario have been proposed in [12], [17]: they focus on low processing gain CDMA or UWB networks (thus linear regime) and show that $0 - P^{MAX}$ power control is optimal for a specific rate-based performance metric, maximizing sum of rates. The optimal power control for an arbitrary metric is not known, nor it is known for an arbitrary physical layer.

Several power adaptation protocols have been proposed for power minimization scenarios. A typical example is given in [18] where the power of a link is adjusted to a minimum necessary to reach a destination, and the routing is chosen to minimize the overall power dissipation.

F. Performance Comparison

For different power control strategies, we are interested in comparing the resulting rate allocations. However, by using different scheduling strategies with one power control strategy, one can obtain different rate allocations. The set of all possible rate allocations that can be obtained with a given power control strategy, and with different schedules, is called the feasible rates set. A feasible allocation where one rate cannot be increased without decreasing another one is called *Pareto efficient*. When maximizing rates, we are clearly interested in Pareto efficient rate allocations. The most general way of comparing performances of two power control strategies is thus to compare the sets of their Pareto efficient allocations, and we will use this method in the analytical part of the paper.

In some numerical examples, when we consider larger networks (Section V), it is numerically complex to calculate Pareto efficient rates for all possible schedules. We will then choose a single schedule which achieves the Pareto efficient rate allocation that maximizes the log-utility of the system, and which is known to have desirable properties in wireless networks [19]. We will use the log-utility of this rate allocation as a performance metric.

Pareto efficiency can be defined in a similar manner for feasible average power consumptions. Precise definitions of all the above terms are given in Section II-E.

G. Modeling of Wireless Networks

We are interested in the fundamental principles in a design of a wireless MAC, and not in designing a specific protocol. Therefore, we assume an ideal, zero overhead MAC protocol, which comprises ideal scheduling and rate adaptation strategies, and we are interested in characterizing properties of an optimal power control strategy.

General models of wireless networks that incorporate various physical layers, MAC and routing protocols, are discussed in [12], [20], [21]. We define a model of a multi-hop wireless network that allows the most general assumptions on a physical layer (including variable rate 802.11, UWB or CDMA) and MAC protocols. We assume arbitrary routing (single-hop or multi-hop), and we assume point-to-point links whose conditions change over time due to random fading. For a given network topology and traffic demand, we characterize the set of feasible average end-to-end rate allocations under given maximum average power constraints, and equivalently the set of feasible average power constraints under minimal average end-to-end rate constraints. We use the model to prove our findings by theoretical analysis and numerical simulations. More detailed assumptions on the network model are given in Section II.

H. Problem Definition and Our Findings

We consider a wireless network where link rates, transmission powers and medium access can be varied. For such system, one can find rate control, power control and theoretical MAC protocols that maximize the performance. This is a joint optimization problem and a change in any of the three components influences the choice of the other two. We consider different power control strategies, for each of them we assume the optimal MAC and rate adaptation, and we compare their performances. The goal is to characterize the optimal power control.

We first analyze the linear regime. We consider the rate maximization scenario and we theoretically prove that every feasible rate allocation can be achieved without power control (power adaptation is useless), and that, if there are no average power constraints, any power control that does not use $0-P^{MAX}$powers control is not Pareto efficient (power adaptation is harmful). We further consider the power minimization scenario and we show that any feasible average power allocation is achievable without power adaptation (power adaptation is useless).

For non-linear regimes, we do not know the optimal power control, scheduling and rate adaptation. We give some conclusions based on numerical simulations. We show that for power minimization, $0-P^{MAX}$power control is far from optimal. The joint optimization problem in this case remains still an open issue. When considering rate maximization in non-linear regime, the conclusions are different. We show on numerical examples that, while $0-P^{MAX}$with scheduling and rate adaptation is not optimal, it is very close to optimal. We then compare the benefits of rate adaptation with $0-P^{MAX}$power control, against power adaptation and $0-R^{MAX}$rates. We show that rate adaptation with $0-P^{MAX}$power control outperforms by far the state-of-the-art power adaptation protocol, CA/CDMA, with fixed rates.

Our findings are based on the assumption that, for every power control protocol of choice, we design an optimal scheduling and rate adaptation protocol. For a fixed scheduling and rate adaptation protocols the findings might not hold, and they depend on properties of those protocols. However, our results indicate what are the design goals that an optimal MAC should try to reach, for the cases that fall in the scope of our conclusions.

I. Organization of The Paper

The next section describes system assumption. In Section III we give a mathematical formulation of the model of a network and present our theoretical findings. In section Section IV we give simple numerical examples that illustrate our findings. In Section V we compare rate adaptation vs. power adaptation. In the last section we give conclusions and directions for further work. Proofs of the propositions can be found in the appendix.

II. System Assumptions

We analyze an arbitrary multi-hop wireless network that consists of a set of nodes, and every two nodes that directly exchange information are called a link. For each pair of nodes we define a signal attenuation, that is a level of signal received at the receiver, assuming the sender is sending with unit power. This attenuation is usually a decreasing function of a link size due to power spreading in all directions, but here we assume it can be an arbitrary number defined for each pair of nodes. We assume the network is located on a finite surface and that all attenuations are always strictly positive, hence every node can be heard by any other node in the network and there is no clustering. Signal attenuation also changes in time due to mobility and different variations of characteristics of paths the signal takes, thus we will model it as a random process. We next give properties of the physical model of communications on links.

A. Physical Model Properties

All physical links are point-to-point, this means each link has a single source and a single destination. There are more advanced models such as relay channel [22] that attain higher performances, but they are not used in most of the contemporary networks, and their performance is in general not known and is still an open research issue.

A node can either send to one next hop or receive from one at a time. There are more complex transmitter or receiver designs that can overcome these limitations. An example is a multi-user receiver that could receive several signals at the time. This would change the performance of links having a common destination, but would not change the interactions over a network. However, these more complex techniques are not used in contemporary multi-hop wireless networks (like 802.11, UWB, bluetooth or CDMA) due to high transceiver complexity, and we do not analyze them here. Still, the model can easily be changed if this assumption is relaxed and our results will still hold.

We model rate as a function $r(\mathrm{SNR})$ of the signal-to-noise ratio at the receiver, which is the ratio of received power by the total interference perceived by the receiver including the ambient noise and the communications of other links that occur at the same time. In case of systems with spreading, such as CDMA, frequency-hopping OFDM or TH-UWB, a receiver does not capture the full power of an interferer, but just a fraction that depends on the correlation of the spreading sequences of the sender and the interferer. The total noise at a receiver can thus be modeled as the sum of the ambient noise and the total interference multiplied by the *orthogonality factor*. The more efficient the spreading is, the smaller is the orthogonality factor.

This model corresponds to a large class of physical layer models, for example:

- Shannon capacity of Gaussian channels [22]: $r(\mathrm{SNR}) = 1/2 \log_2(1+\mathrm{SNR})$.
- Low-power and/or wide-band Gaussian channels [23]: $r(\mathrm{SNR}) \approx K \times \mathrm{SNR}$
- Time-hopping ultra-wide band [10]: $r(\mathrm{SNR}) = K \times \mathrm{SNR}$.
- Moderate processing gain CDMA [12]: $r(\mathrm{SNR}) = K \times \mathrm{SNR}$.
- Fixed rate 802.11b [standard]: $r(\mathrm{SNR})$ is a step function of SNR
- Variable rate 802.11a/g [standard]: $r(\mathrm{SNR})$ is a stair function of SNR.
- CDMA HDR [9]: $r(\mathrm{SNR})$ is a stair function of SNR.

In all the examples except for 802.11b, the rate is variable, and is a function of signal-to-noise ratio at a receiver. This is achieved by adaptive modulation, like in [11], [1], [2], or adaptive coding [3]. Rate as a function of SNR is a concave function. For an efficiently designed system, it usually approaches the Shannon capacity of the system [22], which is a log-like function. However, for low-power (e.g. sensor networks) or high-bandwidth system (e.g. UWB [10] or CDMA systems with moderate processing gain [12]), the total noise is much larger than received powers, and the capacity can be approximated with a linear function of SNR [24], [23]. Also, physical layers with non-linear rate function operate in linear regime when the SNR at the receiver is low. We show in Section IV-A that this is the case for 802.11a/g physical layer.

In this paper we consider both kinds of physical layers: linear and non-linear. We prove our findings analytically for systems with linear rate functions, and extend some of them numerically for a class of systems with non-linear rate functions.

B. MAC Protocol

We further assume a slotted system. In each slot a node can either send data, receive or stay idle, according to the rules defined in Section II-A. Each slot has a power allocation vector associated with it, which denotes what power is used for transmitting by the source of each link. If a link is not active in a given slot, its transmitting power is 0. A schedule consists of an arbitrary number of slots of arbitrary lengths.

The first part of our MAC is a power control strategy. The power control strategy is defined by a set of possible powers that can be allocated to links in any slot. An example of power control strategy is $0 - P^{MAX}$power control where any link in any slot can send with power P^{MAX} or stay idle. This is the simplest strategy where powers are fixed and there is no power adaptation.

The second part of a MAC is the rate adaptation and scheduling. Having chosen a power control strategy, a MAC chooses a schedule and assigns powers that belong to the set of possible powers to links in each slot. Finally, the rate on each link in each slot is adapted to the SNRs at receivers.

We assume that for a given power control strategy we have an optimal MAC protocol that calculates the optimal transmission power of each link out of the set of possible powers defined by power control, and in each slot in a ideal manner and according to a predefined metric. This is equivalent to a network where nodes dispose of an ideal control plane with zero delay and infinite throughput to negotiate schedule and power allocation.

A more realistic MAC protocol would introduce some errors and delays, but a good approximation should be close to the ideal case. Also, by considering an ideal protocol, we focus our analysis on properties of performance metrics, and not artifacts of leaks in protocol design. Our assumption corresponds to neglecting the overhead (in rate and power) of the actual MAC protocol.

We also assume random fading. Since we have an ideal MAC protocol, it can instantly adapt the schedule and the power and rate allocation to any state of the random fading of links. For precise mathematical model of MAC protocol, see Section III-B.

C. *Routing Protocol and Traffic Flows*

We assume an arbitrary routing protocol. Flows between sources and destinations are mapped to paths, according to some rules specific to the routing protocol. At one end of the spectrum, nodes do not relay and only one-hop direct paths are possible. At the other end, nodes are willing to relay data for others and multi-hop paths are possible. There can be several parallel paths. All these cases correspond to different constraint sets in our model, as defined in Section III-B. Sources can send to several destinations (multicast) or to one (unicast).

D. *Power and Rate Constraints*

There are four types of power and rate constraints in a wireless network: peak power constraint, short-term average power constraint, long-term average power constraint and average rate constraint. Here we describe them in detail:

Peak power constraint: Given a noise level on a receiver, a sender can decide which codebook it will use to send data over the link during one time slot. Different symbols in the codebook will have different powers. The maximum power of a symbol in a codebook is then called *peak power*. It depends on the choice of the physical interface and its hardware implementation and we cannot control it. It limits the choice of possible codebooks, and it puts restrictions on the available rate. In our model, the peak power constraint is integrated in a rate function, given as an input.

Short-term average power constraint: We assume a slotted system. In each slot a node chooses a codebook and its average power, and sends data using this codebook within the duration of the slot. We call *transmitted power* the average power of a symbol in the codebook. This is a short-term average power within a slot, since a codebook is fixed during one slot. We assume that this transmission power is upper-bounded by P^{MAX}. This power limit is implied by technical characteristics of a sender and by regulations, and is not necessarily the same for all nodes. For example, this is the only power constraint that can be set by users on 802.11 equipment.

Long-term average power constraint: While transmitting a burst of data (made of a large number of bits), a node uses several slots, and possibly several different codebooks. Each of these codebooks has its transmission power. We call the *consumed power* the average of transmission powers during a burst, and we assume it is limited by $\overline{P}^{MAX}$. Consumed power is related to the battery lifetime in the following way: $T_{\text{lifetime}} \approx \frac{E_{\text{battery}}}{\overline{P}^{MAX} \times u}$ where T_{lifetime} is the battery lifetime, E_{battery} is the battery energy, $\overline{P}^{MAX}$ is the average consumed power constraint and u is the fraction of time a node has data to send (or activity factor, measured in Erlangs). The approximation corresponds to ignoring overhead spent managing the sleep / wakeup phases, etc. $\overline{P}^{MAX}$ is thus set by a node to control its lifetime; it can vary from a node to a node.

Average rate constraint: In networks like sensor or peripheral networks, the goal is to minimize power consumption and to maximize lifetime of nodes rather than maximize the rates of links. Still, there is a lower bound on the rate a node has to transmit. For example, a temperature sensor on a car engine or a computer mouse have a well define rate of information they need to communicate to a central system. This is what we call the *average rate constraint* and we defined it as an average amount of bits a node has to transmit over the network in one second. We assume this average limit is the same on both long and short timescales.

We incorporate explicitly in our model the transmission power constraints, the average consumed power constraints and the average rate constraints. The peak power is incorporated implicitly through the choice of the rate function.

E. Performance Objectives

Design criteria in wireless networks can be divided into two groups: rate maximization and power minimization. We first consider rate maximization. Given a network topology and a family of MAC protocols, one can define a set of feasible rate allocations as the set of all rate allocations that can be achieved on the network with some MAC protocol from the given family. An interesting subset of the feasible rate set is the set of Pareto efficient rate allocations. A rate allocation is *Pareto efficient* if no rate can be increased without decreasing some other rate. When maximizing rates, we are clearly interested only in Pareto efficient rate allocations.

The most general way to compare two families of network protocols on a same network is to compare their Pareto efficient rates' sets. If all Pareto efficient rates of one family of protocols are feasible under the other family of protocol, then one can undoubtedly say that the second family is as good as the first one. If, furthermore, neither of the Pareto efficient rates of the second family is achievable under the first family of MAC protocols, then we can say that the second family is strictly better than the first one. We will use this criterion to compare different power control strategies throughout the paper.

Although the above method for comparing power control strategies is the most general one, it is difficult to use in practice. Namely, for large networks, calculating all feasible rate allocations, and thus all possible schedules, is a difficult and prohibitively expensive numerical problem. Instead, we will use a different approach for numerical comparison of power controls on large networks topologies. We will choose a single scheduling for a given power control that maximize some rate-based performance metric. There are several existing rate-based performance metrics, and all of them yield Pareto efficient rates. Maximizing total capacity is known to be efficient and unfair while max-min fairness is fair but inefficient; proportional fairness represents a good compromise between efficiency and fairness [19]. One can defined *log-utility* of a rate allocation to be the sum of logs of all components in the rate allocation, and the *proportionally fair* rate allocation is the one that maximizes log-utility. For precise mathematical definitions of the therms see Section III-B.

In Section V where we numerically compare rate adaptation with power adaptation, we compare log-utilities of the proportionally fair rate allocations: for a given network topology and MAC protocol, we choose the schedule such that the resulting rate allocation maximizes the log-utility of a network. We use this maximal log-utility to compare protocols, and we say that a MAC protocol that has higher log utility is better than the other MAC.

The above discussion can be similarly put in the context of rate minimization. However, in this case we only need a notion of Pareto efficient power allocation: it is defined as the one where no power can be decreased without increasing some other power. Mathematical definitions of terms are given in Section III-B.

III. Theoretical Findings

A. Notations

We model the wireless network as a set of I flows, L links, O nodes and N time-slots. Flows are unicast or multicast. We assume the network is in a random state S belonging to set $\mathcal{S}$, which defines the attenuations among nodes in the network. Since we analyze a theoretical MAC, we assume for each system state $s \in \mathcal{S}$ that there is a separate instance of the MAC. We give here a list of notations used in this section to describe the model. The precise definitions are given in subsequent subsections.

- $h_{l_1 l_2}(s)$ is the attenuation of a signal from the source of link l_1 to the destination of link l_2 when the system is in state s.
- β is the orthogonality factor that defines how much power of interfering signals is captured by a receiver.
- $\mathbf{f} \in \mathbb{R}^I$ is the vector of average rates achieved by flows.
- $\bar{\mathbf{x}} \in \mathbb{R}^L$ is the vector of average rates achieved on links.
- for every $n \in \{1, \cdots, N(s)\}$, $\mathbf{x}^n(s) \in \mathbb{R}^L$ is the vector of rates achieved on links in time slot n when the system is in state s.
- for every $n \in \{1, \cdots, N(s)\}$, $\mathbf{p}^n(s) \in \mathbb{R}^L, \mathbf{prcv}^n(s) \in \mathbb{R}^L$ are the vectors of transmitted and received powers allocated on links in time slot n, respectively, when the system is in state s.
- $\overline{\mathbf{F}}^{MIN} \in \mathbb{R}^I$ is the vector of minimum average rates achieved by end-to-end flows (every flow may have a different minimum average rate).
- $\mathbf{P}^{MAX} \in \mathbb{R}^L$ is the vector of maximum allowed transmission powers on links, which are assumed

constant in time (every link may have a different maximum power).

- $\overline{\mathbf{P}}^{MAX} \in \mathbb{R}^L$ is the vector of maximum allowed average transmission powers on links (every link may have a different maximum power).
- $\eta_l(s) \in \mathbb{R}$ is the white noise at the receiver of link l when the system is in state s.
- for every $n \in \{1, \cdots, N\}$, $\mathbf{SNR}^n(s) \in \mathbb{R}^L$ is the vector of signal-to-noise ratios at the links' receivers in time slot n, when the system is in state s.
- for every $n \in \{1, \cdots, N\}$, $\alpha^n(s) \in [0, 1]$ is the relative frequency of time slot n in the schedule assigned to the system when in state s.
- R (routing matrix) is such that $R_{l,i} = 1$ if flow i uses link l. We have $\mathbf{f} \leq R\bar{\mathbf{x}}$. The matrix R is defined by the routing algorithm.

B. Mathematical Formulation

We assume that for every state s there is a schedule consisting of time slots $n = 1...N(s)$ of frequency $\alpha_n(s)$. This is an abstract view of the MAC protocol, without overhead. We normalize these lengths such that $\sum_{n=1}^{N(s)} \alpha_n(s) = 1$. Let us call $\mathbf{p}^n(s)$ the vector of transmission powers assigned to links in slot n and state s, and let $\mathbf{SNR}^n(s)$ be the vector of signal-to-noise ratios at receivers of the links, induced by $\mathbf{p}^n(s)$. The rate achievable on link l in slot n and state s is $\mathbf{x}_l^n(s) = K\,\mathbf{SNR}_l^n(s)$. The vector of average rates on the links is thus $\bar{\mathbf{x}} = \mathbb{E}\left[\sum_{n=1}^{N(S)} \alpha_n(S)\mathbf{x}^n(S)\right]$. Since $\mathbf{x}^n(s)$ has dimension L (where L is a number of links), by virtue of Carathéodory theorem, when in state s, it is enough to consider $N(s) \leq N = L + 1$ time slots of arbitrary lengths $\alpha(s)$ in order to achieve any point in the convex closure of points $\mathbf{x}^n(s)$.

Feasible rate and power allocations: Given a network topology and a routing matrix R, we define the *set of feasible average powers and end-to-end rates* $\mathcal{X}$ (without average power or rate constraints). It is the set of $\mathbf{f} \in \mathbb{R}^I$ and $\bar{\mathbf{p}} \in \mathbb{R}^N$ such that there exist schedules $\alpha(s)$, sets of power allocations $\mathbf{p}^n(s)$ and corresponding sets of rate allocations $\mathbf{x}^n(s)$ for all $n = 1 \cdots N$ and all states $s \in \mathcal{S}$, such that the following set of equalities and inequalities are satisfied for all $n = 1 \cdots N, i = 1 \cdots I, l = 1 \cdots L, o = 1 \cdots O$:

$$
\begin{aligned}
\mathbf{f} &\leq R\bar{\mathbf{x}} \\
\bar{\mathbf{p}} &= \mathbb{E}\left[\textstyle\sum_{n=1}^{L+1} \alpha_n(S)\mathbf{p}^n(S)\right] \\
\bar{\mathbf{x}} &= \mathbb{E}\left[\textstyle\sum_{n=1}^{L+1} \alpha_n(S)\mathbf{x}^n(S)\right] \qquad (1)\\
\mathbf{x}_l^n(s) &= K\,\mathbf{SNR}_l(\mathbf{p}^n(s)) \\
\mathbf{SNR}_l(\mathbf{p}^n(s)) &= \frac{\mathbf{p}_l^n(s)h_{ll}(s)}{\eta_l(s)+\beta\sum_{k\neq l}\mathbf{p}_k^n(s)h_{kl}(s)} \\
1 &= \textstyle\sum_{n=1}^{L+1} \alpha_n(s) \\
1 &\geq \textstyle\sum_{l\,:\,l.\mathrm{src}=o} 1_{\{p_l^n(s)>0\}} + \\
&\quad \textstyle\sum_{l\,:\,l.\mathrm{dst}=o} 1_{\{p_l^n(s)>0\}} \\
\mathbf{p}_l^n(s) &\leq P_l^{MAX}
\end{aligned}
$$

where $l.\mathrm{src} = o$ and $l.\mathrm{dst} = o$ are true if node o is the source or the destination of link l, respectively.

We are interesting in comparing average rates and power consumptions with $0 - P^{MAX}$and with arbitrary control. With $0 - P^{MAX}$power control, a node sends with maximum power when sending. More formally this means that in any slot n, power allocation vector $\mathbf{p}^n$ has to belong to the set of extreme power allocations $\mathcal{P}^E = \{\mathbf{p}\,|\,(\forall l = 1 \cdots L)\, p_l \in \{0, P_l^{MAX}\}\}$. In contrast, with an arbitrary power control, any power from the set of all possible power allocations $\mathcal{P}^A$ is possible. The set $\mathcal{P}^A$ is defined as $\mathcal{P}^A = \{\mathbf{p}\,|\,(\forall l = 1 \cdots L)\, p_l \in [0, P_l^{MAX}]\}$.

We say that an average rate allocation $\mathbf{f}$ and average power consumption $\bar{\mathbf{p}}$ is *achievable* with a set of power allocations belonging to $\mathcal{P}$ if for all $n = 1 \cdots N, i = 1 \cdots I, l = 1 \cdots L, o = 1 \cdots O$, it satisfies constraints (1), and for all $n = 1 \cdots N, s \in \mathcal{S}, \mathbf{p}^n(s) \in \mathcal{P}$.

We can similarly define the set of average rate and power allocations $\mathcal{X}(\mathcal{P})$ that is achievable with power allocations belonging to $\mathcal{P}$, as the set of all $(\mathbf{f}, \bar{\mathbf{p}})$ that are achievable using power allocation $\mathcal{P}$. Thus, sets $\mathcal{X}$ and $\mathcal{X}(\mathcal{P}^E)$ represent the sets of all possible average rate allocations and power consumptions with an arbitrary and with $0 - P^{MAX}$power control, respectively.

When we consider rate maximization under constraints on average consumed power, we are interested only in the set of feasible rates. If the average consumed power is limited by $\overline{\mathbf{P}}^{MAX}$, then the set of feasible rates is $\mathcal{F} = \{\mathbf{f}\,|\,(\mathbf{f}, \bar{\mathbf{p}}) \in \mathcal{X}, \bar{\mathbf{p}} \leq \overline{\mathbf{P}}^{MAX}\}$. Similarly, with $0 - P^{MAX}$power control, the set of feasible rate is $\mathcal{F}^E = \{\mathbf{f}\,|\,(\mathbf{f}, \bar{\mathbf{p}}) \in \mathcal{X}(\mathcal{P}^E), \bar{\mathbf{p}} \leq \overline{\mathbf{P}}^{MAX}\}$.

Similarly, when considering power minimization, we focus on the set of feasible average consumed powers. If the average end-to-end flow rate is lower-bounded by $\overline{\mathbf{F}}^{MIN}$, then the set of feasible average consumed powers, under arbitrary power control, is $\overline{\mathcal{P}} = \{\bar{\mathbf{p}}\,|\,(\mathbf{f}, \bar{\mathbf{p}}) \in \mathcal{X}, \mathbf{f} \geq \overline{\mathbf{F}}^{MIN}\}$. Similarly, with $0-P^{MAX}$power control, the set of feasible rate is $\overline{\mathcal{P}}^E = \{\bar{\mathbf{p}}\,|\,(\mathbf{f}, \bar{\mathbf{p}}) \in \mathcal{X}(\mathcal{P}^E), \mathbf{f} \geq \overline{\mathbf{F}}^{MIN}\}$.

Performance Objectives: Finally, we formally define notions of Pareto efficiency and log-utility, that were introduced in Section II-E. Rate vector $\mathbf{f} \in \mathcal{F}$ is Pareto efficient on $\mathcal{F}$ if there exist no other vector $\mathbf{f}' \in \mathcal{F}$ such that for all $i, \mathbf{f}_i' \geq \mathbf{f}_i$ and for some $j, \mathbf{f}_j' > \mathbf{f}_j$. Log-utility of $\mathbf{f} \in \mathcal{F}$ is $\sum_i \log(\mathbf{f}_i)$. Power vector $\mathbf{p} \in \mathcal{P}$ is Pareto efficient on $\mathcal{P}$ if there exists no other vector $\mathbf{p}' \in \mathcal{P}$ such that for all $i, \mathbf{p}'_i \leq \mathbf{p}_i$ and for some $j, \mathbf{p}'_j < \mathbf{p}_j$. Exp-cost of $\mathbf{p} \in \mathcal{P}$ is $\sum_i \exp(\mathbf{p}_i)$.

C. Main Finding - Rate Maximization

In this section we show that any rate allocation that is feasible with an arbitrary power control and under some average power constraint, is also achievable with $0 - P^{MAX}$power control. Moreover, if we consider a scenario without average power constraints, then $0 - P^{MAX}$is the single optimal power control.

We clearly have $\mathcal{F}^E \subseteq \mathcal{F}$, and we want to show that every feasible flow rate allocation can be achieved by a set of extreme power allocation from $\mathcal{P}^E$, that is $\mathcal{F} \subseteq \mathcal{F}^E$.

Theorem 1: For arbitrary values of parameters of constraint set (1), we have that $\mathcal{F}^E = \mathcal{F}$.

The proof of the theorem is in the appendix. We see from this theorem that every feasible rate allocation, thus including the Pareto efficient ones, can be achieved with $0 - P^{MAX}$power control, and with an appropriate scheduling, hence we conclude that $0 - P^{MAX}$is at least as good as any other power control, and power adaptation is useless.

We next consider a scenario where there are no constraints on consumed power (or equivalently $\overline{\mathbf{P}}^{MAX} \geq \mathbf{P}^{MAX}$), and we have the following theorem:

Theorem 2: Consider an arbitrary network, and an arbitrary schedule α and a set of power allocations $\mathbf{p}^n$ for that network. If for some n, $\alpha^n > 0$ and power allocation $\mathbf{p}^n \notin \mathcal{P}^E$ then the resulting average rate allocation $\mathbf{f}$ is not Pareto efficient on $\mathcal{X}$.

The proof of the theorem is in the appendix. We see from the theorem that a Pareto efficient allocation cannot be achieved if in any time slot a power allocation different from $0 - P^{MAX}$is used. Therefore we conclude that in this case, $0 - P^{MAX}$power control is actually the single optimal power control strategy, and any power adaptation is harmful.

D. Main Result - Power Minimization

We show that any average power consumption that is feasible under some average rate constraints is achievable with $0 - P^{MAX}$power control

Theorem 3: For arbitrary values of parameters of the constraint set (1), we have that $\overline{\mathcal{P}}^E = \overline{\mathcal{P}}$.

The proof is in the appendix. All feasible rates, hence all Pareto efficient rates can be achieved with $0 - P^{MAX}$power control, hence it is at least as good as any other power control. Again here, power adaptation is useless. We note here that for power minimization there is no statement analog to Theorem 2. Theorem 2 assumes no average power constraints. In the framework of power minimization, this corresponds to a setting with no average rate constraints, which leads to the trivial solution of having the network silent all the time.

IV. NUMERICAL EXAMPLES

A. Examples With Linear Rate Function

In order to illustrate findings from Section III we give a simple example. Consider a network of 2 links presented on the left of Figure 1. This network is know as the near-far scenario as an interferer is closer to a receiver than the corresponding transmitter. Node S_1 transmits to D_1 and node S_2 transmits to D_2. We introduce two simple MAC protocols. The first MAC protocol assumes $0 - P^{MAX}$power control and arbitrary scheduling. The second assumes no scheduling (constant power allocations through time, like in some cellular systems), and arbitrary power control strategy. The corresponding sets of feasible rates and powers are given on the right of Figure 1.

We see that when maximizing rates, only $0 - P^{MAX}$power control gives Pareto efficient rates. However, when there is an average power limit, there might exist a schedule and a power control strategy, different from $0 - P^{MAX}$, that can achieve Pareto efficient allocations, as discussed in Section III-D. To see this, consider the simple example of a single link. Let P^{MAX} be the maximum transmitting power, $\overline{P}^{MAX} < P^{MAX}$ the maximum average consumed power, h be the fading from the source and η be the power of background white noise. There exist only one Pareto efficient rate allocation which is $R = \overline{P}^{MAX} h/\eta$. It can be achieved by sending $\alpha = \overline{P}^{MAX}/P^{MAX}$ fraction of the time using full power, or by sending all the time using $\overline{P}^{MAX}$ as the transmitting power. The second strategy thus does not have the form of $0 - P^{MAX}$ power control, yet it achieves the Pareto efficient allocation. An analogous construction can be done to show that a non-$0 - P^{MAX}$ power control can achieve Pareto efficient average power allocation.

As already mentioned, the rate functions of 802.11a/g is not linear. Still, for smaller rates, it can be approximated as with a linear function. To show that, we look at the specification of Tsunami 802.11a wireless card in turbo mode [8]. Rate as a function of SNR, for smaller values of SNR, is depicted on Figure 2. We see that this function is almost linear and hence our findings apply to this regime of 802.11a.

B. A 2-node Counter Examples

Rate Maximization: There are physical layers, like high-rates 802.11a [8] or HDR [9], where the rate func-

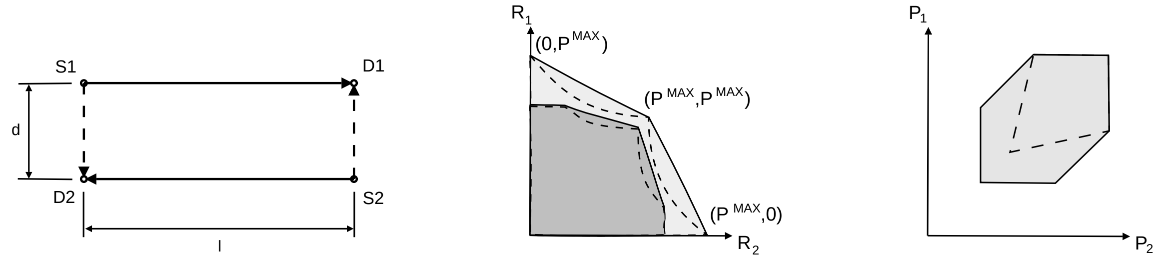

Fig. 1. A simple example of network with 2 links. The topology of the network is given on the left. Node S_1 sends to node D_1 while node S_2 sends to D_2. The feasible rate set for this network is given in the middle. The lighter region in dashed line represent the set of feasible rates that can be achieved without scheduling, only with power adaptation. The lighter region in full lines represent an increase that is achieved by scheduling and without power adaptation ($0-P^{MAX}$ power control). The darker region in dashed lines is the same example without scheduling and with power control, but this time with additional average power constraints. Again the darker region in full lines represents an improvement introduced by scheduling. We see that the second protocol cannot achieve Pareto efficient rates of the feasible rate set, except for the three rate allocations. But these three rate allocations are achieved with power allocations $(0, P^{MAX})$, $(P^{MAX}, 0)$ and (P^{MAX}, P^{MAX}) which belong to $0-P^{MAX}$ power strategy. On the figure on the left, the feasible set of average consumed power under minimum rate constraints is depicted in gray. The region in full lines represent average power consumption achievable with scheduling and without power adaptation, and the region in dashed lines represent average power consumptions achievable without scheduling and with power adaptation. All average powers belonging to this set can be achieved without power adaptation.

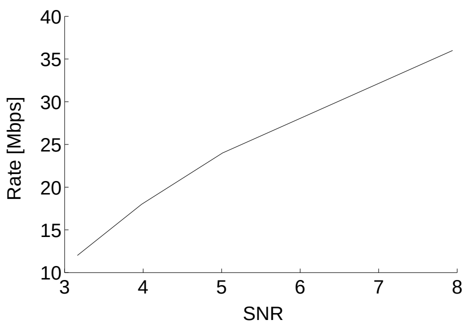

Fig. 2. Link rates that can be achieved on Tsunami 802.11a wireless card in turbo mode [8], as a function of SNR. We assume an adaptive coding technique such as RCPC codes [25] is used in addition to adaptive modulation defined in [8]. For lower SNRs, this function is almost linear.

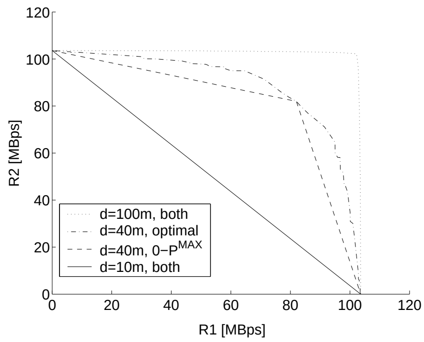

Fig. 3. Near-far scenario from Figure 1 on 802.11a/g network [8]. We set $l = 10m$. In dotted line we plot the feasible rate region for $d = 100m$. In this case $0-P^{MAX}$ and the optimal power control coincide. In dashed-dotted line we plot the feasible rate region for the optimal power control and $d = 40m$, and in dashed line we plot the feasible rate region for $d = 40m$ and $0-P^{MAX}$ power control. We see that the second one is slightly smaller than the first one. Finally, with the solid line we depict the feasible rate region for $d = 10m$. Again, the regions for optimal and $0-P^{MAX}$ power control coincide.

tion is clearly non-linear and our findings do not apply. In order to illustrate this, we consider again the near-far topology from Figure 1. We take the rate function defined in [8], where maximum rate is 108 Mbps. The assumptions on MAC layer and the rest of the network remain as defined in Section II (we do not assume 802.11 MAC protocol). We assume free space path loss model with parameters given in [26].

We carefully handcrafted the example to derive a case where $0-P^{MAX}$ power control is not the optimal strategy. In order to have that, we need to have the two links operating in a high rate regime, that is around 100 Mbps. We put a link distance to be 10m. We next choose interferer distance d. A large d will decrease the effect of the interference, and the feasible rate region will look like a square (Figure 3, dotted line). A small d will force mutual exclusion, and lead to a triangle-like feasible rate region (Figure 3, solid line). In both cases these regions will coincide with a region obtained with $0-P^{MAX}$ power control. Instead, we choose $d = 40$m. The feasible rate region for this case is depicted on Figure 1 (dash-dotted line). As we see from the figure, even for this handcrafted example, the region achieved with $0-P^{MAX}$ power control (dashed line) is only slightly smaller than the one achievable with the optimal power control. Although $0-P^{MAX}$ is close to optimal power control in this example, the optimal power control for an arbitrary network with 802.11a/g physical layer is not known. We obtain a similar result for HDR physical layer [9], but we do not present them here for the sake of brevity.

Power Minimization: In the previous example we saw that rate adaptation with $0-P^{MAX}$ is almost as efficient as the optimal power and rate adaptation for non-linear rate functions. It is not the case for power minimization. To see that, consider again a single link example. Let $R(SNR)$ be a strictly concave rate function, P^{MAX} be the maximum transmitting power, F the minimum rate, h be the fading from the source and η be the power of background white noise. If we use $0-P^{MAX}$power control, we send α fraction of the time such that $F = \alpha R(P^{MAX}h/\eta)$. Otherwise, if we do not restrict to $0-P^{MAX}$we can choose to transmit with some power P such that $F = R(Ph/\eta)$. Since $\alpha R(P^{MAX}h/\eta) = R(Ph/\eta)$ and R is strictly concave, we have that $P < \alpha P^{MAX}$ hence the average dissipated power in the second case is strictly less than in the first case. We quantified numerically this difference for 802.11a/g physical layer and it goes up to two orders of magnitude for larger networks. Therefore we conclude that rate adaptation and $0-P^{MAX}$power control are not efficient for power minimization when rate function is not linear.

V. Rate vs. Power Adaptation

In this section we compare the performances of two design approaches: rate adaptation without power adaptation, and power adaptation with fixed link rates. We focus on rate maximization, as we have already showed in the previous section that the first approach is not optimal for power minimization.

It is difficult to numerically characterize feasible rates' and powers' sets for larger networks. Therefore, instead of comparing feasible sets, we will compare utilities, as explained in Section II-E.

For the case with rate adaptation, we assume $0-P^{MAX}$power control. We then find the optimal schedule that maximizes log-utility of the network. We assume the optimal rate adaptation: for a given schedule and power allocation, in each slot we can calculate SNRs at every receiver, and we select the maximum achievable rate for that SNR, as defined by the rate function of the physical layer.

For the second case, we assume all links have fixed link rates. We use CA/CDMA power control algorithm [7] as the state-of-the-art power control algorithm for networks with fixed links' rates. We again assume ideal scheduling that maximizes utility. The basic principle of CA/CDMA is to allocate slightly higher transmission power than necessary to achieve the fixed link rate. This additional power is called interference margin, and is used to let other nodes transmit during the same slot. For more details about the protocol see [7].

We implemented a centralized version of CA/CDMA that assumes no protocol overhead. Possible power allocations are constructed apriori, and we find a schedule that uses these power allocations and maximizes total log-utility of the network (as discussed in Section II-E). Similarly we have implemented $0-P^{MAX}$power control with rate adaptation and ideal scheduling.

Another important design factor in wireless networks is routing. We considered several routing strategies. We varied maximum hop lengths, and for each predefined maximum hop length we found a route that minimizes the number of hops. This way we varied routes gradually from nearest neighbor routing to direct routing.

We repeat the procedure for a number of uniformly random topologies. For each topology we varied maximum hop distance and constructed several sets of routes. For each of the routes we maximized the total utility by solving the optimization problem. The optimal routing is thus the one for which the total utility is maximized. The simulations are done in MATLAB. We see from the results, depicted in Figure 4, that rate adaptation with $0-P^{MAX}$power control outperforms by far power adaptation with $0-R^{MAX}$rate control.

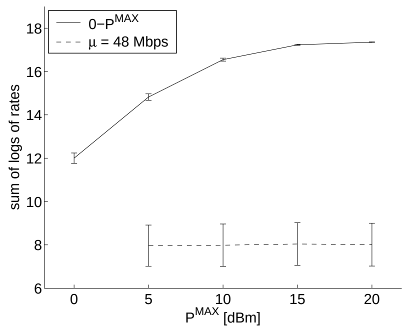

Fig. 4. Comparison between rate adaptation with $0-P^{MAX}$power control and CA/CDMA power adaptation with $0-R^{MAX}$rate control. We put 12 nodes randomly on a 200mx200m grid, and we select randomly source and destination pairs. We see that $0-P^{MAX}$with rate adaptation is always much better and its performance improves with increase in P^{MAX}. On the other hand, CA/CDMA uses fixed rate (in this case 48Mbps). For small powers (e.g. $P^{MAX} = 1mW$) CA/CDMA cannot establish communication (hence the utility is $-\infty$ and is not plotted). For larger powers CA/CDMA cannot benefit from higher link rates and its utility stays fixed.

VI. Conclusion

We have considered multi-hop wireless networks with linear and non-linear rate functions. We have shown that in linear regime, having no power control (or in other words $0-P^{MAX}$power control), is always optimal, and that power adaptation is useless, both for power minimization and rate maximization.

We have also shown that for rate maximization, in the linear regime and without average power limitations, $0-P^{MAX}$is the only optimal power control strategy, and any other power control strategy yields non-Pareto optimal rate allocations, hence there power adaptation is harmful.

For the non-linear regime, the issue of optimal power control, scheduling and rate adaptation remains open. This is especially true for power minimization, where we showed that $0-P^{MAX}$ might be significantly worse than optimal power control. Still, when maximizing rates, we showed that $0-P^{MAX}$ with rate adaptation and scheduling, although not optimal, is very close to the performance of optimal power control.

Then we compared the performance of $0-P^{MAX}$ power control with rate adaptation and scheduling against power adaptation, scheduling and fixed rates. We considered CA/CDMA as a state-of-the-art power control protocol for rate maximization scenarios in fixed link rates networks. We found that a rate adaptation with a simple $0-P^{MAX}$ power control highly outperforms power adaptation with fixed link rates, when maximizing rates. If the number of possible physical link rates is small, one should use power adaptation and scheduling (as for example in [7]), but if the number of possible link rates is large, which is usually the case with adaptive modulation and/or coding, one should adapt rates, use $0-P^{MAX}$ power control and scheduling. The complexity of a protocol should thus be invested in optimizing scheduling and rate adaptation, and not the power adaptation. Another conclusion, that stems from our work is that, unlike common belief, in CDMA or similar data networks with almost-orthogonal links' transmissions, and for rate maximization, it is better to solve near-far problems by scheduling and rate adaptation and to use $0-P^{MAX}$ power control, instead of using power adaptation that tend to equalize received powers.

APPENDIX

Lemma 1: Consider a function $U(\mathbf{f},\bar{\mathbf{p}}) = \sum_i \mu_i \mathbf{f}_i - \sum_i \lambda_i \bar{\mathbf{p}}_i$ for some arbitrary vectors μ, λ. Then:

- There is a unique maximum $U^* = U(\mathbf{f}^*, \bar{\mathbf{p}}^*)$ on set $\mathcal{X}$,
- The maximum $(\mathbf{f}^*, \bar{\mathbf{p}}^*) \in \mathcal{X}^E$,
- If some i, $|\mu_i| > 0$ and for all j, $\lambda_j = 0$, then for arbitrary α and $\{\mathbf{p}^n(s)\}_{1,\dots,N}$, such that for some n, $\alpha_n > 0$ and $0 < p_i^n < P_i^{MAX}$, and the resulting $(\mathbf{f}, \bar{\mathbf{p}})$ we have $U(\mathbf{f}, \bar{\mathbf{p}}) < U(\mathbf{f}^*, \bar{\mathbf{p}}^*)$.

Proof: Both function $U(\mathbf{f},\bar{\mathbf{p}})$ and set $\mathcal{X}$ are convex, hence the maximum is attained in some $(\mathbf{f}^*, \bar{\mathbf{p}}^*) \in \mathcal{X}$. We also know there exist $\alpha^*, \{\mathbf{p}^{n*}(s)\}_{1,\cdots,N}$ that satisfy (1).

Let us first assume there is a single system state $\mathcal{S} = \{s\}$ hence there is no randomness in the system/ We use an approach similar to [12], [17]. Without loss of generality, we fix all $\alpha^*(s), \{\mathbf{p}^{n*}(s)\}_{1,\cdots,N}$ except $p_1^1(s)$, and we consider a function $p_1^1(s) \xrightarrow{V} \sum_i \mu_i \mathbf{f}_i - \sum_i \lambda_i \bar{\mathbf{p}}_i$ as a function of a single free variable $p_1^1(s)$. From the routing equation in (1) we have that $\sum_i \mu_i \mathbf{f}_i = \sum_j \left(\sum_i \mu_i R_{ij}\right) \bar{\mathbf{x}}_j$. Since μ is an arbitrary vector we can further on simplify and assume a single-hop routing ($\mathbf{f}_i = \bar{\mathbf{x}}_i$).

We then have the following derivatives:

$$\frac{\partial V}{\partial p_1^1(s)} = \frac{\mu_1 \alpha_1(s)\, h_{11}(s)}{\eta_1(s) + \beta \sum_{k\neq 1} \mathbf{p}_k^n(s) h_{k1}(s)} - \lambda_1 \alpha_1(s) \quad (2)$$

$$- \sum_{i=2\cdots I} \frac{\mu_i \alpha_i(s)\, p_i^1(s) h_{ii}(s) h_{1i}(s)}{\left(\eta_i(s) + \beta \sum_{k\neq i} \mathbf{p}_k^1(s) h_{ki}(s)\right)^2}, \quad (3)$$

$$\frac{\partial^2 V}{\left(\partial p_1^1(s)\right)^2} = \sum_{i=2\cdots I} \frac{\mu_i \alpha_i(s)\, 2p_i^1(s) h_{ii}(s) h_{1i}^2(s)}{\left(\eta_i(s) + \beta \sum_{k\neq i} \mathbf{p}_k^1(s) h_{ki}(s)\right)^3}. \quad (4)$$

We first suppose that for all i, $\mu_i > 0$. It is easy to see from (4) that regardless of the values of other variables, the second derivative is always positive, $V(p_1^1(s))$ is always convex, hence the maximum is attained for $p_1^1(s) \in \{0, P^{MAX}\}$. Therefore, we have that $\{\mathbf{p}^{n*}(s)\}_{1,\cdots,N} \in \mathcal{P}^E$, and $\mathbf{f} \in \mathcal{F}^E$.

Next we suppose, without loss of generality, that for some m we have $\mu_1 \le 0, \cdots, \mu_m \le 0$. Then clearly the optimal is to have $f_1 = 0, \cdots, f_m = 0$ which is always feasible, regardless of the average rates of links $\bar{\mathbf{x}}$. Then by setting $\mu_1 = 0, \cdots, \mu_m = 0$, the new optimization problem has the same maximum as the old one, and we again have that $\{\mathbf{p}^{n*}(s)\}_{1,\cdots,N} \in \mathcal{P}^E$, and $\mathbf{f} \in \mathcal{F}^E$.

At this point we proved the second claim under assumption that there is no randomness in the system. We next relax this assumption. From the above we know that for every state $s \in \mathcal{S}$ there is a power allocation from $\mathcal{P}^E$ that maximizes the utility. Since averaging over S is a linear operation, the average over S is also going to be maximized, which concludes the proof of the second claim.

Finally, consider the case when $|\mu_1| > 0$ and $\lambda_j = 0$ for all j. We again suppose no randomness ($\mathcal{S} = \{s\}$), and we suppose that $\alpha_1(s) > 0$ and $0 < p_1^1(s) < P_1^{MAX}$. It is easy to verify from (3) that equation $\frac{\partial V}{\partial p_1^1(s)} = 0$ can be transformed into $Q(p_1^1(s)) = 0$ where Q is some polynomial of degree n_Q. Furthermore, one can verify that the coefficient of the polynomial of degree n_Q is strictly positive, hence Q is not identical to 0. Therefore there is only a finite number of values of $p_1^1(s)$ that solves $Q(p_1^1(s)) = 0$, and thus also $\frac{\partial V}{\partial p_1^1(s)} = 0$.

We know from above that the maximum V^* is achieved at one of the extremal points, say P_1^{MAX} without loss of generality. By assumptions, we have $V(p_1^1(s)) = V^*$. Now for some γ we have that $p_1^1(s) = \gamma P_1^{MAX}$ and $V^* = V(p_1^1(s)) \le (1-\gamma)V(0) + \gamma V(P_j^{MAX}) \le V^*$. We thus have $V^* = V(p)$ for all $p \in [0, P_j^{MAX}]$. Now this is impossible since $V'(p)$ has only a finite number of zeros, hence $\{\mathbf{p}^n(s)\}_{1,\cdots,N}$ cannot maximize V.

Now we introduce randomness. Again, due to linearity of averaging it is easy to see that if for any state s with positive probability ($P[S = s] > 0$) we have $\{\mathbf{p}^n(s)\}_{1,\cdots,N} \notin \mathcal{P}^E$, then the utility in that slot is going to be strictly smaller than the maximum achievable, hence the overall utility will be strictly smaller than the maximum, which proves the last claim. ■

Proof of Theorem 1: We clearly have $\mathcal{F}^E \subseteq \mathcal{F}$, and it remains to be shown that $\mathcal{F} \subseteq \mathcal{F}^E$. First, consider the optimization problem $\max \sum_i \mu_i \mathbf{f}_i$ such that $\bar{\mathbf{p}} \le \overline{\mathbf{P}}^{MAX}$, $(\mathbf{f}, \bar{\mathbf{p}}) \in \mathcal{X}$. This is a convex optimization since set $\mathcal{X}$ is convex, hence if the constraint set is not empty there is a unique maximum $(\mathbf{f}^*, \mu^*)$. The dual problem is $\min_{\lambda \geq 0} g(\lambda) + \sum_i \lambda_i \overline{\mathbf{P}}_i^{MAX}$ where $g(\lambda) = \max_{(\mathbf{f}, \bar{\mathbf{p}}) \in \mathcal{X}} \sum_i \mu_i \bar{\mathbf{x}}_i - \sum_i \lambda_i \bar{\mathbf{p}}_i$. According to lemma 1, point $(\mathbf{f}^*, \mu^*)$ that maximizes $g(\lambda^*)$, thus also maximizes the above maximization problem, belongs to $\mathcal{X}(\mathcal{P}^E)$.

We now prove the theorem by contradiction. Suppose there exists a point $\mathbf{f} \in \mathcal{F}$ that is not in $\mathcal{F}^E$. Then by the separating hyperplane theorem [27] there exists a hyperplane defined by $(\mathbf{c}, b)$ that separates $\mathbf{f}$ and $\mathcal{F}^E$, that is $\mathbf{c}^T \mathbf{f} > b$ and for all $\mathbf{g} \in \mathcal{F}^E$, $\mathbf{c}^T \mathbf{g} < b$. This on the other hand means that $\mathbf{f} \notin \mathcal{F}^E$ maximizes the above maximization problem, which leads to contradiction. ■

Proof of Theorem 2: We proceed by contradiction, and assume there exist a schedule α and a set of power allocations $\{\mathbf{p}^n(s)\}_{1,\cdots,N}$ such that the resulting average rate allocation $\mathbf{f}$ is Pareto efficient (and thus on the boundary of set $\mathcal{F}$), and for some n, i, $0 < \mathbf{p}_i^n < P_i^{MAX}$. Since $\mathcal{F}$ is convex, there exists a supporting hyperplane [27] (μ, b) which contains $\mathbf{f}$ (that is $\mu^T \mathbf{f} = b$) and contains $\mathcal{F}$ in one of the half-spaces (that is for all $\mathbf{f}' \in \mathcal{F}, \mu^T \mathbf{f} \leq b$).

Let us first suppose $|\mu_i| > 0$. Then, according to lemma 1 there exists $(\mathbf{f}^*, \bar{\mathbf{p}}^*) \in \mathcal{X}^E$ such that $\mu^T \mathbf{f}^* > \mu^T \mathbf{f}^* = b$, which leads to contradiction. Therefore, we have that $\mu_i = 0$, and $\sum_{j \neq i} \mu_j \mathbf{f}'_j \leq b$ for all $\mathbf{f}' \in \mathcal{F}$. However, it is easy to construct find a counter example. If there exist another $j \neq i$ such that $\mathbf{p}_j^n > 0$, then by setting $\mathbf{p}_i^n = 0$ we increase $\mathbf{f}_j$, thus $\sum_{j \neq i} \mu_j \mathbf{f}'_j > b$, hence the contradiction. On the contrary, if for all $j \neq i$, $\mathbf{p}_j^n = 0$, we then set $\alpha_n = 0$ and increase some other α^m such that for some j, $\mathbf{p}_j^m > 0$. Again, this way we increase $\mathbf{f}_j$, thus $\sum_{j \neq i} \mu_j \mathbf{f}'_j > b$, that also leads us to a contradiction. ■

Proof of Theorem 3: Consider the optimization problem $\min \sum_i \mu_i \bar{\mathbf{p}}_i$ such that $\mathbf{f} \geq \overline{\mathbf{F}}^{MIN}, (\mathbf{f}, \bar{\mathbf{p}}) \in \mathcal{X}$. This is a convex optimization since set $\mathcal{X}$ is convex, hence if the constraint set is not empty there is a unique minimum $(\mathbf{f}^*, \mu^*)$. The dual problem is $\max_{\lambda \geq 0} g(\lambda) + \sum_i \lambda_i \overline{\mathbf{F}}_i^{MIN}$ where $g(\lambda) = \min_{(\mathbf{f}, \bar{\mathbf{p}}) \in \mathcal{X}} \sum_i \mu_i \bar{\mathbf{p}}_i - \sum_i \lambda_i \mathbf{f}_i$. According to lemma 1, point $(\mathbf{f}^*, \mu^*)$ that maximizes $-g(\lambda^*)$, thus also minimizes the above minimization problem, belongs to $\mathcal{X}(\mathcal{P}^E)$. The rest of the proof is the same as in Theorem 1. ■

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