A Lower Bound for Broadcasting in Mobile Ad Hoc Networks *

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Abstract

We give a lower bound of $\Omega(n)$ *rounds* for broadcasting in *mobile* ad hoc networks, where n is the number of nodes in the network. A round is the time taken by a node to successfully transmit a message to all its neighbors. It has been shown by Bruschi *et al.* and Chlebus *et al.* that a minimum of $\Omega(\log n)$ time-slots are required in a round to propagate the broadcast message by one hop. In static networks, this gives us the lower bound for network-wide broadcast to be $\Omega(D \log n)$ time-slots, where D is its diameter. Although this lower bound is valid for a mobile network, we obtain a tighter lower bound of $\Omega(n \log n)$ time-slots by considering explicit node mobility. This result is valid even when $D \ll n$ and the network diameter never exceeds D. This shows that the dominating factor in the complexity of broadcasting in a *mobile* network is the number of nodes in the network and not its diameter.

Keywords: *algorithms, broadcasting, wireless network, distributed systems, lower bound*

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1 Introduction

A Mobile Ad Hoc Network (MANET) is a set of *nodes* communicating with each other via multi-hop wireless links. Each node can directly communicate with nodes within its communication range. Intermediate nodes forward messages to the nodes that are not adjacent to the source. Since the nodes are mobile, the topology of the network is constantly changing.

A fundamental communication primitive in MANET is network-wide broadcast, necessary for disseminating information across the entire network. We consider reliable broadcast, which *guarantees* the delivery of the broadcast message from a single source to *all* the nodes of the network. This is different from *best-effort* solutions such as [5], where message delivery to all the nodes is not guaranteed. In this paper, we study the lower bound on time for the reliable broadcast problem. More specifically, we calculate the asymptotically minimal number of time-slots required by any broadcasting protocol to reliably propagate the broadcast message from a source to all the nodes of the network.

The complexity of broadcast in wireless multi-hop networks is a well investigated problem. In a centralized model, when each node has full knowledge of the network topology, [1] proves a lower bound of $\Omega(\log^2 n)$ slots of transmission. The first lower bound for non-centralized or distributed broadcasting was proposed by Bar-Yehuda *et al.* in 1992 [2]. They give a lower bound of $\Omega(n)$ time-slots, given that the nodes know n and the identities of their neighbors. In 2002, Kowalski and Pelc [9] disprove Bar-Yehuda *et al.*'s linear lower bound by presenting a logarithmic broadcast solution for the graphs considered in [2]. When the knowledge about neighbors and network size is not available, [4] gives a lower bound of $\Omega(D \log n)$ time-slots, when $D \leq n/2$. An equivalent lower bound of $\Omega(D \log n)$ for directed networks is given by Chlebus *et al.* [6] when spontaneous transmissions are allowed. By using selective families, Clementi *et al.* give a lower bound of $\Omega(D\Delta \log(n/\Delta))$ when $\Delta \leq n/D$ [7]. When $\Delta = n/D$, this lower bound reduces to $\Omega(n \log D)$.

The most influential papers that deal with lower bounds for broadcasting in wireless ad hoc networks are summarized in Table 1. These papers derive their lower bound in a static setting, which nevertheless remains a lower bound when the nodes are mobile. In our paper, we improve these results by incorporating mobility directly within the derivation, leading to a tighter lower bound. Moreover, our proof makes no assumption on the network diameter or its maximum degree. Several algorithms have been proposed to solve broadcast in mobile ad-hoc networks. In particular, Basagni *et al.* [3] propose a protocol that completes in $O(D \log n)$

Reference	Proof context			Bound (time-slots)
	Centralized	Mobility	Additional remarks	
[2], 1987[1]	No	Static	D = 2	$\Omega(n)$
[1], 1989	Yes	Static	D = 2	$\Omega(\log^2 n)$
[4], 1997	No	Static	$D \leq n/2$	$\Omega(D\, log\, n)$
[8], 1999	No	Static	linear networks	$\Omega(D + \log^2 R / \log\log R)$
[6], 2000	No	Static	$D < 2n/3$, directed graph	$\Omega(D \log n)$
[9], 2002	No	Static		$\Omega(\sqrt[4]{nD^3})$
[10], 2002	No	Static		$\Omega(n \frac{\log n}{\log(n/D)})$
[7], 2003	No	Static	$\Delta \leq n/D$	$\Omega(D\, \Delta \log(n/\Delta))$
This paper	No	Mobile		$\Omega(n \log n)$

Table 1: Summary of existing lower bounds for deterministic broadcasting

time-slots, when $\Delta = 3$. This upper bound is *lower* than our $\Omega(n \log n)$ lower bound. The reason is because they prove that $O(\log n)$ slots are required to propagate the message by one hop. The one hop message propagation time is then multiplied by the network diameter D to get the overall complexity. Our contribution is to show that when the network is mobile, the uncovered distance does *not* always decrease in each round. As we show in Section 3, $\Omega(n)$ rounds are required for a broadcast to complete in a *mobile* network. The result holds even when the diameter never exceeds D, where D is a constant and $D \ll n$. We consider a discrete grid-based network. This gives us the ability to quantify node mobility. Nodes are situated at grid points and only allowed to move from one grid point to an adjacent grid point. Since node mobility in a grid network is a special case of general MANET node mobility, any lower bound for the grid network also applies to an unconstrained MANET. Our lower bound implies that when node mobility is considered, the dominating factor in the time complexity of a broadcasting algorithm is the number of nodes in the network and not its diameter.

2 System Model

We introduce a model that incorporates node mobility. We assume a grid-based network, with nodes located at the grid points. There are n nodes in the network. The network remains connected all the time and its diameter never exceeds D. We do not make any assumption about the maximum degree of a node. Similar to [4, 10], nodes are not aware of the identities of their neighbors.

[1] Bound proved incorrect by [9].

Nodes send messages in synchronous *time-slots*. Messages can be sent by a node to other nodes that are no more than R inter-grid point distance away, where $R \geq 1$ is the communication range of each node. If a node i is in the communication range of a node j then node i is called a *neighbor* of node j. A node is *covered* if it has successfully received the broadcast message; otherwise, it is *uncovered*. A node i successfully receives a message in a slot if exactly one of its neighbors transmits in that slot. If two or more neighbors of node i transmit in a slot then there is a *collision* at node i and none of the transmitted messages is successfully received by node i. We assume that nodes do not have collision detection capabilities, i.e., they cannot distinguish between background noise and interference noise. A *round* consists of a number of time-slots. It is defined as the time required by all the neighbors of a covered node to get the broadcast message.

Network topology is dynamic. Nodes can move either horizontally or vertically from one grid point to a neighboring grid point at the end of a round. Each node makes this decision independently based solely on its local information.

We would like to emphasize that using a grid-based model does not mean that the lower bound we derive is only applicable to networks that exhibit grid-based mobility. Since node mobility in a grid model is a special case of the more general mobility of a MANET, a lower bound proved for this restricted grid mobility model automatically applies to the more general mobility of a MANET. Similarly, if mobility occurs at arbitrary times, this lower bound still holds. Finally, a lower bound derived using a *synchronous* model remains valid for the more general *asynchronous* model.

3 Lower Bound

In static networks, the added knowledge of network diameter helps in determining the lower bound on the time complexity for broadcast. We investigate if such knowledge is helpful in the context of MANETs. Specifically, given an upper bound D on the diameter of the MANET, is it possible to assert anything about completion time of broadcast? Of course, while broadcast is taking place, from round to round, the diameter of the MANET may change, but never exceed D. We consider an adversary that initially places nodes on grid points and moves them in order to delay as much as possible the termination of broadcast, with the constraints that (1) the network stays connected, and (2) the diameter of the MANET never exceeds D.

Theorem 1. *In a mobile network, the lower bound on time to complete broadcasting is* $\Omega(n)$ *rounds.*

Proof. The result is proved by providing one example where the adversary controls the movement of nodes and it takes in the order of n rounds to broadcast a message in a network of diameter no greater than D.

Consider the network shown in Figure 1, where the distance between neighboring nodes is R grid points. By construction, each row is $2R-2$ hops long, which defines R. Node s is the message source and is located in the lower right-hand position. All other small circles (filled and unfilled) represent two nodes that are collocated. Nodes a, b, c and d serve as visual points of reference. The diameter of the network is equal to $12R-2$ and throughout this proof the network diameter will never exceed this value. Hence, $D = 12R-2$. The total number of nodes in the network, n, is significantly greater than the diameter. Each row, except for the topmost and the bottommost rows, has $4R-2$ nodes.[2] The rows are $2R$ grid distance apart from each other. Successive rows are connected by one node, alternating between the left and the right ends. The remaining nodes are distributed equally among the $2R+1$ positions (represented by large circles) in the top row — the *reserve*.

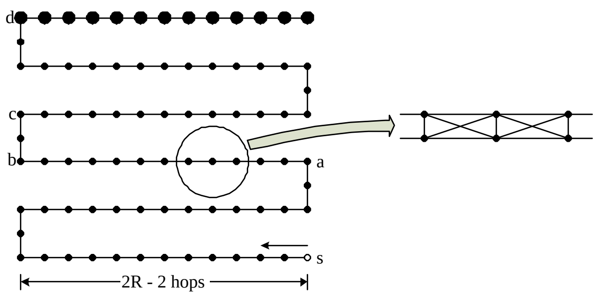
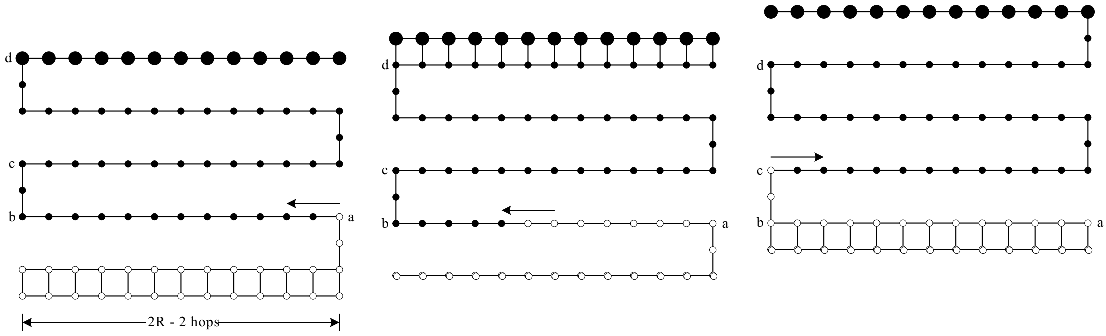


Figure 1: Initial network topology.

Node s initiates the broadcast. On receiving message m in round r, a node tries to forward the message to all its uncovered neighbors in the following rounds. During the first $3R$ rounds there is no node mobility and the message advances by one hop per round. At the end of these $3R$ rounds the message has advanced past the middle of the second row from the bottom. During the next R rounds the nodes in the lowest row move up by one grid distance per round. The network topology at the end of the first $4R$ rounds is shown in Figure 2(a) and the message has advanced up to node a. The unfilled circles represent nodes that have already received the message, while uncovered nodes are represented by the filled circles.

[2]The bottom most row has only $4R-3$ nodes because there is no node collocated with the source s. The topmost row is discussed later.



(a) Beginning of phase

(b) Phase after R rounds.

(c) End of phase after $2R$ rounds.

Figure 2: Mobility adversary in a two-dimensional network.

After this initial step, message propagation happens in a succession of *phases* with each *phase* containing $2R$ rounds. In the first round, nodes at a propagate the broadcast message. While the message is propagated, nodes move in the following fashion:

Lowest row: All nodes in the lowest row move up by one grid distance every round of the phase, covering a total of $2R$ grid distance.

Second lowest row: The nodes remain in place during the first R rounds and move up by one grid distance during the remaining R rounds of the phase.

Top most row: Two nodes stay at each of the occupied positions in the top most row. All other nodes in these positions move up by one grid distance in each of the $2R$ rounds. The only *exceptions* are two nodes in the top row at the right most position which move only during the first R rounds.

Figure 2(b) represents the situation of a phase after R rounds. The lowest and the top most rows moved up by R grid points. The two lowest rows now coincide. During this time, m has been propagated halfway through the (initially) third lowest row.

After the full $2R$ rounds of a phase, represented in Figure 2(c), the broadcast message m has reached node c and a new row has been constructed at the top. The following assertions can be made:

1. By definition, the transformation took place in a phase of $2R$ rounds.

2. The resulting Figure 2(c) is identical (by symmetry and translation) to Figure 2(a). Nodes at c in Figure 2(c) are the starting point for the broadcast in the next phase.

3. The diameter of the graph stays unchanged during the transformation from Figures 2(a) to 2(b) to 2(c).

4. Graph connectivity is maintained throughout the transformation.

5. The remaining distance to cover after the transformation remains the same as before the transformation.

6. $4R$ nodes have been covered in the phase and $4R$ nodes have been consumed from the *reserve* in order to construct the new top-most row.

The same schema can be repeated as long as there are sufficient nodes in reserve in the top-most row, alternating between left most and right most position for the *exception* regarding the top most row. Consequently, in successive phases, the topology changes alternate between (i) Figure 2(a) $\rightarrow$ Figure 2(c), and (ii) Figure 2(c) $\rightarrow$ Figure 2(a). Since in Figure 2(a) there are $n - 8R - 1$ nodes that have not received m, the schema can be repeated for a total of $\lfloor \frac{n-8R-1}{4R} \rfloor$ phases, each of duration $2R$ rounds. Hence, the number of rounds required for m to be received by all nodes is in the order of n. The lower bound for broadcasting is thus $\Omega(n)$ rounds. □

Note that by fixing the number of nodes in each row, we also fix the speed with which the nodes should move in order for the above result to hold. However, with $2k$ ($k \geq 2R - 1$) nodes in each row instead of $2(2R - 1)$ nodes we can still get the same results. In this case, it will take $k + 1$ rounds for the network to transition from Figure 2(a) to Figure 2(c) during which a subset of nodes move R grid distances. As k is independent of R, D and n, our example is not bound to any specific node speed.

Theorem 2. *The lower bound for broadcasting in two-dimensional MANETs is $\Omega(n \log n)$ time-slots.*

Proof. Let nodes located at a vertex of the graph, v_i, be covered at time T. The message is received by nodes at the next vertex, v_{i+1}, in the earliest slot following T when exactly one of the nodes located at v_i transmits, say at time $T + \delta$. So, the duration of this round is δ. In all preceding slots of the round either both nodes at v_i are scheduled to transmit, or neither is scheduled to transmit. As described in [4], if the nodes at v_i have no knowledge of the history of message propagation, *i.e.*, the set of nodes that have already received the message, then for any transmission schedule of length $t \leq \log{(n/2)}$ slots it is always possible to have a network with a pair of nodes at v_i such that in every slot either both the nodes are scheduled to transmit or neither is scheduled to transmit. Hence, for every schedule it is possible to have a network topology where each round takes time greater than $\log(n/2)$ slots. Combined with the lower bound of $\Omega(n)$ given in Theorem 1, we obtain the lower bound of $\Omega(n \log n)$ time-slots, when the message history is not known.

If the message carries with it the history of propagation denoting the set of p nodes that have already received the message then, as stated in [4], for any transmission schedule of length $\leq \log \frac{n-p}{2}$ slots it is always possible to have a network with a pair of nodes at v_i so that the message does not propagate to nodes at vertex v_{i+1} during these slots. We apply Theorem 3.1 of [4] to our construction, in which there are two collocated nodes per grid-point ($h = 2$). Note that by Theorem 1 we do not sum over D but over n. Hence, the time t for the broadcast message to be received by all n nodes can be expressed as:

$$t \geq \sum_{i=1}^{\frac{n-1}{2}} \log \frac{n-1-2(i-1)}{2}$$

Without loss of generality, let $n-1$ be even, and let $(n-1)/2 = x$. Then, $t \geq \sum_{i=1}^{x} \log i = \log(x!)$. By Stirling's approximation, we get: $t \geq x \log x - x$. Substituting for x, we get $t \geq \frac{n-1}{2} \log(\frac{n-1}{2}) - \frac{n-1}{2}$. Hence, we obtain the lower bound of $\Omega(n \log n)$ time-slots for broadcasting the message □

4 Conclusion

We have shown a lower bound of $\Omega(n \log n)$ time-slots for any deterministic distributed protocol for broadcasting a message in *mobile* ad hoc networks. This is an improvement to the lower bounds by Bruschi *et al.* [4] and Chlebus *et al.* [6], which give a lower bound of $\Omega(D \log n)$ time-slots. Our results show that knowledge of an upper bound on the diameter of a MANET does not help in limiting the time to complete a broadcast. Hence, it is not possible, once the number of time-slots per round is determined, to trivially obtain the complexity for reaching the entire network by simply multiplying by a factor of the network diameter. Instead, when mobility is considered, $\Omega(n)$ rounds are required by *any* broadcasting protocol when the network nodes are mobile.

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