# Digital DS-CDMA Receiver Working Below the Chip Rate: Theory and Design

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**Abstract**

We consider a problem of designing low-complexity digital receivers for CDMA systems operating over channels with either one or multiple propagation paths. We specifically exploit a finite rate of innovation property of CDMA signals and develop a method that leads to an efficient solution to a combined problem of multipath channel estimation and signal detection from a low-dimensional subspace of a received signal. Unlike existing schemes that typically resort to chip rate sampling and manipulate with large correlation matrices, our framework allows for digital CDMA receivers to operate at a significantly lower sampling rate, chosen to be close to a rate of innovation of a received signal. Our approach, therefore, can considerably reduce computational requirements compared to existing solutions while providing similar performances.



**Keywords**

CDMA, multiuser channel estimation, multiuser detection, multipath channels, signals of finite rate of innovation.


## I. Introduction

Code-division multiple access (CDMA) has recently gained much attention as an access protocol well suited for voice and data transmission, particularly over wireless communication networks. CDMA possesses many inherent advantages over earlier access techniques, such as TDMA and FDMA, which are a direct consequence of a coded signal format and an expanded bandwidth as a result. In particular, selective addressing capability, low-density power spectra and interference rejection are among properties that have prompted increased interest in CDMA as a flexible and spectrally efficient access strategy.

Bandwidth expansion in CDMA systems is accomplished by means of a spreading code, often called a signature sequence, which is independent of information data to be sent. A communication channel is then accessed by all users simultaneously, while different users are distinguished at a receiver by a unique signature sequence assigned to each user. A conventional CDMA receiver is a bank of matched filters, each matched to a specific user's code. Although the spreading codes are designed to have a low crosscorrelation, in the case when users have widely varying power levels, code design alone may be insufficient to suppress multiple-access-interference and the standard detector becomes almost useless. This is known as the *near-far* problem that had been considered for a long time as an inherent drawback of CDMA systems.

Conventional CDMA systems either ignore the near-far problem or try to alleviate it by employing power control schemes, yet a full benefit of power control can be exploited only in stationary or slowly varying environments. Multiuser detection schemes alleviate the near-far problem, however, all of them deal only with signal detection and assume that timings of users' signals are known. On the other hand, many wireless communication channels are characterized by multiple propagation paths, so that even the strict autocorrelation properties of spreading waveforms are inadequate to suppress multipath interference. This problem is rather severe in urban and indoor environments that are of great interest to cellular mobile radio applications and wireless local area networks. RAKE receivers and adaptive antenna arrays provide different means to cope with the multipath interference. While adaptive antenna arrays improve the performance of CDMA systems in spatial domain by steering beams toward desired users and thus decreasing an interference power level, the RAKE receiver attempts the same goal through temporal operations by coherently combining[1] multipath signals from a desired user. Recently, there has been an increasing interest in the use of 2-D RAKE receivers for wireless CDMA systems, which simultaneously exploit space and time diversity, and that by combining adaptive antennas with RAKE receivers.

While all these schemes result in significant improvement of performances compared to those of the conventional receiver, they typically involve sophisticated signal processing techniques and require exact knowledge of one or several parameters of a channel, such as relative delays of different user's signals, propagation coefficients, direction-of-arrivals, etc. [1] [3] [12], which makes them computationally complex and sometimes unaffordable in a real-time equipment.

In mobile radio applications, as already mentioned, channel estimation represents a major problem due to multipath fading. Most of the early work on channel estimation had focused only on timing acquisition [10] [16]. While those methods yield very good results, they are usually computationally intense since time delays of all users are estimated jointly and thus they involve a multidimensional optimization problem for a large number of parameters. Subspace-based techniques [1] [14], as well as maximum-likelihood methods

[1]An input signal is correlated with delayed replicas of the same desired code, and correlator outputs are subsequently combined to yield a signal estimate

[2] [19] that emerged recently, somewhat reduce computational complexity by decomposing a multiuser optimization problem into a series of single user problems. However, their computational requirements are still rather high since they resort to chip rate sampling (or even fractional sampling) at the receiver and deal with large correlation matrices. This is even more evident in the case of joint delay and angle estimation [7] [11] (required for implementation of 2-D RAKE receivers) and the applicability of those algorithms to real-time systems is often questionable.

We present a new approach to the problem of designing low-complexity digital receivers for CDMA systems which avoids chip rate sampling, that is, all necessary steps are carried out on a sampled lowpass version of a received signal. We extend some of our recent sampling results for signals of a finite rate of innovation [5] [18] to the problem of multiuser channel estimation and present a subspace-based method that estimates channel parameters of all users simultaneously from a low-dimensional subspace of a received signal. In particular, we exploit a finite rate of innovation property[2] of a received signal and show that by choosing a sampling rate to be close to its rate of innovation it is possible to extract the relevant channel parameters, such as time delays, direction-of-arrivals and propagation coefficients. Once a channel-impulse response of each user has been estimated, it is directly used in a detection process, also carried out on a sampled lowpass version of a received signal. In essence, we present a new method that allows for both channel estimation and signal detection from a lowpass version of a received signal, thus resulting in significantly reduced computational complexity compared to existing techniques.

## II. Sampling at the Rate of Innovation

In a recent work [5] [18], it was shown that it is possible to develop sampling schemes for a large class of non-bandlimited signals, that is, certain signals of a finite rate of innovation. A notion of the finite rate of innovation proved to be crucial in developing those new sampling methods that lead to perfect reconstruction from a finite number of samples.

[2] i.e. a finite number of degrees of freedom

For example, consider a set of known functions $\varphi_k(t)_{k=1,...,K}$ and signals of the form

$$x(t) = \sum_{n \in Z} \sum_{k=1}^{K} c_{nk} \varphi_k (\frac{t - t_{nk}}{T}) \tag{1}$$

Since the functions $\varphi_k(t)$ are known, it is obvious that the only degrees of freedom in the signal $x(t)$ are time instants $t_{nk}$ and coefficients $c_{nk}$. If we introduce a function $C_x(t_1, t_2)$ which counts the number of degrees of freedom of $x(t)$ over an interval $[t_1, t_2]$, the *rate of innovaion* $\rho$ can be defined as

$$\rho = \lim_{\tau \to \infty} \frac{1}{\tau} C_x(-\frac{\tau}{2}, \frac{\tau}{2}) \tag{2}$$

A signal of a finite rate of innovation is a signal having parametric representation given by (1) and with a finite $\rho$ as defined in (2).

Although it seems intuitive that any signal of a finite rate of innovation can be represented with a finite number of samples, it is often difficult to propose sampling schemes that would allow for practical reconstruction algorithms. However, for some classes of such signals it is possible to come up with exact sampling theorems that lead to standard computational procedures.

In order to illustrate how the finite rate of innovation property can be exploited for solving some sampling problems, consider the case of a periodic stream of $M$ Diracs:

Let $x(t)$ be a periodic signal of period $\tau = 2\pi/\omega_0$,

$$x(t) = \sum_{q} \sum_{k=0}^{M-1} c_k \delta(t - q\tau - t_k) \tag{3}$$

Denote by $\varphi(t)$ a sinc sampling kernel of bandwidth $[-M\omega_0, M\omega_0]$, and choose a sampling period $T$ such that $N_s = \tau/T \geq 2M + 1$, where $N_s \in N$. Then the samples

$$x_s[p] = < x(t), \varphi(t - pT) >, \;\; p \in [0, N_s - 1]$$

are a sufficient representation of $x(t)$.

Namely, the key is to observe that the Fourier series coefficients of the signal $x(t)$ are given by

$$X[m] = \frac{1}{\tau} \sum_{k=0}^{M-1} c_k e^{-jm\omega_0 t_k} \tag{4}$$

that is, a linear combination of $M$ complex exponentials. Therefore, the parameters $\{c_k, t_k\}_{k=0}^{M-1}$ can be found from the set of $2M$ contiguous coefficients $X[m]$, using either subspace methods for harmonic retrieval [4] [6] or the annihilating filter method [18]. When noise is present, which is the case of our main interest, subspace techniques generally perform better. In order to illustrate a subspace-based solution to this problem, we present an outline of a sampling algorithm that uses the State Space method [4] to estimate the parameters $\{c_k, t_k\}_{k=0}^{M-1}$ from the set of coefficients $X[m]$.

*Sampling Algorithm for a Stream of M Pulses*

- Find the Fourier series coefficients $X[m]$ $m \in [-M, M]$, from the set of samples

$$x_s[p] = < x(t), \varphi(t - pT) >, \quad p \in [0, \tau/T - 1] \tag{5}$$

- Define a $P \times Q$ matix $J$ as

$$J = \begin{pmatrix} X[0] & X[1] & \dots & X[L] \\ X[1] & X[2] & \dots & X[L+1] \\ \vdots & & & \\ X[P] & X[P+1] & \dots & X[P+L+1] \end{pmatrix} \tag{6}$$

where both $L$ and $P$ are greater or equal to $M$.

- Compute the singular value decomposition of $J$

$$J = USV^H + U^{'}S^{'}V^{'H} \tag{7}$$

where the first term consists of the M principal components, and the second term consists of the remaining nonprincipals[3].

- Estimate the signal poles $z_i = e^{-j\omega_0 t_i}$ as the eigenvalues of a matrix $Z$

$$Z = \underline{V}^{+} \cdot \overline{V} \tag{8}$$

where $\overline{(*)}$ and $\underline{(*)}$ denote the operations of omitting the first and the last row of $(*)$ respectively, while $(*)^{+}$ denotes the pseudoinverse of $(*)$.

[3]In the noiseless case we have only the first term

- Find the weighting coefficients $c_i$ from a Vandermonde system

$$\begin{pmatrix} X[0] \\ X[1] \\ \vdots \\ X[M-1] \end{pmatrix} = \begin{pmatrix} 1 & \dots & 1 \\ e^{-j\omega_0 t_0} & \dots & e^{-j\omega_0 t_{M-1}} \\ \vdots & \ddots & \vdots \\ e^{-j(M-1)\omega_0 t_0} & \dots & e^{-j(M-1)\omega_0 t_{M-1}} \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{M-1} \end{pmatrix} \tag{9}$$

## III. Application to CDMA Systems

In this section we consider one possible application these sampling results to a combined problem of multipath channel estimation and signal detection in CDMA systems. We first analyze a problem of channel estimation in multipath fading channels and develop a method that estimates parameters of all users from a set of samples of a received signal taken below the chip rate after smoothing.

Once the channel parameters of all users have been estimated, they are directly used in detection algorithms. We present several detection methods that run on a reduced set of samples as well, and discuss a tradeoff between the sampling rate and the detection performance. We also show how existing multiuser detection schemes, such as a decorrelating detector and an MMSE linear detector, can be implemented on a sampled lowpass version of a received signal, and that without degrading their good performances.

### *A. Channel Estimation*

Channel estimation is crucial in all spread spectrum systems, but at the same time is the most difficult part in the system design. We present a new channel estimation method where channel parameters of all users are estimated simultaneously by considering a series of 1-D or 2-D estimation problems[4], yet the main advantage of our approach is that the estimation of the channel parameters is obtained from a significantly reduced set of samples of a received signal compared to existing schemes. Specifically, our method solves the complex multiuser estimation problem by considering only a low-dimensional subspace of a received signal, obtained by sampling a lowpass approximation of a received signal at a rate determined by its rate of innovation. Besides, our algorithm can be equally

[4]In the case of joint angle and delay estimation, the problem is reduced to a series of 2-D equivalents

well applied to both non-fading and multipath fading channels. In the following, we will first focus on a simpler problem of estimating users' delays and propagation coefficients in multipath fading channels. Later, we will extend our results to the case of joint angle and delay estimation.

### A.1 System Model

Consider the general case of a CDMA system with $K$ users operating over a multipath fading channel with at most $L$ propagation paths for each user. We assume that the channel varies slowly, i.e. it is considered constant over a channel estimation window. A received baseband signal $y_r(t)$ can be therefore represented as a sum of multiple copies of attenuated and delayed signals of $K$ users and noise

$$y_r(t) = \sum_n \sum_{k=1}^{K} b_{k,n} \sum_{l=1}^{L} a_k^{(l)} s_k(t - \tau_k^{(l)} - mT_s) + \eta(t) \tag{10}$$

where $s_k(t)$ is a signature sequence assigned to user $k$, $\tau_k^{(l)}$ denotes a delay of user $k$'s signal along the $l$th path[5], $a_k^{(l)}$ is a complex amplitude that includes a channel attenuation and a phase offset along the $l$th path, $b_{k,n}$ denotes the $n$th bit sent by user $k$, $\eta(t)$ is the additive white Gaussian noise, and $T_s$ denotes a symbol duration.

Assume that $y_r(t)$ is sampled with the sinc kernel of bandwidth $[-M\omega_0, M\omega_0]$[6], where $\omega_0 = 2\pi/T_s$. If the sampling period $T_{smp}$ is chosen such that $T_s/T_{smp} \geq 2M+1$, then from the set of samples we can find the Fourier series coefficients $Y_{r,n}[m]$, $m \in [-M, M]$ [18]

$$Y_{r,n}[m] = \sum_{k=1}^{K} b_{k,n} \sum_{l=1}^{L} a_k^{(l)} S_k[m] e^{-jm\omega_0 \tau_k^{(l)}} + N_n[m], \quad \omega_0 = 2\pi/T_s \tag{11}$$

where $S_k[m]$ and $N_n[m]$ denote the Fourier series coefficients of signature sequences and noise respectively. From (11) it is clear that the relative delays $\tau_k$ appear in phase delays of the Fourier series coefficients, while the complex amplitudes $a_k^{(l)}$ appear as weighting coefficients. We can write (11) in a more compact form as

$$Y_{r,n}[m] = \sum_{k=1}^{K} b_{k,n} c_{mk} + N_n[m] \tag{12}$$

[5]With respect to a reference at the receiver

[6]We will show later that the required bandwidth is related to the number of different propagation paths $L$

where $c_{mk}$ are given by

$$c_{mk} = S_k[m] \sum_{l=1}^{L} (a_k^{(l)} e^{-jm\omega_0 \tau_k^{(l)}}) \tag{13}$$

or written in a matrix form

$$\begin{pmatrix} c_{11} & c_{12} & \dots & c_{1K} \\ c_{21} & c_{22} & \dots & c_{2K} \\ \vdots & & & \\ c_{M1} & c_{M2} & \dots & c_{MK} \end{pmatrix} \cdot \begin{pmatrix} b_{1,n} \\ b_{2,n} \\ \vdots \\ b_{K,n} \end{pmatrix} + \mathbf{N}_n = \begin{pmatrix} Y_{r,n}[1] \\ Y_{r,n}[2] \\ \vdots \\ Y_{r,n}[M] \end{pmatrix} \equiv \mathbf{C} \cdot \mathbf{b}_n + \mathbf{N}_n = \mathbf{Y}_{r,n} \tag{14}$$

Since the matrix $\mathbf{C}$ is of a size $M \times K$, we need at least $K$ such equations to estimate $\mathbf{C}$ (given $\mathbf{b}_n$ and $\mathbf{Y}_{r,n}$). That is, each user $k$ has to send a training sequence $\mathbf{b}_{tk}$

$$\mathbf{b}_{tk} = [b_{tk,1}\ b_{tk,2} \dots\ b_{tk,K}] \tag{15}$$

In the absence of noise, system (14) can be written as

$$\begin{pmatrix} c_{11} & c_{12} & \dots & c_{1K} \\ c_{21} & c_{22} & \dots & c_{2K} \\ \vdots & & & \\ c_{M1} & c_{M2} & \dots & c_{MK} \end{pmatrix} \cdot \begin{pmatrix} b_{t1,1} & b_{t1,2} & \dots b_{t1,K} \\ b_{t2,1} & b_{t2,2} & \dots b_{t2,K} \\ \vdots & & \\ b_{tK,1} & b_{tK,2} & \dots b_{tK,K} \end{pmatrix} = \begin{pmatrix} Y_{r,1}[1] & \dots Y_{r,K}[1] \\ Y_{r,1}[2] & \dots Y_{r,K}[2] \\ \vdots & \\ Y_{r,1}[M] & \dots Y_{r,K}[M] \end{pmatrix} \tag{16}$$

i.e.

$$\mathbf{C} \cdot \mathbf{B}_t = \mathbf{Y}_r \tag{17}$$

The matrix $C$ is then given by

$$\mathbf{C} = \mathbf{Y_r} \cdot \mathbf{B_t^{-1}} \tag{18}$$

Therefore, in the noiseless case, from (18) it is possible to find $C$ exactly, provided $\mathbf{B}_t$ is a full rank matrix. On the other hand, $\mathbf{B}_t$ is the matrix made up of users training sequences, and as long as these sequences are linearly independent, $\mathbf{B}_t$ will be a full rank matrix.

Clearly, the unknown parameters of user $k$ appear only in the $k$th column of the matrix $\mathbf{C}$. However, in order to find those parameters from the estimated values $c_{mk}$, we will first define a new matrix $\mathbf{D}$ as

$$\mathbf{D} = \begin{pmatrix} c_{11}/S_1[1] & c_{12}/S_2[1] & \dots & c_{1K}/S_K[1] \\ c_{21}/S_1[2] & c_{22}/S_2[2] & \dots & c_{2K}/S_K[2] \\ \vdots & & & \\ c_{M1}/S_1[M] & c_{M2}/S_2[M] & \dots & c_{MK}/S_K[M] \end{pmatrix} \tag{19}$$

that is, the elements of $D$ are given by

$$d_{mk} = c_{mk}/S_k[m] = \sum_{l=1}^{L} (a_k^{(l)} e^{-jm\omega_0 \tau_k^{(l)}}) \tag{20}$$

Since $S_k[m]$ are assumed to be known coefficients[7], once the matrix $\mathbf{C}$ has been estimated we can compute all the elements $d_{mk}$. On the other hand, each column $k$ of $\mathbf{D}$ defines a new matrix equation

$$\begin{pmatrix} e^{-j\omega_0\tau_k^{(1)}} & e^{-j\omega_0\tau_k^{(2)}} & \dots & e^{-j\omega_0\tau_k^{(L)}} \\ e^{-j2\omega_0\tau_k^{(1)}} & e^{-j2\omega_0\tau_k^{(2)}} & \dots & e^{-j2\omega_0\tau_k^{(L)}} \\ \vdots & & & \\ e^{-jM\omega_0\tau_k^{(1)}} & e^{-jM\omega_0\tau_k^{(2)}} & \dots & e^{-jM\omega_0\tau_k^{(L)}} \end{pmatrix} \begin{pmatrix} a_k^{(1)} \\ a_k^{(2)} \\ \vdots \\ a_k^{(L)} \end{pmatrix} = \begin{pmatrix} d_{1k} \\ d_{2k} \\ \vdots \\ d_{Mk} \end{pmatrix} \tag{21}$$

If $M$ is chosen such that $M \geq 2L$, then the $k$th column of $\mathbf{D}$ provides a sufficient information to solve uniquely for the parameters $\tau_k^{(l)}$ and $a_k^{(l)}$ of user $k$.

From the above analysis it becomes clear that we have decomposed the problem of multiuser channel estimation into a series of single user estimation problems that can be efficiently solved using 1-D subspace methods for harmonic retrieval [6] [13]. However, unlike other subspace-based channel estimation techniques [1] [14], we estimate the unknown parameters from a lowpass version of the received signal, which obviously leads to a considerably reduced computational complexity of the estimator.

The number of samples $M$ we need to take per symbol depends only on the number of multipaths $L$, while the number of symbols in each training sequence depends on the number of active users. In the noiseless case, our method leads to a perfect estimation of the channel parameters $a_k^{(l)}$ and $\tau_k^{(l)}$ by taking only $M = 2L$ samples per symbol. As already mentioned, existing algorithms typically require sampling at the chip rate.

At this point, it is important to relate the minimum sampling rate more explicitly to the rate of innovation of the received signal. During the training phase, unknown parameters

[7]We assume that the receiver has the information on signature sequences of all users

are the delays $\tau_k^{(l)}$ and the propagation coefficients $a_k^{(l)}$. For a system with $K$ users, the total number of unknown parameters is $2KL$. Since the channel parameters are constant over the entire ($KT_s$ long) training phase, the rate of innovation $\rho$ of the received signal is thus $\rho = 2L/T_s$. According to our above analysis, the required sampling rate is $f_s \geq 2L/T_s$, that is, we have demonstrated that the sampling rate must be greater or equal to the rate of innovation $\rho$.

In the noisy case, however, in order to obtain more accurate estimates of the channel parameters, the sampling rate must be increased above the critical rate $\rho$, while the length of the training sequence can remain the same. A choice of the sampling rate clearly depends on a signal-to-noise ratio (SNR), yet we will show that in most cases encountered in practice a required sampling rate is still far below the chip rate. A block scheme of the proposed channel estimator is sketched in Figure 1.

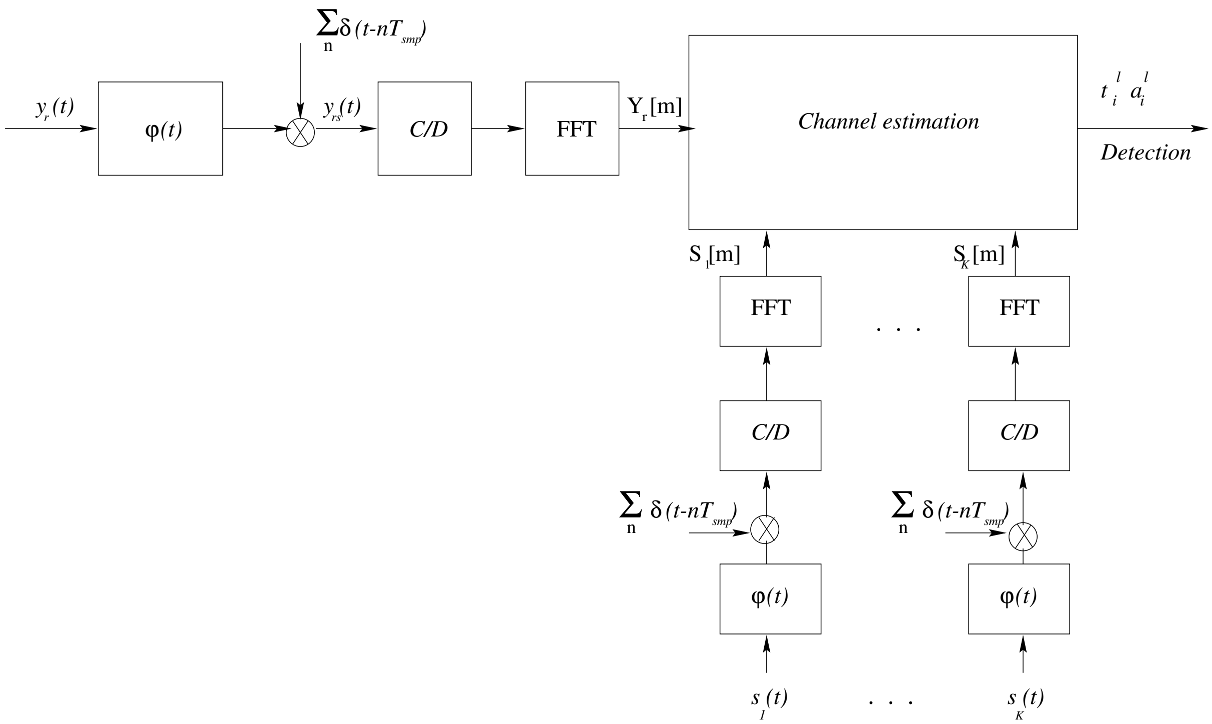


Fig. 1. *Channel Estimator*

Note that the presented algorithm is rather general and can be applied to asynchronous systems as well, where the receiver has an arbitrary timing reference that is not aligned to transmitted bit boundaries. In the general case, however, the sample values will in fact incorporate the information about two consecutive bits in the training sequence. As a result, the estimated Fourier series coefficients may not correspond to their actual values, specifically in the case when adjacent bits have different signs. It is thus desirable to repeat every beat in the training sequence twice, provided that a maximum delay spread of each user is within the symbol duration.

### A.2 Joint angle and delay estimation

Recently, there has been an increasing interest in the use of 2-D RAKE receivers for wireless CDMA systems, which simultaneously exploit space and time domain structure of received multipath signals. Although theoretically sound, implementation of 2-D RAKE receivers in not trivial in practice. Most of the existing solutions resort to complicated spatial and temporal processing involving large correlation matrices, which makes them inefficient or sometimes even inapplicable, particularly in fast varying environments. In particular, estimation of direction-of-arrival (DOA) and relative time delays of multipath signals, required by many of the existing schemes, still presents a bottleneck in the system design due to high computational complexity of such schemes.

In the following, we will further extend our sampling results to a problem of joint delay and angle estimation in multipath fading CDMA channels using antenna arrays. We will show that by applying the developed channel estimation algorithm at each antenna, it is possible to estimate all the channel parameters without modifying the receiver structure. However, in order to estimate direction-of-arrivals of users' signals, we exploit spatial diversity by combining outputs of different antennas.

Consider a uniform linear antenna array system consisting of omnidirectional elements with equal interelement spacing D. We will assume that a carrier frequency if relatively high compared to a bandwidth of the transmitted signal and that a channel is slowly varying. A direction-of-arrival $\theta_k^{(l)}$ of user $k$'s signal propagating along the $l$th path, is assumed to be the same for all antennas in the array. However, there will be a fixed phase difference between signals received at each two consecutive elements of the array, given by $e^{j\omega_c \frac{D sin\theta_k^{(l)}}{c}} = e^{j\phi_k^{(l)}}$, where $\omega_c$ denotes an angular frequency of the carrier. Therefore, there will be a phase difference of $e^{j\phi_k^{(l)}}$ between corresponding Fourier series coefficients of signals received at each two consecutive sensors.

Consider next a set of matrices $D$, given by (19) and (20), estimated separately at each antenna, and denote them as $D_1,\ D_2,\dots\ D_S$. The elements $d^s_{mk}$ of matrices $D_s$, $s = 1, 2, \dots S$, are thus given by

$$d^s_{mk} = \sum_{l=1}^{L} a_k^{(l)} e^{-jm\omega_0 \tau_k^{(l)}} A_g e^{-j(s-1)\phi_k^{(l)}} \tag{22}$$

that is, a linear combination of two-dimensional complex exponentials, where $A_g$ denotes an antenna gain which is the same for all antennas. Note that the previous equation can be also written in the form

$$d^s_{mk} = \sum_{l=1}^{L} A_k^{(l)} e^{-jm\omega_0 \tau_k^{(l)}} e^{-js\phi_k^{(l)}} \tag{23}$$

where $A_k^{(l)} = a_k^{(l)} A_g e^{-j\phi_k^{(l)}}$

In order to estimate the unknown parameters $\tau_k^{(l)}$ and $\theta_k^{(l)}$ of user $k$, consider a matrix $F_k$ obtained by stacking the $k$th columns of matrices $D_s$, $s = 1, 2, \dots S$,

$$F_k = [D_1(:,k)\ D_2(:,k)\ \dots\ D_S(:,k)]_{M\times S} \tag{24}$$

or more explicitly,

$$F_k = \begin{pmatrix} d^1_{1k} & d^2_{1k} & \dots & d^S_{1k} \\ d^1_{2k} & d^2_{2k} & \dots & d^S_{2k} \\ \vdots & & & \\ d^1_{Mk} & d^2_{Mk} & \dots & d^S_{Mk} \end{pmatrix} \tag{25}$$

Therefore, the elements $f^k_{ms}$ of the matrix $F_k$ are given by

$$f^k_{ms} = d^s_{mk} = \sum_{l=1}^{L} A_k^{(l)} e^{-jm\omega_0 \tau_k^{(l)}} e^{-js\phi_k^{(l)}} \tag{26}$$

In the case when the time delays $\tau_k^{(l)}$ of each user $k$ are different, we need only two columns of the matrix $F_k$ in order to solve for all the unknown parameters of user $k$. In other words, it suffices to have only two antennas in the array system. By considering only the first column we can estimate the parameters $\tau_k^{(l)}$ and $A_k^{(l)}$, while from the second column of $F_k$, we can solve for $A_k^{(l)} e^{-j\phi_k^{(l)}}$, and thus uniquely specify $\theta_k^{(l)}$. This result is also in agreement with our previous analysis on the relation between the minimum required sampling rate and a rate of innovation of the received signal. Namely, since the unknown channel parameters are $\tau_k^{(l)}$, $\theta_k^{(l)}$ and $a_k^{(l)}$, the total number of parameters is $3KL$, which are constant over the entire training phase. If we choose the sampling rate to be $f_s = 2L/T_b$, we require at least two antennas in the system[8].

In the noisy case, however, this method is not desirable. While with 1-D subspace methods we can estimate delays $\tau_k^{(l)}$ quite accurately, estimation of the weighting coefficients $A_k^{(l)}$ and $A_k^{(l)} e^{-j\phi_k^{(l)}}$ is much more sensitive to noise, which in turn results in imprecise estimation of the angles $\theta_k^{(l)}$ as well. One possible way to overcome this problem is to use 2-D subspace algorithms for harmonic retrieval (such as ACMP, 2-D ESPRIT etc.) since the elements of the matrix $F_k$ have the form of a linear combination of 2-D complex exponentials. Therefore, similar to the approach discussed in the previous section, we can decompose a multiuser parameter estimation problem into a series of 2-D single user problems where the parameters of each user are estimated separately. It is important to note, however, that in order to apply 2-D subspace algorithms successfully, the number of antennas $S$ in the array cannot be chosen arbitrarily. The minimum number of sensors $S_{min}$ required for a unique solution[9] is $S_{min} = 2L$, that is, it depends only on the number of multipaths and not on the number of users. In the noisy case, as the number of antennas increases the angles can be estimated more precisely. On the other hand, as the number of samples per symbol $M$ increases, estimates of time delays become more accurate, as we will show in the sequel.

### *B. Signal Detection*

The developed channel estimation algorithm is rather general and allows for implementation of different detection strategies, such as multiuser detection schemes, RAKE receivers or 2-D RAKE. In the following, we will focus on multiuser detection schemes and show how they can be efficiently implemented in digital receivers where the sampling rate is determined by a rate of innovation of a received signal.

Multiuser detection has been studied extensively over the past fifteen years, due to

[8]Note that the sampling rate could be lower, but in that case we wouldn't have a practical algorithmic solution

[9]i.e. required by most 2-D spectral estimation methods

the fact that it outperforms the conventional receiver in the near-far scenario. Optimum multiuser detector, developed by Verdu [17], was the first such scheme that exploited a structure of multiuser interference in order to improve the receiver performances. His work was followed by many suboptimal detection schemes of lower computational complexity, such as decorrelating detectors or maximum-likelihood (ML) detectors. In order to take a full advantage of multiuser detection in digital receivers, it is typically assumed that a received signal must be sampled at the chip rate. We will show, however, that with some of these multiuser schemes it is possible to carry out detection on the lowpass version of the received signal, while retaining their good performances.

B.1 Decorrelating Detector

Consider the case of a synchronous channel with only one propagation path for each user. A received baseband signal can then be modeled as a superposition of $K$ active users' signals and noise

$$y_r(t) = \sum_{k=1}^{K} a_k b_k s_k(t) + \eta(t) \tag{27}$$

where $\eta(t)$ is a zero mean white Gaussian noise of variance $\sigma^2$. After passing $y_r(t)$ through a bank of $K$ matched filters, the output vector becomes

$$\mathbf{y} = \mathbf{R}\mathbf{A}\mathbf{b} + \eta \tag{28}$$

where

$$\mathbf{A} = diag(a_1\ a_2, ..., a_K)$$

$$\mathbf{b} = (b_1\ b_2, ..., b_K)$$

while $\mathbf{R}$ is the normalized cross-correlation matrix whose elements are given by

$$\rho_{kj} = \int_0^{T_s} s_k^*(t) s_j(t) dt \tag{29}$$

If the output of the matched filter is multiplied by $\mathbf{R^{-1}}$, then in the absence of noise the following relation holds

$$\mathbf{R}^{-1}\mathbf{y} = \mathbf{A}\mathbf{b} \tag{30}$$

Therefore, in order to recover transmitted bits, we can simply take the sign of each components in (30), i.e.

$$\hat{b_k} = \mathrm{sgn}(\mathbf{R^{-1}y})_k = \mathrm{sgn}(\mathbf{Ab})_k = b_k \tag{31}$$

If one wants to implement the decorrelating detection scheme in digital receivers, a common approach is to sample the received signal $y_r(t)$ at the chip rate, pass the sampled signal through a bank of chip-matched filters and multiply the output of such a filter bank by a digital version of the cross-correlation matrix (29), whose elements are a discrete correlation coefficients between chip-sampled signature sequences. However, if we want to avoid the chip rate sampling and yet obtain the same or at least comparable performances, one possible approach would be to carry out all the above steps on signals sampled below the chip rate with a sinc sampling kernel. As we will show later, in order to obtain performances essentially equivalent to those of chip-rate sampling schemes, a required sampling rate is typically close to the rate of innovation.

Note that the sampling kernel doesn't necessarily have to be the sinc kernel, and we can use a Gaussian sampling kernel as well. Namely, in order to simplify our analysis we have ignored a potential problem of intersymbol interference due to a tail of the sinc function. By using the Gaussian kernel, which has an exponential decay, this problem can be avoided.

### B.2 Minimum Mean-Square Error Linear Detector

Minimum Mean Square Error (MMSE) linear detector is aimed at improving the performances of the decorrelating detector by incorporating information about a received signal-to-noise ratio. In particular, the MMSE linear detector replaces the transformation $\mathbf{R}^{-1}$ of the decorrelating detector by

$$[\mathbf{R} + \sigma^2\mathbf{A}^{-2}]^{-1}$$

where

$$\sigma^2\mathbf{A}^{-2} = diag(\frac{\sigma^2}{a_1^2}, ..., \frac{\sigma^2}{a_K^2})$$

Similar to the previous case, we will run the MMSE detection algorithm on samples of signals taken after smoothing. As we will see later, the MMSE detector has better performances compared to the decorrelating detector for low sampling rates.

### B.3 "Direct" Detection

(*or a better term for this...*)

It is interesting to note that in the case of quasi-synchronous CDMA systems, where a maximum delay spread of each user is relatively small compared to the symbol duration $T_s$, it is possible to use equation (14) directly to solve for bits sent by each user, without even knowing specific users' codes. In this case the receiver only needs to estimate the matrix $\mathbf{C}$ in the training phase. In the detection phase, bits sent by users $1, 2, \ldots K$ are then given by

$$\mathbf{b}_n = \mathbf{C}^{-1} \cdot \mathbf{Y}_{r,n} \tag{32}$$

It is clear from our analysis that the receiver doesn't require a knowledge of user's signature sequences, nor it has to estimate propagating coefficient $a_i^{(l)}$. However, (32) can be successfully applied to systems where approximate timing of users is a priori known to the receiver.

## IV. Numerical Results

We present some simulation examples that demonstrate the performance of the developed algorithms. In all our simulations we used pseudo-random sequences of length 1023. Figure 2(a) shows an average estimation error of a relative time delay (normalized to a chip duration $T_c$) for the first user in a multiuser scenario versus the number of samples $N_s$ taken per symbol, and that for different values of a signal-to-noise ratio (SNR). We assumed a non-fading channel and considered only the effect of a sampling rate on the estimation error of user's timing[10]. Obviously, as the sampling rate increases we can estimate the time delays more accurately, however, since the error exhibits an exponential decay it is unnecessary to resort to very high sampling rates. For example, by taking only 100 samples the average estimation error is less than one tenth of the chip duration (for $SNR \geq 8dB$), and the performance of the estimator doesn't considerably improve by further increasing the sampling rate. Figure 2(b) illustrates average estimation errors for systems with 5 and 10 users versus SNR, for different values of $N_s$. The error depends both on a sampling rate and a value of SNR, however, it doesn't depend on the number of active

[10] with respect to the reference in the receiver

users in the system. This is a somewhat expected result, since our method for channel estimation decomposes the multiuser estimation problem into a series of 1-D problems, where the unknown parameters of only one user need to be estimated. A similar result holds for multipath fading channels as well. This is illustrated in Figure 2(c), where we considered systems with 5 and 10 users and 3 propagation paths for each user. However, for very low sampling rates (e.g. less than 15 samples per symbol) the estimation error is slightly higher in the case of a fading channel, since in this case there are more parameters that need to be estimated simultaneously. In Figure 2(d) we illustrate the near-far performance of our estimator for a system with 5 users. In particular, we show average estimation errors for the first user versus SNR, and that for different values of $r = P_i/P_1$, $i = 2, ..., 5$, where $P_i$ denotes a received power of user $i$. Clearly, the error doesn't depend on $r$ since the parameters of different users are estimated separately.

We next consider the performance of our scheme in the case of a joint angle and delay estimation. Figures 3(a) and 3(b) show an average angle estimation error (normalized to $2\pi$) and a delay estimation error (normalized to the chip duration $T_c$) for user 1 in a system with 10 users. In order to estimate time delays and angles of all the signals, we used a 2-D subspace based algorithm for harmonic retrieval, the so-called ACMP (Algebraic Coupling of Matrix Pencils) algorithm. The error is plotted as a function of the number of antennas $S$ in the array, for two different values of the signal-to-noise ratio (SNR=8dB and SNR=10dB) as well as for two different sampling rates ($N_s = 40$ and $N_s = 60$), and that for both non-fading channels (Figure 3(a), (b)) as well as for fading channels with 3 propagating paths for each user (Figure 3(c),(d)). Similar to the previous case, the estimation error decays exponentially as the number of sensors increases. However, this does not significantly affect a delay estimation error, since by increasing the number of antennas we add in more information only about a structure of a received signal in spatial domain. On the other hand, as a sampling rate increases, we can better estimate timings of different users, while the angle estimation error is much less affected in this case. The same conclusions hold for multipath channels, yet all the estimation errors are somewhat higher compared to the case of non-fading channels.

We next consider several detection schemes where detection is carried out on a sampled lowpass version of a received signal. In particular, we analyze the effect of a sampling rate on the performance of the decorrelating detector, the MMSE detector and the direct detection sheme (32) in synchronous systems, while a similar analysis can be done for other detection schemes as well. Figures 4(a) and 4(b) illustrate the performance of the decorrelating detector, specifically an average bit-error-rate (BER) for systems with 5 and 10 users respectively versus the number of samples, while Figures 4(c) and 4(d) illustrate the performance of the MMSE linear detector. We assumed that timings of all users are perfectly estimated, and that both schemes are implemented on samples taken with the sinc kernel. Obviously, as we increase the sampling rate the performance of both detectors improves. The two schemes have basically the same performance, except for low sampling rates when the MMSE linear detector performs better. Figure 4(e) illustrates the effect of a delay estimation error (i.e. synchronization error) on the performance of a system with 10 users and the MMSE detector, where we assumed that timings of all users are estimated with an error of $0.1T_c$[11]. Our results indicate that the synchronization error somewhat degrades detection performances, however, the BER increases by approximately 2dB compared to the case with perfect synchronization.

However, a more interesting result is illustrated in Figure 5, which shows average bit-error-rates for a synchronous CDMA system with 5 and 10 users, versus the number of samples (Figures 5(a)-(d)), as well as versus SNR (Figures 5(e) and 5(f)). We present the results for the decorrelating and subspace detectors, although a similar analysis can be done for other detection schemes as well. Obviously, in order to achieve the good detection performance it is unnecessary to resort to chip rate sampling, that is, almost the same error rate is obtained when all the detection steps are carried out on the set of signal samples taken far below the chip rate. For example, from Figures 5(a) and 5(c) it is clear that in the case of a system with five users, it suffices to take only 40 samples in order to achieve nearly the same BER as with 1023 samples (which corresponds to the chip-rate sampling). For a system with ten users that number is roughly 80, etc. It is interesting to note that in the case when a sampling rate is very low, the direct detection scheme outperforms decorrelating detector. As the sampling rate increases both schemes have

[11] This is a more realistic assumption than the one of a perfect synchronization

nearly the same performance.

In order to explain an exponential decay of the detection error, consider a bit-error-rate of user $k$, given by

$$P_k(\sigma) = Q(\frac{a_k}{\sigma\sqrt{\mathbf{R}_{kk}^{-1}}}) \tag{33}$$

where $\mathbf{R}$ is a normalized cross-correlation matrix, while $Q$ is the error function given by

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

A detailed discussion on the performances of multiuser detection schemes is already presented in [17], however, note that in our case all the necessary steps[12] are implemented on the samples of a signal lowpass version. Specifically, in (33) the matrix $\mathbf{R}$ is the normalized cross-correlation matrix whose elements are correlation coefficients of sampled signature sequences, etc. Although (33) doesn't provide an explicit dependence of a bit-error-rate on a sampling rate, that dependence is included implicitly in the term $\mathbf{R}_{kk}^{-1}$. For example, if the signal is sampled at the chip rate, the $k$th user is orthogonal to other users[13] and $\mathbf{R}_{kk}^{-1} = 1$. On the other hand, when the sampling rate is very low, the matrix $\mathbf{R}$ is close to singular and the system becomes ill-conditioned, which in turn results in high BER. In practice, when computational complexity is one of the main constraints in the system design, there obviously must be a tradeoff between the complexity of the receiver and its performance.

## V. Conclusion

We have presented a new approach to the problem of multiuser channel estimation and signal detection in CDMA systems, where all the necessary steps are carried out on a sampled lowpass version of a received signal. We extended some of the recent sampling results for signals of a finite rate of innovation to the problem of multiuser channel estimation and developed a method that estimates the unknown channel parameters of all users simultaneously, by sampling a received signal at a rate determined by its rate of innovation, that is, below the chip rate. Once the channel parameters of each user have

[12] e.g. filtering, decorrelation etc.

[13] Provided that the signature sequences are chosen from a set of orthogonal sequences

been estimated, they are used in the detection process, carried out on a lowpass version of the received signal as well. For simplicity, we have considered only multiuser detection schemes in synchronous and quasi-synchronous systems, however, the presented channel estimation method is rather general and allows the implementation of different schemes such as RAKE receivers, 2-D RAKE etc. Our approach leads to algorithms that have lower computational requirements compared to existing methods, while providing similar performances.

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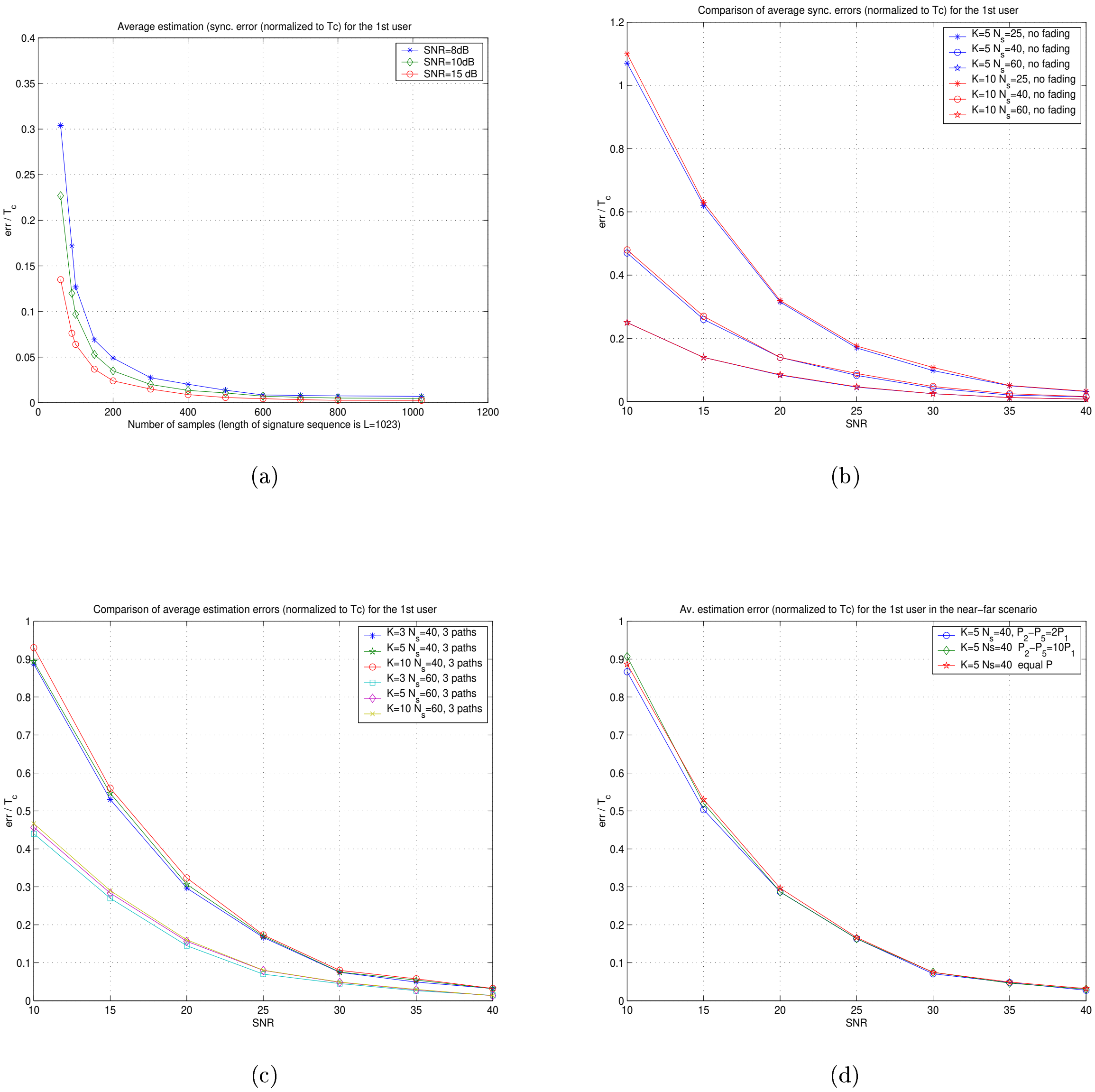



Fig. 2. **(a)** *Average timing synchronization error (normalized to $T_c$) for user 1 in the multiuser case (10 users) vs. number of samples $N_s$, for different values of SNR. We assumed a non-fading channel. Signature sequences of all users are of length 1023.* **(b)** *Comparison of the synchronization errors in non-fading channels for systems with 5 and 10 users* **(c)** *Average synchronization errors for 3, 5 and 10 users in multipath fading channels. The number of samples $N_s$ is chosen to be 40 or 60, and we assumed three propagation paths for each user* **(d)** *Average synchronization errors for a system with 5 users in the near-far scenario.*

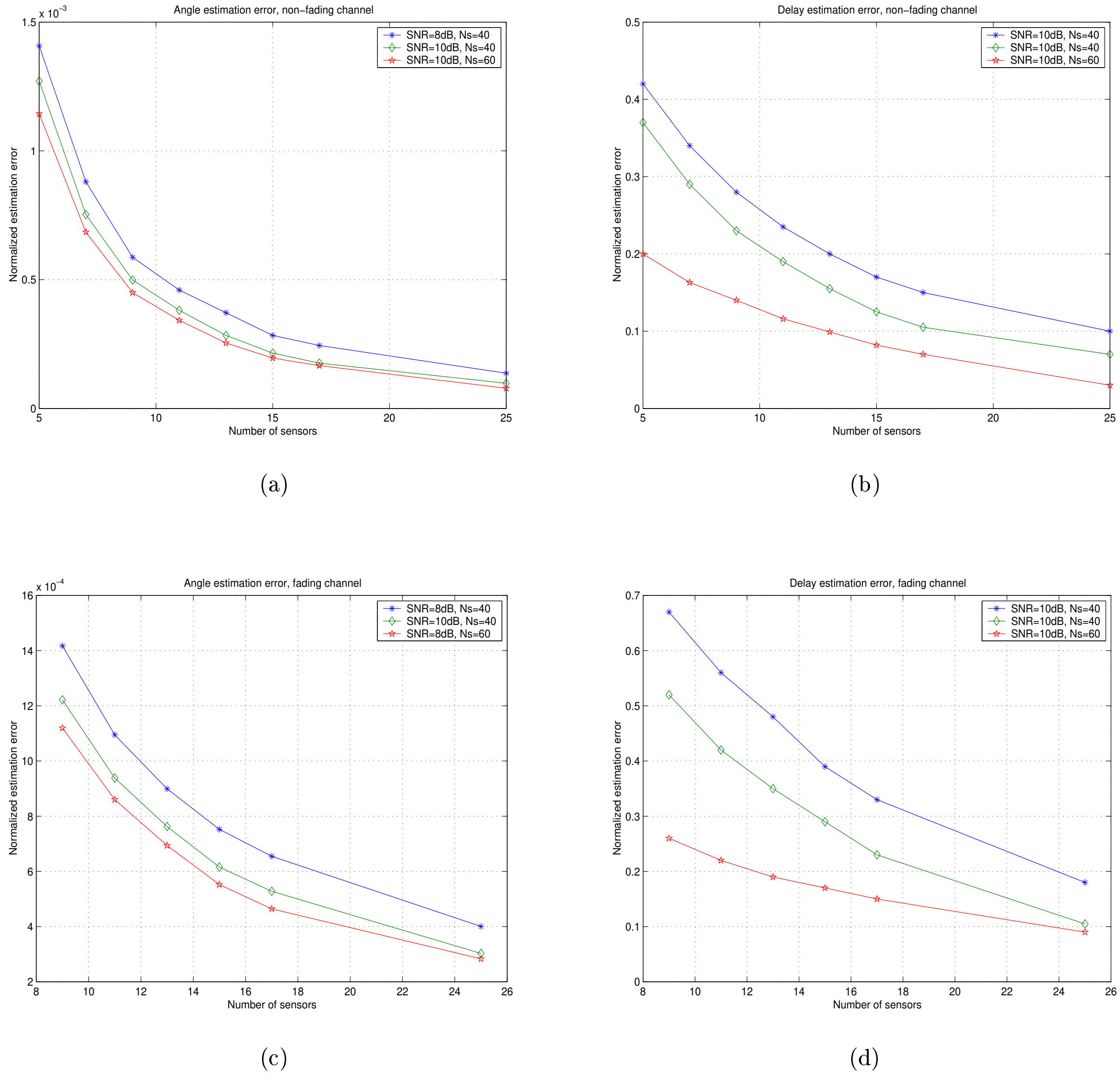
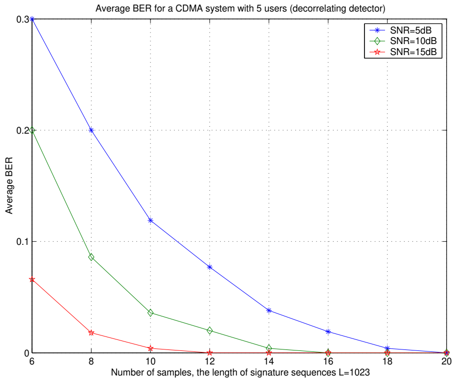
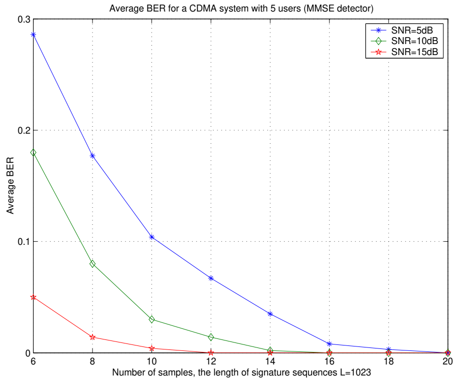



Fig. 3. **(a)** *Normalized angle estimation errors for the first user in the multiuser case (10 users) vs. number of sensors in the antenna array $S$, for different values of SNR and $N_s$. A channel is assumed to be non-fading. Each signature sequence is of length 1023.* **(b)** *Normalized delay estimation errors for user 1 in the multiuser case (10 users) vs. $S$ for non-fading channels.* **(c)** *Normalized angle estimation errors in fading channels. We assumed 3 propagation paths for each user.* **(d)** *Normalized delay estimation errors in fading channels, with 3 propagation paths for each user.*




(a)

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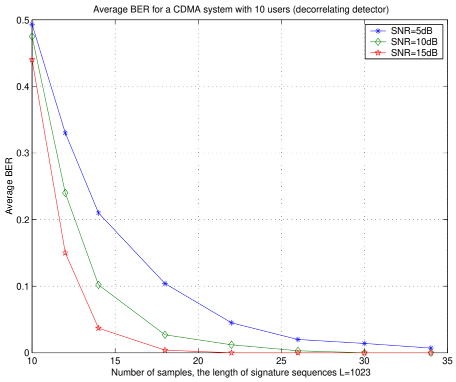


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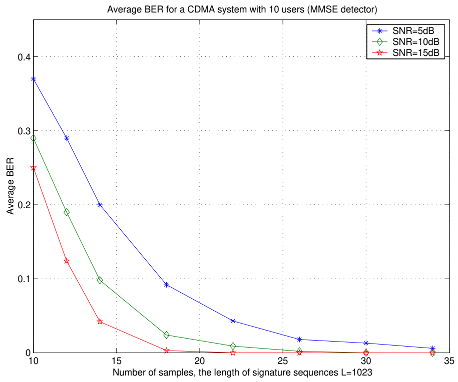


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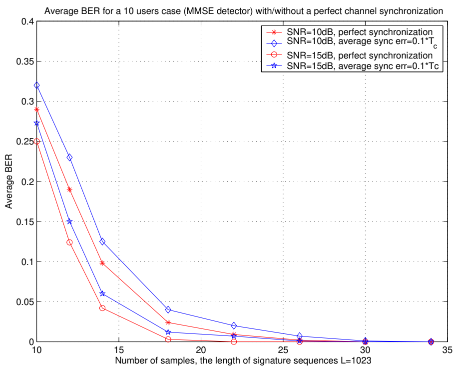


(e)

Fig. 4. *Average BER in the case of non-fading synchronous CDMA systems with 5 and 10 users. The sampling scheme is used along with the decorrelating detector (a),(b) or the MMSE linear detector (c) and (d).* **(a)** *Average BER of the decorrelating detector in a system with 5 users* **(b)** *Average BER of the MMSE detector in a system with 5 users* **(c)** *Average BER of the decorrelating detector and 10 user case* **(d)** *Average BER of the MMSE detector and 10 user case* **(e)** *The effect of timing synchronization error (assumed to be* $0.1 \cdot T_c$ *for all users) on the BER.*

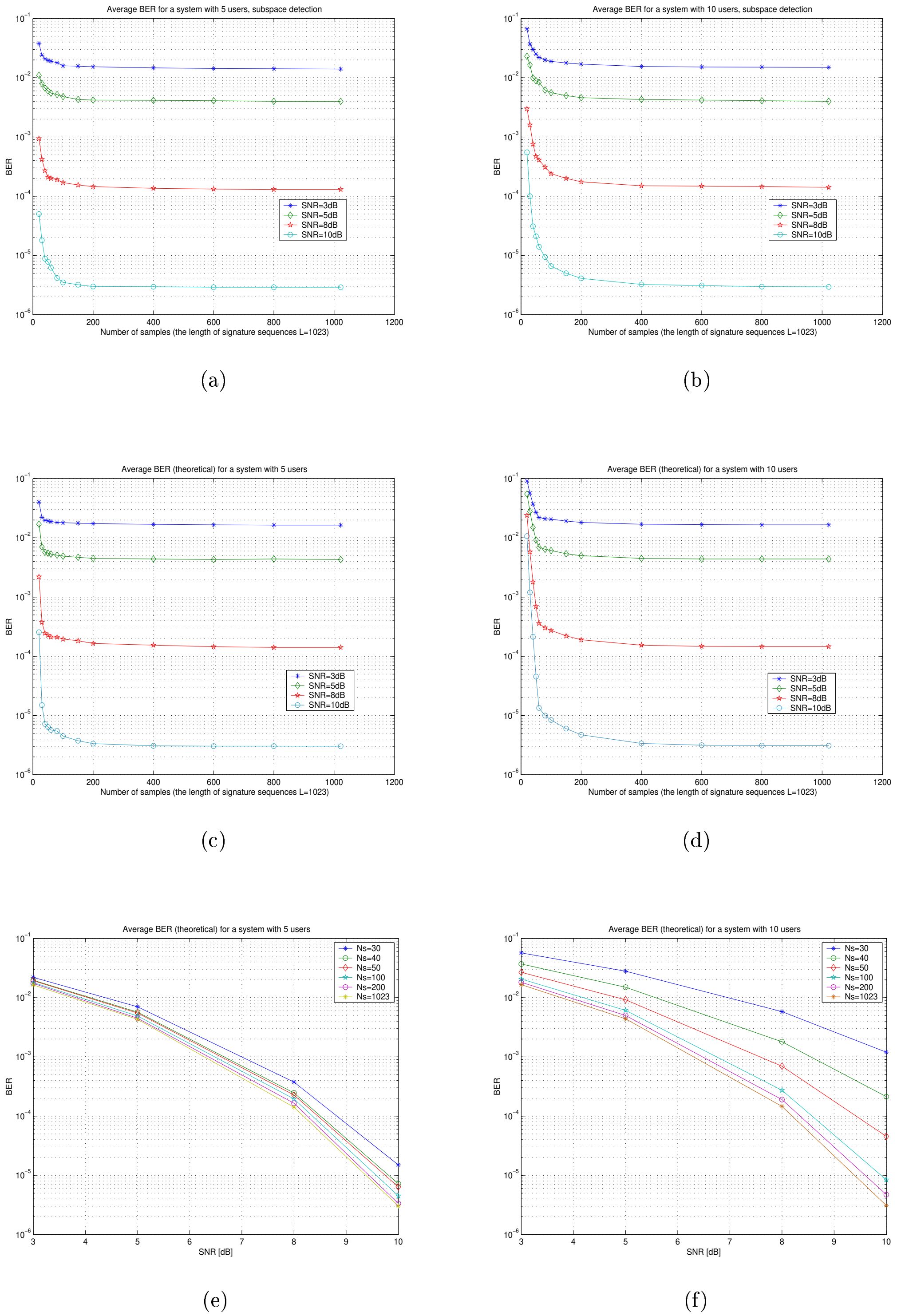


Fig. 5. *Average bit-error-rates for a non-fading synchronous CDMA system with 5 and 10 users. All signature sequences are of length 1023.* **(a), (b)** *BER of subspace detector vs. number of samples $N_s$ for different values of SNR, for systems with 5 and 10 users respectively* **(c), (d)** *BER of decorrealting detector vs. $N_s$ for different values of SNR in systems with 5 and 10 users* **(e), (f)** *BER of decorrelating detector vs. SNR for different sampling rates in systems with 5 and 10 users respectively.*