Current problems for the transmission of information using a chaotic signal

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Abstract

We discuss the importance of reducing the influence of channel disturbances and parameter mismatch on the synchronization of chaotic systems, if information transmission systems are to be used in practice. We explain in this context some new mathematical results that give a better understanding of the synchronization properties of coupled chaotic systems.

1. Introduction

Since the first papers [1-4] on the synchronization of chaotic systems have appeared, many methods have been proposed to achieve synchronization and to apply it to the transmission of information. In this paper we shall discuss some problems that are linked to them. The purpose is not to present any new results, but to put various previously published results into perspective. We have given several overviews before [5-7], but here we intend to shed a slightly different light onto selected issues.

2. Transmission of information using chaos

A system of information transmission can be decomposed according to Fig.1. An information signal s(t) is injected into a chaotic dynamical system, which produces a chaotic output signal y(t). The transmission of y(t) through some medium, called the channel, degrades it, so that some modified, chaos-like signal $\hat{y}(t)$ enters the receiver. The receiver extracts by a suitable procedure the information from $\hat{y}(t)$. This procduces a signal $\hat{s}(t)$ which should be a copy of the original information signal s(t).

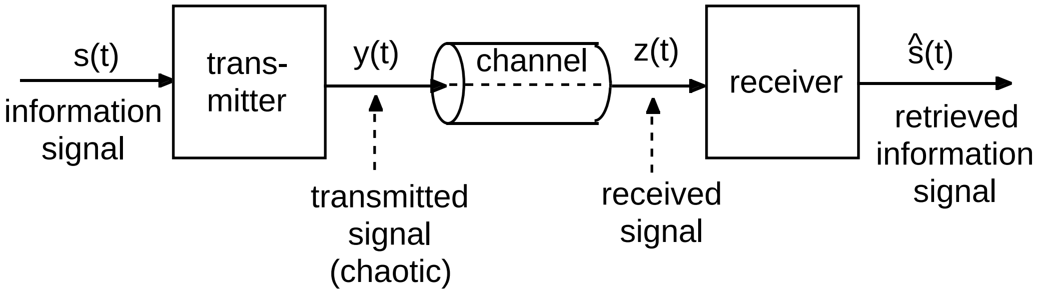

Fig.1. Transmission system

The motivation to use chaos for transmission is two-fold:

- to transmit a wide-band signal, as in conventional spread-spectrum communications, to avoid signal fading and other interferences.
- to hide the information signal to achieve privacy of the communcation.

There are highly developed conventional methods for both of the above goals. However, they are rather complex in nature which implies some limitations concerning cost and/or speed. Maybe it is possible to develop simple systems based on chaotic circuitry that achieve simultaneously both goals to a sufficient degree for lower-end applications.

From a communication engineering point of view, what matters is the end-to-end behavior, i.e. the quality of $\hat{s}(t)$ when compared to s(t), measured against the cost of transmission, and taking into account some other features, like the degree of privacy. The quality of the retrieved signal is measured, for digital transmission, as the bit error rate, i.e. the proportion of wrong information bits among all transmitted information bits. The cost of transmission is measured by the signal energy per information bit, divided by the channel noise intensity. Hence, different communication systems are compared by plotting the bit error rate against this measure of the transmission cost. Communication systems using chaos still have to be optimized with this figure of merit in mind. In fact, to the authors knowledge, it has hardly ever been tried to compute this quantity for chaos communication systems.

An exception is [8], where the results of the numerical simulations indicate that systems based on the synchronization of chaotic systems cannot yet compete with conventional systems, whereas systems based on correlation techniques come close. The latter do not really use the nonlinear dynamics of the chaotic transmitter, but only the statistical properties the chaotic signal. Furthermore, the privacy of the communication is not achieved. For these reasons, we believe that chaos synchronization still has a lot of potential for the transmission of information, but synchronization in the presence of disturbances has to be improved substantially.

There have been some attempts to realize prototypes of mobile chaos communication systems under realistic conditions [9,10]. With respect to these realizations, the system of Fig.1 is a *baseband model* of the communication system. It hides the fact that the frequency band of transmission lies orders of magnitude higher than the frequency band of the produced chaotic signal y(t). Thus, in addition to the chaotic circuit, the transmitter needs circuits for amplification and modulation, and the reciever for amplification and demodulation. It appears that at the current state of development all circuits need to be of rather good quality and the channel not too noisy in order to enable transmission using synchronization.

For some applications it would be temptating to design a chaotic oscillator that produces a signal with a frequency band centered at the desired transmission frequencies and to inject a wideband information signal into this oscillator in a suitable manner. Some of the work on chaotic oscillators is directed to this goal [11,12]. The problem at present is to obtain sufficiently well reproducible chaotic behavior that does not change over time.

Finally, while many applications target mobile communications, it should kept in mind that optical fibers constitute a very important medium for information transmission. Contrary to mobile communications, the disturbances in an optical fiber are very low, and thus, systems based on optical fiber transmission may well constitute promising candidates for high speed transmission on a chaotic carrier signal. In this case, the main advantage could be to realize some privacy at a reasonable cost. Optoelectronic circuits with chaotic behavior have been reported for some time, and recently synchronization of two chaotic optoelectronic circuits has been achieved in the laboratory [13, 14] and information has been transmitted using synchronization [15]. These experimental systems involve conventional electronics in the feedback loop of the chaos generator and therefore are very slow compared to the bandwidth of the optical fibers. However, it is not unrealistic to imagine an all-optical device which would then speed up the chaotic motion by orders of magnitude.

3. Synchronization of chaotic systems

If we exclude systems of information transmission based on chaos that use only statistical properties of the chaotic signal without referring to the nonlinear dyncamical system that has produced the signal, then the main problem for information retrieval using chaos is to achieve synchronization of the receiver with the transmitter under non-ideal conditions. We can consider this problem in the framework of Fig.2.

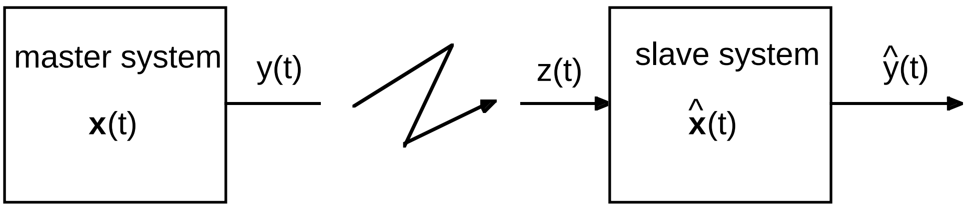

Fig.2. Master-slave system with perturbed interaction, for the study of synchronization

The master and the slave system are similar nonlinear analog or discrete dynamical systems, described by state equations. We give here the form for analog systems. Discrete time systems are described similarly.

$$\begin{array}{llllll} \dfrac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}) & \qquad \dfrac{d\hat{\mathbf{x}}}{dt} = \hat{\mathrm{F}}(\hat{\mathbf{x}}, z) & \qquad \mathbf{F} : \Re^N \to \Re^N, \ \hat{\mathbf{F}} : \Re^{N+1} \to \Re^N \\ y = g(\mathbf{x}) & \qquad \hat{y} = \hat{g}(\hat{\mathbf{x}}, z) & \qquad g : \Re^N \to \Re, \ \hat{g} : \Re^{N+1} \to \Re \end{array} \tag{1}$$

If the channel disturbances were absent, i.e. z = y, and if the master and the slave systems matched ideally, i.e.

$$\hat{\mathbf{F}}\big(\mathbf{x}, g(\mathbf{x})\big) = \mathbf{F}(\mathbf{x}) \qquad \text{and} \qquad \hat{g}\big(\mathbf{x}, g(\mathbf{x})\big) = g(\mathbf{x}) \tag{2}$$

then we should have synchronization in the form

$$\left| \hat{y}(t) - y(t) \right| \xrightarrow[t \to +\infty]{} 0 \tag{3}$$

for any solution of (1), i.e. for arbitrary initial conditions $\mathbf{x}(0)$ and $\hat{\mathbf{x}}(0)$. Depending on the example, this is easy or difficult, if not impossible, to prove. A typical example where the proof is easy is the following coupling of two Lure's systems (Fig.3)

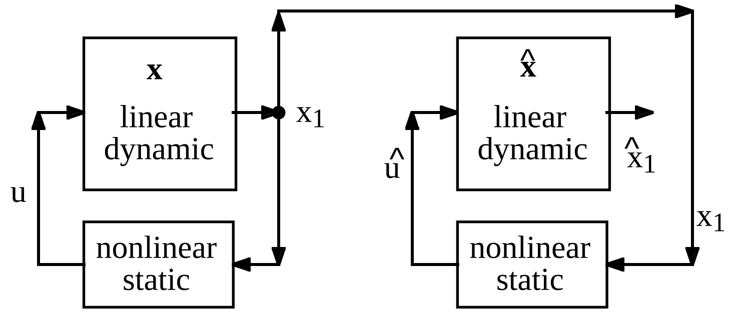

Fig.3 Coupled Lure's systems

They are described by

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{A}\mathbf{x}+\mathbf{b}u & \frac{d\hat{\mathbf{x}}}{dt} &= \mathbf{A}\hat{\mathbf{x}}+\mathbf{b}v \\ u &= f(x_1) & v &= f(x_1) \end{aligned} \tag{4}$$

Clearly, u = v and the synchronization error $\mathbf{x}-\hat{\mathbf{x}}$ satisfies the linear equation

$$\frac{d(\mathbf{x}-\hat{\mathbf{x}})}{dt} = \mathbf{A}(\mathbf{x}-\hat{\mathbf{x}}) \tag{5}$$

Thus, if all eigenvalues of **A** have negative real parts, the slave system synchronizes with the master system.

Unfortunately, for many master-slave systems it is much more difficult, if not impossible, to prove synchronization, even though numerical experiments appear to confirm synchronization. Some recent mathematical papers shed a new light on this fact [16-21]. There are systems, or parameter values of systems, that have a high probability of synchronization when the initial conditions of the master and the slave system are chosen at random. There remains, however, a small probability of non-synchronization. It may be that in numerical experiments this fact is not observed. However, when disturbances are introduced, then such systems often show large desynchronization bursts. In the following, we try to explain this phenomenon.

In papers [16-21] the master-slave system is considered to be a single dynamical system in R^{2N}, which has, due to (2), the invariant N-dimensional linear subspace $S = \left\{ (\mathbf{x},\hat{\mathbf{x}}) \mid \mathbf{x} = \hat{\mathbf{x}} \right\}$, the *synchronization subspace*. The trajectories in S are *synchronized* and the dynamical system restricted to S is equivalent to the master system. The question is now, whether S is a globally attracting set, i.e. whether all trajectories are attracted to S. This is our notion of synchronization (3), and this is the case for system (4) when **A** has all eigenvalues in the open left half plane. One can be less demanding and require only that trajectories that are sufficiently close to S are attracted to S. More precisely, suppose that $\Omega \subset S$ is an attractor in the sense that it contains a dense trajectory and it attracts all trajectories starting in a neighborhood of Ω in S. The question is then whether it is also an attractor for the whole dynamical system in R^{2N}. It amounts to asking whether points close to Ω, **but not in S**, are also attracted to S. From the application point of view, one could be satisfied if almost all points in a neighborhood of Ω were attracted to Ω.

One seemingly obvious way to approach this question is to compute the Lyapunov exponents for the trajectories in Ω, considered as trajectories of the 2N-dimensional dynamical system. Among the 2N Lyapunov exponents, N belong to the dynamical system restricted to S. The remaining N are called *transversal Lyapunov exponents*. If they are all negative, we expect synchronization to take place for almost all trajectories in a neighborhood of Ω. Unfortunately, this is not necessarily the case!

Underlying the computation of the Lyapunov exponents is the assumption that there exists a "natural" invariant ergodic measure on Ω, which assures the existence of the Lyapunov exponents and their independence of the particular trajectories, for almost all trajectories in Ω. Some of the remaining trajectories may well have a positive transversal Lyapunov exponent, but why should we care? The point is that trajectories that approach Ω get close to every point of Ω, and thus also to the points of trajectories that have positive Lyapunov exponents. In particular, there may be periodic points with positive Lyapunov exponents. Ususally these points are, if they exist, even dense in Ω. In a neighborhood of such a point there is an open set of points that move away from S. Hence, in any neighborhood of Ω there is a set of points of positive measure which move away from Ω and thus Ω is not an attractor. However, the measure of this set may be very small, so that synchronization is achieved with a high probability.

Let us illustrate this phenomenon with a simple discrete-time system described by the following recursion [21]

$$\begin{aligned} x(k+1) &= f(x(k)) \\ y(k+1) &= f((y(k)) + \varepsilon(x(k)-y(k)) \end{aligned} \qquad (6)$$

where f is the skew tent map

$$f(x) = \begin{cases} \dfrac{x}{a} & \text{if} \quad 0 \le x \le a \\ \dfrac{1-x}{1-a} & \text{if} \quad a < x \le 1 \end{cases} \qquad (7)$$

represented in Fig.4. For convenience, we suppose $0.5 \le a < 1$.

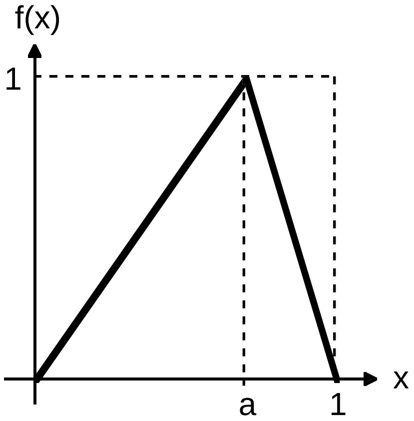

Fig.4. Skew tent map

The first equation of (6), the master system, has chaotic behavior, since the map f is everywhere expanding and its natural ergodic invariant measure is given by the constant density in [0,1]. The corresponding Lyapunov exponent can be computed explicitly by Birkhoff's ergodic theorem [22]:

$$\lambda_{||} \;=\; \int_0^a \ln\left|\frac{1}{a}\right| dx \;+\; \int_a^{1-a} \ln\left|\frac{1}{1-a}\right| dx \;=\; -a\ln(a) \;-\; (1-a)\ln(1-a) \qquad (8)$$

The notation $\lambda_{||}$ is inspired by the fact that this value is also the Lyapunov exponent of the synchronized trajectories of the coupled system corresponding to perturbations parallel to the synchronization subspace S. Note that this value is always positive, expressing the fact that the system has chaotic behavior. The Jacobian matrix of the RHS of (6) has eigenvectors that are independent of the point (x,x) in S, except at the point (a,a) where the RHS is not differentiable. Because of this special feature, the "natural" transversal Lyapunov exponent can also be computed explicitly

$$\lambda_{\perp} \;=\; \int_0^a \ln\left|\frac{1}{a}-\varepsilon\right| dx \;+\; \int_a^1 \ln\left|-\frac{1}{1-a}-\varepsilon\right| dx \;=\; a\ln\left|\frac{1}{a}-\varepsilon\right| \;+\; (1-a)\ln\left|\frac{1}{1-a}+\varepsilon\right| \qquad (9)$$

It is represented in Fig.5 as a function of ε. There are two intervals of ε where it is negative.

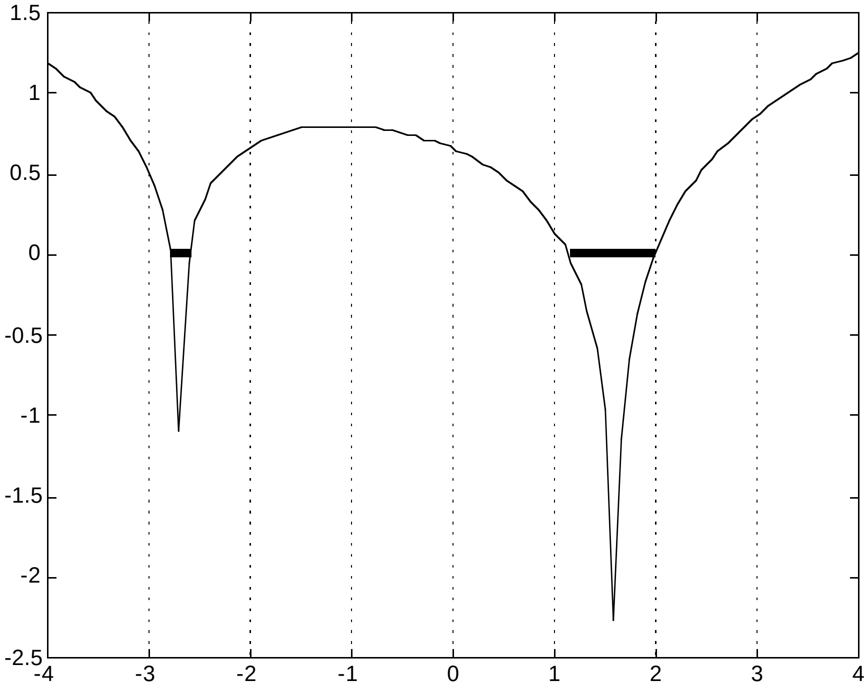

Fig.5. Transversal Lyapunov exponent as a function of the coupling constant ε, for a = 0.63. The intervals where it is negative are indicated in bold.

Since the slope of the skew tent map is either 1/a or -1/(1-a), it is not difficult to show that the transversal Lyapunov exponent of any trajectory satisfies

$$\ln\left|\frac{1}{a}-\varepsilon\right| \;\le\; \lambda_{\perp} \;\le\; \ln\left|\frac{1}{1-a}+\varepsilon\right| \qquad (10)$$

or the opposite inequality. Furthermore, the extremal values are reached at the fixed points x = 0 and x = 1/(2-a). The two bounds are shown, as functions of ε, in Fig.6. For any given value of the coupling constant ε, the upper limit is positive. Thus, there are always trajectories with positive transversal Lyapunov exponents.

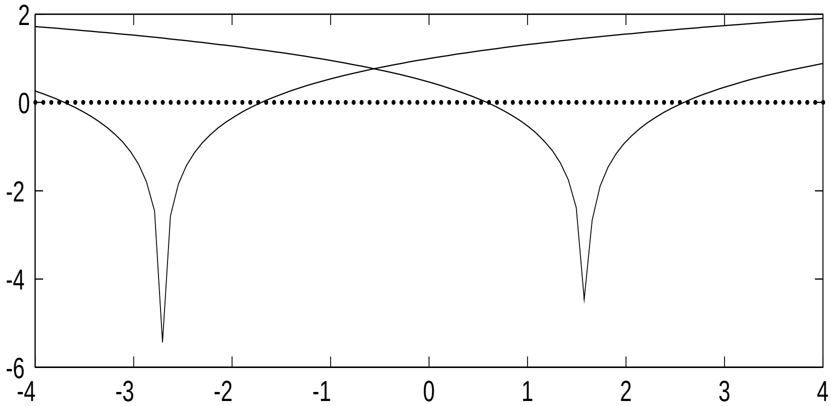

Fig.6. Bounds for the transversal Lyapunov exponents, for a = 0.63

According to the general reasoning given before, choosing initial conditions at random, there is a positive probability for trajectories not to synchronize, even when the "natural" transversal Lyapunov exponent (9) is negative. In this latter case, we expect a certain proportion of the trajectories to get close to the synchronization subspace S and then to be pushed away from S. Our numerical simulations did not confirm this directly; when the transversal Lyapunov exponent (9) was negative all computed trajectories converged to S. We think that quantization effects close to S are responsible to for this. In order to reduce the influence of quantization, we added to the RHS of the second equation of (6) a term of about 10^{-16} with random sign. We noticed that this very small perturbation has little influence and the trajectories still synchronized, as long as the transversal Lyapunov exponent (9) was sufficiently negative. On the contrary, when the transversal Lyapunov exponent was only slightly negative, sporadic bursts of the synchronization error appeared (Fig.7).

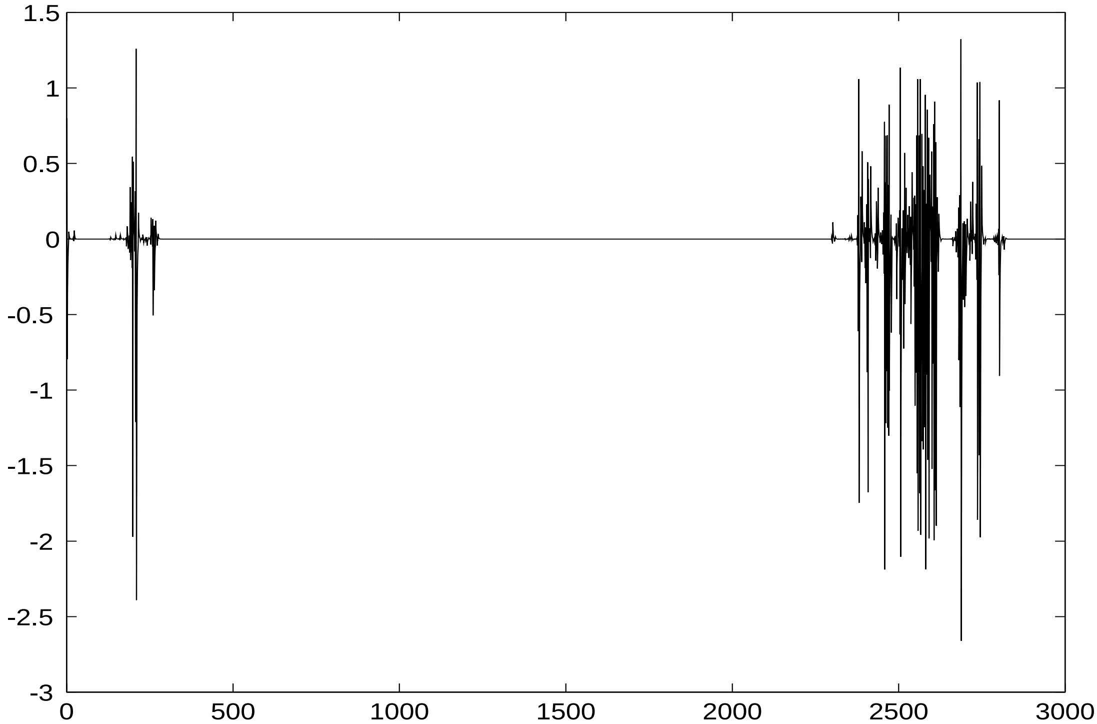

Fig.7. Synchronization error of a trajectory perturbed by a term of order 10^{-16}. Here a = 0.63, e = 1.135, which gives a transversal Lyapunov exponent of -0.0022 according to (9). Note that for almost 2000 iterations the trajectory was practically synchronized, before a big synchronization error burst took place.

The reasoning with the transversal Lyapunov exponents is purely local, in a neighborhood of the synchronization subspace S. It does not give any information about the dynamics far from S. The global behavior will determine how important the desynchronization bursts are. In the example of Fig.7, the burst are very large, in other examples, they are so small that from a practical point of view synchronization is not distroyed. Note also, that there are dynamical systems that completely synchronize, as e.g. the example of Fig.3 with an asymptotically stable linear subsystem. In these cases, the transversal Lyapunov exponents of **all trajectories in S**, not only of almost all trajectories, are negative.

So far, we have discussed the case when the transmitted signal is not distorted by the channel and the master and slave systems are perfectly matched. In practice, neither is the case, and it is important to be able to obtain approximate synchronization even in a non ideal environment. The tent maps coupled according to (6) do not satisfy this requirement. Even small perturbations and mismatches lead to large desynchronization bursts (Fig.8).

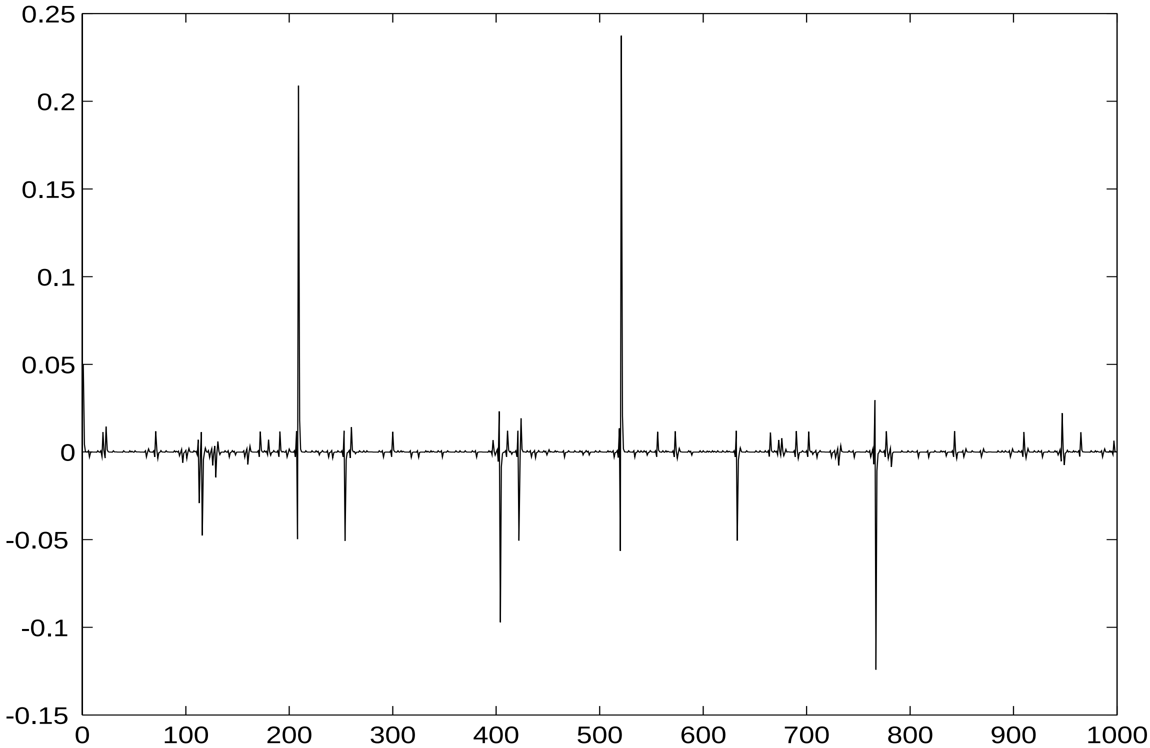

Fig.8. Synchronization error of a trajectory of the system (6) with slightly mismatched parameters of the master and the slave system (a = 0.63 and a = 0.6301). The coupling parameter is ε = 1.5, which corresponds to a "natural" transversal Lyapunov exponent (9) of value -1.005 for a = 0.63.

The amplitude of the desynchronization bursts depends on the global behavior of the system. If we couple the skew tent maps according to

$$\begin{aligned} x(k+1) &= f(x(k)) \\ y(k+1) &= f\left[y(k) + \varepsilon\left(x(k) - y(k)\right)\right] \end{aligned} \tag{11}$$

instead of (6), it can be shown that for a = 0.63 the resulting dynamical system completely synchronizes for $0.63 < \varepsilon < 1.37$ and that the "natural" transversal Lyapunov exponent is negative for $0.4826... < \varepsilon < 1.5174...$ .In Fig.9 the synchronization error of a trajectory is shown for 10 times larger mismatch between the master and the slave system and a 10 times less negative transversal Lyapunov exponent than in the example of Fig.8. Despite of this, the desynchronization bursts are much smaller.

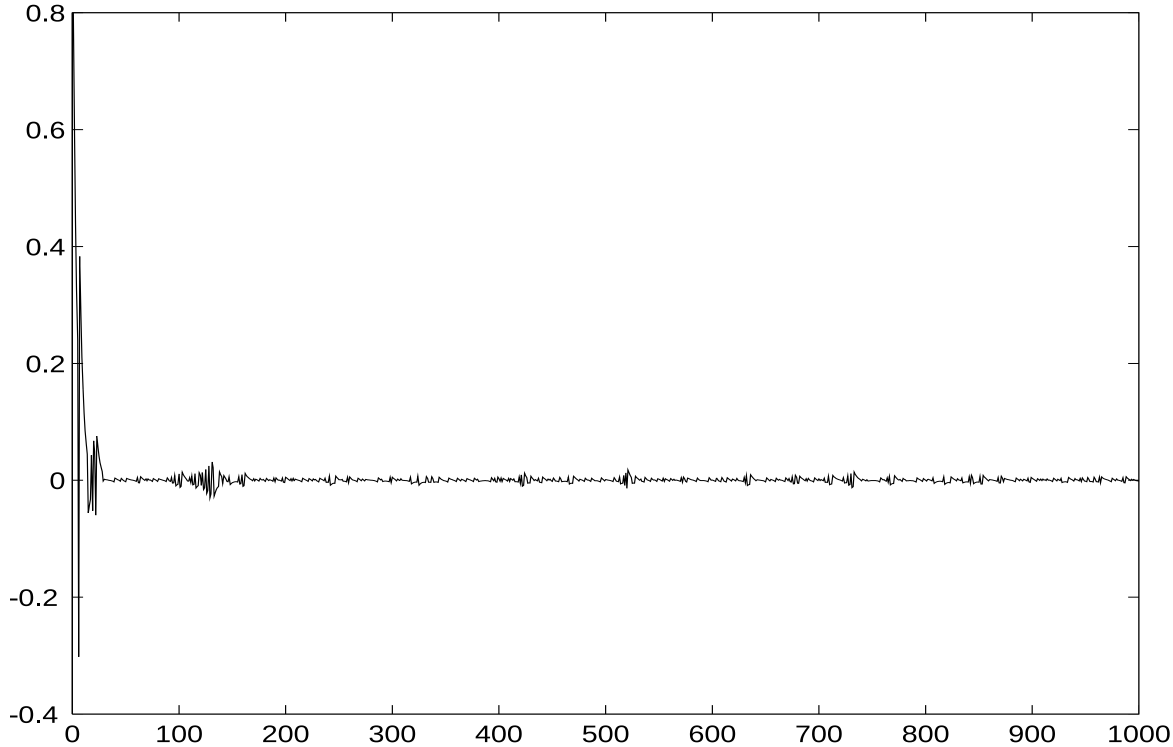

Fig.9. Synchronization error of a trajectory of the system (11) with mismatched parameters of the master and the slave system (a = 0.63 and a = 0.631). The coupling parameter is ε = 0.532, which corresponds to a "natural" transversal Lyapunov exponent of value -0.1003 for a = 0.63.

For practical purposes, even the choice of a chaotic system with good global synchronization properties may not be sufficient because of a large mismatch between the master and the slave system, or because of channel distorsions and noise that are too large, i.e. the signal z(t) is too different from y(t) in Fig.1. Some methods havee been proposed to improve synchronization by compensating channel disturbances. In [23] synchronization is achieved when the channel transforms y(t) into z(t) by a linear filter. The drawback of the method is the assumption that the transfer function of the filter is known and invariant in time. In [24,25] adaptive circuits compensate for an unknown and possibly time varying channel gain factor. In addition, the antisynchronization property introduced in [26] could be used to recover a perturbed chaotic signal.

Finally, in analog systems, the weaker notion of phase synchronization could be used [27]. Receivers that extract the information only by using phase synchronization may well be less sensitive to disturbances than those that require synchronization of both the amplitude and the phase.

4. Conclusion

The various problems that arise when information is transmitted via a chaotic carrier signals have been discussed. It is pointed out that for practical applications synchronization should be substantially improved in the presence of disturbances. Some new mathematical papers give a better understanding of the synchronization property. It is not sufficient for complete synchronization that the "natural" transversal Lyapunov exponent (with respect to the synchronization subspace) is negative. Even if only a set of measure zero of synchronized trajectories have a positive Lyapunov exponent, the phenomenon of intermittent desynchronization bursts appears. This phenomenon may be too rare to be observable in practice under ideal conditions, but it is likely to cause frequent and large desynchronization bursts when the transmitted signal is distorted and corrupted by noise and when there is parameter mismatch between the transmitter and the receiver. The task of future research is to minimize the influence of such disturbances on synchronization.

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