Wireless Operators in a Shared Spectrum

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Abstract*— So far, cellular networks have been operated in "private" frequency bands. But recently, several researchers and legislators have argued in favor of a more flexible and more efficient management of the spectrum, leading to the possible coexistence of several network operators in a *shared frequency band*. In our paper, we study this situation in detail, assuming that mobile devices can freely roam among the various operators. Free roaming means that the mobile devices measure the signal strength of the *pilot signals* (i.e., beacon signals) of the base stations and attach to the base station with the strongest pilot signal. We model the behavior of the network operators in a game theoretic setting in which each operator decides the power of the pilot signal of its base stations. We first identify possible Nash equilibria in the theoretical setting in which all base stations are located on the vertices of a two-dimensional lattice. We then relax this topological assumption and show that, in the more general case, finding the Nash equilibria is an NP-complete problem. Finally, we prove that a socially optimal Nash equilibrium exists and that it can be enforced by using punishments.

***Index Terms*— Wireless networks, shared spectrum, cooperation, game theory, Nash equilibrium, NP-completeness**

I. Introduction

Cellular networks are notoriously difficult to design and operate; in particular, defining the optimal location of the base stations and fine tuning their configuration parameters is very challenging. For this reason, government agencies (such as the FCC for the US) have sold or rented, for example by auction, each operator a frequency band for its exclusive usage in a given country or region. Only a small part of the whole spectrum is allocated as a *shared spectrum*, in which networks function in the same (unlicensed) frequency band.

With the progress of technology and the fast growing demand for ubiquitous high-speed wireless services, it is clear that the pressure towards more flexibility of the usage of the spectrum will only increase. Therefore, the government agencies are likely to adapt the current regulations in order to increase the proportion of the unlicensed spectrum as discussed in [3], [7].

The evolution towards unlicensed frequency bands can lead to a better usage of the spectrum. Yet, it would also create a novel situation, in which the base stations of different operators would interfere with each other. An operator may be tempted to let its base stations transmit at the maximum authorized level. But by doing this, it would maximize interference not only to its own base stations, but also to the base stations of the other operators, and to all mobile devices in the power range of its base stations; in addition, it would face the danger that the other operators retaliate by behaving in the same way.

In our paper, we assume that mobile users can *freely roam* across the base stations located in their neighborhood, attaching to the one offering the most favorable signal quality (i.e., the base station with the strongest pilot signal), irrespectively of the operator to which the base station belongs[1]. From the interference perspective, this operating principle is much more efficient than the current practice, because it enables mobile devices to find the "closest" base station in the area and hence mobile devices and base stations can significantly decrease their transmission power. This free roaming could be beneficial for both operators and users, because the former could serve an increased set of users, whereas the latter could enjoy various services across several operators.

We also assume that the operators want to cover the largest possible area by increasing the transmission range of their base stations. At the same time, they want to minimize interference. We assume that these two contradictory goals correspond to maximizing the number of users who attach to their base stations. We model this situation as a game in terms of power control of the base stations.

Note that the general problem of power control of base stations is hard to solve (i.e., NP-complete); it is characterized by the following dimensions: (i) the size of the base station sets, (ii) the geographic locations of the base stations and (iii) their possible radio ranges.

Game theory was already used to study the power control of user devices in wireless networks, notably in cellular systems as studied in [1], [9], [12], [13], [14], [16], [26] and [28]. Game theory was also used to study cooperation in wireless ad hoc networks, for example in [5], [15] and [23], in particular for cooperative power control [17]. A general framework for resource allocation in wireless network was addressed in [6].

Recently, the coexistence of multiple Internet Service Providers (ISPs) was studied in [22]. The authors consider both transit and customer prices for the ISPs. They show that if the number of ISPs competing for the same customers is large, then it can lead to price wars. In addition to this work in wired networks, the coexistence of wireless operators in a non-shared spectrum was addressed in two contributions. In [10], Halldórsson *et al.* study channel assignment strategies for Wi-Fi operators. They use the maximum graph coloring problem

[1]The users might have other attachment preferences based on subscription type, past experience, etc. We will consider the extension of user attachment behavior in our future work.

to identify Nash equilibria and they also provide a bound on the price of anarchy of these equilibria. In another paper [27], Zemlianov and de Veciana consider the scenario, in which users are able to choose between a cellular network and a Wi-Fi network. They show that congestion sensitive strategies are better than proximity-based strategies. None of these works considers the power control of the base stations.

Our paper addresses the problem of pilot signal power control in shared spectrum networks. Haykin provides a comprehensive overview [11] of the current tendencies and research challenges in shared spectrum communications in general. One of the challenges, namely opportunistic spectrum access is addressed in the paper of Wang *et al.* [25].

This paper is organized in the following way. In Section II, we describe the system model and the corresponding power control game. We solve this game on a two-dimensional lattice topology in Section III. In Section IV, we present our results in the case of a general topology of base stations. We extend our study with a repeated game model in Section V. Finally, we conclude in Section VI.

II. Model

A. System Model

In our paper, we make the following assumptions with respect to the communication network. We assume two wireless communication networks, each operated by an *operator* and we call the operators A and B. Operator $i \in \{A, B\}$ controls a set of *base stations (BS-s)* denoted by $\mathcal{B}_i$. We denote the union of all base stations by $\mathcal{B}$. We also assume several *users* equipped with *mobile devices* to access the communication network. The networks reside in a given *area*, where the operators want to provide wireless access for the users. We consider two operators to provide an insight in the basic principles of cooperation in a multi-operator context. Note that our results for a general network topology presented in Sections IV and V hold for more than two operators as well.

We assume that the radios of the base stations and the mobile devices are compatible, meaning that any user is able to access the network via any of the base stations. Base stations and mobiles operate on the same unlicensed band of the frequency spectrum. Each might perform power control to optimize its transmission power and reduce interference. This optimization can be realized in three ways: the power control of the pilot signal of the BS-s, downlink (BS to mobile) and uplink (mobile to BS) power control. In this paper, we focus on the first problem and we postpone the investigation of the other two problems to our future work. To mitigate interference, the shared frequency band is usually split up into channels (i.e., separated frequency sub-bands), but the pilot signal is typically emitted on a single shared channel for all the base stations, which results in mutual interference of the pilot signals (in CDMA networks, the interference of the pilot signals is referred to as the *pilot pollution* [21]).

According to the *physical model* of signal propagation [21], the pilot signal of a base station $b_i \in \mathcal{B}$ can be received by a user device u if its *signal-to-interference-noise ratio (SINR)* exceeds a reception threshold β:

$$\frac{P_i \cdot d_{iu}^{-\alpha}}{N_0 + \sum_{j \in \mathcal{B}, j \neq i} P_j \cdot d_{ju}^{-\alpha}} \geq \beta \tag{1}$$

where $2 \leq \alpha \leq 5$ is the path loss exponent that characterizes the radio signal propagation properties of the environment, P_i is the transmission power of BS b_i, d_{iu} is the Euclidian distance between the BS b_i and user device u and N_0 is the Gaussian thermal noise. Assuming specific antenna characteristics, (1) corresponds to the Friis free space radio signal propagation equation (see [21] Equation (4.1)). It captures how the reception power depends on the most important factors, namely on the transmission power and the distance between transmitter and receiver. Note that we consider the local average of the received pilot signal as described in [21]. In a small time scale, the pilot power signals have a time-varying property due to fading. In our future work, we will consider a more realistic radio signal modelling that incorporates fading and more realistic path loss models.

We assume that (1) holds for every point in the area for at least one base station and the user device u attaches to a base station b_i with the best SINR. Thus, we can write that:

$$\frac{P_i \cdot d_{iu}^{-\alpha}}{N_0 + \sum_{j \in \mathcal{B}, j \neq i} P_j \cdot d_{ju}^{-\alpha}} \geq \frac{P_l \cdot d_{lu}^{-\alpha}}{N_0 + \sum_{m \in \mathcal{B}, l \neq m} P_m \cdot d_{mu}^{-\alpha}} \tag{2}$$

for any other base station b_l.

We abstract away the mobiles and assume that their expected position is uniformly distributed over the area. Let us assume that reception is possible everywhere in the area and that $\alpha = 2$ (this means that the pilot signals propagate in an open area). Then (2) defines a Multiplicatively Weighted Voronoi power diagram (MW power diagram) [19], which defines the set of points in the area (potential places of user devices) that belong to a given base station. In the MW power diagram, a point belongs to a base station if it is "closer" to it than to any other base station, where the distance is defined as follows:

Definition 1: The *multiplicatively weighted power distance* between the points u and b_i is defined as:

$$d_{mpw}(u, b_i; w_i) = \frac{d_{iu}^2}{w_i} \tag{3}$$

where d_{iu} is the Euclidian distance between the points u and b_i and w_i is a weight assigned to point i.

We can define the *Voronoi region* $V(b_i)$ around a base station $b_i \in \mathcal{B}$ as the set of points u that are "closer" to point b_i than to any other point b_j, where $b_i \neq b_j$. Hence, we can write $V(b_i)$ as:

$$V(b_i) = \{u | d_{mpw}(u, b_i; w_i) \leq d_{mpw}(u, b_j; w_j) \ for \ i \neq j\} \tag{4}$$

We can write the *Voronoi diagram* of $\mathcal{V}$ $(\mathcal{B})$ as:

$$\mathcal{V}(\mathcal{B}) = \bigcup V(b_i) \tag{5}$$

where $b_i \in \mathcal{B}$.

Due to the complex shape of the Voronoi diagram with multiplicatively weighted distances, it is difficult to derive analytical solutions for the pilot power control problem. Hence, we apply a *radio range model* that is widely used in the literature. We will show in Section III-E that the principles derived from the range model hold for the physical model as well.

Let us derive from (1) the *radio range* of the pilot signal of the BS b_i as the Euclidian distance within which the users are able to attach to this base station if there is no interference:

$$r_i = \sqrt[\alpha]{\frac{P_i}{\beta N_0}} \tag{6}$$

Assuming an open area propagation (i.e., $\alpha = 2$), we can define the Additively Weighted Voronoi power diagram (AW power diagram) [19]. In the AW power diagram, the distance is defined as follows:

Definition 2: The *additively weighted power distance* between the points u and b_i is defined as:

$$d_{apw}(u, b_i; w_i) = d_{iu}^2 - w_i \tag{7}$$

where d_{iu} is the Euclidian distance between the points u and b_i and w_i is a weight assigned to point b_i.

In this paper, we substitute $w_i = r_i^2$ and hence we obtain a *Voronoi diagram in the Laguerre geometry* [19]. This model corresponds to a Voronoi diagram, where the distance is defined as a tangential Euclidean distance to circles with a center of the base stations and the radius of their radio ranges, respectively.

We assume that the base stations are placed on the vertices of a two-dimensional lattice in an alternating way such that any BS that belongs to operator A has four neighboring BS-s that belong to operator B (a small part of the network is shown in Figure 1). Let us call d the smallest Euclidian distance between base stations. In Section IV and V, we will extend our model to general network topologies.

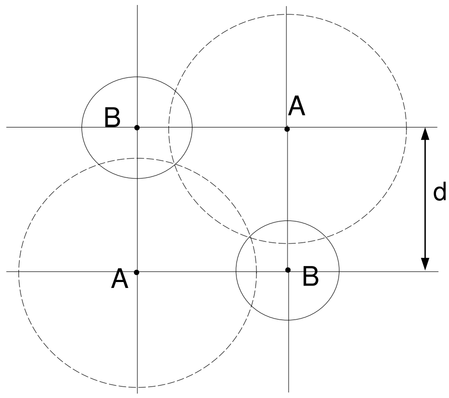


Fig. 1. Base stations on the vertices of a two-dimensional lattice. Here A is the operator with a larger radio range.

To further specify our model, we assume that:

- **A1:** Operators want to provide wireless access service everywhere. Thus, no place remains uncovered in the area.
- **A2:** Operators can estimate or measure their coverage.
- **A3:** Each BS belonging to the same operator has the same radio range. We show in Section IV that relaxing this assumption makes the power control problem NP-complete.
- **A4:** There exists a limitation P_{MAX} on the transmission power of the base station, which is defined by the regulator of the wireless spectrum. Then, the maximum radio range R_{MAX} can be derived from (6) by substituting $P_i = P_{MAX}$. Furthermore, if the radio ranges of all base stations $b_m \in \mathcal{B}$ are equal, we denote the minimum radio range for which **A1** holds by $R_{MIN} = \frac{\sqrt{2}}{2}d$.
- **A5:** The users can freely roam between any of the base stations (i.e., the operators do not disable roaming between their networks).
- **A6:** The base stations and the mobile devices have omnidirectional antennae. The investigation of the effect of directional antennas is part of our future work.

These assumptions ensure an open spectrum environment, in which users enjoy ubiquitous wireless connectivity. In particular, we make Assumption **A3**, as well as the assumption that the base stations are placed on the vertices of a grid, to make the model tractable. This special scenario is reasonable for a small number of base stations, such as for a small city network. We show stability points for this special model. We were motivated to study this special model, because we wanted to provide some quantitative insights into the power control problem. We believe our paper to be one of the first steps towards a deeper understanding of the tradeoffs of operating cellular networks in shared spectrum. The general problem is very involved: We show that if operators can set an arbitrary radio range for their base stations (i.e., **A3** does not hold), then the power control problem is hard to solve. Then, assuming that **A3** holds, we prove a condition for which cooperation can be enforced.

B. Power Control Game

We model the power control problem with two operators as a two-player, nonzero-sum game. We refer to the two operators as *players* A and B, respectively. Due to **A3**, we designate the radio ranges of the pilot signal of the players by r_A and r_B. The *strategy* of the players defines their best radio range. The goal of the players is to maximize the area they cover with their pilot signal as expressed by their *utility* function. To express the utility of the players formally, let us introduce the following concepts.

Assume that the two players choose a different radio range. Let us call the player with the larger radio range *heavy* and the player with the smaller range *light*. In the following, we assume that A is the heavy player and B is the light player; note, however, that it can be the opposite due to the symmetric situation. Let us denote the radio range of the heavy player by r_H and the one of the light player by r_L (recall from **A3** that a player has the same range for all of its base stations). Since the placement of the BS-s is symmetric and the players apply the same radio range to all of their BS-s, we can analyze the game considering two neighboring base stations, as shown in Figure 2.

We define the *coverage area* (O_i) for any BS b_i as its Voronoi region $V(b_i)$ in the radio range model (i.e., in the

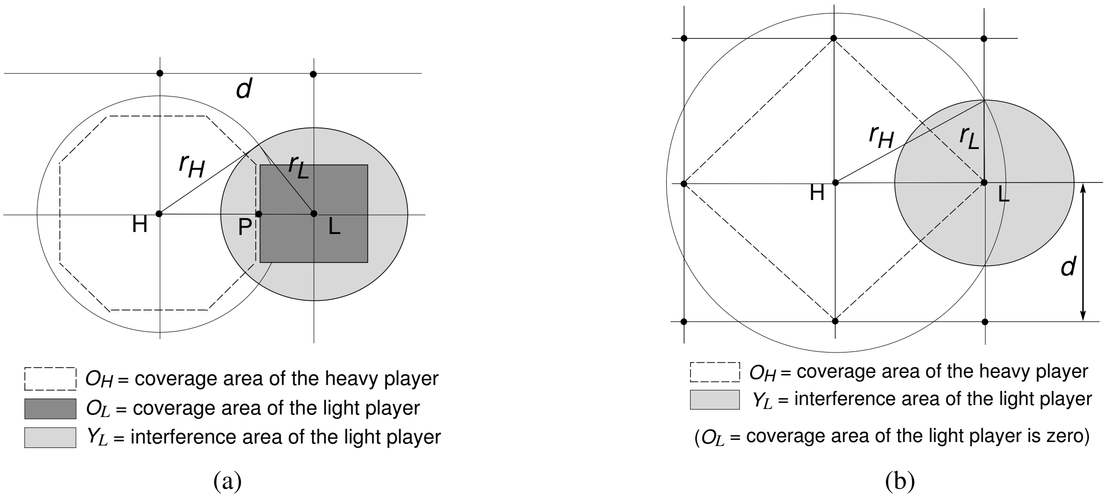


Fig. 2. Coverage and interference area of a base station, illustrated with two base stations: (a) both BS-s have a coverage area; (b) the BS-s of the light player are overwhelmed by the BS-s of the heavy player; the light player has no coverage area at all.

Voronoi diagram in the Laguerre geometry). We define the *interference area (Y_i)* for a BS b_i as:

$$Y_i = T_i - O_i = r_i^2 \cdot \pi - O_i \tag{8}$$

where T_i is the total area covered by the radio range and r_i denotes the radio range of BS b_i.

Note that the coverage area of a player depends on the radio range of the other player. Accordingly, we can distinguish two cases as follows.

In the first case, both players have a non-empty coverage area as presented in Figure 2a. For this case, the following condition holds:

$$r_H < \sqrt{r_L^2 + d^2} \tag{\textbf{C1}}$$

We can express the coverage area of the heavy player by calculating the area of the octagon. As shown in Figure 2, this area can be calculated based on the distance d of the two base stations, the distances $\overline{HP}$ and $\overline{LP}$ and the ranges r_H and r_L. Thus, we can write the coverage area as follows:

$$O_H = \frac{d^4 + 2d^2(r_H^2 - r_L^2) - (r_H^2 - r_L^2)^2}{d^2} \tag{9}$$

The coverage area of the light player is as follows:

$$O_L = \frac{(d^2 - r_H^2 + r_L^2)^2}{d^2} \tag{10}$$

If Condition **C1** does not hold, then the light player is overwhelmed by the heavy player, meaning that the pilot signal of the heavy player is the strongest everywhere (as presented in Figure 2b). If the heavy player overwhelms the light player, the coverage area functions are as follows:

$$O_H = (\sqrt{2}d)^2 = 2d^2 \tag{11}$$

$$O_L = 0 \tag{12}$$

In addition to **C1**, we can derive a condition for the radio ranges of the two players from **A1** as follows:

$$r_H^2 \geq (\frac{\sqrt{2}}{2}d - r_L)^2 + (\frac{\sqrt{2}}{2}d)^2 = d^2 - \sqrt{2}dr_L + r_L^2 \tag{\textbf{C2}}$$

In the limit case, in which the equality holds in (**C2**), they just cover the area (as shown in Figure 1).

From (**C2**) and **A4**, we can derive the definition interval for r_H:

$$\sqrt{d^2 - \sqrt{2}dr_L + r_L^2} \leq r_H \leq R_{MAX} \tag{13}$$

Similarly, from (**C2**) we get the bounds on r_L knowing that it is positive and smaller than r_H:

$$\max\{0, \frac{\sqrt{2}}{2}(d - \sqrt{-d^2 + 2r_H^2})\} \leq r_L \leq r_H \tag{14}$$

The upper bound comes from the fact that $r_H \leq \frac{\sqrt{2}}{2}(d + \sqrt{-d^2 + 2r_H^2})$ for all values of r_H. Note that the expressions in (13) and (14) always take real values.

We assume that the goal of the players is to maximize their utility, in other words to increase their coverage area while minimizing their interference area (i.e., the area, which is in their radio range, but they do not cover eventually). We define the utility per base station for player i playing r_i given that the other player j plays r_j at its BS-s as follows[2]:

$$U_i(r_i, r_j) = O_i - \gamma_i \cdot Y_i = (1 + \gamma_i) \cdot O_i - \gamma_i \cdot r_i^2 \cdot \pi \tag{15}$$

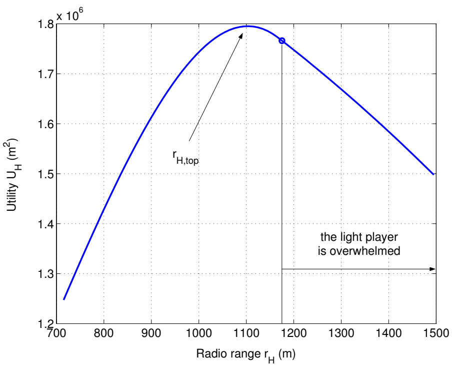
where $\gamma_i \geq 0$ is a *sensitivity* parameter that defines how much player i cares about the size of its interference area.

Let us graphically present the utilities of the players based on expression (15). Figure 3a presents an example for the utility of the heavy player for a fixed value of r_L and Figure 3b presents the utility of the light player for a fixed value of r_H. In the next section, we derive stability points in the game using these utility functions.

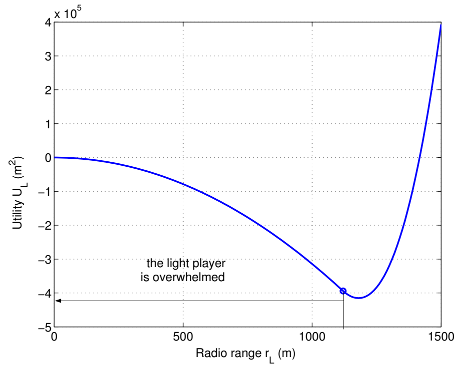
III. SINGLE-STAGE GAME

In this section, we consider a *single-stage game*, where both players simultaneously choose their radio range once and

[2]Note that due to the specific scenario, the utility of player i can be calculated by multiplying U_i with the number of its base stations. In this scenario, we refer to the utility per base station as the utility of the player.



(a)



(b)

Fig. 3. Utility function (a) of the heavy player for $d = 1km$, $\gamma_H = 0.1$, $r_L = 0.6km$ and $R_{MAX} = 1.5km$ (defined by the regulator); and (b) of the light player for $d = 1km$, $\gamma_L = 0.1$ and $r_H = 1.5km$.

forever. This corresponds to the case in which the base stations are not able to perform power control during the operation of the network, thus the radio power has to be set manually at the installation of the base stations. We use this basic scenario to study the basic equilibria of the power control game. We extend our investigation to more complex scenarios in the following sections.

We make use of the concept of Nash equilibrium [8], [18], [20] to show stability points in the game. Let us denote the strategy of player i by $s_i \in S$ and the strategy of the other player by $s_j \in S$, where S is the set of strategies (i.e., the set of possible radio ranges). Then, we can define the *best response function* of player i as follows (as presented in [20] Equation (15.1)):

Definition 3: For any $s_j \in S$, define $BR_i(s_j)$ to be the set of player i's best strategies given s_j.

$$BR_i(s_j) = \{s_i \in S : U_i(s_i, s_j) \geq U_i(s_i^{'}, s_j),\ \forall s_i^{'} \in S\}$$

Based on this definition, we can formulate the Nash equilibria as follows (corresponds to Equation (15.2) in [20]):

Definition 4: In a Nash equilibrium, in which the players play $\hat{s}_i$ and $\hat{s}_j$, we have:

$$\hat{s}_i \in BR_i(\hat{s}_j),\ i \in \{A, B\}$$

Hence, in a Nash equilibrium, none of the players is motivated to change its strategy. This formulation shows us a method to find Nash equilibria: we first find the best response function for each player, then we identify a set of strategies for which Definition 4 holds.

A. Best Response of the Heavy Player

From the utility functions of the heavy player presented in Figure 3a, we see that the utility is a concave function with a maximum point $r_{H,top}$. We can derive $r_{H,top}$ by maximizing (15) with the coverage area defined in (9).

$$r_{H,top} = \frac{\sqrt{2(1+\gamma_H)(d^2+r_L^2) - d^2\gamma_H\pi}}{\sqrt{2}\sqrt{1+\gamma_H}} \quad (16)$$

We can identify different best response strategies, corresponding to the interval in which the utility function is defined, as follows.

1) If $r_{H,top} < \sqrt{d^2 - \sqrt{2}dr_L + r_L^2}$, then the best response of the heavy player is the lower bound in (13), because the utility function is strictly decreasing:

$$BR_H(r_L) = \sqrt{d^2 - \sqrt{2}dr_L + r_L^2} \quad (17)$$

2) If $\sqrt{d^2 - \sqrt{2}dr_L + r_L^2} < r_{H,top} < R_{MAX}$, then the best response is (this corresponds to Figure 3a).

$$BR_H(r_L) = r_{H,top} \quad (18)$$

3) Finally, if $r_{H,top} > R_{MAX}$, then the best response of the heavy player is:

$$BR_H(r_L) = R_{MAX} \quad (19)$$

The resulting functions are shown in Figure 4a. Note that the shape of the second part depends on the value of γ_H. If $\gamma_H < \frac{2}{\pi-2}$, then the second part of the function is convex; if $\gamma_H = \frac{2}{\pi-2}$, the second part is linear; otherwise it is concave. If $\gamma_H > \frac{2}{\pi-2}$, then the function is limited by $r_L = R_{MAX}$ and the third part does not exist.

B. Best Response of the Light Player

We can now derive the best response strategies for the light player. From the utility function, we see that the best response strategy should be one of the bounds as defined in (14).

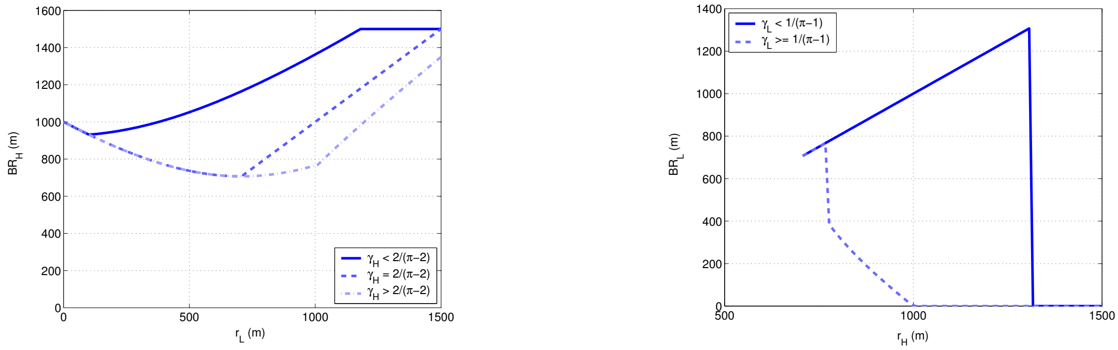
Let us define the *knockout (KO) range* of the heavy player as the range r_H for which the light player's utility playing the upper bound in (14) equals to the utility playing the lower bound. If the heavy player plays a radio range larger than the knockout range of the light player, then the light player should play its minimum range (it is knocked out from the game).

If $\gamma_L > \frac{1}{\pi-1}$, we can write the knockout range as:

$$r_H^* = \sqrt{\frac{5 + \gamma_L(10-4\pi) + \gamma_L^2(5-4\pi+\pi^2)}{2(1+\gamma_L)}} \cdot d < d \quad (20)$$

and if $\gamma_L \leq \frac{1}{\pi-1}$ the knockout range is:

$$d \leq r_H^{**} = \sqrt{\frac{1+\gamma_L}{\gamma_L\pi}} \cdot d \quad (21)$$



(a) (b)

Fig. 4. Best response function (a) BR_H of the heavy player for various γ_H values for $d = 1km$; and (b) BR_L of the light player for different γ_L values for $d = 1km$.

Thus, we can write the best response of the light player as:

1) If $\gamma_L > \frac{1}{\pi-1}$ and $\sqrt{d^2 - \sqrt{2}dr_L + r_L^2} \leq r_H \leq r_H^*$, then the best response is:

$$BR_L(r_H) = r_H \tag{22}$$

2) If $\gamma_L > \frac{1}{\pi-1}$ and $r_H > r_H^*$, then the best response is:

$$BR_L(r_H) = \frac{\sqrt{2}}{2}(d - \sqrt{-d^2 + 2r_H^2}) \tag{23}$$

3) If $\gamma_L \leq \frac{1}{\pi-1}$ and $\sqrt{d^2 - \sqrt{2}dr_L + r_L^2} \leq r_H \leq r_H^{**}$, then the best response is $BR_L(r_H) = r_H$.
4) If $\gamma_L \leq \frac{1}{\pi-1}$ and $r_H > r_H^{**}$, then the best response is $BR_L(r_H) = 0$.

Figure 4b shows the possible best response functions for the light player.

For the first and third case, the maximum radio ($r_L = r_H$) range results in the optimal utility:

$$U_L(r_H, r_H) = (1 + \gamma_L)d^2 - \gamma_L r_H^2 \pi \tag{24}$$

If the second case is true, then the minimum radio range is the optimal one. We can write the optimal utility if $BR_L(r_H) = \frac{\sqrt{2}}{2}(d - \sqrt{-d^2 + 2r_H^2})$ as :

$$\begin{aligned} &U_L(BR_L(r_H), r_H) = \\ &= (2 - \gamma_L(\pi - 2))(r_H^2 - d\sqrt{-d^2 + 2r_H^2}) \end{aligned} \tag{25}$$

Finally, in the fourth case, the optimal utility is zero.

C. Nash Equilibria in the Single Stage Game

Using the best response functions derived for both players, we can identify Nash equilibria in the single stage game.

1) An infinite number of Nash equilibria exist if
   a) $\gamma_L > \frac{1}{\pi-1}$ or;
   b) $\gamma_H = \frac{2}{\pi-2}$.
2) Two Nash equilibria exist if $\gamma_L = \frac{1}{\pi-1}$ and $\gamma_H > \frac{2}{\pi-2}$.
3) There is a unique Nash equilibrium for the following sensitivity values:
   a) $\gamma_L = \frac{1}{\pi-1}$ and $\gamma_H < \frac{2}{\pi-2}$;
   b) $\gamma_L < \frac{1}{\pi-1}$, $\gamma_H < \frac{2}{\pi-2}$ and $r_H^{**} \geq R_{MAX}$;
   c) $\gamma_L < \frac{1}{\pi-1}$, $\gamma_H > \frac{2}{\pi-2}$;
4) No Nash equilibrium exists if $\gamma_L < \frac{1}{\pi-1}$, $\gamma_H < \frac{2}{\pi-2}$ and $r_H^{**} < R_{MAX}$.

Table I shows the number and the types of different equilibria as a function of the sensitivity values of the players. In parentheses we write R_{MIN} or R_{MAX} if a Nash equilibrium exists in which both players play (R_{MIN}, R_{MIN}) or (R_{MAX}, R_{MAX}), respectively. We write *knockout (KO)* if the heavy player forces the light player to play its minimum radio range and we write *equal* if they have the same radio range.

The theorem can be proven for each possible value of γ_L and γ_H by solving the corresponding equations (17), (18), (19), (22) and (23) and the crossing points of the best response functions result in the Nash equilibria as defined in Definition 4.

D. Equilibrium Selection

From Table I, we can observe that there is a variety of Nash equilibria depending on the parameters (i.e., sensitivity, maximum radio range) in the power control game. In order to assess the success of the players in these Nash equilibria, we use the concept of *Pareto-optimality*.

Definition 5: A pair of radio ranges is Pareto-optimal (or socially optimal), if none of the players can increase its utility unless the utility of another player decreases.

In order to assess the feasible region (all possible values of the utilities) of the radio ranges, we show the utilities for each possible values of r_A and r_B in Figure 5. Furthermore. let us distinguish the KO state in which the light player plays $r_L = 0$ and denote it by KO_0.

The following theorem shows that, depending on the parameter values, two states can be socially optimal.

Theorem 1: If several Nash-equilibria exist in the grid scenario, then the Pareto-optimal (socially optimal) Nash-equilibria are as follows:

1) Both players playing R_{MIN} is Pareto-optimal, if:
   a) $\gamma_H > \frac{2}{\pi-2}$ or
   b) $\gamma_H = \frac{2}{\pi-2}$ and $\gamma_L < \frac{2}{\pi-2}$.
2) The R_{MIN} and KO_0 solutions are both Pareto-optimal, if $\gamma_H = \gamma_L = \frac{2}{\pi-2}$.

TABLE I

NUMBER OF NASH EQUILIBRIA IN THE SINGLE STAGE GAME AS A FUNCTION OF THE SENSITIVITY VALUES.

	$\gamma_H < \frac{2}{\pi-2}$	$\gamma_H = \frac{2}{\pi-2}$	$\gamma_H > \frac{2}{\pi-2}$
$\gamma_L < \frac{1}{\pi-1}$	0 or 1 (R_{MAX})	∞ (*equal* or R_{MIN})	1 (R_{MIN})
$\gamma_L = \frac{1}{\pi-1}$	1 (*KO*)	∞ (*KO* or *equal* or R_{MIN})	2 (*KO* or R_{MIN})
$\gamma_L > \frac{1}{\pi-1}$	∞ (*KO*)	∞ (*KO* or *equal* or R_{MIN})	∞ (*KO* or R_{MIN})

TABLE II

THE BEST (PARETO-OPTIMAL) NASH EQUILIBRIA IN THE REPEATED GAME AS A FUNCTION OF THE SENSITIVITY VALUES.

	$\gamma_H < \frac{2}{\pi-2}$	$\gamma_H = \frac{2}{\pi-2}$	$\gamma_H > \frac{2}{\pi-2}$
$\gamma_L < \frac{1}{\pi-1}$	R_{MAX}	R_{MIN}	R_{MIN}
$\gamma_L = \frac{1}{\pi-1}$	KO_0	R_{MIN}	R_{MIN}
$\frac{1}{\pi-1} < \gamma_L < \frac{2}{\pi-2}$	KO_0	R_{MIN}	R_{MIN}
$\gamma_L = \frac{2}{\pi-2}$	KO_0	KO_0 or R_{MIN}	R_{MIN}
$\gamma_L > \frac{2}{\pi-2}$	KO_0	KO_0	R_{MIN}

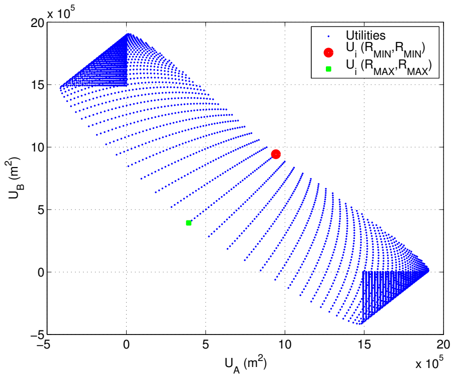


Fig. 5. The utilities for the possible values of r_A and r_B if $d = 1km$ and $\gamma_A = \gamma_B = 0.1$.

3) The KO_0 solution is Pareto-optimal, if:
   a) $\gamma_H < \frac{2}{\pi-2}$ or
   b) $\gamma_H = \frac{2}{\pi-2}$ and $\gamma_L > \frac{2}{\pi-2}$

We provide the proof of the theorem in Appendix I-A.

This theorem shows that if several Nash equilibria exist in the grid scenario, then playing either R_{MIN} or KO_0 is a good choice, from the social point of view, for given sensitivity values. Except for $\gamma_H = \gamma_L = \frac{2}{\pi-2}$, the given equilibrium is the unique Pareto-optimal solution.

Based on Theorem 1, we can identify the most beneficial Nash equilibria from Table I. We express this modified solution in Table II.

Table II shows that if the operators are sensitive to interference, then they should play R_{MIN}. However, if one of the players is not sensitive to interference and the other is sensitive, then the non-sensitive player can increase its radio range to force the sensitive player out of the game. If none of the players cares about interference, then they will end up in both playing the maximum radio range.

E. Discussion

Our model based on the Voronoi diagram in the Laguerre geometry results in coverage areas with straight separation lines. We adopted this model, because if we applied the physical radio model based on (1), it would be difficult to derive a closed-form expression for the coverage and hence for the utility of the players. We use a numerical method to compare the radio range model to the physical model and to show that the principles derived in our model hold for the physical model as well.

We compare the coverage areas in both models as follows. We transform the continuous area into a discrete area by substituting it with a grid of side ϵ as shown in Figure 6. In our numerical study we use a grid of 100x100 points.

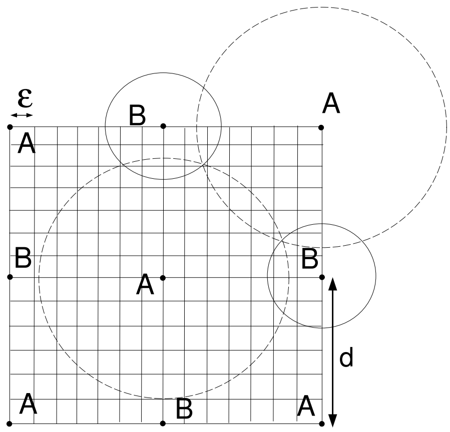


Fig. 6. Discrete area model with points in ϵ distance.

For a given set of radio ranges, we determine the number of points that belong to the base station in the middle of the considered area in each of the radio models. This results in an empirical value of the coverage area. We substitute this coverage area value into (15) to obtain the utility of the players in both cases and then we calculate the best responses from the utility function. Figure 7 shows an example of the best response of player i who controls the base station in the middle of Figure 6 for each of the radio models.

We can observe that the best response functions are very similar for the two models. We performed our numerical

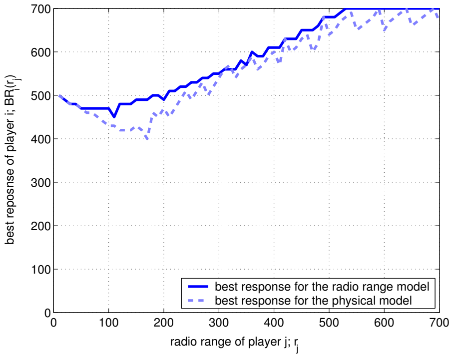


Fig. 7. Best response function of player i for $\gamma_i = \gamma_j = 0.1$, $d = 500m$ and $R_{MAX} = 700m$.

analysis for various values of γ_i and γ_j and it resulted in the same conclusion. Hence, the players playing R_{MIN}, R_{MAX} or being in the KO_0 state are the Nash equilibria in the physical model as well. However, the derivation of the precise values of γ_H and γ_L requires an extensive set of numerical calculations. This fact motivated us to study the problem based on the radio range model.

IV. NP-COMPLETENESS OF THE GENERAL PROBLEM

In this section, we analyze the power control problem for general network topologies and for general values of radio ranges in the single stage game.

The goal of player i is to allocate the radio ranges such that their overall utility $U_i = \sum_{b_m=1}^{|\mathcal{B}_i|} U_m$ is maximized, where $|\mathcal{B}_i|$ is the number of bases stations that belong to player i and the utility per base station U_m is as follows (derived from (15)).

$$U_i = \sum_{m=1}^{|\mathcal{B}_i|} \left[(1+\gamma_i) \cdot O_m - \gamma \cdot r_m^2 \cdot \pi\right] \tag{26}$$

where O_m is the coverage area and r_m is the radio range of base station b_m.

We can now formulate the following theorem.

Theorem 2: Finding the maximum utility of player i for general values of radio ranges is NP-complete.

We provide the proof in Appendix I-B.

Since finding the maximum utility for an operator is NP-complete in general, it is impossible to calculate best responses for a given player in polynomial time. Thus, we can state the following result.

Corollary 1: Finding Nash equilibria in the power control game for general values of radio ranges is NP-complete.

V. REPEATED GAME

In the previous section, we assumed that the radio range of the base stations has to be set in advance and no power adjustment is possible. In this section, we consider the possibility of an iterative power control in a *repeated game*. We assume that the operators do not know the end of the game, hence we study the problem in an infinite repeated game model with discounting [2], [8]. Note that we consider a two-player game with a general topology of base stations.

We extend the single-stage game as follows: We assume that the game is split up into steps denoted by t. In each step, player $i \in \{A, B\}$ adjusts the radio range of its base stations according to its strategy s_i.

If the optimal solution is unknown, then the players might be tempted to play R_{MAX} at each of the base stations. In this case, the coverage areas of their base stations define the ordinary Voronoi diagram of the area [19] (i.e., a Voronoi diagram with Euclidian distances). Let us consider an arbitrary base station b_m and its neighbors as shown in Figure 8. Assuming that the base stations have the same radio range over their whole network (i.e., **A3** is fulfilled and the radio ranges of the two players are equal). We define $R_{m,MIN}$ as the *smallest common range* with which they can cover the whole space around b_m. Obviously, playing $R_{m,MIN}$ is more beneficial than playing R_{MAX}[3]. Let us denote $R_{MIN} = \max_m[R_{m,MIN}]$ (meaning that it is the smallest common range which enables the coverage of the total area).

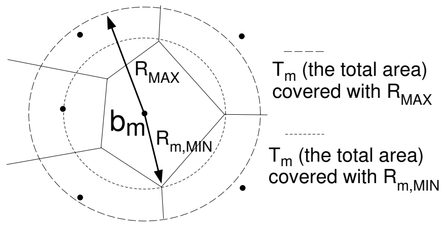


Fig. 8. Voronoi tesselation for a general topology, where **A3** applies (i.e., the radio ranges are the same). R_{MAX} is defined by the regulator and $R_{m,MIN}$ is the smallest range with which they can cover the whole space around base station b_m.

Furthermore, let us define the discounted cumulative utility in $k < \infty$ time steps as:

$$\bar{U}_i(k) = \sum_{t=0}^{k} U_i(t) \cdot \omega^t \tag{27}$$

where $0 < \omega < 1$ is the *discounting factor*, which expresses the value of future utilities for the players. The discounting factor is sometimes interpreted as a value related to the probability that the game ends in the subsequent time slot[4].

We now prove a theorem to show the conditions for which cooperation can be enforced. Then we calculate specific values for the grid scenario.

Theorem 3: If both players always play R_{MAX}, it is a Nash equilibrium if $R_{MAX} < r_H^{**}$ holds.

Recall that r_H^{**} is defined in (21).

Proof: Let us assume that player i plays R_{MAX} all the time. Since the decision of the other player does not affect its radio range, we can analyze the game by time steps. In any time step, player i necessarily becomes the heavy player (or they are of equal weight). If $R_{MAX} < r_H^{**}$, then the best strategy of the other player is $r_L = r_H = R_{MAX}$ in every time step. ■

[3]Recall that R_{MAX} is defined in **A4**.

[4]Based on this interpretation, we assume that the discounting factor is the same for both players.

In this case, the players are in a socially non-optimal equilibrium. We prove in this section that they can do better, by applying a strategy called *Punisher*.

Definition 6: If player i plays the *Punisher* strategy, it plays R_{MIN} in the first time step. For any further time steps, it plays:

- R_{MIN} in the next time step if the other player played R_{MIN} in the previous time step, or
- R_{MAX} for the next k_i time steps, if the other player played anything else.

The parameter k_i (also called the *punishment interval*) defines the number of time steps for which player i punishes the other player. Note that the Punisher strategy is similar to the well-known *Tit-For-Tat (TFT)* strategy [2][5]. The major difference is that it retaliates any defection by playing R_{MAX} instead of copying the same behavior.

If both players cooperate, they play R_{MIN}. In this case they both have the *cooperative utility* $C_i = U_H(R_{MIN}, R_{MIN})$[6]. If player i defects, while the other player does not, the defecting player has a *cheating gain* $G_i = U_H(BR_i(R_{MIN}), R_{MIN})$. If both players defect, they have a *defection utility* $D_i = U_H(R_{MAX}, R_{MAX})$.

Using this notation, we can prove a specific equilibrium is socially optimal.

Theorem 4: A Nash equilibrium based on R_{MIN} is enforceable with the *Punisher* strategy (i.e., player i is able to punish the defection of the other player j) if

$$\frac{G_j - D_j}{C_j - D_j} \cdot (1 - \omega) < 1 \tag{28}$$

where $\omega \leq 1$, $\gamma_i \neq 0$, $\gamma_j \neq 0$ and $R_{MAX} < r_H^{**}$. If the above condition holds, the punishment interval is defined by:

$$k_i \geq \log_\omega \left(1 - \frac{G_j - D_j}{C_j - D_j} \cdot (1 - \omega)\right) - 1 \tag{29}$$

We refer to Appendix I-C for the proof of the theorem. Note that for $\omega = 1$, cooperation can always be enforced using the Punisher strategy. This principle is expressed in general by the Nash folk theorem [8].

In general topologies, the value of the punishment interval k_i is hard to compute. In the grid scenario (i.e., for the basic scenario presented in Section III), however, we can calculate the necessary value of k_i as follows. Substituting γ_j to γ_H, we can obtain this value from (16):

$$BR_j(R_{MIN}) = \frac{d^2(3 - (\pi - 3)\gamma_j))}{2(1 + \gamma_j)} \tag{30}$$

If we substitute $r_i = r_L = R_{MIN}$ into (15), we get the cheating gain $G_j = U_H(BR_j(R_{MIN}), R_{MIN})$:

$$G_j = \frac{d^2(8 + (16 - 6\pi)\gamma_j + (8 - 6\pi + \pi^2)\gamma_j^2)}{4(1 + \gamma_j)} \tag{31}$$

In the following k_i time steps, player i plays R_{MAX}, because it plays the *Punisher* strategy. Consequently, player j has to play R_{MAX} in the subsequent time slots as well (given that $R_{MAX} < r_H^{**}$). Its utility for the next k_i time slots is the defection utility D_j:

$$D_j = (1 + \gamma_j)d^2 - \gamma_j R_{MAX}^2 \pi \tag{32}$$

Otherwise, if it played R_{MIN}, it would have a cooperation utility C_j for all the k_i time slots:

$$C_j = \frac{d^2(2 - (\pi - 2)\gamma_j)}{2} \tag{33}$$

The typical value of k_i is small (for $d = 1km$, $\gamma_j = 0.1$, $R_{MAX} = 1.5km$ and $\omega = 0.1$, the value is $k_i = \lceil 1.23 \rceil = 2$). For higher values of γ_j, R_{MAX} and ω, the punishment interval is one time slot (i.e., there is an immediate punishment). Figure 9 illustrates the average per time slot utility of a player for both cooperation and defection. One can observe that cooperation is more beneficial, because defection is quickly retaliated by the other player.

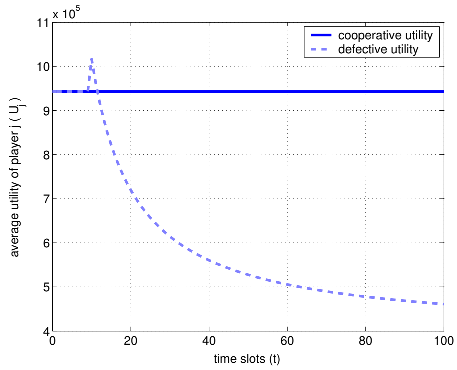


Fig. 9. Average utility for $d = 1km$, $\gamma_j = 0.1$, $R_{MAX} = 1.5km$ and $\omega = 0.1$ if both player apply the Punisher strategy. Defection is quickly retaliated and hence cooperation is the best choice.

Based on Theorem 4, we state the following result.

Corollary 2: If both players play the *Punisher* strategy and the conditions of Theorem 4 hold, then it results in a Nash equilibrium.

VI. Conclusion

In this paper, we have investigated the problem of co-existing wireless operators in a shared spectrum. We have assumed that the operators apply power control at the base stations to mitigate interference, while providing a permanent service to the users. To the best of our knowledge, our paper is the first to investigate this problem.

The contribution of this paper is threefold. First, we have shown that Nash equilibria exist if the operators set the power of their base stations at the beginning of the operation of the network. We have identified different equilibrium situations depending on the sensitivity of the operators to interference. Second, we have shown that the solution of the power control problem is NP-complete for a general topology of base stations. Third, we have proved a condition for which a socially optimal Nash equilibrium exists and that it can be enforced using punishments. This result holds in both the grid scenario (in which we derived the necessary "amounts of punishment")

[5]TFT defines the choice of a given player in the next time slot, while the Punisher strategy defines the punishment interval as a set of subsequent time slots.

[6]Note that for $r_A = r_B$, $U_H(r_A, r_B) = U_L(r_A, r_B)$. Hence we can apply any of the two utility functions.

and in the general topology with common radio ranges. In general, our results show which operation points are beneficial for the players and how these should be achieved.

In terms of future work, we will solve the power control problem by designing an approximation algorithm that converges to a desirable equilibrium situation for a general set of radio ranges; in particular, we will study the properties of the convergence by simulations. Furthermore, we will consider power control on the data channels as well. We also intend to study the effect of other techniques to mitigate interference, such as directional antennae and mobile devices with multiple antennae.

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APPENDIX I
PROOFS

A. Proof of Theorem 1

If any of the players increases its radio range, then it becomes a heavy player (note that the case is symmetric, so the proof holds no matter which player increases its radio range). For the light player, it is sufficient to prove the theorem for the best response strategies of the light player, because the utility of the light player is maximized for these values.

First, let us consider the case in which $r_H < r_H^*$ or $r_H < r_H^{**}$, thus the light player should play the same radio range. We see that the utility function of the light player as presented in (24) is a decreasing function of r_H, hence the theorem is true for this case. Note that in this case the utility of the heavy player does not increase either.

For the knockout state, in which $r_H \geq r_H^*$, the best response of the light player is to play its minimum radio range, which is different from zero. The utility for the minimum radio range is defined by (25). If $\gamma_L < \frac{2}{\pi-2}$, then this utility is smaller than the utility in the R_{MIN} state, thus the theorem holds. If $\gamma_L \geq \frac{2}{\pi-2}$, then the utility of the light player is maximized for $r_L = 0$, which is Pareto-superior to the utility achieved in KO states with other ranges.

If the utility function of the light player is zero (for $\gamma_L \leq \frac{1}{\pi-1}$), we have to solve

$$U_L(R_{MIN}) \geq U_L(0) \tag{34}$$

Inequality (34) holds if $\gamma_L \leq \frac{2}{\pi-2}$. Thus, it holds for all cases, where $\gamma_L \leq \frac{1}{\pi-1}$.

Finally, let us consider the case $\gamma_L \geq \frac{2}{\pi-2}$, in which the utility of the light player is increased compared to the case where both play R_{MIN}. Let us compare the utility of the heavy player as presented in (15) in the two situations. In the first case (KO_0 state), we have $r_H = d$ and $r_L = 0$, and in the second case, we have $r_H = r_L = \frac{\sqrt{2}}{2}d$. The results show that if $\gamma_H > \frac{2}{\pi-2}$, then the utility of the heavy player playing in the R_{MIN} state is higher than in the KO_0 state. For $\gamma_H = \frac{2}{\pi-2}$ the utilities in the two states are equal and for $\gamma_H < \frac{2}{\pi-2}$ the utility in the KO_0 state is higher.

B. *Proof of Theorem 2*

To prove the theorem, let us consider the special case of finding the optimal radio range allocation in the presence of a single operator. In this case, operator i has the utility:

$$U_i = \sum_{b_m=1}^{|\mathcal{B}_i|} \left[(1+\gamma_i) \cdot O_m - \gamma \cdot r_m^2 \cdot \pi\right] \tag{35}$$

Let us denote the total area by $O_{tot} = \sum_{b_m=1}^{|\mathcal{B}_i|} O_m$. Since the γ_i values are the same for all base stations, we can reformulate the utility as:

$$U_i = (1+\gamma) \cdot O_{tot} - \gamma_i \cdot \pi \sum_{b_m=1}^{|\mathcal{B}_i|} r_m^2 \tag{36}$$

Under the assumption that $\alpha = 2$, the power is proportional to the square of the radio range. Chamaret *et al.* [4] as well as Värbrand and Yuan [24] have proven that finding the minimum power allocation in the network of a cellular operator while maintaining the total coverage is NP-complete. Hence, the minimum value of U_i cannot be determined in polynomial time. Because the problem is NP-complete for the special case of one operator, we conclude that it is NP-complete in the general game as well.

C. *Proof of Theorem 4*

Let us assume that player j deviates in time step t_0. Let us assume that it applies the best option, hence it plays $BR_j(R_{MIN})$. The Punisher strategy played by player i reduces the discounted cumulative utility of player j for the time interval from t_0 to $t_0 + k_i$ if:

$$G_j + D_j \cdot \sum_{t=1}^{k_i} \omega^t \leq C_j \cdot \sum_{t=0}^{k_i} \omega^t \tag{37}$$

If $\omega = 1$, we can write (37) as follows:

$$G_j + D_j \cdot k_i \leq C_j \cdot (k_i + 1) \tag{38}$$

Hence, we obtain the following bound on the punishment interval:

$$k_i \geq \frac{G_j - C_j}{C_j - D_j} \tag{39}$$

Note that if $\omega = 1$, then cooperation is always enforceable.

Now if $\omega < 1$, we can transform the sums in (37) to the same intervals:

$$G_j - D_j + D_j \cdot \sum_{t=0}^{k_i} \omega^t \leq C_j \cdot \sum_{t=0}^{k_i} \omega^t \tag{40}$$

Since the sums are geometric sequences, we can write that:

$$G_j - D_j \leq (C_j - D_j) \cdot \frac{1 - \omega^{k_i+1}}{1-\omega} \tag{41}$$

Given that $C_j - D_j > 0$ and $1 - \omega > 0$, we can rewrite the inequality:

$$\frac{G_j - D_j}{C_j - D_j} \cdot (1-\omega) \leq 1 - \omega^{k_i+1} \tag{42}$$

Reordering the inequality gives us:

$$\omega^{k_i+1} \leq 1 - \frac{G_j - D_j}{C_j - D_j} \cdot (1-\omega) \tag{43}$$

This gives the condition on k_i, because the left side is strictly positive. Thus the inequality cannot be fulfilled if the right side is non-positive.

$$\frac{G_j - D_j}{C_j - D_j} \cdot (1-\omega) \leq 1 \tag{44}$$

If the condition in (44) holds, we can take the logarithm of both sides in (43). Since $\omega < 1$, the logarithm function is strictly decreasing and hence the direction of the inequality changes.

$$k_i \geq \log_\omega \left(1 - \frac{G_j - D_j}{C_j - D_j} \cdot (1-\omega)\right) - 1 \tag{45}$$

Due to the symmetric situation, the same arguments apply for the opposite case that defines the punishment interval for player j.

Appendix II
Glossary

We provide the list of symbols used in the paper in Table III.

TABLE III

TABLE OF SYMBOLS USED IN THE PAPER

Symbol	*Definition*	*Section*
A and B	operators / players	Section II
$\mathcal{B}_i$	set of the base stations of operator i	Section II
$\mathcal{B}$	set of all base stations	Section II
α	path loss exponent	Section II
β	required signal-to-interference-noise ratio for a successful reception at a receiver	Section II
P_i	transmission power of base station b_i	Section II
N_0	Gaussian thermal noise	Section II
d_{iu}	Euclidian distance between base station b_i and user u	Section II
$d_{mpw}(u, b_i; w_i)$	multiplicative power distance between between base station b_i and user u with the weight w_i	Section II
$d_{apw}(u, b_i; w_i)$	additive power distance between between base station b_i and user u with the weight w_i	Section II
d	smallest Euclidian distance between base stations in the grid scenario	Section II
$V(b_i)$	Voronoi region of base station b_i	Section II
$\mathcal{V}(\mathcal{B})$	Voronoi diagram of all base stations	Section II
r_i	radio range of base station b_i	Section II
R_{MAX}	maximum radio range allowed by the regulator	Section II
$R_{MIN} = \frac{\sqrt{2}}{2}d$	minimum common radio range to cover the whole area	Section II
r_A and r_B	radio ranges of the operators	Section II
r_H	radio range of the player with the higher power	Section II
r_L	radio range of the player with the lower power	Section II
O_i	coverage area of player i	Section II
Y_i	interference area of player i	Section II
T_i	total coverage area of player i	Section II
U_i	utility of player i	Section II
γ_i	sensitivity of player i to interference	Section II
S	the set of strategies	Section III
s_i	strategy of player i	Section III
$BR_i(r_j)$	best response strategy of player i to r_j	Section III
$r_{H,top}$	maximum utility of player i as defined in (16)	Section III
r_H^* and r_H^{**}	knockout (KO) ranges for the heavy player	Section III
KO_0	knockout state in which the light player plays $r_L = 0$	Section III
$\bar{U}_i(k)$	cumulative utility of player i in k time steps	Section V
k_i	punishment interval of player i in the repeated game	Section V
U_{tot}	total utility of player i for all its BS-s	Section IV
O_{tot}	total area of player i for all its BS-s	Section IV
$R_{m,MIN}$	minimum radio range to cover the area around BS b_m	Section IV
C_i	cooperative utility of player i	Section V
G_i	cheating gain of player i	Section V
D_i	defective utility of player i	Section V