Equilibrium Analysis of Packet Forwarding Strategies in Wireless Ad Hoc Networks – the Dynamic Case

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***Abstract*— Ad hoc networks are expected to be used in a number of very different situations. But a common characteristic is that the nodes have to cooperate with each other. This problem is particularly crucial, if each node is its own authority. Reckoning the relevance of this issue, several groups of researchers have proposed different *incentive mechanisms*, in order to foster cooperation between the nodes, notably for packet forwarding. However, the need for these incentives was not formally justified. In this paper, we address the problem of cooperation without incentive mechanisms and propose a simple model based on game theory. We then prove several theorems about the equilibrium conditions in a simple scenario. We investigate by simulation a more realistic scenario, which includes a real network topology as well as a mobility model. We show that the level of contribution of the nodes to reach cooperation is much higher than in the theoretical model, and we quantify the relationship between mobility and cooperation. We conclude that spontaneous cooperation is easier to reach when mobility is higher.**

I. Introduction

Ad hoc networks have the potential to increase the flexibility of wireless communication systems. They, however, also require novel operating principles. In particular, due to the absence of fixed infrastructure, most of the functions (routing, mobility management, in some cases even security) must rely on the cooperation between the nodes.

The most fundamental of these functions is packet forwarding. Cooperation is straightforward if all the nodes are under the control of a single authority, as is usually the case in military networks or for rescue operations: in these cases, the interest of the mission by far exceeds the vested interest of each participant.

However, if each node is its own authority, the situation changes dramatically: The most reasonable assumption is then to consider that each node will try to maximize the benefit it gets by using the network, even if this means adopting a selfish behavior. This selfishness can mean not participating in the unfolding of mechanisms of common interest, notably to spare resources, including battery energy.

Over the last few years, several researchers have proposed incentive techniques to encourage nodes to collaborate, be it by circumventing misbehaving nodes [20], by making use of a reputation system [5], [21], or by relating the right to benefit from the network to the contribution to the common interest of a node provided thus far [7]. These proposals have been based on heuristics, and are therefore rather difficult to compare with each other.

Very recently, Srinivasan *et al.* [27] have proposed a formal framework, based on game theory, to study cooperation without incentives. They have identified the conditions under which cooperation is a Nash-equilibrium[1]. In order to do this, their system model is quite simple: For each connection to be set up, they randomly select

[1]In a *Nash-equilibrium* none of the nodes can increase its utility by unilaterally changing its strategy.

several nodes to be part of it; as a result, their approach does not take the topology of the network into account (we discuss their work in more detail in Section VI).

Our own approach has essentially the same goal as this seminal work; however, we believe that the network topology is important, and we therefore include it in our model. In a previous work [10], we have already studied the *static* case, meaning that we have assumed that nodes do not move. We have identified the network topologies under which cooperation can be an equilibrium, and we have shown that the likelihood for these topologies to exist is extremely small.

In this paper, we pursue exactly the same ambition, but we now consider that the nodes can move. As a consequence, we have to adopt a different model. Due to the complexity of the problem, we deliberately devote a substantial part of the paper to a simplified scenario (all nodes are located - and shuffled - on a ring). In this way, we are able to formulate and to prove several theorems. Then, by means of simulations, we study the more general (and more realistic) case where the nodes move on a plane; thus, we can easily assess to what extent the situation differs from the ring scenario.

Our main contribution is to show that cooperative Nash-equilibria are much more likely to happen with mobile than with static nodes. In addition, we quantify how much "generosity" the nodes should grant in order to make these equilibria feasible.

The work presented in this paper is part of the Terminodes project [13]. The rest of the paper is organized as follows. In Section II we introduce a game theoretical model for packet forwarding. In Section III and IV we present our analytical results for connections with a single relay and multiple relays, respectively. Section V contains our simulation results for the ring and for a more realistic scenario. We give an overview of the related work in Section VI and conclude the paper in Section VII.

II. Modeling packet forwarding as a game

A. System model

We assume a network of N nodes. Each node uses an omnidirectional antenna with the same radio range. Hence, there is a bidirectional communication link between two nodes if they reside within the radio range of each other.

We assume that the packets are sent via multiple nodes that are expected to relay the packet. We call a *connection* the communication path defined by the source, the relays and the destination. We assume that each node is the source of one connection. We also assume that the connections last for the duration of the game. The study of routing behavior is out of the scope of this paper, so we assume an ideal routing protocol that establishes a connection between a given source and a given destination.

We assume an end-to-end mechanism that enables a source to detect the loss of a packet (e.g., at the transport layer), hence, we do not require an additional acknowledgement from the relays to the source. This means that the source can observe the fact that a packet is lost, but it cannot tell where, when and how it happened.

We introduce the following notation to identify our investigation scenarios:

Scenario-xR

where *Scenario* stands for the given scenario and x (or $\bar{x}$) stands for the constant (or average) number of relays of each connection.

B. Game theoretical model

In this section we present a game theoretical framework to investigate the conditions of cooperation for packet forwarding. We model packet forwarding as a game of infinite duration, where each node as a player interacts with the rest of the network (the concept is shown in Figure 1). It is important to mention that in our approach, the node interacts with the rest of the network without identifying the players it interacts with. In this way, we avoid the problem of authentication of nodes (authentication in ad hoc networks is still an open research problem).

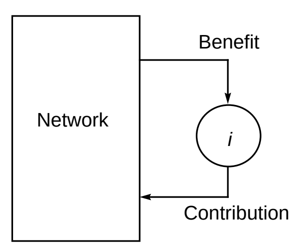

Fig. 1. The abstract representation of the game: any node (denoted by i) plays with the rest of the network

If a node runs out of energy, it is removed from the network. We assume that the nodes are not able to estimate at the beginning of the game when the game ends for them, because their energy usage depends on the particular unfolding of the game. Thus, we assume that a node perceives the game as being of infinite duration. An adaptive strategy that takes the current battery level into account during the game is part of our future work.

In our model, we assume that the source benefits from the arrival of a packet at the destination. But the model can be adapted to the case in which the destination benefits if a packet successfully arrives at it.

We split up the time in discrete steps. At the end of each time step (denoted by k), each node evaluates the results of its interaction with the network in the following way.

Each node maintains two variables, which are the basis for its strategy function:

- $\beta_i(k)$ represents[2] the number of packets until step k that were originated at node i and were successfully received at the destinations. This number represents the *benefit* for the node.
- $\gamma_i(k)$ represents the number of packets until step k that node i forwarded for other nodes. This number represents the *contribution* of node i to the operation of the network.

We define the *interaction ratio* at step k as the ratio of these two values ($\rho_i(k) = \frac{\beta_i(k)}{\gamma_i(k)}$). If $\gamma_i(k) = 0$, we set $\rho_i(k) = \Omega$, where Ω is an arbitrarily large number with $\Omega < \infty$.

Each node decides for each packet whether to forward it or not, using its own strategy. The strategy of node i is defined in the following way:

- The initial step of node i (for the value $\rho_i(0)$) is: Forward or Drop
- For each subsequent packet:
  - If $\rho_i(k) \geq \kappa_i$, then Forward.
  - Otherwise, Drop.

  The value κ_i is a constant that characterizes the strategy of node i.

In Table I, we show that specific values of κ_i correspond to strategies identified in the literature of game theory [2]. In particular, we call *TFT* the strategy that imposes the benefit for node i to be equal to its contribution to the network (taking into account the average number of relaying nodes ($\bar{\ell}$) on its connections).

Strategy	*Initial step*	κ_i
AllD (always defect)	Drop	∞
AllC (always cooperate)	Forward	0
TFT (Tit-for-Tat)	Forward	$\frac{1}{\bar{\ell}}$

TABLE I

THREE HIGHLIGHTED STRATEGIES

In principle, a node could decide to update its behavior after each packet processing; however, this would be too fine grained. Therefore, we assume that a node reconsiders its decision only at the end of each step. This means that we evaluate simultaneously all packets that the node sends and relays in each step.

We define two constants for each node: (i) B_i stands for the benefit from a single packet for node i, if the packet reaches the destination and (ii) C_i is the forwarding cost at node i for a single packet. For the sake of simplicity, we assume that $B_i = B,\ \forall i$ (i.e., each node enjoys the same benefit if a packet successfully goes through) and $C_i = C, \forall i$ (i.e., each node suffers the same cost for each packet it forwards).

In this paper, we assume that the overall utility of the node is linearly dependent of the benefit of the node. The aim of each node is to maximize its expected average utility per step over an infinite game:

$$max\{\lim_{k\to\infty} E[\bar{U}_i(k)]\} \tag{1}$$

where

$$\bar{U}_i(k) = \frac{B \cdot \beta_i(k) - C \cdot \gamma_i(k)}{k} \tag{2}$$

The node can maximize its utility by decreasing its contribution. However, this might not be beneficial, because this selfish behavior might negatively affect the behavior of other nodes that might be relays for the considered node.

In our model, each source sends a small amount of information at each step that corresponds to a unit of information. For better understanding, we refer to this unit of information as a *packet*[3].

In order to mimic mobility, at the end of each step, we randomly shuffle the nodes on the ring. We assume that the time for a topology change is much higher than the time to send a packet from the source to the destination. Thus, the network is considered to be static during the sending of a single packet.

In the following sections we provide analytical and simulation results for the cooperation of nodes in different scenarios.

III. Analysis of Nash-equilibria with a single relay

To illustrate our approach, we begin with the analysis of a simple and deliberately unrealistic scenario.

[2]We provide the list of symbols used in this paper in the appendix in Table V.

[3]Note that our concept of *packet* is general in the sense that it does not correspond to any specific protocol packet, but it contains a given number of protocol packets. The number of protocol packets is limited by the fact that the time to send a *packet* must be much shorter than the time for topology change.

A. Investigation scenario

We assume that the nodes are organized in a ring (an example network with four nodes is shown in Figure 2). Each node is the source of one connection. We also assume that each connection has one relay (i.e., $\ell = 1$), which is the next node in clockwise direction from the source (according to our notation introduced in Section II-A, this scenario is *Ring-1R*). In general, we assume that $B > C$ (meaning that the node has a "natural" incentive to send packets).

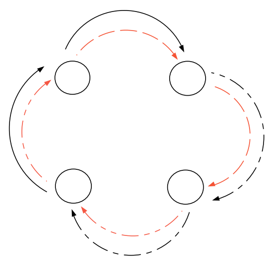

Fig. 2. A ring network with four nodes and four connections. Each node is a source of one connection and each connection has one relay.

B. Equilibrium of TFT strategies

In the following we show that nodes playing the TFT strategy constitute a scenario with stable cooperation.

If any node i drops a packet, then there will be a node r whose packet does not arrive. Concerning the balance of the packets sent and forwarded in the network, this means that every node $j \neq r$ will have $\rho_j = 1$. Since node r forwards in this step, its interaction ratio will drop below the strategy constant ($\rho_r \leq \kappa_r = 1$). In the next step it drops a packet and its interaction ratio will be again equal to one ($\rho_r = \kappa_r = 1$). Now, another node's interaction ratio decreases below one. Because every node applies the TFT strategy, this packet dropping behavior propagates through the network until it gets back to node i. We refer to this propagation as the *contamination* of the defection, in the rest of the paper.

Lemma 1: In Ring-1R, if any node i defects once (and otherwise always cooperates) and all other nodes $j \neq i$ permanently play TFT ($\kappa_j = 1$)[4], then the defection affects node i in expectedly $N-1$ steps (meaning its benefit is reduced because of its own defection).

We provide the proof of the lemma in the appendix.

Our aim is to identify the number of defections of a given node i that are beneficial for it. If this number is equal to 0, it means that it is better for node i to never defect. Let us denote by *x(k)* the number of packets dropped by node i until step k. We denote by *y(k)* the number of packets that were generated at node i and were dropped by other nodes until step k.

[4]Note that the number of relays is one in the considered case.

We refer to nodes that play a strategy whose output is independent of the input as *sinks*. Nodes playing AllC or AllD are examples of sinks. These nodes do not propagate defections. In our approach, we want to define a sequence of actions that result in the highest benefit for node i, thus we assume that its output is *a priori* independent of its input. Thus, we consider node i as a sink.

Because we assume that every node except node i plays TFT, all the defections in the network are consequences of defections done by node i. The number of propagating defections in the network (denoted by *c(k)*) is given by:

$$c(k) = x(k) - y(k) \quad (3)$$

Since there are N nodes in the network, we can state that:

$$E[c(k)] \leq N - 1 \quad (4)$$

This means that the number of propagating defections is upper bounded by the number of nodes on the ring excluding node i.

Lemma 2: In Ring-1R, if node i defects a finite number of times (and otherwise always cooperates) and all other nodes $j \neq i$ play TFT ($\kappa_j = 1$), then $\lim_{k\to\infty} E[c(k)] = 0$.

The proof of the lemma is provided in the appendix.

Now let us formulate a theorem for cooperation for a single node:

Theorem 1: In Ring-1R, if every node $j \neq i$ plays TFT, then the best strategy for node i is a strategy that results in full cooperation (meaning a strategy with $\kappa_i \leq 1$).

For the proof the user is referred to the appendix. Since TFT results in full cooperation in this specific scenario, we can state the following corollary.

Corollary 1: In Ring-1R, if every node plays TFT, it is a Nash-equilibrium.

C. Other Nash-equilibria

Now we focus on the following question: If some of the nodes choose the strategy AllC instead of TFT, does it undermine the Nash-equilibrium among the nodes?

Theorem 2: In Ring-1R, if s nodes play AllC and the other nodes play TFT, it is a Nash-equilibrium if $\frac{B}{s} > C$.

Because of space limitations we provide only the sketch of the proof as follows. If any node i defects in the network, then this defection is sunk by any of the AllC players or by the node itself. Because of the random shuffling used in the scenario, the proportion of the defections that affect node i is:

- $\frac{1}{s-1}$, if node i belongs to the AllC players, and
- $\frac{1}{s}$, if node i belongs to the TFT players.

This fraction of the defections causes a reduction in the benefit of node i. This expected reduction must be greater than the cost a node is able to save.

We finally mention the "worst case" situation in the following theorem:

Theorem 3: In Ring-1R, if every node plays AllD, then it is a Nash-equilibrium.

The proof is trivial: In this case a node does not receive any benefit, no matter what strategy it plays. Hence, it is beneficial for it to defect as well.

IV. Cooperation with multiple relays

Now we extend our analysis for connections with several relays, still considering the nodes placed on a ring. We assume that each node is a source of one connection. Each connection has exactly ℓ relay nodes (i.e., we denote the scenario by *Ring-ℓR*). We also assume that $B > \ell \cdot C$.

As an example, let us assume that the number of relays on each connection is two. Thus, each node i has two relay nodes that are the two nodes clockwise from node i (an example is presented in Figure 3).

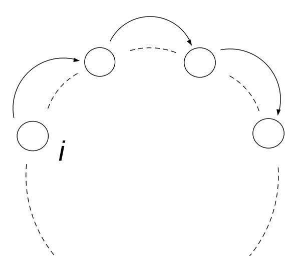

Fig. 3. Ring of nodes with an example connection from node i ($\ell = 2$ in this case)

As in the previous case, at the end of each step, we randomly shuffle the nodes on the ring. We assume that the number of relays (ℓ) is a known parameter for each node.

Theorem 4: In Ring-ℓR, if every node except node i plays TFT, then the best strategy for node i is a strategy that results in full cooperation (play TFT as well, or an even more generous strategy ($\kappa_i \leq \frac{1}{\ell}$)).

The proof is provided in the appendix.

From this theorem we can conclude that:

Corollary 2: In Ring-ℓR, if every node plays TFT, it results in a Nash-equilibrium.

If some of the nodes play AllC, then they might be a sink for the contamination. We can now formulate a theorem for sinks in the multi-hop relaying case.

Theorem 5: In Ring-ℓR, if s nodes play AllC and the rest of the nodes except node i play TFT, then the best strategy for node i (i does not belong to the set of AllC players) is to always cooperate if $s < \ell$.

The proof of the theorem is provided in the appendix.

Corollary 3: In Ring-ℓR, if s nodes play AllC in the network, where $s < \ell$ and the other nodes play TFT, then it is a Nash-equilibrium.

The corollary expresses that the cooperative equilibrium is resistant to the phenomenon of *drift* [12], provided that the number of sinks is below a threshold given by the number of relays at the connections.

Note that the above mentioned analysis does not apply for sinks playing AllD. If at least one such node exists in the network, it might contaminate everyone, independently of the behavior of node i. In this case, the best strategy for node i is to defect in every step.

V. Simulation results

In this section we present simulation results where we vary the connection length: Instead of having a constant number of relays for a connection, we choose the number of relays between two values. We first investigate the ring network and then a more realistic network scenario.

Our analysis presented in Sections III and IV relies on the fact that each connection has the same number of relays. This enables the TFT strategy to constitute a Nash-equilibrium. The idea is that each node contributes as much as it receives from the network. If the number of relays varies, this balance can be undermined. In order to tolerate this possible difference of interaction at each node, we introduce a new strategy.

Inspired by [2], we call *Generous Tit-For-Tat (GTFT)* a strategy that overestimates the required contribution to the network. Thus, a node playing this strategy is *generous*, because it is willing to contribute more to the network than to benefit from it. If a node i plays the GTFT strategy, it uses the following strategy constant:

$$\kappa_i = \frac{1}{\bar{\ell} + g_i} \tag{5}$$

where $\bar{\ell}$ stands for the average number of relays for all the connections of the network during its whole lifetime and g_i stands for the *generosity* of the node. For the sake of simplicity, we choose $g_i = g, \ \forall i$. Note that we get the usual TFT strategy if $g = 0$.

A. Simulations on a ring network

We performed simulations on a ring network with the parameters provided in Table II. We performed each simulation as follows. First, we place nodes with uniform probability on the ring. Then, we generate a connection for every node with the given average number of relays. Then, we let every node send a packet on the connection for which it is the source. At the end of the step, we release the connections. We repeated this procedure for the number of steps.

Parameter	*Value*
Number of nodes	100
Number of relay nodes	1-3
Distribution of the number of relays	uniform
Mobility	random shuffling
Number of simulations	200

TABLE II

PARAMETER VALUES FOR THE SIMULATION ON THE RING

We performed simulations for the average number of relays equal to two (we denote the scenario by *Ring*-$\bar{2}R$). We observe that the network always converges to one of the two extreme states: either all nodes cooperate or all nodes defect. Figure 4 shows the proportion of simulations that result in full cooperation as a function of the generosity. We can observe that the generosity must be reasonably high (compared to the average number of relays) to have full cooperation in the network. If the generosity is above a given threshold (in the example this threshold is equal to 1.6), all simulations result in full cooperation.

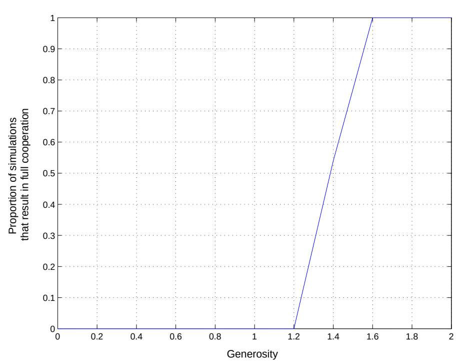

Fig. 4. The proportion of simulations on the ring that end with full cooperation between the nodes (in *Ring*-$\bar{2}R$)

B. Simulations on a realistic network

We simulated a realistic network with the parameters provided in Table III. We performed each simulation as follows. First, we place nodes with uniform probability in the simulation area. Then, we generate a connection for every node with the given average number of relays (we denote the scenario by *Plane*-$\bar{\ell}R$). Then, we let every node send a packet on the connection for which it is the source. We repeat this procedure for the number of steps. In order to improve our simulation scenario further, we introduce a more realistic connection generation model. Instead of generating a connection for each node at each step, we generate a new connection only if the old one breaks because of mobility.

Parameter	*Value*
Number of nodes	100
Number of steps per simulation	500
Duration of one step	variable (1-1024 s)
Area type	Toroid plane
Area size	1500 m x 1500 m
Number of relay nodes	1-3
Distribution of the number of relays	uniform
Number of simulations	200
Radio range	250 m

TABLE III

PARAMETER VALUES FOR THE SIMULATION ON THE REALISTIC NETWORK

We used the random waypoint model with the parameters presented in Table IV. Note that we chose the speed of the nodes as suggested in [31]. In our first simulation, we set the duration of a step to 1024 seconds. In this case, the expected time a node travels to a destination (given the average speed and pause time presented in Table IV) is much shorter than the duration of one step. Thus, with this setting, we approximate the random shuffling of nodes (between two steps the network topology changes completely).

Parameter	*Value*
Mobility model	Random waypoint
Speed	1-19 m/s
Distribution of speed	uniform
Average pause time	10 s

TABLE IV

PARAMETER VALUES FOR THE RANDOM WAYPOINT MOBILITY MODEL

Figure 5 shows the proportion of simulations that result in full cooperation as a function of the generosity. We can see that the realistic connection generation introduces an additional difference among the nodes in terms of required contribution to the network, thus a much higher generosity is needed to ensure full cooperation. The reason for this is that generosity is required to cope with the worst case situation. In the worst case, a node is a relay in a number of connections that is higher than the average number of relays on a connection ($\bar{\ell}$). In the realistic scenario – compared to the situation on the ring – the worst case situation means more connections, where a node has to relay. Hence, more generosity is required to ensure full cooperation.

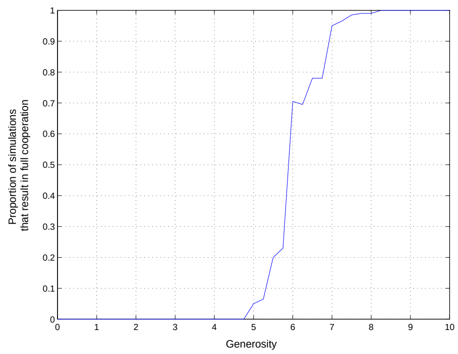

Fig. 5. The proportion of simulations on a realistic scenario that end with full cooperation between the nodes (with *Plane-$\bar{\ell}R$*); step duration is 1024 seconds

In the same model, we investigate the effect of mobility on cooperation. We increase the step duration exponentially (2 to the power of x seconds, where $x = 0, 1, \ldots$), and we observe the required generosity level that ensures that 95 % of the simulations result in full cooperation (we call this value the *generosity threshold*). Figure 6 presents the generosity threshold as a function of the duration of a step (which represents the effect of mobility). We see that if the length of one step is small (meaning that mobility is small), then a higher generosity threshold is required. The higher the mobility, the lower the generosity threshold. This result is fully consistent with our previous work [10]: The absence of mobility is a major hurdle for "spontaneous" cooperation.

As explained before, generosity is needed for nodes that are relays in a high number of connections compared to the average number of relays in a connection. This situation represents the worst case for a node. If the duration of the step is small, then this worst case situation is valid for several steps and the node has to be more generous to cope with the cumulative effect of the situation. If mobility increases (meaning that the topology of the network changes more between the steps), then the duration of a worst case situation is shorter and less generosity is required to cope with its cumulative effect. For a detailed investigation of the effect of mobility on the duration of paths, the reader is referred to [25].

VI. Related work

A. Cooperation without incentive mechanisms

An approach that addresses cooperation in the absence of any incentive mechanism is provided by Srinivasan

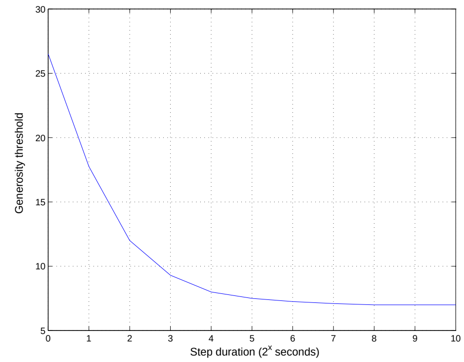

Fig. 6. Generosity threshold ensuring full cooperation as a function of the duration of one step (i.e., the effect of mobility)

et al. [27]. Their work focuses on the energy-efficient aspects of cooperation. In their solution, the nodes are classified in different energy classes. The nodes differentiate among the connections based on the energy classes of the participants and apply different behaviors according to the type of the connection. This framework relies on an ideal mechanism that distributes class information.

A solution based on [27] would require a secure mechanism to prevent malicious nodes from cheating with the class information provided by the relays to the source. We introduce a game theoretical model that does not rely on any additional mechanism, thus we believe our investigations to be more generic. Srinivasan *et al.* also make use of time slots, but they generate only one communication session for the whole network in each time slot. They randomly choose the participating nodes for this session. They show that the GTFT strategy results in a stable cooperation for any positive value of generosity.

Urpi *et al.* [29] propose a general framework for cooperation without any incentive mechanism. Their solution is based on the idea that each node monitors the behavior of other nodes in the neighborhood.

In our previous work [10], we addressed the problem of cooperation in *static* ad hoc networks. Using a framework based on game theory, we were able to identify the necessary and sufficient conditions for cooperation. We showed that cooperation is strongly influenced by the topology of the network. By simulations we assessed the likelihood that the conditions for cooperation will be fulfilled.

B. Incentive mechanism in ad hoc networks

Marti *et. al.* [20] consider an ad hoc network where some misbehaving nodes agree to forward packets but then fail to do so. They propose a mechanism, called *watchdog*, in charge of identifying the misbehaving nodes, and a mechanism, called *pathrater*, that deflects the traffic around them. However, misbehaving nodes are not punished, and thus there is no motivation for the nodes to cooperate. To overcome this problem, Buchegger and Le Boudec [5] as well as Michiardi and Molva [21] define protocols that are based on a reputation system. In both approaches, the nodes observe the behavior of each other and store this knowledge locally. Additionally, they distribute this information in reputation reports. According to their observations, the nodes are able to behave selectively (e.g., nodes may deny forwarding packets for misbehaving nodes).

Zhong *et al.* [33] present a solution, where an off-line central authority collects *receipts* from the nodes that relay packets and remunerates them based on these receipts. Another solution, presented by Buttyan and Hubaux [6], [7], is based on a virtual currency, called *nuglets*: If a node wants to send its own packets, it has to pay for it, whereas if the node forwards a packet for the benefit of another node, it is rewarded.

C. Charging and rewarding in multi-hop cellular networks

An incentive mechanism is proposed for multi-hop cellular networks by Jakobsson *et al.* [14]. They use the concept of lottery tickets to remunerate the forwarding nodes in a probabilistic way. They consider an asymmetric scheme where the uplink (from the initiator to the base station) is multi-hop and the downlink (from the base station to the initiator) is single-hop. Ben Salem *et al.* [4] investigate the symmetric scheme where both uplink and downlink are multi-hop. They use the concept of sessions to authenticate the nodes involved in a given communication and to correctly perform the charging and rewarding mechanism. Lamparter *et al.* [18] consider a charging scheme for ad hoc stub networks that relies on the presence of an Internet Service Provider.

D. Application of game theory to networking

Game theory has been used to solve problems in ad hoc, fixed and cellular networks. Qiu and Marbach [19] define a price-based approach for bandwidth allocation in wireless ad hoc networks. Jin and Kesidis [15] propose a generic mechanism for rate control and study Nash-equilibria in a networking game. Alpcan *et al.* [1] apply game theory for uplink power control in cellular networks. In [32], Xiao *et al.* describe a utility-based power control framework for a cellular system. In [11], Goodman and Mandayam introduce the concept of network assisted power control to equalize signal-to-interference ratio between the users. Korilis *et al.* [16] address the problem of allocating link capacities at routing decisions. In [17], they suggest a congestion-based pricing scheme. Roughgarden [24] quantifies the worst-possible loss in network performance arising from non-cooperative routing behavior. In [30], Yaïche *et al.* present a game theoretical framework for bandwidth allocation. They study the centralized problem and show that the solution can be distributed in a way that leads to a system-wide optimum.

E. Cooperation studies in other areas of science

The emergence of cooperation has also been studied in a biological [9], a sociological [22] and an economical [26] context. Most of these studies use the *Iterated Prisoner's Dilemma (IPD)* game as their underlying model (see e.g., Axelrod [2], [8], Rapaport and Chammah [23] or Trivers [28]). The simplicity of the IPD makes it an attractive model, but it is not appropriate for modeling packet forwarding because it involves only two players that, in addition, have symmetric roles. Consequently, in this paper, we have defined a multi-player, asymmetric game that better suits our purposes. In [2], Axelrod identifies *Tit-for-Tat (TFT)* as a robust strategy that performs surprisingly well (in terms of maximizing the player's payoff) in many situations. In [3], Axelrod gives an overview of scenarios with imperfect information. He identifies that the *Generous TFT (GTFT)* strategy results in equilibrium in this case.

VII. Conclusion and future work

In this paper, we have studied the level of cooperation in packet forwarding, where the nodes have a selfish (but not malicious) behavior. We have adopted a game theoretical approach, in which a node considers that it plays against the rest of the network. In this model, the node does not need to distinguish between the behavior of the different other nodes, which has the benefit of avoiding the intricacies of node authentication or the burden of complex schemes based on reputation. With this new model, we have stated and proved several theorems, expressing the conditions for the existence of cooperation; we have quantified the tolerance to the phenomenon of drift, as well as the level of generosity required, in the case the connections have varying lengths.

We have then considered a more realistic model, where the nodes are on the plane and move according to the random waypoint mobility pattern. We have shown that the generosity required to reach cooperation is much higher in this case, and we have quantified the relationship between mobility and cooperation. We have concluded that cooperation is easier to reach when mobility is higher.

In terms of future work, we intend to relax the assumption according to which each node is the source of exactly one connection; we would also like to take the battery levels of the nodes into consideration. Finally, we plan to study the impact of mainstream ad hoc routing protocols (e.g., DSR and AODV) on the conditions under which spontaneous cooperation may exist.

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APPENDIX

Proof of Lemma 1:

Let us consider the expected number of steps (denoted by t) after which this dropping affects node i (meaning that another node drops its packet).

We assume that in step K node i dropped someone's packet for the first time and in step $K+t$ someone drops the packet of node i (as all the other nodes play TFT). It means that in all steps $k < K+t$, other nodes forwarded for node i, but in step $K+t$ it is not the case.

Given the scenario, there is one node at each step that is contaminated with defection. The probability p that the contaminated node is relaying for node i in any step $k > K$ is given by:

$$p = \frac{1}{N-1} \tag{6}$$

where N stands for the number of nodes in the network.

Thus, the probability (q) that the contaminated node in step $K+t$ will relay for node i in that step and no contaminated node relayed for node i in the previous steps ($K < k \leq K+t-1$) is given by:

$$q = (1-p)^t \cdot p \tag{7}$$

This corresponds to a geometric distribution with respect to t. The expected value of the geometric distribution is (substituting the given p value):

$$E[t] = \frac{1}{p} = N-1 \tag{8}$$

This means that, on the average, the defection comes back to node i in $N-1$ steps. ■

Proof of Lemma 2:

Let us assume that the node defects D times, where $D < \infty$, and cooperates in all the other steps. For the sake of simplicity, we assume that the node defects in the first D steps and consider the expected value of $c(k)$ in the subsequent steps. The general case can be proven in a similar way.

For any step $k > D$, we can write the expected number of propagating defections:

$$E[c(k)] = \sum_{i=1}^{D} (1-p)^{k-i} \tag{9}$$

$$= (1-p)^k \cdot \sum_{i=1}^{D} (1-p)^{-i} \tag{10}$$

where p is the probability that a contaminated node is relaying for node i. But in (10) the second factor is a finite number. The first factor goes to zero if $k \to \infty$, because $0 < 1-p < 1$. Hence:

$$\lim_{k\to\infty} E[c(k)] = 0 \tag{11}$$

■

Proof of Theorem 1:

We assume that each node wants to maximize its expected average utility per step over an infinite game as expressed in (1).

Let us compute the expected average utility until step k:

$$\begin{aligned} E[\bar{U}_i(k)] &= E[\frac{B\cdot\beta_i(k) - C\cdot\gamma_i(k)}{k}] \\ &= E[\frac{B\cdot(k-y(k)) - C\cdot(k-x(k))}{k}] \\ &= E[\frac{(B-C)\cdot k - (B\cdot y(k) - C\cdot x(k))}{k}] \\ &= (B-C) - E[\frac{(B\cdot y(k) - C\cdot x(k))}{k}] \end{aligned} \tag{12}$$

Let us assume that node i cooperates at each step; then $y(k) = x(k) = 0$ for any step k (because the other nodes play TFT). In this case, the average utility for node i (until any step k) is given by:

$$\bar{U}_i(k) = B - C \tag{13}$$

To be superior to the always cooperating strategy, an alternative strategy should be such that the second term in (12) is positive (taking also the minus sign into account):

$$\begin{aligned} -E[\frac{(B\cdot y(k) - C\cdot x(k))}{k}] &\geq 0 \\ E[-B\cdot y(k) + C\cdot x(k)] &\geq 0 \\ E[-B\cdot(x(k)-c(k)) + C\cdot x(k)] &\geq 0 \\ E[(C-B)\cdot x(k)] + E[B\cdot c(k)] &\geq 0 \\ B\cdot E[c(k)] &\geq (B-C)\cdot E[x(k)] \\ E[x(k)] &\leq \frac{B\cdot E[c(k)]}{B-C} \qquad (14) \\ E[x(k)] &\overset{1)}{\leq} \frac{B\cdot(N-1)}{B-C} \qquad (15) \end{aligned}$$

1) From (4) we know that the expected number of propagating defections *c(k)* is upper bounded by $N-1$.

Hence, if node i wants to play a strategy that outperforms TFT until step k, the expected number of beneficial defections is upper bounded by a constant.

According to Lemma 2, if the node defects a constant number of times, then the expected number of propagating defections goes to zero with the number of steps. Using this statement in (14):

$$\lim_{k\to\infty} E[c(k)] = 0 \Rightarrow \lim_{k\to\infty} E[x(k)] = 0 \tag{16}$$

Symbol	*Definition*	*Section*
$\beta_i(k)$	Number of packets originating at node i that have reached the destination until step k	Section II
$\gamma_i(k)$	Number of packets relayed by node i until step k	Section II
$\rho_i(k)$	Interaction ratio for node i	Section II
κ_i	Strategy constant of node i	Section II
ℓ	Number of relays on each connection	Section II
$\bar{\ell}$	Average number of relays on the connections, if the connection length varies	Section V
B	Benefit enjoyed by the source for one packet reaching the destination	Section II
C	Relaying cost of one packet	Section II
$x(k)$	Number of packets dropped by a considered node i until step k	Section III
$y(k)$	Number of packets others drop for a considered node i until step k	Section III
$c(k)$	Number of propagating defections at step k in the single-hop case	Section III
g	Generosity for each node	Section V

TABLE V

TABLE OF SYMBOLS USED IN THE PAPER

Thus, if the node wants to maximize its expected utility for the whole duration of the game, its best strategy is to cooperate in every time step. ■

Proof of Theorem 4:

We prove the theorem for $\ell = 2$ which corresponds to the example of Figure 3; as we will see, the proof can be extended to any value of l. Let us denote the number of contaminated nodes at step k by $n(k)$.

Let us assume that node i defects in an arbitrary step K. It contaminates the nodes whose packets are dropped (in this case two nodes, because $\ell = 2$ and the topology is a ring). In the next step, these two nodes contaminate other nodes, and so on. We will show that the number of contaminated nodes is non-decreasing: If we have $n(k)$ contaminated nodes in step k (where $k > K$), then the number of contaminated nodes in step $k+1$ is $n(k+1) \geq n(k)$. An example for $n(k) = 2, \ell = 2$ is presented in Figure 7. If the contaminated nodes in step k happen to become neighbors on the ring (Figure 7a), then they drop the packets for three nodes. Since node i is a sink for contamination, if node i is among the three nodes, then $n(k+1) = 2$, otherwise, $n(k+1) = 3$. If they are not neighbors (Figure 7b), then $n(k+1) = 3$ or $n(k+1) = 3$ depending on whether node i is among the contaminated nodes or not.

One can see that this contamination continues until it reaches all nodes in the network. During this procedure, node i suffers more and more decrease in its benefit as more and more nodes defect in the network. After the contamination reaches every node in the network (we call $K^{'}$ the step at which it happens), the benefit of node i becomes zero.

Before node i defects in step K, none of its packets are dropped ($y(k) = 0$). During the contamination (in the steps $K < k < K^{'}$), a number of packets, denoted by a, is dropped for node i. After step $K^{'}$, all nodes in the network, except node i, defect.

In step $k > K^{'}$, the number of packets that were originating at node i and are dropped by other nodes is:

$$y(k) = 0 + a + (k - K^{'}) = k - (K^{'} - a) \qquad (17)$$

We know that $x(k) \leq \ell \cdot k$. Furthermore, $x(k)$ is not a random variable, but it is determined by the sequence of actions of node i. Thus, $E[x(k)] = x(k)$.

We can express the expected average utility for $k > K^{'}$ as follows:

$$\begin{aligned}
E[\bar{U}_i(k)] &= E[\frac{B \cdot \beta_i(k) - C \cdot \gamma_i(k)}{k}] \\
&= E[\frac{B \cdot (k - y(k)) - C \cdot (\ell \cdot k - x(k))}{k}] \\
&= E[\frac{(B - \ell \cdot C) \cdot k - (B \cdot y(k) - C \cdot x(k))}{k}] \\
&= (B - \ell \cdot C) - E[\frac{(B \cdot y(k) - C \cdot x(k))}{k}] \\
&= (B - \ell \cdot C) - B \cdot \frac{k - (K^{'} - a)}{k} + C \cdot \frac{x(k)}{k} \\
&= (B - \ell \cdot C) + C \cdot \frac{x(k)}{k} - B \cdot (1 - \frac{(K^{'} - a)}{k}) \\
&= -\ell \cdot C + C \cdot \frac{x(k)}{k} + B \cdot \frac{(K^{'} - a)}{k} \\
&\leq B \cdot \frac{(K^{'} - a)}{k}
\end{aligned}$$

Hence, for $k > K^{'}$, we have:

$$\lim_{k \to \infty} \bar{U}_i(k) \leq 0 \qquad (18)$$

Since the game is going to infinity, the expected average utility converges to a number that is not greater than zero.

Hence, the best strategy for the node is to cooperate in all step.

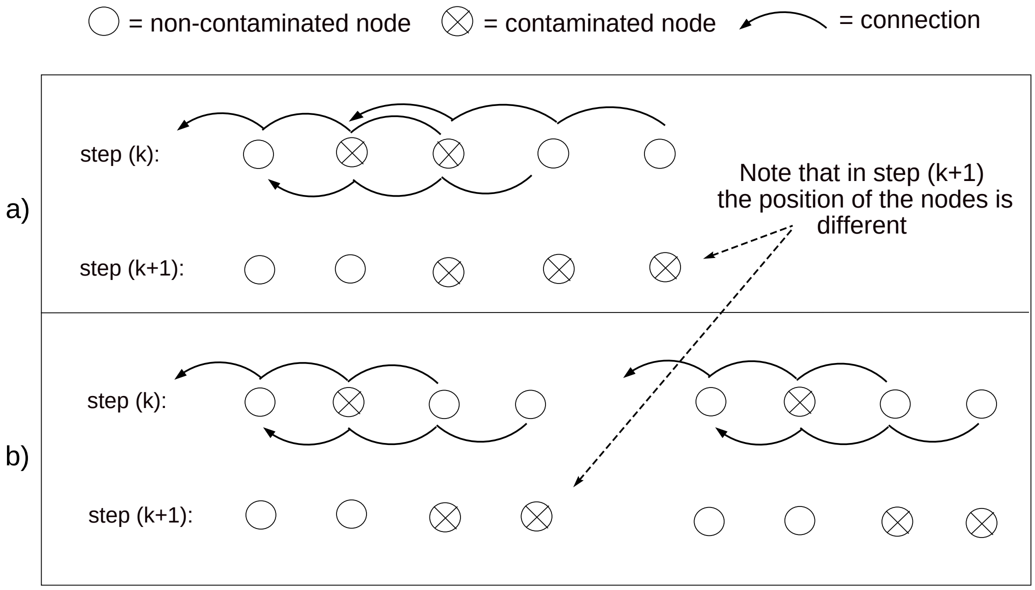

Fig. 7. *Contamination effect on the ring for $\ell = 2$, $n(k) = 2$:* In a) the two contaminated nodes are neighbors on the ring in step k; in this case, they contaminate at least two other nodes (note that if node i is among the nodes whose packets were dropped, then it sink the contamination). In b) the contaminated nodes are not neighbors; thus, they contaminate four other nodes.

The theorem can be proved for any value of ℓ in a similar way. ■

Proof of Theorem 5:

Because of the multi-hop scenario, node i is a relay in ℓ connections. If it defects, it contaminates the ℓ nodes that are sources on these connections. We distinguish two cases:

1) If $s < \ell - 1$ or at any step some of the ℓ sinks do not relay for node i. In this case, more than one node will be contaminated in the first step. This implies that in the subsequent steps, the contamination will reach the whole network. Let us consider step K when all nodes are contaminated except node i and the nodes that play the AllC strategy. Because the number of relays ℓ is greater than the number of AllC players s, there is at least one relay node on the connection originating from node i that drops the packet. Thus, after step K, no packet of node i reaches the destination. Hence, the proof for Theorem 4 applies for this case. Again, the best strategy for node i is to cooperate in every step.
2) If $s = \ell - 1$ and the s sinks are within the ℓ nodes contaminated by defection in each and every step. Clearly, because of the random shuffling at each step, the probability of this sequence of events is extremely small. In this case, only one node will contaminate in the next step. In the subsequent steps, the contamination of this one node continues until it is sunk by node i itself. Hence the proof in Theorem 1 applies (we are back to the case where one defection propagates in the network, because the other defections are constantly sunk by the AllC players). Thus, in this case, defection is not beneficial for the node.

We can thus conclude that the best strategy for node i is to always cooperate. ■