The Role of Tarski's Declarative Semantics in the Design of Modeling Languages

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Abstract. This paper focuses on Tarski's declarative semantics and their usefulness in the design of a modeling language. We introduce the principles behind Tarski's approach to semantics and explain what advantages this offers in the context of modeling languages. Using sentential logic we demonstrate the necessity and sufficiency of Tarski's semantics for effectively addressing several issues that arise in the design of modeling languages. We explain what role Tarski's semantics play in the organization of a modeling language. This role is compared to the analogous roles of denotational semantics and operational semantics. We show that in the context of a modeling language Tarski's semantics are complementary to the other two kinds of semantics. The paper is intended to assist modeling language researchers and designers, particularly in connection with the UML - a language that in its current form does not feature Tarski's declarative semantics.

1 Introduction

For designers and users of the UML alike, "semantics" in the context of the UML is today associated with the concept of denotational semantics, or in some cases operational semantics. This situation can be explained by the fact that the UML evolved from software systems modeling where denotational and operational kinds of semantics are the traditional subjects of research and practice. In defining the semantics of the UML, therefore, it was natural for this community to transfer the concepts associated with "language semantics" from the traditional domain of programming languages. However, in the latest official UML specification [6] available from OMG the scope of the language is defined as follows: "*The Unified Modeling Language (UML) is a language for specifying, visualizing, constructing, and documenting the artifacts of software systems, as well as for business modeling and other non-software systems*" (see [6] section 1.1). Thus the UML is defined as a general system modeling language and not as a programming language, and therefore for UML as for any other modeling language denotational and operational semantics are not sufficient.

In this paper we will explain why Tarski's declarative semantics, introduced by Alfred Tarski in 1935, and which are absent in the current UML specifications, are necessary for the definition of a modeling language. In Section 2 we introduce the notion of Tarski's semantics. In Section 3 readers are acquainted with the advantages that Tarski's semantics bring to the definition of modeling languages, and are introduced to the formal logical justifications for the guarantees of these advantages. In Section 4 we review two other kinds of semantics that are relevant in the context of modeling languages: denotational semantics and operational semantics. The reader is then shown the differences between Tarski's semantics and these two more traditional forms of language semantics. In particular, by reviewing the application contexts for these different kinds of semantics we explain how the role of Tarski's semantics is complementary to the role of denotational and operational semantics in the definition of modeling languages.

The primary purpose of this paper is to clarify the foundations of modeling languages design, and to clarify the important contribution that Tarski's declarative semantics can play. At present this potential has not been recognized by the UML modeling community and, judging from the official UML specifications [6], by the UML designers.

2 Tarski's Declarative Semantics

2.1 Universe of Discourse versus Representation Domain

A modeling language provides a vehicle for a modeler to create a representation of some subjects of interest. Thus, from the outset we can differentiate two domains:

- Universe of Discourse, that is the domain where the subjects of modeling interest are positioned;
- Representation Domain, that is the domain where the representation means as well as the results of representations are positioned.

These two domains are defined assuming the presence of an interested party (e.g. a modeler, a modeling community, a modeling language designer, etc) who has a representation-related interest with regard to some subject. Obviously if there is nobody who would have this interest in representation, then it is impossible to introduce this differentiation between the Universe of Discourse and the Representation Domain. The principal difference between these two domains is that the latter is under the responsibility and control of the interested party, while for the former in the general case we can affirm neither the presence nor absence of the responsibility and control of the interested party. Indeed, in some cases the Universe of Discourse is beyond the responsibility and control of the interested party, while in the other cases (in particular, when the subjects of modeling interest are themselves the results of a prior represen tation) the Universe of Discourse is under the responsibility and control of the interested party.

Thus, a modeling language itself belongs to the Representation Domain, and at the same time its purpose is to serve as the means to represent the Universe of Discourse. So a modeling language *must* have some links that would relate the Representation Domain with the Universe of Discourse, otherwise it will be impossible to make a reference to the Universe of Discourse from within the Representation Domain and thus – impossible to make any representation. Obviously, these links are unidirectional relations, that is they are a part of the modeling language (which in turn belongs to the Representation Domain), and they are not a part of the Universe of Discourse.

Tarski's Theory of Truth introduced in 1935 [7] allows the definition of such links. In the context of a modeling language these links form the foundation for Tarski's declarative semantics for the modeling language.

2.2 Basic Principles of Tarski's Declarative Semantics

One of the first tasks in designing a modeling language is to identify the scope of the subjects that the language should be able to represent; in other words (as explained in the previous section) - to identify the Universe of Discourse. Tarski's declarative semantics therefore have to be considered right from the beginning of the design process, before other forms of semantics are considered. In this process of identification, the Universe of Discourse has to be differentiated from within the universal scope. That is, the Universe of Discourse containing all the things to be represented by the modeling language must be separated from, and thus related to, all the other domains containing things of no representation interest. In this way the conceptual limits of the Universe of Discourse are defined, and thus, automatically, an initial conceptualization of the Universe of Discourse is performed. By identifying what the Universe of Discourse is, and thus relating it to what it is not, the Universe of Discourse is automatically conceptualized.

According to Tarski's Theory of Truth [7] this conceptualization process is the first step in the definition of Tarski's declarative semantics. The second step is then to designate a term in the modeling language to be related to the constructed conceptualization and to declare that this given term is relevant in the Representation Domain if and only if the respective conceptualization is relevant in the Universe of Discourse.

At the most abstract level of the conceptualization is the Universe of Discourse itself. We can define that there is the concept of the Universe of Discourse for which the language designer has a representation interest and automatically assume that everything else (for which the designer does not have the representation interest) is not the Universe of Discourse. We can then designate a term in the designed modeling language that would be bound to the Universe of Discourse by the rules of Tarski's declarative semantics. For example, in many languages the term 'Model' represents the Universe of Discourse at the most abstract level, so we may declare that in these languages Model is relevant for consideration in the Representation Domain if and only if the Universe of Discourse is relevant for consideration.

More specialized conceptualizations are created in order to increase the representation capabilities of modeling languages. For example, as it was done in [2], we may be interested in representing the things that happen in the Universe of Discourse, and to designate the term 'Action' in a modeling language for the representation of 'something that happens' within the Universe of Discourse. In this case we will have an Action in the Representation Domain if and only if there is 'something that happens' in the Universe of Discourse. In this way we can proceed to define Tarski's declarative semantics for the terms of any modeling language.

3 Benefits of Tarski's Declarative Semantics

3.1 Sufficiency of Tarski's Semantics

In this section we explain why Tarski's declarative semantics are important for a modeling language. We can identify two main reasons:

I) they allow explicit conceptualizations of the Universe of Discourse to be presented;

II) they allow unambiguous references from the terms of a modeling language to the defined conceptualizations of the Universe of Discourse to be presented.

The 'I)' factor allows the users of a particular modeling language to understand the representation capabilities of the language. Such an understanding is very important because it removes the temptation to apply the language in cases for which it is not designed. That is, such an understanding prevents the kind of failures that occur in modeling projects when modelers try to resolve modeling problems using an inadequate conceptual toolkit.

The unambiguity in 'II)' factor captures the necessity and sufficiency of the Universe of Discourse conceptualization to enable terms within a particular modeling language to participate in affirmative assertions within the Representation Domain. Thus the 'II)' factor gives to the users of a particular modeling language the possibility to reason formally about the adequateness of representations and about the coherency of interpretations within representations of the Universe of Discourse.

The *adequacy of representations* and the *coherency of interpretations* within the representations are two essentially different things. For example, a result of a coherent interpretation can be an inadequate representation (as illustrated later in the paper).

To confirm the value of the 'II)' reason let us look at the original example used by Tarski to illustrate his declarative semantics definition [7]: 'It snows' is true (in the Representation Domain) if and only if it snows (in the Universe of Discourse), - see Figure 1. In this example we declare the term of some hypothetical modeling language 'It snows' to correspond unambiguously to the conceptualization "it snows" that is part of the Universe of Discourse.

Firstly, this declaration makes it possible to formally qualify a representation chosen in a particular modeling language as either adequate or as inadequate. Indeed, in the "if and only if" conditions of Tarski's declarative semantics the first part ("if") guarantees the sufficiency and the second part ("only if") – the necessity of the relevance of the Universe of Discourse conceptualization for the corresponding assertion in the Representation Domain. Thus, because of the sufficiency, when it snows in the Universe of Discourse the representation <<'It snows' is true>> will be adequate. If we define the Tarski's semantics for any other term that would correspond to a different (from "it snows") conceptualization of the Universe of Discourse (e.g. the term 'It does not snow' and the semantics: 'It does not snow' is true if and only if it does not snow) then because of the necessity of this differing from "it snows" conceptualization a positively affirmed representation (e.g. <<'It does not snow' is true>>) will be inadequate for the case when it snows in the Universe of Discourse.

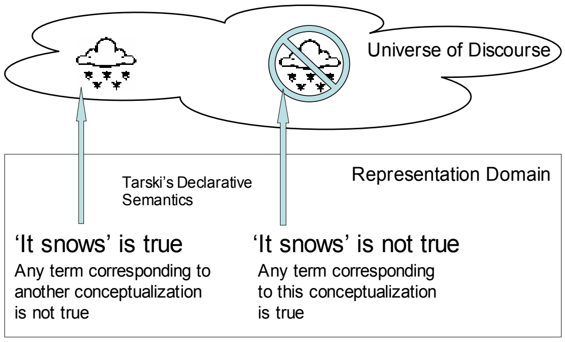

Fig. 1. Example of Tarski's declarative semantics definition for 'It snows'

As we will show later in this paper, without Tarski's declarative semantics different mutually contradictory models can falsely appear to adequately represent the same subject of modeling and a single model can falsely appear to adequately represent different mutually conflicting subjects of modeling. Thus, using Tarski's declarative semantics it is possible to formally distinguish adequate and inadequate representations – a fact that is very important.

In addition to the unambiguously determined adequacy of representations, Tarski's semantics declarations also allow the particular interpretations of the Universe of Discourse to be unambiguously interpreted either as coherent or as incoherent. This promotes coherent interpretations of the Universe of Discourse in a given modeling project (and respectively hinders incoherent interpretations). Indeed, conceptualizations of the Universe of Discourse are necessary and sufficient for their respective modeling terms to be true within a particular representation in the Representation Domain. Thus, should a conceptualization of the Universe of Discourse be incoherently interpreted in several individual cases within the representation (that is, should the conceptualization be mapped to several conflicting assertions in the representation), then the resulting incoherency formally described and immediately highlighted.

To summarize, the *presence* of Tarski's declarative semantics:

A: encourages an understanding of the application limits of a modeling language by providing explicit conceptualizations of the Universe of Discourse;

B: makes it possible to reason formally about the adequateness of representations of the Universe of Discourse;
C: makes it possible to reason formally about the coherency of interpretations within representations of the Universe of Discourse.

Tarski's declarative semantics therefore clearly provide a sufficient basis for endowing a modeling language with these three highly desirable properties.

3.2 Necessity of Tarski's Semantics

Let us now turn to the necessity of Tarski's semantics as the basis for modeling language definition. In other words, let us check whether or not the *absence* of Tarski's declarative semantics hinders the attainment of the three aforementioned properties. The absence of Tarski's declarative semantics basically means that one of the following three cases is realized:

- Case 1: there is no conceptualization of the Universe of Discourse;
- Case 2: there is some conceptualization of the Universe of Discourse, but there are no references from the language terms to that conceptualization;
- Case 3: there is some conceptualization of the Universe of Discourse that is referred to by the language terms, but in these references there is no fulfillment of the condition of necessity and sufficiency of the Universe of Discourse conceptualizations for an assertion containing the respective language terms.

In the following parts of this section we examine the necessity of Tarski's semantics for each of the three desirable properties (A, B and C). Figure 2 shows the sections of the paper in which the examinations will be presented.

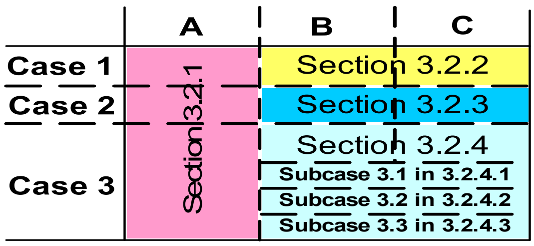

Fig. 2. Justification of necessity of Tarski's semantics

3.2.1 Necessity for the Universe of Discourse Conceptualizations. The second and the third of the three cases introduce some conceptualizations of the Universe of Discourse, so in these cases the absence of Tarski's semantics does not necessarily mean that it is impossibile to understand the application limits of modeling languages. Therefore Tarski's declarative semantics are formally sufficient but not formally necessary for such an understanding (i.e. such an understanding is also possible with other techniques that satisfy either the second or the third case).

Let us now check if in all of the three cases it is impossible to guarantee the unambiguity in adequateness of representations and the coherency of interpretations within the representations of the Universe of Discourse.

3.2.2 Necessity for the Unambiguity of Adequateness and of Coherency: Case 1. In the first case the conceptualization of the Universe of Discourse is not defined, and thus here it is even impossible to identify what should be adequately represented and coherently interpreted. Any representation of an unidentified subject is baseless and thus in this case there is no ground to guarantee the unambiguity in adequacy and in coherency.

3.2.3 Necessity for the Unambiguity of Adequateness and of Coherency: Case 2. Let us now explore if in the second of the three cases it is also impossible to guarantee the unambiguity of adequacy and coherency. To enhance the presentation of our reasoning let us first introduce a simple example of the definition of Tarski's declarative semantics.

In this example we decide to differentiate two conceptualizations of colors: black and white in the Universe of Discourse. We introduce the two modeling terms: 'Black' and 'White', respectively, and define the Tarski's semantics as follows:

'Black' is true (in a representation) if and only if there is black (in the Universe of Discourse);

'White' is true (in a representation) if and only if there is white (in the Universe of Discourse).

These semantics are shown on the left part of Figure 3.

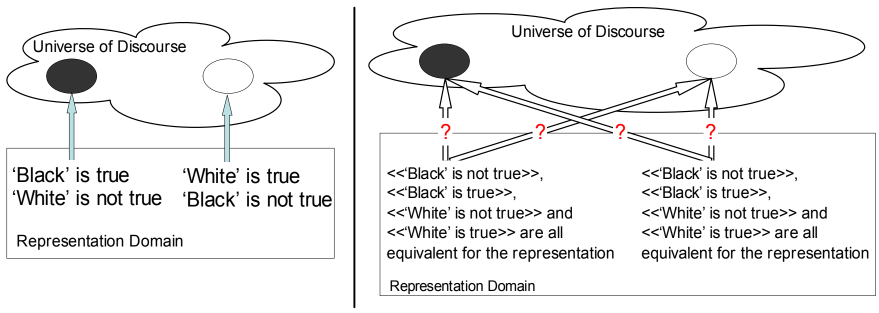

Fig. 3. Example: Tarski's semantics definition for 'Black' and 'White'

Now let us return to the second of our three cases, where the correspondence of the language terms to the conceptualization of the Universe of Discourse is not defined, as it is illustrated on the right part of Figure 3 using the example of our introduced black and white colors conceptualizations and 'Black' and 'White' modeling terms.

In this situation we do not have the two relations that we defined in our example; moreover we do not have any other relation between the modeling terms and the Universe of Discourse conceptualizations. Because of this absence of relations we cannot formally affirm the truth of any representation of the Universe of Discourse that uses our introduced modeling terms.

So in this case all of the possible assertions containing the modeling terms ‘Black’ and ‘White’ are equally valid (also equally invalid) for a representation. Thus, in the case of two mutually contradicting representations of the same Universe of Discourse conceptualization (e.g. <<‘Black’ is true>> and <<‘Black’ is not true>>) it is not possible to formally qualify their adequacy (or inadequacy). Also in the case presented on the right hand side of Figure 3 it is possible that coherency of interpretations within a representation cannot formally be resolved. E.g. within a single representation, the simultaneous truth of the statements <<‘Black’ is true>> and <<‘Black’ is not true>> could be the result of an incoherent interpretation (if they interpret the same conceptualization of the Universe of Discourse) but could also be the result of a coherent interpretation (if they interpret different conceptualizations of the Universe of Discourse). In our case, since there are no relations defined between the modeling terms and the conceptualizations, we cannot formally qualify the coherency (or incoherency) of the interpretation and therefore both are possible.

Thus we have demonstrated that in the second of the three cases of the absence of Tarski’s semantics (when the correspondence of the language terms to the conceptualization of the Universe of Discourse is not defined) it is impossible to guarantee unambiguity in the adequateness of representations and in the coherency of interpretations within the representations.

3.2.4 Necessity for the Unambiguity of Adequateness and of Coherency: Case 3. Let us now consider the last of the three cases of the absence of Tarski’s semantics, namely the case when there are some relations defined between the modeling language terms and the Universe of Discourse conceptualizations, but there is no fulfillment of the necessity and sufficiency conditions of the conceptualizations for the assertions containing the language terms within a representation. With regard to the necessity and sufficiency conditions we can divide this case into the following three subcases:

- Subcase 3.1: the conceptualizations are necessary but not sufficient for the positive assertions containing the language terms within a representation;
- Subcase 3.2: the conceptualizations are sufficient but not necessary for the positive assertions containing the language terms within a representation;
- Subcase 3.3: the conceptualizations are not sufficient and not necessary for the positive assertions containing the language terms within a representation.

Let us first reformulate the conditions for these three subcases, expressing them differently, using the sentential logic (e.g. see in [1] chapter 4), which may facilitate their understanding. Let us introduce the following denotations:

C stands for a conceptualization of the Universe of Discourse.

A stands for a modeling language term.

Then Tarski’s declarative semantics assume that <<***A*** is true>> if and only if ***C***. In other words, ***C*** is necessary and sufficient for <<***A*** is true>>.

Necessity: “***C*** is necessary for <<***A*** is true>>” means that if there is no ***C*** then <<***A*** is true>> is not valid. In logical symbols: (¬***C*** => ¬<<***A*** is true>>).

Sufficiency: “***C*** is sufficient for <<***A*** is true>>” means that if there is ***C*** then <<***A*** is true>> is valid. In logical symbols: (***C*** => <<***A*** is true>>).

Tarski's semantics: "***C*** is necessary and sufficient for <<***A*** is true>>" means that if there is no ***C*** then <<***A*** is true>> is not valid, and if there is ***C*** then <<***A*** is true>> is valid. In other words: <<***A*** is true>> is valid if and only if there is ***C***. In logical symbols: ((¬***C*** => ¬<<***A*** is true>>) & (***C*** => <<***A*** is true>>))

Absence of necessity: "***C*** is not necessary for <<***A*** is true>>" means that it is not true that if there is no ***C*** then <<***A*** is true>> is not valid. In logical symbols: (¬(¬***C*** => ¬<<***A*** is true>>)).

Absence of sufficiency: "***C*** is not sufficient for <<***A*** is true>>" means that it is not true that if there is ***C*** then <<***A*** is true>> is valid. In logical symbols: (¬(***C*** => <<***A*** is true>>)).

Thus we can reformulate our subcases (3.1-3.3) as follows:

Subcase 3.1 (presence of necessity and absence of sufficiency): "***C*** is necessary and is not sufficient for <<***A*** is true>>" means that if there is no ***C*** then <<***A*** is true>> is not valid, and that it is not true that if there is ***C*** then <<***A*** is true>> is valid. In logical symbols: ((¬***C*** => ¬<<***A*** is true>>) & ¬(***C*** => <<***A*** is true>>)).

Subcase 3.2 (presence of sufficiency and absence of necessity): "***C*** is sufficient and is not necessary for <<***A*** is true>>" means that that if there is ***C*** then <<***A*** is true>> is valid, and it is not true that if there is no ***C*** then <<***A*** is true>> is not valid. In logical symbols: ((***C*** => <<***A*** is true>>) & ¬(¬***C*** => ¬<<***A*** is true>>)).

Subcase 3.3 (absence of necessity and absence of sufficiency): "***C*** is not necessary and is not sufficient for <<***A*** is true>>" means that it is not true that if there is no ***C*** then <<***A*** is true>> is not valid, and that it is not true that if there is ***C*** then <<***A*** is true>> is valid. In logical symbols: (¬(¬***C*** => ¬<<***A*** is true>>) & ¬(***C*** => <<***A*** is true>>)).

3.2.4.1 Necessity for the Unambiguity of Adequateness and of Coherency: Case 3, Subcase 3.1. Let us consider an example for Subcase 3.1 (the existing relations are necessary but not sufficient). Suppose that we have two conceptualizations: "black" and "white", two modeling terms: 'Black' and 'White', and the following defined correspondence: *'Black' is true (in a representation) only if there is black (in the Universe of Discourse), but if there is only black (in the Universe of Discourse) then it is not true that 'Black' is true (in a representation)*. In other words, the fact that we know about the correspondence between the positive assertion <<'Black' is true>> within a representation and the "black" conceptualization of the Universe of Discourse is that the conceptualization is necessary but not sufficient for the positive assertion.

In accordance with the previously explained conditions of the presence of necessity and absence of sufficiency in this example the following relations are valid:

If there is no black (in the Universe of Discourse) then <<'Black' is true>> is not valid (in a representation), and it is not true that if there is black (in the Universe of Discourse) then <<'Black' is true>> is valid, - see the left part of Figure 4.

Let us present two different instances for the introduced correspondence between <<'Black' is true>> and "black". The first is: *'Black' is true (in a representation) if and only if there is black and there is no white (in the Universe of Discourse)*, - see the middle part of Figure 4. The second is: *'Black' is true (in a representation) if and* *only if there is black and there is white (in the Universe of Discourse)*, - see the right part of Figure 4.

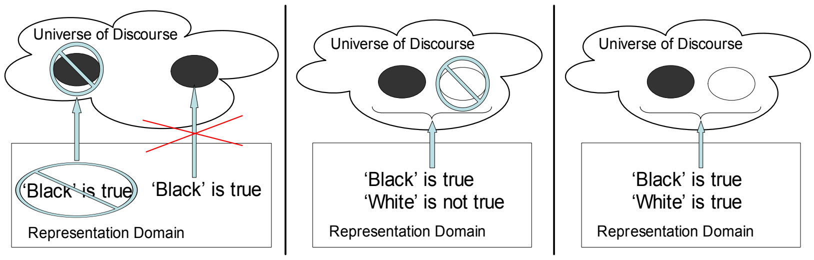

Fig. 4. Example of the absence of Tarski's semantics, case 3 of 3, subcase 3.1

Obviously in our subcase (the left part of Figure 4), without knowing which of the instances is realized, we cannot guarantee unambiguity in the adequacy of representations. E.g. we cannot know which of the representations is adequate, the one where <<'Black' is true>> and <<'White' is true>> can be valid simultaneously (as it can be with the definitions presented in the right part of Figure 4), or the one where <<'Black' is true>> cannot be valid if <<'White' is true>> is valid at the same time (as it can be with the definitions presented in the middle part of Figure 4).

Analogously in this subcase we cannot have a formal guarantee for the unambiguity in coherency of interpretations within a representation. E.g. within a single representation the simultaneous truth of the statements <<'Black' is true>> and <<'White' is not true>> can be the result of an incoherent interpretation (as it can be with the definitions presented in the right part of Figure 4) and also can be the result of a coherent interpretation (as it can be with the definitions shown in the middle part of Figure 4).

Thus we have shown that the conditions of Subcase 3.1 are not sufficient to reason formally about the adequateness of representations and about the coherency of interpretations within a representation of the Universe of Discourse.

3.2.4.2 Necessity for the Unambiguity of Adequateness and of Coherency: Case 3, Subcase 3.2. Let us now examine Subcase 3.2 (the existing relations are sufficient but not necessary). Suppose that we have conceptualizations "black" and "no black", a modeling term 'Black', and the following defined correspondence: *'Black' is true (in a representation) if there is black (in the Universe of Discourse), but if there is no black (in the Universe of Discourse) then it is not true that 'Black' is true (in a representation) is not valid*. That is the conceptualization "black" of the Universe of Discourse is sufficient but not necessary for the positive assertions containing the modeling term 'Black' within a representation. This correspondence is in agreement with the previously explained conditions of the absence of necessity and presence of sufficiency; indeed, here the following relations are valid:

If there is black (in the Universe of Discourse) then <<'Black' is true>> is valid (in a representation), and it is not true that if there is no black (in the Universe of Discourse) then <<'Black' is true>> is not valid, - see the left part of Figure 5.

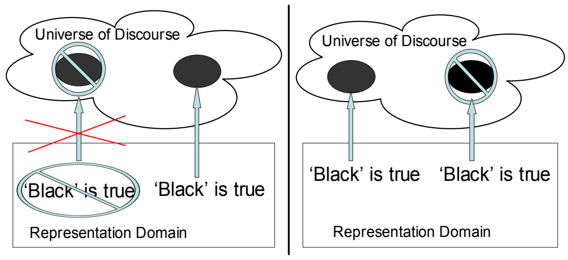

Fig. 5. Example of the absence of Tarski's semantics, case 3 of 3, subcase 3.2

In this subcase, <<'Black' is true>> can falsely appear to adequately represent the conflicting conceptualizations of the Universe of Discourse (i.e. the mutually contradicting conceptualizations, when the presence of one implies the absence of the other). For example, as shown in the right part of Figure 5, <<'Black' is true>> can appear to be valid for the conflicting conceptualizations: "black" and "no black", having the respective relations:
'Black' is true (in a representation) if there is black (in the Universe of Discourse);
'Black' is true (in a representation) if there is no black (in the Universe of Discourse).
Thus, under these definitions it may seem that a single assertion <<'Black' is true>> indeed adequately represents either of the two conceptualizations. But this is a delusion because with the two defined relations we have no means to represent the conflict of the two Universe of Discourse conceptualizations, therefore the relations themselves are defined inadequately with regard to the conceptualizations. In general, relations linking the modeling terms in the Representation Domain to the Universe of Discourse conceptualizations are *the reason* for assertions in concrete representations within the Representation Domain. Thus, when the relations themselves are inadequate (with regard to the possibilities of representation for the Universe of Discourse conceptualizations), they unavoidably prevent any successive reasoning about the adequacy of representations derived from them. So, in this case the adequacy of representations is formally unidentifiable.

It is interesting that in this Subcase 3.2 formal reasoning about coherency of interpretations within a representation is achievable. Indeed, because of the presence of sufficiency, here we have a direct logical implication (which we didn't have in Subcase 3.1 due to the absence of sufficiency), and thus any of representations here can be unambiguously classified either as coherently interpreting the Universe of Discourse or as incoherently interpreting the Universe of Discourse. As we demonstrated, in contrast to Subcase 3.2, in Subcase 3.1 when attempting to formalize such classification we necessarily had an ambiguity that was due to the absence of awareness about the sufficient conditions for an interpretation.

Thus we have shown that the conditions of Subcase 3.2 are not sufficient to provide the possibility to reason formally about the adequateness of representations of the Universe of Discourse. And we also explained that the conditions of Subcase 3.2 are sufficient to support formal reasoning about the coherency of interpretations within a representation of the Universe of Discourse.

3.2.4.3 Necessity for the Unambiguity of Adequateness and of Coherency: Case 3, Subcase 3.3. Let us now examine the last of the three subcases, Subcase 3.3 that assumes the absence of necessity and the absence of sufficiency of the Universe of Discourse conceptualization for a positive assertion containing a modeling language term.

As in Subcase 3.1 suppose that we have two conceptualizations: “black” and “white”, and two modeling terms: ‘Black’ and ‘White’. For Subcase 3.3 we will have the following situation:

It is not true that if there is black (in the Universe of Discourse) then <<‘Black’ is true>> is valid (in a representation), and it is not true that if there is no black (in the Universe of Discourse) then <<‘Black’ is true>> is not valid (see left part of Figure 6).

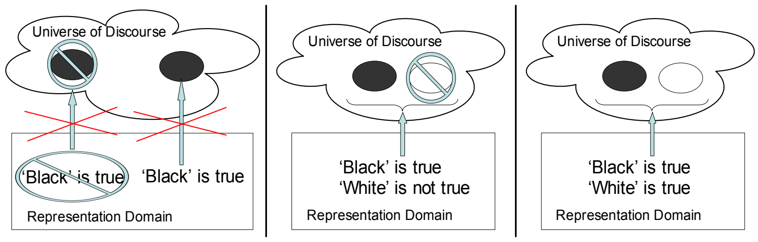

Fig. 6. Example of the absence of Tarski’s semantics, case 3 of 3, subcase 3.3

Since in Subcase 3.1 we based our conclusions only on the absence of sufficiency (and not on the presence of necessity), the same reasoning and the same examples that we used in Subcase 3.1 will also be valid in Subcase 3.3. Therefore, we can immediately conclude that the conditions of Subcase 3.3 are not sufficient to support formal reasoning about the adequateness of representations and about the coherency of interpretations within a representation of the Universe of Discourse.

3.3 The Usefulness of Tarski’s Semantics for Modeling Languages

Based on the logical arguments we presented in the sections 3.1 and 3.2 we can conclude that the use of Tarski’s semantics in the definition of a modeling language leads to the following valuable properties:

- *if* Tarski’s declarative semantics are defined for a modeling language, then this modeling language has formally defined scope of applications and thus the users of the modeling language have the possibility to understand the application limits of the language.
- *if and only if* Tarski’s declarative semantics are defined for a modeling language, then the users of this modeling language can formally compare their models for the adequacy of representations.
- *if* Tarski’s declarative semantics are defined for a modeling language, then the users of this modeling language can formally check their models for the coherency of interpretations.

Thus, for the first and third of these properties Tarski’s semantics are sufficient, while for the second - Tarski’s semantics are necessary and sufficient.

Although the realization of the first valuable property is assured by Tarski's semantics, in principle it is also possible without the Tarski's semantics. This property is of practical value because having formally understood the application limits of a modeling language the users of this language will be able to classify their modeling problems into those for which the language is appropriate and those for which the language is not appropriate. Users will therefore be able to avoid the use of the language for problems where it is not suitable, thus preventing all the potential negative consequences of such applications.

The realization of the second property is not only assured by Tarski's semantics it is impossible without it. This is highly significant for users of modeling languages. Indeed, if it is not possible to affirm the adequacy of a modeling representation, the representation can have no theoretical value and its practical value is a matter of chance rather then a matter of founded reason.

The third property is useful because it formally guarantees the coherency of modeling interpretations. The realization of the third implication is assured by Tarski's semantics, although in principle it is also possible without the Tarski's semantics. However, as clarified in section 3.2.4.2 of this paper, the realization of the third property does not have much of practical meaning in the case of absence of the Tarski's semantics. As we explained, in this case the achieved unambiguity in the coherency of interpretations is necessarily accompanied by the formally unidentifiable adequacy of representations, which diminishes the practical benefits resulting from the coherency.

4 Relationship to Other Kinds of Semantics

In this section we will first consider two other kinds of semantics which are relevant in the context of modeling languages: denotational and operational semantics. Then we will compare the application areas of these two kinds of semantics with the application area of the Tarski's semantics.

Denotational semantics and operational semantics were transferred to the context of modeling languages from the context of programming languages. These notions are explained in a variety of sources (e.g. see [3] for the denotational semantics and [4] for the operational semantics) and, judging from our experience, are reasonably well understood by UML designers and researchers, in contrast to Tarski's semantics. So, since our primary concern in this paper is to explain the role of Tarski's semantics in the design of modeling languages, here we will limit ourselves to the brief descriptions both for denotational and for operational semantics. These descriptions will be sufficient for us to make a relation between the three kinds of semantics in the context of a modeling language.

4.1 Denotational Semantics

In *programming* language theory denotational semantics provide mathematical models either for a language or for the applications of the languages (i.e. for programs). The basic rational behind denotational semantics is that it is easier to unambiguously un derstand some mathematical representation of a construction of programming language terms than it is to understand the construction itself. Indeed, since the representation repeats all the relations found in the construction (and thus unambiguously corresponds to the construction), with the defined denotational semantics it is possible to understand the constructions of programming languages and their applications without learning the peculiarities of every particular programming language. The only thing required for such understanding is to have the necessary level of expertise in the mathematical technique used to represent the constructions. In denotational semantics the domain containing programming language constructions is called the syntactic domain, and the domain containing the respective mathematical representations is called semantic domain.

Analogous reasoning can be applied for *modeling* languages. That is, for the complicated constructions of a modeling language (either for the semantic interrelations of modeling terms of the language, or for applications of the modeling language on a concrete application context) it can be easier to understand their mathematical representation than to understand the constructions themselves. This application of the idea of denotational semantics for the modeling languages is shown on Figure 7.

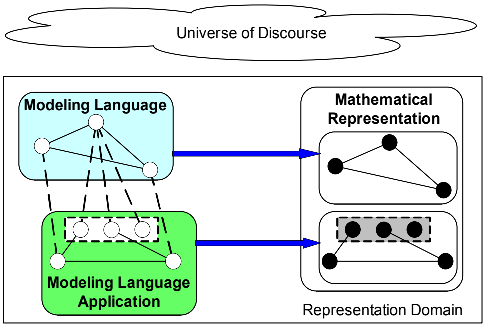

Fig. 7. Denotational semantics for modeling languages

4.2 Operational Semantics

Operational semantics, being defined in the context of programming languages, are meant for programs (that is, for concrete applications of programming languages) and not for the languages themselves (i.e. not for the interrelations of a language terms). The purpose of operational semantics is to show how the programs are executed, more precisely, how the states of a machine that can be used for the executions of the programs are modified during the execution [4]. In this way, a program is unambiguously mapped to its machine execution and looking at the state changes during the execution it is possible to understand the meaning that was put into the program by its programmer (assuming that the machine itself is well understood).

While operational semantics are undoubtedly relevant in the context of programming languages, their value for the modeling languages is less important. Indeed, the applications of programming languages (programs) are destined to be executed on a machine, while modeling languages applications are generally meant to be just interpreted by humans and, generally, not executed. However for those cases when applications of modeling languages are supposed to be executed on a machine we can use the notion of operational semantics in the context of modeling languages.

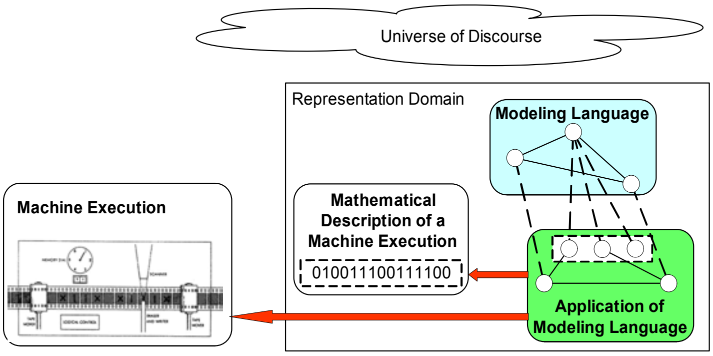

Fig. 8. Operational semantics for modeling languages

4.3 Comparison of Application Contexts for the Tree Kinds of Semantics

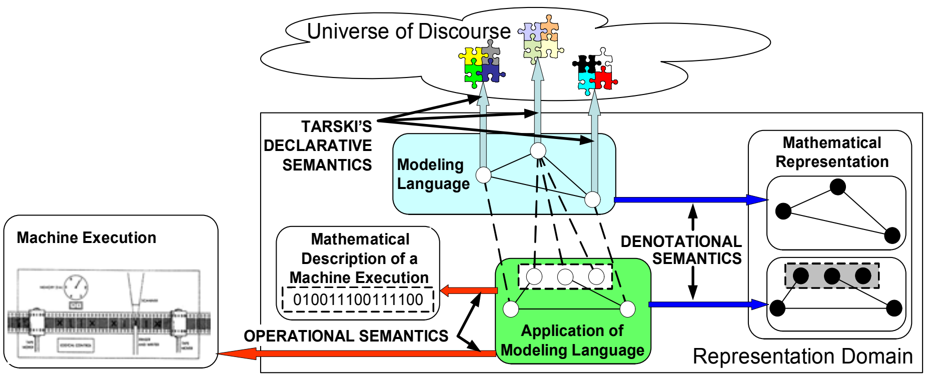

Fig. 9. Operational, Denotational and Tarski's semantics for modeling languages

Figure 9 relates the three kinds of semantics presented in this paper. Let us look here at the three respective application contexts for these semantics. From the explanations that we presented we can conclude that in the scope of modeling languages:

- Tarski's declarative semantics are applicable for the particular terms of a modeling language;
- Denotational semantics are applicable either for modeling languages (i.e. for the complete or incomplete structures of the semantic interrelations between the terms of a modeling language), or for the contextual applications of a modeling language;
- Operational semantics are applicable for the contextual applications of a modeling language.

Thus the application context of Tarski's semantics is different from the application contexts of denotational and operational semantics. Thus, we can conclude that for a modeling language Tarski's semantics are complementary to denotational and operational semantics.

4 Conclusions

In this paper we described the role that Tarski's declarative semantics play in the organization of a modeling language. We provided logical arguments justifying that:

- Tarski's semantics are *sufficient* for the definition of the application scope of a modeling language;
- Tarski's semantics are *sufficient* for unambiguity in coherency of interpretations within modeling representations;
- Tarski's semantics are *necessary and sufficient* for unambiguity in adequateness of modeling representations;

We explained the benefits that these features provide for modeling languages. We demonstrated how Tarski's semantics are positioned in the scope of modeling languages in relations to denotational and to operational semantics. Finally, we showed that in this respect Tarski's declarative semantics are complementary to denotational semantics and to operational semantics.

The conclusions of this paper are valid for modeling languages in general. However, we believe that these conclusions are particularly important for UML, since as has been discussed previously [5], Tarski's declarative semantics are conspicuous by their absence from the latest official version of the UML.

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