Nonseparable Multidimensional Perfect Reconstruction Filter Banks and Wavelet Bases for $\mathcal{R}^n$

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Abstract

Although filter banks have been in use for more than a decade, only recently have some results emerged setting up the theory of general nonseparable multidimensional filter banks. At the same time, wavelet theory emerged as a useful tool in many different fields of pure and applied mathematics as well as in signal analysis. Recently, it has been shown that the two theories were closely related. Not only does the filter bank perform a discrete wavelet transform but also under certain conditions it can be used to construct continuous bases of compactly supported wavelets.

We start by examining filter banks in polyphase and modulation domains and we give conditions for perfect reconstruction. The orthogonal case is analyzed showing the orthogonality relations between the filters in the bank and their shifts with respect to the sampling lattice. A linear phase condition, valid for the whole filter bank, follows, as a tool for testing or building banks containing linear phase (symmetric) filters. We then discuss the possibility of obtaining any of the combinations separable/nonseparable lattice/filters/polyphase components leading to the observation that a nonseparable filter can sometimes be implemented using separable polyphase components. Due to its importance the two-channel case in multiple dimensions is studied in detail and the form of the general orthogonal polyphase matrix is given. An analysis of the structure of possible linear phase solutions for the multidimensional two-channel case is presented and attractive cascade structures yielding perfect reconstruction filter banks with specific properties (orthogonality and linear phase) are proposed. Next, we make the connection to nonseparable wavelets through the concept of iterated filter banks. We show the orthogonality relations between the scaling function and wavelets. An iterated filter leading to an interesting "dragon", which by construction tiles the space, is shown. For the quincunx case that is used as a case study, we prove that the wavelet obtained previously together with its shifts on the quincunx lattice and scales by a particular, well-behaved matrix $\mathbf{D}$, constitute a basis for $L^2(\mathcal{R}^2)$ and give design examples.

References

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