Signal Expansions for Compression

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Abstract — **Signal expansions play a key role in practical compression schemes, from audio/image/video coding standards to current adaptives bases. Recent developments, especially related to wavelets series expansions, are reviewed, and current work on "best bases" is discussed.**

I. Introduction

Transform coding, together with predictive coding, is a key technique used in many practical compression systems. Its foundation is based on the Karhunen-Loève transform, which is the optimal transform under certain constraints [3]. In practice, approximations like the discrete cosine transform (DCT) are used, both for computational efficiency, and the fact that it is signal independent but still efficient for many practical signals. The transform coefficients are then quantized (usually in a scalar fashion) and entropy coded. This three block system [transform, quantization, entropy coding] raises some interesting questions:

- what are the best, possibly adaptive, transforms?
- what is the interplay of the three components?
- can successive approximation or multiresolution be efficiently achieved?

Recently, wavelets and their generalizations have appeared as alternatives to the more classic Fourier and DCT expansions [2]. In particular adapted expansions, and related algorithms to find the best bases, are an interesting extension.

II. Wavelet Series Expansions

Classically, windowed Fourier transforms have been used to obtain time-frequency representations of signals, and such representations are useful for source coding as well. Alternatively to local Fourier transforms, wavelet series have gained popularity. In this case, a particular prototype "mother" wavelet $\psi(t)$ is used to generate an orthonormal basis $\{\psi_{m,n}(t)\}$ for $L_2(R)$ by shifts and scales

$$\psi_{m,n}(t) = \frac{1}{2^{m/2}}\ \psi(t/2^m - n) \quad m,n \in Z.$$

A main difference between local Fourier expansions and wavelet series is that they provide a different tiling of the time-frequency plane. For example, at high frequencies or small scales, the wavelet is very sharp in time and acts like a mathematical microscope. In discrete-time, subband coding and filter banks permit the computation of sampled equivalents of local Fourier and wavelet transforms [7].

III. Adaptive Best Bases

Obviously, short-time Fourier transforms and wavelet series are only two out of a myriad of possible useful tilings. In particular, wavelet packets [1] and their time-varying generalizations [4] provide signal adaptive orthonormal bases. When the basis selection criterion is based on operational rate-distortion, we effectively have an adaptive transform coding algorithm, where quantization and entropy coding are included in the cost function. Since such transforms are adapted on the fly, computational efficiency is a must. In [4], a tree based pruning algorithm is used, while in [8] a dynamic programming procedure is applied.

IV. Overcomplete Representations

While orthonormal bases have many desirable properties as expansions for compression, a major drawback is their lack of shift-invariance. Overcomplete representations or frames overcome this problem, but the redundancy hurts compression. A recent result from oversampled analog to digital conversion [6] indicates that fine quantization in an orthonormal basis can be traded for coarse quantization in an overcomplete representation. Then, we discuss the use of matching pursuits [5] for compression applications. In matching pursuit, a very redundant dictionary is used together with a greedy algorithm to find a best approximation to a given signal. Choices of dictionaries and applications in video coding are considered.

V. Conclusion

A survey of signal expansions in the context of transform-type coding was given, with an emphasis of wavelets, adaptive and overcomplete representation. Expansions that adapt to the signals to be coded are a step towards *universal transforms* for compression.

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