Source coding and transmission of signals over time-varying channels with side information

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Abstract — **We look at the problem of transmitting information over time-varying channels with side information, where for time-varying channels the statistics of the channel change with time and by channel side information we mean the current state of the channel. We show that when this side information is available at both the transmitter and the receiver, then for the power-constrained channel, the power allocation policy that achieves minimum end-to-end distortion is not necessarily the same as the one required for maximum transmission rate.**

I. Introduction

A new challenge in telecommunication is the transmission of information over time-varying channels where the statistics of the channel change with time. Examples of such time-varying channels are wireless links where due to multi-path fading and interference from other users, the received signal strength can vary within a few orders of magnitude. Traditionally, the preferred transmission method has been to make the channel behave or look like a channel with uniformly distributed error - e.g. through use of interleaving. Achieving this, then the problem of communication is no harder than it used to be and all the classical methods and tools can be used. It is well-known that this "average channel" method is inherently sub-optimal [1][2]. However, to achieve higher channel capacity, it is required to provide channel *state* side information to either the transmitter or the receiver.

II. Time-varying channels with side information

We consider the state process with sample space $\mathcal{I}$ where at each time instant the channel is at one of these states and hence has different statistics. For example, consider an AWGN channel, where the noise power is modulated in accordance with the channel state. Based on the availability of the current channel state side information, we can distinguish the following four different cases: **(I)**: Informed receiver and transmitter, **(II)**: Informed receiver, **(III)**: Informed transmitter and **(IV)**: Average channel. In this paper, we concentrate on case **I**. Note that providing the current channel state does not imply a knowledge about the distribution of the states. In fact, we assume that neither the receiver nor the transmitter is aware of this distribution. It is well-known that the capacity of the channel is given by $C = \sum_{i\in\mathcal{I}} q_i I(X_i, Y_i)$ where q_i is the probability of the channel being at state i and $I(X_i, Y_i)$ is the mutual information between the channel input and output processes at this state. Note that the policy that achieves this capacity is independent of the channel state distribution (q_i). Also since the distribution of the states is unknown, the capacity of the channel is also not known. By policy, here, we mean the distribution of the input channel alphabets that maximizes $I(X_i, Y_i)$.

We can then show that the minimum end-to-end distortion is given by:

$$D_m = \sum_{i\in\mathcal{I}} q_i D(I(X_i, Y_i)). \qquad (1)$$

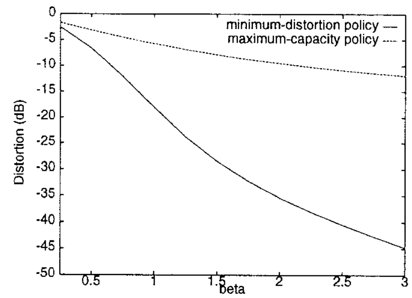

Fig. 1: Performance of minimum distortion and maximum capacity policies vs. β ($D(R) = 2^{-\beta R}$) over a narrow-band Rayleigh fading channel.

Note that had the channel state distribution been also provided to the transmitter then the channel capacity would have been known and $D_m = D\left(\sum_{i\in\mathcal{I}} q_i I(X_i, Y_i)\right)$. In the following section, we look at the power-constrained channels and show that the power allocation policy that achieves minimum end-to-end distortion is not necessarily the same as the one required for maximum transmission rate.

III. Power-constrained Channel

We are considering channels with constraint on the average transmitted power $\bar{S} = \sum_{i\in\mathcal{I}} q_i S_i$ where S_i is the transmission signal power at state i. Moreover, we characterize the channel states based on the received signal to noise ratio (γ). It is then straightforward to show that the following policy results in channel capacity: $S(\gamma)/\bar{S} = 1/\gamma_c - 1/\gamma$ if $\gamma \geq \gamma_c$ and 0 otherwise [2], where γ_c is the cut-off signal to noise ratio which is set so that the constraint on average signal power is met. If we now assume that the source has the distortion rate function $D(R) = 2^{-\beta R}$ then the optimum policy that results in minimum end-to-end distortion is given by:

$$\frac{S(\gamma)}{\bar{S}} = \begin{cases} \left(\frac{1}{\gamma^{\beta}\gamma_c}\right)^{\frac{1}{\beta+1}} - \frac{1}{\gamma} & \gamma \geq \gamma_c \\ 0 & \gamma < \gamma_c \end{cases} \qquad (2)$$

which is dependent on the source through β [3]. In fact, the more convex the distortion-rate function (the higher the value of β) the more dissimilar the above policies become. Figure 1 shows the performance of these two policies over narrow-band Rayleigh fading channel where the received SNR γ has exponential distribution ($f(\gamma) = 1/\gamma_s \exp(-\gamma/\gamma_s)$).

References

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