LAYERED TRANSMISSION OF SIGNALS OVER POWER-CONSTRAINED WIRELESS CHANNELS

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ABSTRACT

Transmission of layered information over power-constrained channel is considered. We address the question of power allocation policy where the goal is to minimize the overall end-to-end distortion. A layer containing significant information is more heavily protected against channel error through allocating more power to that layer. We define an appropriate coding gain and show that the KLT is the best unitary transform in the sense of maximizing this gain. Furthermore, we look that the embedded transmission of information and compare this method to orthogonal methods such as multi-carrier transmission. We also consider end-to-end distortion optimization for broadcast channels.

1. INTRODUCTION

Layered coding has proven to be an important concept in transmission of information over channels with time-varying impairments. In layered coding, the input signal is split into two or more streams where each stream can now be treated separately. For example, different streams can be protected differently against errors caused by the channel noise leading to what is known as unequal error protection (UEP). Also, if the current channel status is available to the transmitter then using this side information only a fraction of the layers can be transmitted at a time [1].

In the case of channels where the power is the main constraint (e.g. wireless channels), unequal error protection of each stream can be achieved through transmitting each stream with different power. Those streams that are transmitted with higher transmitted power experience lower channel bit error rate (BER) than those that are transmitted with smaller transmission power. One method to achieve this is through use of multi-carrier transmission where different layers are transmitted using different carrier frequencies [2] [3]. An alternative approach is to embed enhancement layer information into layers containing basic or more important information [4]. In this paper, we investigate both of these approaches.

In general, there are two sources of distortion. The first component is due to lossy compression done at the source coding stage. The other source of distortion is because of the possibility of the channel coding failure where some of the erroneously received bits cannot be corrected contributing to the overall distortion. These two components are contradictory. For example, one can reduce the source coding rate and hence provide the possibility of more protection against channel impairments; decreasing the second component at the expense of increasing source coding distortion. Therefore, by joint design of source and channel coding blocks it may be possible to minimize the overall distortion. This is indeed the case if the channel encoder used has finite complexity and delay. The separation principle, however, states that by allowing complex enough source and channel encoders, it is possible to transmit information at as closely as possible to the channel capacity and hence eliminate the second distortion component. In this paper, we first consider the case where the channel distortion due to channel errors is assumed to be negligible and then provide a generalization to includes the effect of channel errors.

2. PROBLEM STATEMENT AND OPTIMUM POWER ALLOCATION POLICY

Let us consider an information stream $x(n)$ which is split into M streams $\{x_k(n)\}$ having distortion rate functions $D_k(R)$, $k = 1, \cdots, M$. Also, let P_k denote the transmission power used to transmit the kth layer information ($x_k(n)$). The problem addressed here is then to minimize the total distortion $d = \sum_k d_k$ subject to the constraint on the average transmission power $\bar{P} = 1/M \sum_k P_k$ where d_k is distortion incurred while transmitting the kth layer [1]. To this end it is necessary to find a relationship between distortion d_k and P_k - the transmitted power used to send this stream. In general, the channel capacity is an increasing concave function of the transmission power which we denote by $C(P)$. We can then define distortion power function $D(P)$ as $D(C(P))$ which provides a lower bound on the transmission power given a required end-to-end distortion.

We first assume that the source compression is done using a high-rate quantizer. In this case it is well known that the distortion rate function can be closely approximated as:

$$D_k(R) = \epsilon_k \sigma_k^2 2^{-2R}, \tag{1}$$

where ϵ_k is a parameter depending on the source distribution, σ_k^2 is the variance of the kth layer signal and R is the rate at which the source is encoded at. An implication of this assumption is that distortion incurred at the kth layer cannot be greater that $\epsilon_k \sigma_k^2$. Assuming that the channel is additive white Gaussian noise (AWGN) channel and it is possible to at channel capacity given by $C = 1/2 \log(1 + P/N)$ then

$$D_k(P) = \frac{\epsilon_k \sigma_k^2}{1 + P/N}. \tag{2}$$

[1]Note that it is implicitly assumed that the layers are uncorrelated.

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Note that it is assumed that orthogonal transmission of layers, e.g. through multi-carrier transmission, is used and hence the signal power of the other layers do not interfere. Then, to find the optimal power allocation policy, we have to solve the following convex optimization problem:

$$\begin{cases} \min \quad d = \sum_{k=1}^{M} \frac{\epsilon_k \sigma_k^2}{1 + P_k/N} \\ \text{subject to} \quad \frac{1}{M} \sum_{k=1}^{M} P_k = \bar{P} \end{cases} \tag{3}$$

The solution to the above convex optimization problem is:

$$P_k = \gamma_k \bar{P} + (\gamma_k - 1)N, \qquad k = 1, \cdots, M \tag{4}$$

where γ_k is defined as

$$\gamma_k \triangleq \frac{\sqrt{\epsilon_k \sigma_k^2}}{1/M \sum_{l=1}^{M} \sqrt{\epsilon_l \sigma_l^2}}. \tag{5}$$

The above power allocation policy and hence the rate at which each layer is encoded is dependent on the noise level of the channel. Using the above policy the distortion incurred at band k is:

$$d_k = \frac{1}{M} \sqrt{\epsilon_k \sigma_k^2} \left(\sum_{l=1}^{M} \sqrt{\epsilon_l \sigma_l^2} \right)^2 (1 + \bar{P}/N)^{-1}, \tag{6}$$

which is not same for different bands. Indeed, the bands with higher amount of energy experience more distortion. This is in contrast to the customary rate allocation policy where the optimal policy results in equal distortion in all the bands. In the following sections, we will show that embedded transmission results in distortion at each bands to be the same. One can then show that the total end-to-end distortion resulting from the optimum power allocation policy (4) is:

$$d = \frac{\frac{1}{M} \left(\sum_{l=1}^{M} \sqrt{\epsilon_l \sigma_l^2} \right)^2}{(1 + \bar{P}/N)} \tag{7}$$

By further investigation of the power allocation policy (4), one can distinguishes two different classes of layers, namely those with $\gamma_k \geq 1$ and those with $\gamma_k < 1$. Note that γ_k is an indication of the energy of that layer. As the power noise level of channel (N) increases more power is allocated to those layers with higher energy at the expense of those layers having smaller energy.

If the channel has a time-varying nature and the noise level of the channel changes with time, then the allocation policy should also change with time. One method is to allocate power based on the channel worst case condition (maximum N). In this case the transmission data rate for the high energy layers ($\gamma_k \geq 1$) is always below the capacity provided for those layers and is therefore guaranteed. The situation is however different for the low energy layers ($\gamma_k < 1$).

2.1. Coding Gain

It is possible to define coding gain in a similar fashion as is done in the case of rate allocation policy for transform or subband coding methods [5]. If we assume that the same power ($\bar{P}$) is allocated to all the layers, then from (2) we can show that the end-to-end distortion is

$$d = \frac{\sum_{l=1}^{M} \epsilon_l \sigma_l^2}{1 + \bar{P}/N}. \tag{8}$$

Comparing the above to (7), we can define the coding gain G as the ratio of the distortions resulted from these two policies

$$G = \frac{\frac{1}{M} \sum_{l=1}^{M} \epsilon_l \sigma_l^2}{\left(\frac{1}{M} \sum_{l=1}^{M} \sqrt{\epsilon_l \sigma_l^2} \right)^2}, \tag{9}$$

which is the ratio of the arithmetic of $\{\epsilon_l \sigma_l^2, l = 1, \cdots, M\}$ to the square of the arithmetic mean of $\{\sqrt{\epsilon_l \sigma_l^2}, l = 1, \cdots, M\}$ and as a result $G \geq 1$. The equality holds if and only if $\epsilon_l \sigma_l^2$ is the same for all the bands.

An interesting observation is since the arithmetic mean of positive number is lower bounded by their geometric mean,

$$\left(\prod_{l=1}^{M} \epsilon_l \sigma_l^2 \right)^{1/M} \leq \left(\frac{1}{M} \sum_{l=1}^{M} \sqrt{\epsilon_l \sigma_l^2} \right)^2 \tag{10}$$

and hence

$$G \leq \left(\frac{1}{M} \sum_{l=1}^{M} \epsilon_l \sigma_l^2 \right) / \left(\prod_{l=1}^{M} \epsilon_l \sigma_l^2 \right)^{1/M} \tag{11}$$

where the right-hand side of the above inequality is the coding gain had the separation between source and channel coding been invoked. In other words, fundamentally, it is more advantageous to consider the information of all the layers as one stream and allocate all the power to maximize the transmission rate for this stream than unequal power allocation for different layers. Practical implementation aspects of both methods should, however, be taken into consideration. Moreover, for the class of time-varying channels - where the statistics of the channel noise changes with time - the capacity of the channel and hence the rate that being allocated among the bands is not necessarily known a-priori.

2.2. Best Unitary Transform

It is well known that the Karhunen-Loeve transform (KLT) is the best unitary transform in the sense of maximizing the coding gain expression given by the right hand side of (11) [5]. This is done by showing that the geometric mean of the variance(energy) of the output signals is lower bounded by the geometric mean of the eigenvalues of the input signal correlation matrix. Therefore, the best transform is the one that make the correlation matrix to become diagonal - i.e.

the KLT transform. We now show that the same is true for the coding gain expression (9).

We assume that $\epsilon_1 = \cdots = \epsilon_M$, which is indeed the case if the input signal has Gaussian distribution. In the appendix we show that for an $M \times M$ positive definite matrix $\mathbf{R}$

$$\sum_{l=1}^{M} \sqrt{\lambda_l(\mathbf{R})} \leq \sum_{l=1}^{M} \sqrt{(\mathbf{URU}^*)_{ll}}, \qquad (12)$$

where $\lambda_i(\mathbf{R})$ is the ith eigenvalue of $\mathbf{R}$, $\mathbf{U}$ is a unitary transform and $(\mathbf{URU}^*)_{ll}$ denotes the lth diagonal element of $\mathbf{URU}^*$. Note that the right hand side of the above inequality corresponds to the denominator of (9). Therefore by minimizing this expression the maximum coding gain can be achieved. This is indeed the case if the unitary transformation used diagonalizes the correlation matrix $\mathbf{R}$ resulting in the equality to hold in (12). As a result, the best strategy is to de-correlate the signal among the bands through use of the KLT transform.

3. DISTORTION-POWER OPTIMIZATION

The analysis of the previous section is based on the assumption that high-rate quantization is used and the distortion at band k is always less than or equal to $\epsilon_k \sigma_k^2$. This assumption implies certain constraints on the value of γ_k's. Using the fact that in (4) $P_k \geq 0$ in (4), we can find the following bounds:

$$\begin{aligned} \gamma_k &\geq \frac{N}{N+\bar{P}} \qquad k = 1, \cdots, M, \\ \Gamma_l &\leq \frac{lN + M\bar{P}}{N+\bar{P}} \qquad l = 1, \cdots, M \end{aligned} \qquad (13)$$

where Γ_l is the sum of l arbitrary γ_k's. As the noise level of the channel, N, increases the above constraints implies that for the analysis of the previous section to be valid, it is necessary for all the bands to contain almost the same amount of energy.

We define the distortion power function which relates the distortion incurred at each band to the transmission power used. Note that no assumption on the error-free transmission or high-rate source coding is being made. This function characterizes the lower bound on the transmission power necessary to achieve a certain level of distortion or conversely the lower bound on end-to-end distortion given a specific level of transmission power. This function is a non-increasing function since as one increases the allocated power, the overall distortion cannot increase. It is also convex since otherwise it would be possible to use time-sharing and achieve better performance than the one specified by the function which is clearly a contradiction. We can now rewrite the optimization problem (3) as

$$\begin{cases} \min & d = \sum_{k=1}^{M} D_k(P_k) \\ \text{subject to} & \frac{1}{M}\sum_{k=1}^{M} P_k = \bar{P} \end{cases} \qquad (14)$$

where $D_k(P)$ is the distortion power of the kth band. Since all $D_k(P)$ are convex, d is a convex function and invoking

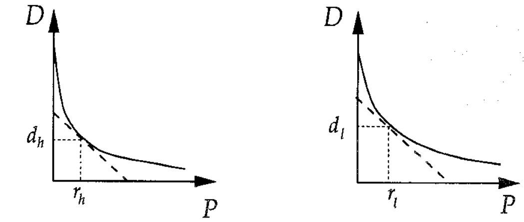

Figure 1: Constant slope policy for power allocation

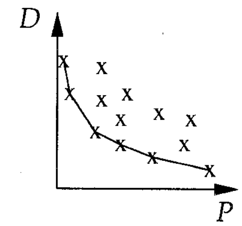

Figure 2: Operational distortion power function

Kuhn-Tucker theorem, the solution to the above problem $\{P_k^*\}$ is given by:

$$\begin{cases} \frac{\partial d}{\partial P_k}|_{P_k = P_k^*} = \theta & \text{if } P_k^* > 0 \\ \frac{\partial d}{\partial P_k}|_{P_k = P_k^*} < \theta & \text{if } P_k^* = 0 \end{cases} \qquad (15)$$

for some $\theta < 0$. The above policy which is also known as *constant slope policy* has been also proposed for rate allocation [7] in the rate constrained optimization.

We can also define operational distortion power function for each layer. For example consider a binary symmetric channel where the probability of error is dependent on the transmission power being used. We also consider a family of quantizers (not necessarily scalar) where the representation levels are not necessarily the centroid of their respective Voronoi region and as a result the source coding distortion and the distortion due to channel errors cannot be decoupled [6]. Now for each power transmission level, we can find the corresponding distortion (X's in Figure 2) and define the operational distortion power as a convex hull of these operating points as is shown in Figure 2. We then use (15) to find the operational point of each layer such that the constraint on total transmission power is met.

4. EMBEDDED TRANSMISSION

Through embedded transmission the information of one layer is embedded in the other layers - e.g. embedded modulation proposed in [4] for transmission of HDTV signal over broadcast channels. In this section, we consider two layers where as before P_i ($i = 1, 2$) is the power allocated to transmission of layer i where $P_1 + P_2 = 2\bar{P}$. It is well-known that

$$\begin{aligned} r_1 &= \frac{1}{2}\log\left(1 + \frac{P_1}{N}\right) \\ r_2 &= \frac{1}{2}\log\left(1 + \frac{P_2}{P_1 + N}\right) \end{aligned} \qquad (16)$$

where r_i ($i = 1, 2$) is the maximum transmission rate for the ith layer [8]. Assuming that the distortion rate function of both bands is given by (1), the following power allocation can be shown to minimize the overall end-to-end distortion d:

$$P_1 = \sqrt{\frac{\epsilon_1\sigma_1^2}{\epsilon_2\sigma_2^2}}\sqrt{N(N+\bar{P})} - N \quad (17)$$

$$P_2 = 2\bar{P} - P_1 \quad (18)$$

resulting in

$$d = 2d_1 = 2d_2 = \frac{2\sqrt{\epsilon_1\sigma_1^2\ \epsilon_2\sigma_2^2}}{\sqrt{1+2\bar{P}/N}}, \quad (19)$$

where a necessary condition for the above analysis (high-rate quantization) to hold is:

$$\frac{1}{1+2\bar{P}/N} \leq \frac{\epsilon_1\sigma_1^2}{\epsilon_2\sigma_2^2} \leq 1 + \frac{2\bar{P}}{N}. \quad (20)$$

Note that in contrast to the power allocation policy (4) for multi-carrier transmission, the above policy results in equal distribution of distortion at each band. Also, generalization to arbitrary distortion rate functions can be made in a similar fashion as was done in the previous section.

4.1. Broadcast Channel

In the broadcast channel, the transmitter tries to send information (possibly the same) simultaneously to two or more receivers [8]. For example, this is the situation in the downlink (base to mobile) channel of wireless access networks. Among possible strategies are time sharing, frequency sharing and embedded transmission. It is well-known that embedded transmission always outperform either time or frequency sharing methods [8].

Let us assume that the same information stream is intended to two receivers, where each receiver experiences different noise level $N_1 < N_2$. The ith receiver ($i = 1, 2$) receives information at rate r_i and experiencing distortion level d_i. If we assume $N_1 < N_2$, then $r_1 > r_2$, $d_1 < d_2$. For AWGN r_1 and r_2 are given by:

$$r_2 = \frac{1}{2}\log\left(1 + \frac{P_2}{P_1 + N_2}\right)$$

$$r_1 = r_2 + \frac{1}{2}\log\left(1 + \frac{P_1}{N_1}\right). \quad (21)$$

The goal is to allocate power to each stream (choose P_1 and P_2) subject to the constraint $P_1 + P_2 = 2\bar{P}$ such that the overall distortion ($d_1 + d_2$) is minimum. Assuming high-rate distortion power function, the optimum power allocation policy is given by:

$$P_1 = \sqrt{\frac{\epsilon_1\sigma_1^2}{\epsilon_2\sigma_2^2}N_1(N_2 - N_1)}$$

$$P_2 = 2\bar{P} - P_1. \quad (22)$$

Note that P_1 is independent of the average power $\bar{P}$. Extension to the general distortion rate functions can be made in a similar fashion as that of Section 3.

5. APPENDIX

In this appendix, we show that (12) holds for all positive definite matrices $\mathbf{R}$. Let us, as before, denote the lth row and kth column of matrix $\mathbf{X}$ by $\mathbf{X}_{lk}$. Then clearly

$$\sum_l \mathbf{X}_{ll} \leq \sum_l \sqrt{\sum_k \mathbf{X}_{lk}^2} = \sum_l \sqrt{(\mathbf{X}\mathbf{X}^*)_{ll}} \quad (23)$$

where $\mathbf{X}^*$ is the Hermitian transpose of $\mathbf{X}$. Then we have

$$\sum_l \sqrt{\lambda_l(\mathbf{R})} = \sum_l \lambda(\mathbf{R}^{1/2}) \quad (24)$$

$$= \sum_l (\mathbf{U}\mathbf{R}^{1/2}\mathbf{U}^*)_{ll}$$

$$\leq \sum_l \sqrt{(\mathbf{U}\mathbf{R}\mathbf{U}^*)_{ll}} \quad (25)$$

where the second equality holds because of the invariance of the trace of a matrix under unitary transformation and the last inequality results from (23) by substituting $\mathbf{X} = \mathbf{U}\mathbf{R}^{1/2}\mathbf{U}^*$. This is possible since $\mathbf{R}$ is a positive definite matrix and its square root $\mathbf{R}^{1/2}$ is well-defined. □

6. REFERENCES

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