

# Analysis of Optimal Filter Banks for Multiple Description Coding

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## Abstract

*We study the problem of Multiple Description (MD) coding of stationary Gaussian sources with memory. First, we compute an approximate rate distortion region for these sources, which we prove to be asymptotically tight at high rates: this region generalizes the standard MD rate distortion region for memoryless sources. Then we develop an algorithm for the design of optimal biorthogonal filter banks for MD coding. Finally, we present some experimental results, where we measure the deviation from optimality of our proposed system. For almost uncorrelated sources the gap between the performance of our proposed system and the ideal bounds is quite high, on the other hand for highly correlated sources this gap is reduced, due to the ability of our system to take advantage of the memory in the source. In this case, in realistic scenarios where finite complexity/ delay is an issue, the subband coding approach is competitive with other approach like decorrelating transform followed by MD Scalar Quantizers.*

## 1 Introduction

Recently, the problem of transmitting data over unreliable networks in which, e.g. due to real-time delay constraints, it is not possible to retransmit lost data, has received considerable attention. Multiple Description (MD) coding offers a potentially attractive framework in which to develop coding algorithms for such scenarios: a MD coder encodes an information source into multiple bit streams (descriptions) having a property known as *mutual refinability*: each individual description provides an approximation to the original message, and multiple descriptions refine each other, to produce a better approximation than that attainable by any single one alone. The simplest formulation of the problem of coding into MDs involves only two descriptions, at rates  $R_1 = R_2$ , that are sent over two erasure channels. If both descriptions are received then the decoder can reconstruct the source at some small distortion value  $D_0$  (the *central* distortion), but if either one is lost, the decoder can still reconstruct the source at some higher *side* distortion  $D_1 = D_2 \geq D_0$ .

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The basic idea of MD coding is that of introducing dependencies among the two descriptions so that, in case of an erasure, the decoder still has access to some information about the lost piece of data. On the other hand, this redundant information reduces the coding efficiency. The resulting excess rate  $\rho = R_1 + R_2 - R(D_0)$  ( $R(\cdot)$  is the rate/distortion function for the source) is called *redundancy*, and represents the price that needs to be paid in order to attain graceful degradation in the presence of losses. One formulation of the problem of MD coding consists then of minimizing the distortion in the presence of loss of data, given some allowed redundancy  $\rho$ .

Early papers on MD coding are information theoretic in nature, and deal with the problem of finding sets of achievable values for the quintuple  $(R_1, R_2, D_0, D_1, D_2)$  [3, 7]. More recent papers however consider the problem of designing practical MD systems. MD quantizers are designed in [8, 10]. A suitable blockwise transform is applied to an input vector before coding to obtain the MD property in [4, 6]: the input vector is usually a jointly Gaussian vector, the basic idea being to decorrelate the vector components and then to introduce again correlation between coefficients but in a known and controlled manner so that erased coefficients can be statistically estimated from the received ones.

In this paper we investigate the more general case of arbitrary stationary Gaussian processes. In this case the MD rate/distortion region was not known: here we present a region which asymptotically approximates the real one arbitrarily well at high rates. We then develop an algorithm for designing two-channel biorthogonal filter banks for optimal MD coding of Gaussian sources. The approach used is similar to the one in the blockwise transform context: we construct a first filter bank to decorrelate the two input sequences, and then we use a second filter bank to efficiently recorelate them, with the frequency response of this second set of filters depending on the allowed redundancy. Finally, the case of a first order Gauss-Markov process is examined in detail: the performance of this system is compared against the generalized bound. The results show that for memoryless sources, there is a large gap between the performance of the the proposed filter banks and that of the MD rate region; however, this gap is reduced as the memory in the source increases.

## 2 The Multiple Description Rate Region

For a memoryless Gaussian source with variance  $\sigma^2$ , Ozarow [7] has found an explicit characterization of the set of achievable distortions  $(d_0, d_1, d_2)$  for a given pair of rates  $R_1, R_2$ . The inverse of the functions presented in [7], are the following [5, 7]:

$$R_1 \geq \frac{1}{2} \log \left( \frac{\sigma^2}{d_1} \right) \quad (1)$$

$$R_2 \geq \frac{1}{2} \log \left( \frac{\sigma^2}{d_2} \right) \quad (2)$$

$$R_1 + R_2 \geq \frac{1}{2} \log \left( \frac{\sigma^2}{d_1} \right) + \frac{1}{2} \log \left( \frac{\sigma^2}{d_2} \right) + \delta \quad (3)$$

$\delta$  is defined by:

$$\delta = \begin{cases} \frac{1}{2} \log \left( \frac{1}{1-\rho^2} \right), & d_0 \leq d_0^{\max} \\ 0, & d_0 > d_0^{\max} \end{cases}$$

where:

$$d_0^{\max} = \frac{d_1 d_2}{d_1 + d_2 - (d_1 d_2 / \sigma^2)}$$

and

$$\begin{aligned} \rho &= -\frac{\sqrt{\Pi\epsilon_0^2 + \gamma} - \sqrt{\Pi\epsilon_0^2}}{(1-\epsilon_0)\sqrt{\epsilon_1\epsilon_2}} & \gamma &= (1-\epsilon_0)[(\epsilon_1 - \epsilon_0)(\epsilon_2 - \epsilon_0) + \epsilon_0\epsilon_1\epsilon_2 - \epsilon_0^2] \\ \Pi &= (1-\epsilon_1)(1-\epsilon_2) & \epsilon_i &= d_i/\sigma^2 (i = 0, 1, 2) \end{aligned}$$

Now we are interested in finding the rate region for a stationary Gaussian sequence  $x[n]$ . As a first step we can take  $N$  successive components of the sequence and apply a KLT to them to get uncorrelated (and so independent) components. Each of this component has a different variance ( $\lambda_i$   $i = 1..N$ ). We aim at finding the MD rate region for this N-sequence; so the question is how we should allot the rates  $R_1, R_2$  to minimize the total distortions  $d_0, d_1, d_2$ . If the slopes of the three equations (1, 2, 3) are independent of the variances ( $\lambda_i$ ), then the optimum allocation of the rates to the various component results in equal distortions for each random variable and the rate-region for the N-sequence is:

$$R_1 \geq \frac{1}{2N} \sum_{i=1}^N \max \left[ 0, \log \left( \frac{\lambda_i}{d_1} \right) \right] \quad (4)$$

$$R_2 \geq \frac{1}{2N} \sum_{i=1}^N \max \left[ 0, \log \left( \frac{\lambda_i}{d_2} \right) \right] \quad (5)$$

$$R_1 + R_2 \geq \frac{1}{2N} \sum_{i=1}^N \max \left[ 0, \log \left( \frac{\lambda_i}{d_1} \right) \right] + \frac{1}{2N} \sum_{i=1}^N \max \left[ 0, \log \left( \frac{\lambda_i}{d_2} \right) \right] + \frac{1}{N} \sum_{i=1}^N \delta \quad (6)$$

where we have used the Kuhn-Tucker conditions when necessary.

Now passing to the limit of infinite  $N$  and using the fact that the  $N \times N$  correlation matrix is a symmetric Toeplitz matrix [2], we can find the MD rate region for the complete sequence:

$$R_1 \geq \frac{1}{4\pi} \int_{-\pi}^{\pi} \max \left[ 0, \log \left( \frac{S(\omega)}{d_1} \right) \right] d\omega \quad (7)$$

$$R_2 \geq \frac{1}{4\pi} \int_{-\pi}^{\pi} \max \left[ 0, \log \left( \frac{S(\omega)}{d_2} \right) \right] d\omega \quad (8)$$

$$\begin{aligned} R_1 + R_2 \geq \frac{1}{4\pi} \left\{ \int_{-\pi}^{\pi} \max \left[ 0, \log \left( \frac{S(\omega)}{d_1} \right) \right] d\omega + \int_{-\pi}^{\pi} \max \left[ 0, \log \left( \frac{S(\omega)}{d_2} \right) \right] d\omega \right. \\ \left. + \int_{-\pi}^{\pi} \delta(\omega) d\omega \right\}, \end{aligned} \quad (9)$$

where  $S(\omega)$  is the power spectral density (p.s.d.) of the input sequence.

Unfortunately, the hypothesis that the slope of the rate-distortion curves does not depend on the variances does not hold for the equation (3). So the proposed bounds are not tight. On the other hand it can be shown that in the limit of high resolution coding (so in the limit of small distortions)  $\delta$  and  $d_{0max}$  do not depend on the variances anymore [5], so in this case the hypothesis that the slopes are independent of the variances holds also for equation (3) and consequently the proposed bounds are tight in the limit. In the rest of the paper we will consider as bounds for a Gaussian source the one presented in (7, 8, 9), while keeping in mind that the smaller the distortions  $d_0, d_1, d_2$  the closer our bounds are to the real ones.

### 3 Optimal Filter Bank for MDC \*

Consider the classic two-channel filter banks scheme shown in Fig. 1. Here the input  $x[n]$  is assumed to be a stationary Gaussian process with known statistics, and is fed through an analysis filter bank. The two output sequences are then separately quantized and sent over two different erasure channels. The two output sequences are then separately

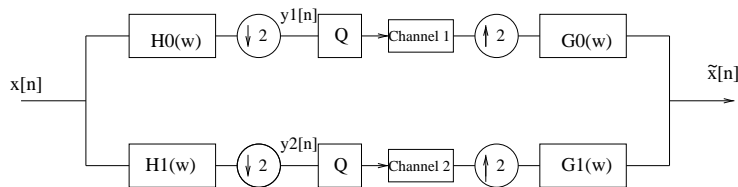


Figure 1: Two channel filter banks

We suppose that the channels are independent, that they have the same erasure probability, and that  $R_1 = R_2$  (which is equivalent to requiring to transmit equal power over the two channels). For convenience we will formulate our problem in the polyphase domain [12]. In this case, the input-output relation can be expressed in matrix notation introducing the analysis polyphase matrix  $H(\omega)$ :

$$\begin{pmatrix} Y_1(\omega) \\ Y_2(\omega) \end{pmatrix} = \begin{pmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{pmatrix} \begin{pmatrix} X_1(\omega) \\ X_2(\omega) \end{pmatrix} \quad (10)$$

$R_x(\omega)$  is the  $2 \times 2$  polyphase power spectral density (p.s.d.) matrix of the input process, so  $R_{x_{ij}}(\omega)$  is the auto or cross p.s.d. between the i-th and j-th polyphase components. Likewise  $R_y(\omega)$  is the p.s.d. matrix of the outputs, but here  $R_{y_{ij}}(\omega)$  is the p.s.d. between the i-th and j-th channel signals (as in Fig 2). The synthesis part of the system can be analyzed in a similar fashion.

Without loss of generality we decompose the matrix  $H(\omega)$  into the product of two matrices  $M(\omega)$  and  $T(\omega)$ :<sup>†</sup>

$$H(\omega) = T(\omega)M(\omega). \quad (11)$$

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\*During the writeup of this work we became aware of the still unpublished work of Yang and Ramchandram [13], where the authors approach the same problem of optimal subband decomposition for MD coding, but in a slightly different way.

<sup>†</sup>It can be shown that this factorization does not reduce the generality of the solution. This is mainly because  $M(\omega)$  is unitary and it does not depend on the redundancy.

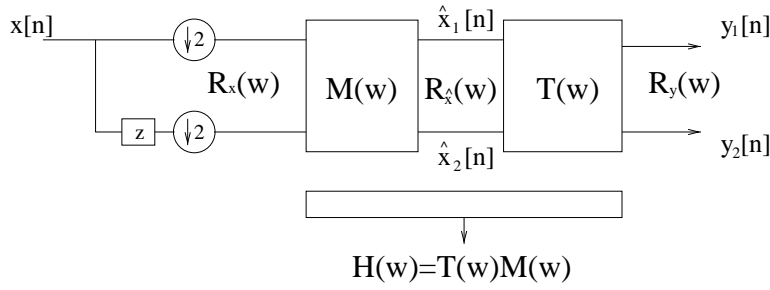


Figure 2: The polyphase matrix decomposition into  $M(\omega)$  and  $T(\omega)$ .

Here,  $M(\omega)$  is the matrix that has to decorrelate the two input sequences and its frequency response depends only on the statistics of the input signal. For a stationary input process, the decorrelating matrix can be found analytically and has the following shape [9]:

$$M(\omega) = \frac{\sqrt{2}}{2} \begin{bmatrix} e^{j\omega/2} & 1 \\ -1 & e^{-j\omega/2} \end{bmatrix}. \quad (12)$$

$M(\omega)$  is clearly unitary and represents the solution we will use in the rest of the paper.  $T(\omega)$  is the “recorelation” matrix, whose frequency response depends on the redundancy and on the p.s.d. of the decorrelated sequences. This is the matrix that, given an allowed amount of redundancy, has to be optimized.

We call  $R_{\hat{x}}$  the p.s.d. matrix of the input process after decorrelation,  $\sigma_1^2(\omega)$  the p.s.d. for the sequence  $\hat{x}_1[n]$ , and  $\sigma_2^2(\omega)$  the p.s.d. for the sequence  $\hat{x}_2[n]$  (clearly from Fig. 2, the cross p.s.d.’s are equal to zero).

Given a target central distortion  $D_0$ , we define the redundancy to be the difference between the minimum bit rate  $R(D_0)$  necessary to code the two output sequences  $y_1[n], y_2[n]$ , and the bit rate  $\hat{R}(D_0)$  necessary to code the two decorrelated sequences  $(\hat{x}_1[n], \hat{x}_2[n])$ . Under a fine quantization assumption, then the redundancy is given by the following formula [2]:

$$\rho = R(D_0) - \hat{R}(D_0) = \frac{1}{8\pi} \int_{-\pi}^{\pi} \log \left( \frac{R_{y_{11}}(\omega) R_{y_{22}}(\omega)}{\sigma_1^2(\omega) \sigma_2^2(\omega)} \right) d\omega. \quad (13)$$

Notice that  $\rho$  does not depend on  $D_0$ .

Since the two channels have the same erasure probability, the expected distortion due to an erasure is the average of the distortions  $D_1$  and  $D_2$ , divided by 2 since each distortion is related only to a downsampled version of the input sequence :

$$D = \frac{1}{4}(D_1 + D_2), \quad (14)$$

where  $D_1 = D_2$ , and:

$$D_2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (H_{12}^*(\omega) H_{12}(\omega) + H_{22}^*(\omega) H_{22}(\omega)) \cdot \left( R_{y_{11}}(\omega) - \frac{|R_{y_{12}}(\omega)|^2}{R_{y_{22}}(\omega)} \right) d\omega. \quad (15)$$

A derivation of this formula must be omitted due to lack of space.

In our formulation we have only considered the distortion due to erasure and have neglected the one due to quantization, since it is usually much smaller. Thus our optimization problem consist in constructing the matrix  $T(\omega)$  to minimize the distortion  $D$  for a given redundancy  $\rho$ .

To develop our formulation we refer to the results published in [4], where Goyal and Kovačević show that the optimal transform, in case of transmission of two Gaussian decorrelated variables over two independent channels with the same erasure probability and at the same rate ( $R_1 = R_2$ ), is given by:

$$T = \begin{bmatrix} a & \frac{1}{2a} \\ -a & \frac{1}{2a} \end{bmatrix}, \quad (16)$$

where the value of  $a$  depends on the redundancy  $\rho$ :

$$a = \sqrt{\frac{\sigma_2}{2\sigma_1(2^{2\rho} - \sqrt{2^{4\rho} - 1})}}. \quad (17)$$

$\sigma_1^2$  and  $\sigma_2^2$  are the variances of the two Gaussian components, with the usual assumption that  $\sigma_1^2 > \sigma_2^2$ . Finally the side distortion is given by:

$$D = \frac{1}{2} \sigma_1^2 - \frac{1}{4 \cdot 2^{2\rho}(2^{2\rho} - \sqrt{2^{4\rho} - 1})}(\sigma_1^2 - \sigma_2^2). \quad (18)$$

It is interesting to notice that if the source has a circularly symmetric probability density, i.e.,  $\sigma_1 = \sigma_2$ , then the distortion is independent of  $\rho$ . We would like to generalize these results to our case, where the two variances change with the frequency. Without loss of generality let us suppose that  $\sigma_1^2(\omega) \geq \sigma_2^2(\omega)$ ,  $\forall \omega$ .

As a first approximation we divide the frequency axis into  $N$  equal sub-intervals, and suppose that the two p.s.d.'s are constant over each of these intervals. Now we have only  $N$  possible values for the p.s.d.  $\sigma_1^2(\omega)$  and  $\sigma_2^2(\omega)$ :  $(\sigma_{1i}^2, \sigma_{2i}^2)$ ,  $i = 1..N$ . With this approximation we can apply the results of [4] on each interval, and claim that the optimal transform for the generic  $i$ -th interval, given a redundancy  $\rho_i$  for that interval, is the one given by (16), where the value of  $a$  is given by (17); the resulting side distortion is:

$$D_i = \frac{1}{2} \sigma_{1i}^2 - \frac{1}{4 \cdot 2^{2\rho_i}(2^{2\rho_i} - \sqrt{2^{4\rho_i} - 1})}(\sigma_{1i}^2 - \sigma_{2i}^2). \quad (19)$$

However we want to minimize the global side distortion  $D = \frac{1}{N} \sum_i D_i$ , given a global redundancy budget  $\rho = \frac{1}{N} \sum_i \rho_i$ . So the problem now is to find an optimal strategy for the allocation of the redundancy over the  $N$  intervals to accomplish this. This is a typical problem of constrained minimization. We define a new cost function  $L$  which combines the distortion and the redundancy through a positive Lagrange multiplier  $\lambda$ :  $L = D + \lambda\rho$ .

Finding a minimum of  $L$  (which now depends on  $\lambda$  too) amounts to finding minima for each  $L_i = D_i + \lambda \rho_i$  (because the costs are additive). If we suppose that the redundancy budget is sufficiently large and that  $\sigma_{1i}^2$  is never equal to  $\sigma_{2i}^2$  then it is possible to give a closed form expression for the allocation problem. In fact, writing the distortion as a function of the redundancy  $D_i(\rho_i)$ , and taking derivatives, we get:

$$\frac{\partial D_i}{\partial \rho_i} = -\frac{\ln 2(\sigma_{1i}^2 - \sigma_{2i}^2)}{4 \cdot 2^{2\rho_i}(\sqrt{2^{4\rho_i}} - 1)} \approx -\frac{\ln 2}{4}(\sigma_{1i}^2 - \sigma_{2i}^2)2^{-4\rho_i} = -\lambda, \quad (20)$$

The constant-slope solution forces the redundancies to be of the following form:

$$\rho_i = \alpha + \frac{1}{4} \log(\sigma_{1i}^2 - \sigma_{2i}^2). \quad (21)$$

Using the redundancy constraint  $\rho = \frac{1}{N} \sum_i \rho_i$ , we can find  $\alpha$  and finally

$$\rho_i = \rho + \frac{1}{4} \log(\sigma_{1i}^2 - \sigma_{2i}^2) - \frac{1}{4N} \sum_i \log(\sigma_{1i}^2 - \sigma_{2i}^2). \quad (22)$$

The approximation in (20) holds if  $\rho_i$  is sufficiently large. Its value depends of course on the total redundancy budget  $\rho$ , but also on the difference  $\sigma_{1i}^2 - \sigma_{2i}^2$ . If this difference is zero the corresponding side distortion (19) will not change with the redundancy and in this case it is better not to allocate any redundancy in this interval ( $\rho_i = 0$ ). So when  $\sigma_{1i}^2 = \sigma_{2i}^2$  the hypothesis of high redundancy budget is not enough to guarantee that the closed form (22) holds.

Now that we know the optimal strategy of redundancy allocation, we can let the number  $N$  of intervals go to infinity (which means reducing the size of the intervals to zero) and find, in this way, the optimal spectral distribution of the redundancy for the two real p.s.d.'s ( $\sigma_1^2(\omega), \sigma_2^2(\omega)$ ):

$$\rho(\omega) = \rho + \frac{1}{4} \log(\sigma_1^2(\omega) - \sigma_2^2(\omega)) - \frac{1}{8\pi} \int_{-\pi}^{\pi} \log(\sigma_1^2(\omega) - \sigma_2^2(\omega)) d\omega. \quad (23)$$

Once we know the spectral distribution of the redundancy, we can automatically find the shape of the side distortion for the “recorrelating” transform  $T(\omega)$  and of  $a(\omega)$ :

$$\begin{aligned} D &= \frac{1}{2\pi} \int_{-\pi}^{\pi} D(\omega) d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{2} \sigma_1^2(\omega) - \frac{(\sigma_1^2(\omega) - \sigma_2^2(\omega))}{4 \cdot 2^{2\rho(\omega)}(2^{2\rho(\omega)} - \sqrt{2^{4\rho(\omega)}} - 1)} d\omega. \end{aligned}$$

Note that when the approximation (20) cannot be applied, the only way to find the spectral distribution of the redundancy and of the other variables is via numerical optimization.

To conclude this section, we want to point out that a generalization of these results to the case of three or more channels has not been found yet. In fact while it is possible to analytically construct the decorrelating matrix  $M(\omega)$  for the case of more than two channels [9], an analytical solution for the matrix  $T(\omega)$  for the general case is not known.

## 4 Application to Gauss-Markov Sources

In this section we show our optimization results for a first order Gauss-Markov or Gauss autoregressive source  $x[n] = \alpha x[n-1] + w[n]$ . The p.s.d. of this process is:

$$S_x(\omega) = \frac{1}{|1 - \alpha e^{-j\omega}|^2}. \quad (24)$$

After downsampling, the two polyphase subsequences  $x_1[n], x_2[n]$  are still Gauss-Markov processes, but with the regression coefficient  $\alpha$  replaced by  $\alpha^2$  and the i.i.d. original Gaussian source  $w[n]$  replaced by a new i.i.d. Gaussian source with zero mean and variance  $1 + \alpha^2$ . Hence the p.s.d. for these two processes are given by

$$R_{x11}(\omega) = R_{x22}(\omega) = \frac{1 + \alpha^2}{|1 - \alpha^2 e^{-j\omega}|^2}, \quad (25)$$

whereas the cross p.s.d.  $R_{x12}(\omega)$  is given by

$$R_{x12}(\omega) = \frac{\alpha(1 + e^{(-j\omega)})}{1 + \alpha^2} R_{x11}(\omega), \quad (26)$$

with  $R_{x21}(\omega) = R_{x12}^*(\omega)$ . The p.s.d. matrix after decorrelation is

$$R_{\hat{x}}(\omega) = \begin{bmatrix} R_{x11}(\omega) \left(1 + \frac{2\alpha \cos(\omega/2)}{(1+\alpha^2)}\right) & 0 \\ 0 & R_{x11}(\omega) \left(1 - \frac{2\alpha \cos(\omega/2)}{(1+\alpha^2)}\right) \end{bmatrix} \quad (27)$$

Observe that the two p.s.d.'s after decorrelation are equal only at  $\pi$  (and of course at  $-\pi$ ). The next step entails the construction of the matrix  $T(\omega)$ . As previously stated, at the points closest to the frequency values where  $\sigma_1^2(\omega) = \sigma_2^2(\omega)$  it is not possible to use the closed-form (22) even in the high redundancy hypothesis. So, for the Gauss-Markov source,  $a(\omega)$  (and consequently  $T(\omega)$ ) can only be found numerically. The final polyphase matrix  $H(\omega)$  is given by the product of the matrix  $T(\omega)$  with the matrix  $M(\omega)$ .

To analyze the performance of this filter bank, we compare it against the ideal bounds found in Section 2. In this experiment we have fixed the target central distortion  $D_0$  and the total amount of bits ( $R_1 + R_2$ ) that we are allowed to transmit over the two channels, given these constraints we have computed the side distortion for the two systems. We have varied the value of the memory coefficient  $\alpha$  of the Gauss



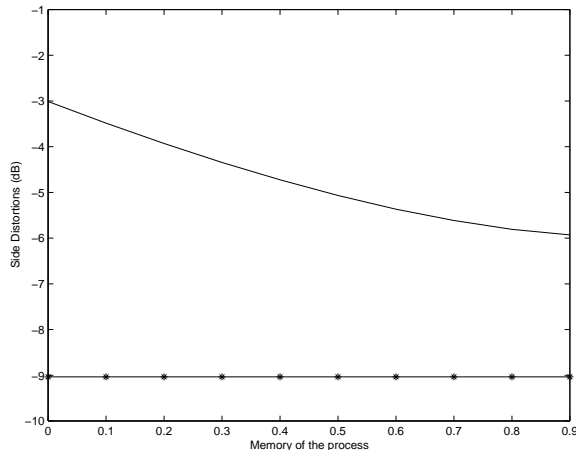


Figure 3: Solid: MDC using filterbanks, dashed-star: ideal bound.

Markov process to see the effect of the memory of the source on the performance of the two systems. The results are shown in figure 3. We can see that for small values of  $\alpha$  (so for an almost memoryless source) the performance of the proposed system is quite poor. (For  $\alpha = 0$  the gap between the two cases is of 6.02dB while in this case an entropy constrained MD Scalar Quantizer performs only 3.06dB worse than the ideal bounds[11]) On the other hand for highly correlated sources the gap between the optimal filter banks and the MD bounds is reduced in this experiment to 3.10dB. So in this case the filter bank system could effectively compete with Multiple Description Transform codes based on decorrelating the input sequence with a unitary transform and then in applying a different MD Scalar Quantizer on each decorrelated component [1].

An important question raised by our work relates to trade offs between coding performance and delay/complexity of the encoders. The filters obtained with our proposed design procedure are in general of infinite length, but in practical settings, we may be forced to consider approximate FIR solutions. Analogously in the case of MD transform codes, the size of the unitary transform (Karhunen-Loeve transform for the Gaussian case) cannot be arbitrarily large. Then the issue is to understand which of the two systems performs better for a fixed filter/block length. This is the focus of our current work on this topic.

## 5 Conclusions

In this work we addressed the problem of MD coding in a subband coding framework. We have generalized previous results, which apply only to finite length input vectors, to the more general case of input sequences and subband decompositions. We have shown how to design filter banks that can minimize the side distortion given a certain amount of redundancy. Two important contributions of this paper are: (a) the characterization of a region which asymptotically is the MD rate/distortion region for general stationary Gaussian sources, and (b) the identification of conditions (i.e.,

MD coding of highly correlated sources) under which subspace-based methods are potentially competitive with quantizer-based methods. This is important because, in the context of scalar quantization, MD transform codes attain optimal performance [1].

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