SIG AL TRAN FORMS FOR THE E E AN IDE TIFICATION F SIG ALS

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ABSTRACT

In this paper the authors address the problem of detection and identification of signals buried in high level noise. Frequency domain techniques with long integration intervals are particularly well suited to perform this task if the signal is composed of a mixture of stationary sinusoidal terms. One can achieve reliable detection even in the very low SNR case. However when the signal itself exhibits time-varying features, even when these are known in advance, detection and identification is reliable only over time intervals where the signal is approximately stationary. The limited integration time puts a lower bound to the allowable SNR. In this paper we propose the use of an adaptive signal transformation previously introduced by the authors, which reverts the time-varying signal to a simpler stationary one. Constant features over time allow longer integration times -- up to the duration of the total event -- thus granting proper detection even in the extremely low SNR case. This is for example the case of a simple monochromatic or narrow-band signal, whose frequency varies over time, with a known frequency law over time, such as a chirp signal. In the paper we review the relevant features of the proposed transformation and detail our method providing significant bounds for its numerical performance and examples.

1 INTRODUCTION

A recently introduced approach to signal transformations [1] claims the advantages of signal transforms, e.g., frequency warping, aimed at enhancing some features of the signal. This enhancement may render its identification easier and allows for a straightforward way to the detection of relevant signal features, or to the detection of the signal itself. Also, the approach may facilitate proper filtering for the removal of noise or artifacts by working in the transformed domain. Inverse transformation in this case will recover the source signal after proper enhancement of selected features.

When the signal is buried in high-level noise, noise removal techniques employing frequency-domain averaging are largely used. Let us consider the simple case of a complex exponential signal with frequency ω_0 buried in WSS zero-mean white noise $w(n)$ with variance σ_w^2:

$$s(n) = \sqrt{P_s}\, e^{j(n\omega_0+\varphi)} + w(n) .$$

For detection one takes the magnitude square DFT of the signal over a sufficiently large interval of N samples:

$$Y(\omega_k) = \frac{1}{N}\left|\sum_{k=0}^{N-1} s(n) e^{-j\omega_k n}\right|^2 ,$$

where $\omega_k = \frac{2\pi k}{N}$. It is easy to show that the expected value of the frequency bin k nearest to the ω_0 value, in the worst case of $\omega_0 = \frac{\pi}{N}(2k \pm 1)$, equals

$$E\{Y(\omega_k)\} = \frac{P_s}{N\sin^2\left(\frac{\pi}{2N}\right)} + \sigma_w^2 \geq \frac{4NP_s}{\pi^2} + \sigma_w^2 .$$

By means of the DFT estimator, which in this case proves to be optimal, estimation is possible if the SNR ρ satisfies the condition:

$$\rho = \frac{P_s}{\sigma_w^2} > \frac{\pi^2}{4N} .$$

We can therefore argue that the allowed SNR decreases in inverse relationship with the length N of the signal or of the analysis window. Using the dB equivalent Q of the SNR ρ, we may state that for reliable detection we need to meet the condition $N > .25 \times 10^{1-Q/10}$. As a consequence, if our signal spreads its energy over more than a single bin, e.g., on a narrow-band due to a slowly time-varying frequency of a frequency sweep, the performance of the DFT detector is degraded. It is also demonstrated that by reverting the signal to a single line one can approach the optimal method of detection. Also, if we are willing to reduce the variance of the frequency estimate, we may resort to averaging techniques such as the periodogram approach. In these cases the described optimal detection consists in a peak-picking algorithm. Finally, if detection must be performed with a fixed degree of confidence, we may also resort to statistical peak detection, which provide exact estimates for the probability of detection, false alarm and miss rate. In all of the above cases there is a need for performing a suitable frequency warping with the described features. Concentrating the energy of the signal on a narrow frequency band greatly enhances detection.

2 FREQUENCY WARPING AND THE TIME-VARYING LAGUERRE TRANSFORM

The authors of this paper recently developed a technique for altering in a controlled way the frequency content of a signal by warping its spectrum in a time-varying fashion [10]. This is achieved by means of a generalization of the discrete Laguerre transform. It is well known [2] that projection of a signal on the complete orthogonal set of discrete Laguerre sequences can be computed by means of the usual building blocks of discrete processing. Expansion on a related biorthogonal set [3] can be implemented by means of a time-reversal operation and a chain of identical all-pass sections sampled at time k=0. The structure is shown in the block diagram of Fig. 1, where the $A_i(z)$ transfer functions, which determine the basis, are copies of the first order all pass section

$$A_i(z) = \frac{z^{-1} - b_i}{1 - b_i z^{-1}},$$

whose real pole b_i=b is constant at each stage.

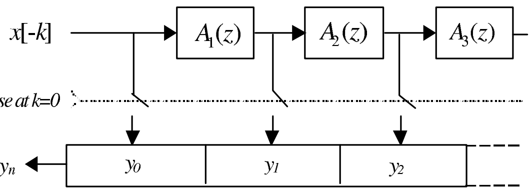

Fig. 1 Analysis structure for the biorthogonal Laguerre transformation.

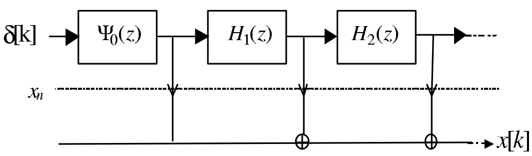

Fig. 2 Synthesis structure for the inverse transformation.

The sequence of the orthogonal projections produced in this way is the transformed signal; it exhibits a frequency spectrum that is a warped version of the original signal spectrum [3][4]:

$$X(e^{j\omega}) = Y(e^{j\vartheta(\omega)}).$$

Here $\vartheta(\omega)$, the warping law, is the sign-reversed phase response of the first order all-pass section. The pole of the all-pass section controls the warping law for each frequency bin of the signal. This transformation can be orthogonalized while preserving the computational structure based on rational filters. This is achieved by means of a proper low-pass or high-pass first order section [4] used as a first stage in the all-pass chain of Fig. 1. The unitary property implies energy preservation: in any warped frequency band the warped signals has the same energy as the original signal in the original bands. Orthogonality and completeness of the Laguerre set allows cascading the transform to a second unitary transform, mainly a wavelet transform, while preserving orthogonality and completeness. Cascading the two transforms results in a new and interesting class of wavelet transforms that allows to overcome the drawback of octave band frequency resolution of the ordinary discrete dyadic wavelet transform [4][5]. This property is useful in various DSP environments [6][7] [8] [9].

As a further development the authors introduced an interesting generalization of the Laguerre transform to a time-varying version. Starting from the idea underlying the Laguerre Transform it is possible to provide different values of the Laguerre parameter at each stage of the chain. Since to each value of the parameter corresponds a different curve in the family of the warping curves, this approach provides a time-varying frequency warping in which each output sample is influenced by the history of the warping curves traversed along the chain. Thus, by properly setting the sequence of Laguerre parameters we may produce an arbitrary frequency law for a specified signal bin. The frequency contents of segments of the signal are modified according to the time-varying warping characteristic, moving from a curve to the next one in the family. Obviously, under these conditions the transform looses some of its features: in the orthogonal Laguerre transform, in fact, the inverse transform is again an orthogonal Laguerre transform based on a sign-reversed value of the warping parameter [4]. In the time varying case this feature is lost. The authors have found a set of sequences which is biorthogonal to the time varying warping set [10]. The biorthogonal sets pair $\varphi_n(k)$ and $\psi_n(k)$ whose Z-transforms are:

$$\Phi_n(z) = \begin{cases} 1 & if\ n=0 \\ \prod_{k=1}^{n} \frac{z^{-1} - b_k}{1 - b_k z^{-1}} & if\ n>0 \end{cases}$$

$$\Psi_n(z) = \begin{cases} \frac{1}{1 - b_1 z^{-1}} & if\ n=0 \\ \frac{1 - b_n b_{n+1}}{(1 - b_n z)(1 - b_{n+1} z^{-1})} \Phi_n(z) & if\ n>0 \end{cases}$$

allow perfect reconstruction of the source signal (invertibility). The block diagram of the direct transformation is given in Fig. 1 and is similar to the biorthogonal Laguerre case except that the b_i parameters are different at each stage. The synthesis diagram for the inverse transform has the structure of a Laguerre filter [10] shown in Fig. 2, where

$$H_n(z) = \frac{1 - b_n b_{n+1}}{1 - b_{n-1} b_n} \frac{z^{-1} - b_{n-1}}{1 - b_{n+1} z^{-1}}.$$

3 WARPING A MONOCHROMATIC SIGNAL

The warping structure in Fig. 1 may be analyzed in time domain as a special type of time-varying filter. Each of the y_n lines, n=0,1,…, carries a filtered version of the input signal $x(-k)$, the transfer function at index n being the cascade of the sections up to n. The signals $y_n(k)$ are therefore filtered versions of the input signal by the transfer functions

$$\Phi_n(z) = A_1(z) \cdot A_2(z) \cdots A_n(z).$$

For a real signal $x(k)$ we have in the frequency and time domains, respectively, we have:

$$Y_n(\omega) = X^*(\omega) \cdot \Phi_n(\omega),$$

$$y_n(k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X^*(\omega) \left(\prod_{m=1}^{n} A_m(\omega) \right) e^{jk\omega} d\omega$$

The n-th sample of the output sequence stored in the shift register is given by the sample of the $y_n(k)$ sequences, at time k=0.

$$y_n = y_n(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X^*(\omega) \left(\prod_{m=1}^{n} A_m(\omega) \right) d\omega = \langle x, \varphi_n \rangle$$

where,

$$\langle x, \varphi_n \rangle = \sum_k x(k)\, \varphi_n(k)$$

is the orthogonal projection coefficient of the signal over the analysis set.

The $A_n(\omega)$ are all-pass functions characterized by a pure phase response: $A_n(\omega) = e^{-j\vartheta_n(\omega)}$, where for $|\omega| < \pi$

$$\omega + 2\arctan\frac{b_r \sin\omega}{1 - b_r \cos\omega} = 2\arctan\left(\frac{1+b_r}{1-b_r}\tan\frac{\omega}{2}\right). \quad (1)$$

At the n-th stage we obtain:

$$\Phi_n(\omega) = \prod_{r=1}^{n} e^{-j\vartheta_r(\omega)} = e^{-j \cdot \sum_{r=1}^{n} \theta_r(\omega)}.$$

Consider the simple case of single complex exponential tone $x(k) = e^{jk\omega_0}$; we have

$$X(\omega) = 2\pi\delta(\omega - \omega_0) \text{ for } |\omega| < \pi.$$

Therefore, by inversion, we obtain for the output sample:

$$y_n = e^{j \cdot \sum_{r=1}^{n} \theta_r(\omega_0)} \quad (2)$$

This formula has a simple meaning when the all-pass sections are identical, $\theta_r(\omega)=\theta(\omega)$, $r=1,2\ldots$. In this case we have: $y_n = e^{j \cdot n \cdot \theta(\omega_0)}$ and the output is a complex exponential whose frequency is the image of the source frequency ω_0 via the warping map $\theta(\omega)$. This warping map is the sign-reversed phase response of the replicated all-pass sections.

If different warping laws are applied at each stage by using different values of the parameters b_r, then (2) corresponds to a phase-modulated signal, whose features and spectrum depend on the $\theta_r(\omega)$ laws. Each of these laws shows a linear part plus a deviation:

$$\vartheta_r(\omega) = \omega + \Delta_r(\omega)$$

obtaining a complex oscillating signal

$$y_n = e^{j \cdot (n \cdot \omega_0 + \sum_{r=1}^{n} \Delta\theta_r(\omega_0))}.$$

However, in general, the terms $\Delta_r(\omega)$ can have the same order of magnitude as the carrier.

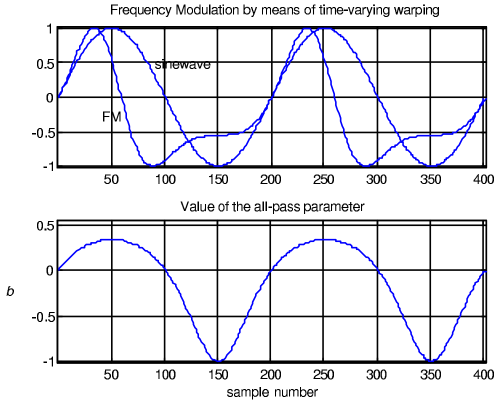

Fig. 3 Frequency modulation by means of warping

This signal corresponds to a frequency modulated sinusoid depending on $\Delta\theta_r(\omega)$, i.e., on the choice of b_n, as shown in Fig. 3. The defined transformation may be seen as a "phase transformation". Interestingly, the ability to control the phase of the signal allows us, under some simple conditions, to reduce oscillating signals to a simple sinusoidal model. For example, by inverse transforming the frequency modulated signal one is able to reduce a complex time signal to a simple oscillatory term. The features of the basis set of the warping transform will then simplify the Fourier analysis of the transformed signal.

4 FREQUENCY LAW COMPENSATION

Returning to the problem under investigation, the above-described transform may be used in order to reduce a time-varying monochromatic signal to an almost perfectly periodic one. We must select the proper sequence of Laguerre parameters b_n in order to equalize the frequency law of the signal. This choice is dictated by (1). Suppose that ω_n is the instantaneous frequency of the signal and we desire a constant output instantaneous frequency ϖ. Then we must set $\varpi = \theta_n(\omega_n)$, which gives

$$b_n = \tan\left(\frac{\pi}{4} - \frac{\tan\frac{\omega_n}{2}}{\tan\frac{\varpi}{2}} \right)$$

In warping or unwarping a signal, we must take into account the fact that time-varying warping is a time-frequency operation, which results in a modification of time duration, as well as of frequency. This can be achieved by delaying the variation of the parameters by an amount approximately equal to the phase delay introduced by the transform.

Once we build the proper b_r law compensating for the modulation law of the source signal, we are able to revert it to a fairly monochromatic signal. The results of the operation are as follows: Fig. 4 shows the spectrogram of the source linear frequency sweep and Fig. 5 shows the spectrogram of the same signal after warping with the proper frequency law. We can clearly recognize that the sweep signal has been reduced to a signal whose frequency is constant over time.

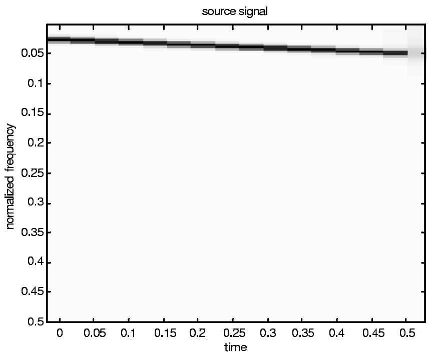

Fig. 4 Spectrogram of the source sweep signal

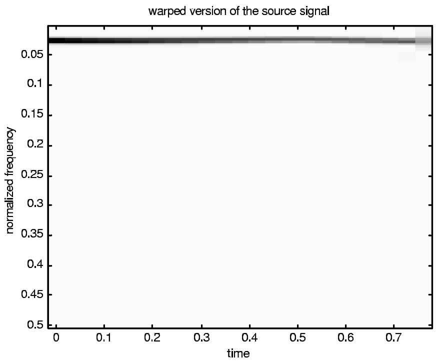

Fig. 5 Spectrogram of the frequency compensated sweep by means of time-varying warping

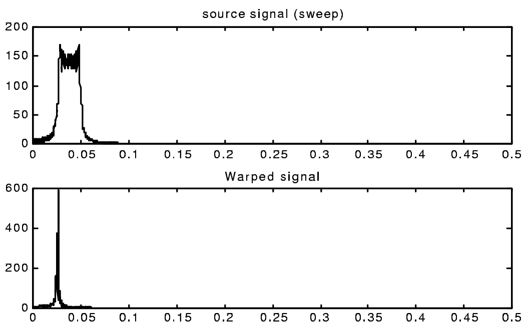

Fig. 6 Magnitude Fourier transform of a frequency sweep and of its time-varying warped version.

This is also illustrated in Fig. 6 where both the source linear sweep and its time-varying warped are shown in the frequency domain. It is clear seen that the energy of the signal, which was originally spread over a broad interval, is now concentrated almost in a single frequency bin. This is the basis for efficient detection. In fact, if we add noise to the signal (see Fig. 7) we see that while in the source spectrum no frequency bin shows prominence over the others, in the warped domain the sinusoidal components add coherently. This allows for reliable detection, even under severe SNR conditions, by using sufficiently long integration times.

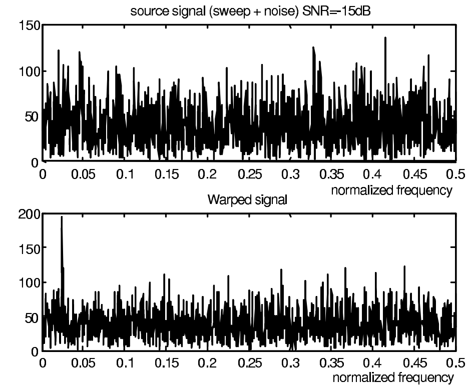

Fig. 7 Source frequency spectrum and warped version (SNR=-15dB)

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