# On adaptive non-linearity for color discrimination and chromatic adaptation

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## Abstract

Assuming that the photoreceptor response of the human visual system is adaptive and non-linear, we can derive mathematical properties that can account for both color discrimination and chromatic adaptation. This could be due to the photoreceptors' response to illumination, which is non-linear and varies according to the adaptation state. Assuming the Naka-Rushton non-linear function and an automatic gain control function, we can derive color discrimination and chromatic adaptation data. We extend the discussion to a three-layer model of retinal color processing, and show how we could predict corresponding color data.

## Introduction

Many studies have shown that the response of the human visual system to color stimuli is non-linear and adaptive. Color discrimination and chromatic adaptation are two visual phenomena that underline these properties. First, to derive a perceptually uniform color space, for example CIELab, a non-linear function is applied to XYZ tristimulus values. Second, the appearance of a color is not constant over space and time; rather it is dependent on the illuminant and surround, i.e. the state of adaptation of the visual system at a given moment in time.

We know from the physiology literature that the photoreceptors' response to light is non-linear and adaptive. However, we can assume that they are not the only elements of the visual system responsible for color vision. But to understand what their role is remains an interesting and unsolved question. The functionality of photoreceptors is known from physiology, while color experiments are based on psychophysical tests. It is still unclear how a particular neuron function can influence the whole behavior of color vision.

Color discrimination data is usually based on *simultaneous* psychophysical experiments, and reveal the non-linearity of the visual system. On the other hand, chromatic adaptation data, i.e. corresponding colors, are usually based on *successive* tests, and reveal the visual system's response to adaptation. In this paper, we make the hypothesis that the photoreceptor's non-linearity is modified by the adaptation state. However, for a given adaptation state, non-linearity remains constant. This allows us to consider only the non-linearity in case of a fixed adaptation state. Additionally, the relationship is linear with respect to a change in the state of adaptation. To simplify the discussion, we restrict our study to one particular non-linearity function and one particular adaptation scheme.

We demonstrate how non-linearity and adaptation of a photoreceptor function is compatible with line elements of a color space and the von Kries hypothesis. We also explain in detail why we need more than the photoreceptor function to explain color discrimination data, and we compare adaptation in photoreceptors with the classical model of chromatic adaptation.

Once we have established that photoreceptor adaptive non-linearity accounts for both sets of experimental data, we extend our analysis to the functionality of the whole retina. In particular, we present a three-layer model that includes a non-linear encoding and two adaptation levels. The advantage of this approach is that such a model is more meaningful from a physiological point of view. We show how this model could predict corresponding color data.

## Adaptive non-linearity

Photoreceptors are the light-sensitive elements of the human visual system. They are responsible for the spectral sensitivities of color vision, and they also respond non-linearly and adaptively to allow for a large dynamic response under different illumination conditions. As the regulation of input light is very important for matching neuron dynamics, we could think intuitively that this behavior strongly relates to adaptation and color discrimination. Practically, we will show in this part that assuming an adaptive non-linearity for the photoreceptor's light transduction is compatible with both the von Kries hypothesis, i.e. the model for chromatic adaptation, and line elements, used to account for color discrimination data.

Photoreceptor non-linearity is modeled by the Naka-Rushton law [5], a generalization of the Michaëlis-Menten homographic function, which was designed for enzyme kinetics. This model is very similar to the behavior found regulating the GMP (guanosine monophosphate) responsible for transforming light into

electrical current in the photoreceptors [6]. Finally, it is confirmed by the microelectrode suction response [7].

The Naka-Rushton law is given by the following function $f$ :

$$f_{X_0}(X) = \frac{X^\alpha}{X^\alpha + X_0} \qquad (1)$$

Where $X = \{L, M, S\}$ represents the excitation of the photoreceptors, meaning color values in LMS-cone space. $X_0 = \{L_0, M_0, S_0\}$ is the adaptation state of a particular photoreceptor, $\alpha$ is a parameter (in this study, we assume $\alpha = 1$). We can show that the adaptation parameters modify the curvature of the non-linear function.

The way adaptation operates is often explained as *automatic gain control*. This means applying the function in $X / X_0$ instead of $X$. $X_0$ is a scaling factor equal to the adaptation state calculated from the environment, such as the illuminant contribution or spatio-temporal statistics. $X_0$ modifies the gain of the function and is calculated automatically from the input statistics.

The Naka-Rushton law can therefore be rewritten as follows:

$$f_{X_0}(X) = f\left(\frac{X}{X_0}\right) \ \text{ with } \ f(X) = \frac{X}{X+1} \qquad (2)$$

This formulation shows that the adaptation state $X_0$ operates as a scaling factor. As already shown in [4], assuming a constant adaptation state for the photoreceptor non-linearity is a reasonable hypothesis for MacAdam ellipses.

Actually, color discrimination data are represented by ellipsoids in LMS space, which represent the loci of just noticeable difference (JND) with the reference color in the center of the ellipsoid. In fact, ellipsoids in cone space define a Riemanian metric rather than a Euclidian metric. The Euclidian space is a particular case of a Riemanian space. In color science, a Euclidian space is known as a perceptually uniform color space, where as a Riemanian space is characteristic of color discrimination data representation. Line elements [13] is a mathematical formalism that allows to construct a relation between these metrics. This relation is known to be non-linear and depends on the location in the color space. If we take the linear approximation[1] of the Naka-Rusthon function around the center of ellipsoid, we can write:

$$\left.\frac{\partial f(X/X_0)}{\partial X}\right|_{X=X_c} = \frac{X_0}{\left(X_c + X_0\right)^2} \qquad (3)$$

Consequently, adaptive non-linearity operates as a scaling factor, depending on the adaptation condition $X_0$ and the position in color space $X_c$. This transformation allows converting ellipsoids into spheres as shown in figure 1.
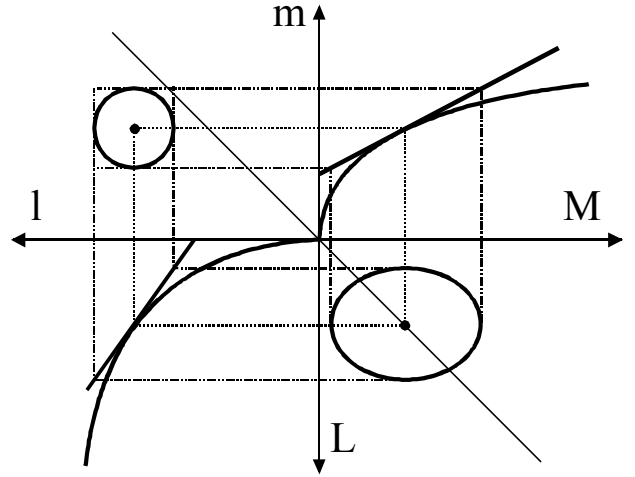
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[1] This approximation is correct if we consider small thresholds of color difference



*Figure 1: Adaptive non-linearity allows the transformation of a discrimination ellipsoid into a sphere.*

When considering chromatic adaptation, we wish to map the appearance $x$ of an object having color $X$ under one adaptation condition $X_0$ to the appearance $x'$ under another condition $X_0'$ :

$$x = f\left(\frac{X}{X_0}\right) \quad x' = f\left(\frac{X'}{X_0'}\right) \qquad (4)$$

If the appearance of the two colors is the same, then $x = x'$, which is equivalent to:

$$\frac{X'}{X_0'} = \underbrace{\left(f^{-1} \circ f\right)}_{Id}\left(\frac{X}{X_0}\right) \qquad (5)$$

Since $f^{-1} \circ f$ is the identity function, the ratio between the colors $X$ and $X'$ is equal to the ratio of the adaptation state. This is equivalent to the von Kries hypothesis, where we assume that the linear gain factors depend on the ratio of the illuminants. Note that in case of incomplete adaptation, this model still applies. However, the adaptation parameters are not equal to the illuminant values anymore.

Thus the adaptive, non-linear behavior of the photoreceptors explains both color discrimination and chromatic adaptation. In the next two sections, we will see how accurate this model is.

## Color Discrimination

We assume that non-linearity applies in each color channel L, M and S independently, and that the adaptation state is constant (equal to illuminant C in the MacAdam experiment) during all the measures. We call the uppercase letter $\mathbf{X} = \begin{bmatrix} \Delta L & \Delta M & \Delta S \end{bmatrix}^T$ the differential element in LMS space and lowercase letter $\mathbf{x} = \begin{bmatrix} \Delta\ell & \Delta m & \Delta s \end{bmatrix}^T$ its transformation through the Naka-Rushton non-linearity.

Defining $\mathbf{x}$ as a perceptually uniform color space is equivalent to express color discrimination as a sphere of unitary diameter:

$$(\ell - \ell_c)^2 + (m - m_c)^2 + (s - s_c)^2 = 1 \ or \ \mathbf{x}^T\mathbf{I}\mathbf{x} = 1 \qquad (6)$$

with $\mathbf{x}^T = \begin{bmatrix} \ell - \ell_c & m - m_c & s - s_c \end{bmatrix} = \begin{bmatrix} \Delta\ell & \Delta m & \Delta s \end{bmatrix}$ and $\mathbf{I}$ the identity matrix.

Since the threshold of color differences is small, we can approximate non-linearity by a linearity around the ellipse centers so that:

$$\Delta x = \left. \frac{\partial f(X/X_0)}{\partial X}\right|_{X=X_c} \Delta X \qquad (7)$$

where $X_c$ is the color center around which color discrimination is measured, and $\Delta X$, $\Delta x$ are the distances of each point of the ellipsoid with respect to the center in LMS space and $\ell ms$ space, respectively.

Ellipsoid $i$ in LMS space is defined by the following equation:

$$\mathbf{X}^T\mathbf{G}_i\mathbf{X} = 1 \qquad (8)$$

The non-linear model transforms the ellipsoid such that it becomes:

$$\mathbf{x}^T\left(\mathbf{T}_i^{-1}\right)^T \mathbf{G}_i\mathbf{T}_i^{-1}\mathbf{x} = 1$$

*with*

$$\mathbf{T}_i = \begin{bmatrix} \dfrac{L_0}{\left(L_0 + L_c[i]\right)^2} & 0 & 0 \\ 0 & \dfrac{M_0}{\left(M_0 + M_c[i]\right)^2} & 0 \\ 0 & 0 & \dfrac{S_0}{\left(S_0 + S_c[i]\right)^2} \end{bmatrix} \qquad (9)$$

where $L_c[i]$, $M_c[i]$, $S_c[i]$ represent the LMS center for ellipse $i$.

Otherwise said, the model predicts that $\left(\mathbf{T}_i^{-1}\right)^T \mathbf{G}_i\mathbf{T}_i^{-1} = \mathbf{I}$. We have computed $\mathbf{g}_i = \left(\mathbf{T}_i^{-1}\right)^T \mathbf{G}_i\mathbf{T}_i^{-1}$ for all the 25 ellipses of MacAdam's original data, assuming a constant adaptation state equal to illuminant C. MacAdam's data is given in xy chromaticity coordinates; the complete transformation into the LMS space is given in Annex A.

All $\mathbf{g}_i$ are not equal to the identity matrix. Therefore, we have chosen to define a mean matrix parameter as $\mathbf{g} = \overline{\mathbf{g}_i}$ and reconstruct ellipses from $\mathbf{g}$, through $\tilde{\mathbf{G}}_i = \mathbf{T}_i^T\mathbf{g}\mathbf{T}_i$, which results in a good approximation [14].

One reason why $\mathbf{g}_i$ is not equal to identity is that the ellipsoids in LMS space are not oriented along the LMS axis. However, the transformation matrix $\mathbf{T}_i$ (a diagonal matrix) applies a scaling factor only along the L, M and S axis. Therefore, assuming that only photoreceptor non-linearity is responsible for the color discrimination ellipses is not enough to fully account for the experimental data.

We can argue that photoreceptors are not the only site of adaptation and non-linearity in the visual system. For example, Vos and Walraven [13] have already suggested applying line elements additionally on luminance and opponent colors. There is other evidence in the literature that this kind of non-linearity and

adaptation is only a first order approach to the reel phenomenon and that a second mechanism is needed [12]. In discrimination tasks, Yeh, Smith and Pokorny [9] argue for two non-linearities. Webster and Mollon [10] also proposed a second adaptive process for taking into account all adaptation properties. Sharpley and Enroth [12] call this second adaptation site the *contrast gain control*.

Ganglion cells, which form the relay between the retina and the cerebral cortex, should also be modeled with adaptive non-linearity. These cells also need to guarantee maximum dynamic range response for the entire signal carried by previous neurons in the retina. The maximum of the difference of signals is in general less than the maximum of each. Additionally, ganglion cells are known to carry color opponent signals. For these reasons, ganglion cells should adapt to the dynamic range. We therefore assume that the ganglion cells also function with adaptive non-linearity.

We suppose that color signals (L, M and S cone responses) are coded into luminance and two color opponent channels, and then the adaptive ganglion cell operates non-linearly. Thus, the complete retinal processing model we propose is composed of three layers and is as follows:
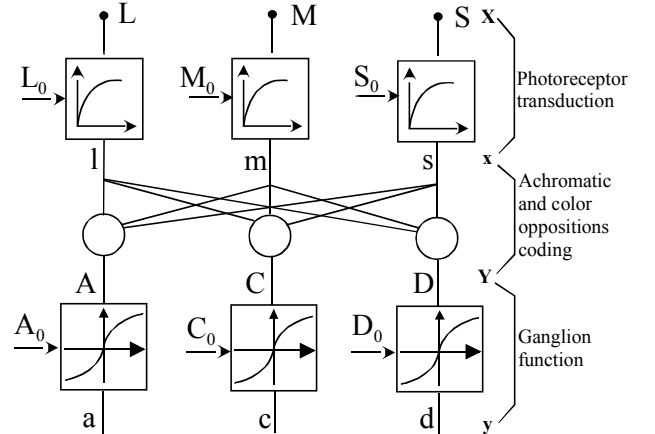


*Figure 2: Three-layer model of the retina.*

This model is able to account for ellipsoids in every orientation, as shown in [11]. Moreover, the model successfully predicted several discrimination data sets for several observers in different experimental conditions.

In the following section, we will demonstrate why it is interesting to consider this model in the case of chromatic adaptation.

## Chromatic Adaptation

Chromatic adaptation models and transforms are usually evaluated on sets of experimental corresponding color data, which is available encoded in CIE 1931 XYZ tristimulus values [15]. If $\mathbf{P}$ is a (Nx3) matrix of color triplets of $N$ color surfaces under illuminant $\mathbf{i}$, $\mathbf{S}$ (Nx3) is another set of color triplets corresponding to the same appearance perceived under a second illuminant $\mathbf{j}$. $\mathbf{i}$ and

**j** are [3x1] vectors containing illuminant tristimulus values.

According to the von Kries chromatic adaptation model, the relationship between corresponding colors is linear. **S** values can therefore be derived as a linear combination of **P** values. In matrix notation, this is given by:

$$\mathbf{S} = \mathbf{TP} \qquad (10)$$

Many studies exist that have tested and confirmed the relevance of the linear model to account for chromatic adaptation. However, the question remains as to what exactly the meaning of this linear relation is. We have already seen that the photoreceptor adaptive non-linearity is compatible with this model. However, what is the role of the photoreceptors, and what can we say about the other retinal encoding parameters?

A way for estimating the matrix **T** is to minimize the least-squares error between the estimate $\tilde{\mathbf{S}}$ given by the linear model and **S** :

$$\min\left\|\tilde{\mathbf{S}} - \mathbf{S}\right\| = \min\left\|\mathbf{TP} - \mathbf{S}\right\| \Rightarrow \mathbf{T} = \mathbf{SP'(PP')^{-1}}$$

*with*

$$\left\|\tilde{\mathbf{S}} - \mathbf{S}\right\| = \frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{s}\left(\tilde{S}_{i,n} - S_{i,n}\right)^2 \qquad (11)$$

For all known corresponding color data sets, this gives a full [3x3] matrix **T** . This matrix can be decomposed as:

$$\mathbf{T} = \mathbf{MDM^{-1}} \qquad (12)$$

Where **D** is a diagonal matrix and **M** a full [3x3] matrix. **M** is the linear transformation from XYZ space to LMS cone space, or the "response space" where von Kries scaling applies, and **D** is composed of scaling factors for the individual channels.

There are several decompositions possible with this formalism. The one that results in minimum error (see Table 1) is given by taking **M** and **D** as eigenvector and eigenvalue matrices, respectively, of the space spanned by **T** . However, this decomposition is neither related to human vision, such as XYZ to LMS transforms, nor to the ratio of adapting illuminants.

The Hunt, RLAB, and Nayatani et. al. color appearance models [16] use the *HPE* (Hunt-Pointer-Estevez) linear transform, which relates XYZ to relative LMS cone responses. The diagonal matrix is composed of the ratios of the illuminants encoded in LMS:

$$\tilde{\mathbf{S}} = \mathbf{M^{-1}DMP}, \; \mathbf{D} = diag(\mathbf{Mj}./\mathbf{Mi}) \qquad (13)$$

(./) denotes an element-by-element division. Giving such constraints to the choice of **M** and **D** allows to give more meaning to the linear transformation, in the sense of relating it to the human visual system and the illuminants. As a matter of fact, equation (13) describes the linear chromatic adaptation transform (CAT) model most often applied today. The differences between the CATs are in the response space the scaling operates. The diagonal matrix, consisting of the ratios of the two adapting illuminants, is called the von Kries coefficient matrix.

The HPE CAT is widely used today, but it generates a larger error than the eigenvalue decomposition (see Table 1).

Lam [2] chose **M** by minimizing perceptual error ($\Delta E$) between predicted and actual corresponding colors. He optimized his matrix so that the transform is reversible. The resulting *Bradford* CAT also includes a non-linear exponent for the blue. However, in many imaging applications, this non-linearity is neglected and the linear form of the Bradford CAT is used. The interpretation is that we don't know where the von Kries scaling coefficients operate, but that it is in a space that minimizes perceptual error.

Finlayson and Süsstrunk [3] based their transform on white-point preserving data-based spectral sharpening of Lam's corresponding color data set. They also apply eigenvalue decomposition to the general matrix **T** . They conclude that scaling is applied in a more decorrelated response space than LMS.

| Method | Error |
|---|---|
| Eigenvalue decomposition | 6.95 |
| HPE | 14.51 |
| Linear Bradford | 9.53 |
| Sharp | 8.30 |
| 3-layer model | 6.76 |

*Table 1: XYZ least squares errors for several chromatic adaptation models, applied to Lam's corresponding color data.*

Applying the photoreceptor model, as outlined above, corresponds to the HPE transform. While it has the advantage of taking into account the physiological plausible XYZ to LMS transform and to relate adaptation parameters to the illuminant values, it results in a larger error than other chromatic adaptation transforms.

As in the previous section, we can therefore test if the three-layer model better predicts corresponding color data than photoreceptor adaptive non-linearity by itself. Note that the three-layer model is not linear, and that it is not possible to determine all model parameters by a linear optimization, such as eigenvalue decomposition.

Conforming to the notation defined in figure 2, LMS data is transformed to $\ell ms$ through the Naka-Rusthon function. Then, $\ell ms$ is transformed to ACD, which is representative to encoding in luminance A and two opponent colors C and D. The data is then transformed to acd through a second non-linearity.

We have chosen to compute the corresponding color data in $\ell ms$ space. We use the HPE matrix to transform XYZ into LMS. We then apply the Naka-Rushton law on LMS, with the adaptation parameters equal to the (LMS) illuminant values. Now, the correspondence between $\ell ms$ and $\ell 'm's'$ can be expressed as a linear relation. This linear relation could be decomposed as in equation (12). However, this decomposition is not possible with Lam's data as the eigenvalues become complex. We use the singular value decomposition instead, so we write:

$$[\ell' \quad m' \quad s']^{\mathbf{T}} = \mathbf{U}\mathbf{D}\mathbf{V}^{\mathbf{T}}[\ell \quad m \quad s]^{\mathbf{T}}$$

*Where*

$$[A \quad C \quad D']^{\mathbf{T}} = \mathbf{V}^{\mathbf{T}}[\ell \quad m \quad s]^{\mathbf{T}}$$

$$[A' \quad C' \quad D']^{\mathbf{T}} = \mathbf{U}^{-1}[\ell' \quad m' \quad s']^{\mathbf{T}}$$

$$[A' \quad C' \quad D']^{\mathbf{T}} = \mathbf{D}[A \quad C \quad D]^{\mathbf{T}} \qquad (14)$$

The inability to use eigenvalue decomposition on Lam's data can be interpreted that the encoding into luminance and opponent color is not constant and varies with the state of the adaptation. The singular value decomposition allows the mathematical formulation of the modified encoding and to find the two encoding transformations in the two adaptation conditions:

$$\mathbf{U} = \begin{bmatrix} 0.1912 & -0.2188 & -0.9569 \\ 0.9662 & -0.1299 & 0.2228 \\ -0.1731 & -0.9671 & 0.1866 \end{bmatrix}$$

$$\mathbf{V}^{\mathbf{T}} = \begin{bmatrix} -0.1647 & 0.9815 & -0.0979 \\ -0.2470 & -0.1372 & -0.9593 \\ 0.9549 & 0.1338 & -0.2650 \end{bmatrix}$$

$$\mathbf{D} = diag(1.3808 \quad 0.9442 \quad 0.6811) \qquad (15)$$

We found that the error in $\tilde{\mathbf{S}}$ is significantly smaller than with HPE, and even less than with the generalized linear method (see Table 1).

$\mathbf{D}$ contains the adaptation scaling factors applied in the opponent color encoding and can be considered, borrowing the notation of Sharpley and Enroth [12] as the contrast gain control.

## Conclusion

We have shown that a model of adaptive non-linearity in photoreceptors can explain both visual phenomena of color discrimination and chromatic adaptation. An adaptive, non-linear photoreceptor model, which is equivalent to line element and von Kries adaptation in LMS space, can be considered as a first order approximation of the reel phenomenon.

The von Kries chromatic adaptation model does not give good results when applied in cone space, as already shone in [8]. However, a three-layer model of the retina that includes the relationship of encoding into cone space and afterwards into opponent color space could relate the physiological processes with the von Kries chromatic adaptation model. It could also confirm why the Sharp transform, which predicts a much more de-correlated response space, performs so well.

However, we still have to investigate if the three-layer model has properties that fully explain chromatic adaptation. For example, we have to show that it is possible to predict corresponding colors for illuminants other than A and D65. Additionally, as this model works well for color discrimination data, we can investigate how to combine both experiments to be able to predict perceptually corresponding colors.

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## Annex A

For transforming ellipses in CIE-xy space to Smith and Pokorny LMS space, we first transform the xy center coordinates of the ellipse into XYZ through the following transformation:

$$X = \frac{xY}{y} \quad Y = 100 \quad Z = \frac{(1-x-y)Y}{y} \qquad (16)$$

Because MacAdam realized his experiment at constant luminance, we can assume $Y = 100$. Then, we compute the LMS values from XYZ:

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \mathbf{M} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad (17)$$

For transforming the ellipse parameter $\mathbf{H_i}$, such that $\mathbf{X^T H_i X} = 1$ with $\mathbf{X^T} = \begin{bmatrix} dx & dy \end{bmatrix}$ and

$$\mathbf{H_i} = \begin{bmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{bmatrix} \qquad (18)$$

we first extend arbitrary ellipses into an ellipsoid as follows:

$$\mathbf{H_i} = \begin{bmatrix} h_{11} & h_{12} & 0 \\ h_{12} & h_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (19)$$

The Jacobian is then computed as follows:

$$J = \begin{bmatrix} \partial X/\partial x & \partial X/\partial y \\ \partial Y/\partial x & \partial Y/\partial y \\ \partial Z/\partial x & \partial Z/\partial y \end{bmatrix} = \begin{bmatrix} Y/y & -xY/y^2 \\ 0 & 0 \\ -Y/y & (1-x-2y)Y/y^2 \end{bmatrix} \qquad (20)$$

which we extend artificially to a 3x3 matrix:

$$J = \begin{bmatrix} Y/y & -xY/y^2 & 0 \\ 0 & 0 & 1 \\ -Y/y & (1-x-2y)Y/y^2 & 0 \end{bmatrix} \qquad (21)$$

And finally, we compute the parameters of the ellipsoid in LMS space:

$$\mathbf{G_i} = (\mathbf{M'})^{-1} (\mathbf{J'})^{-1} \mathbf{H_i J^{-1} M^{-1}} \qquad (22)$$

Once the model estimates the parameter $\tilde{\mathbf{G}}_i$, we can return to xy coordinates in the following way:

$$\tilde{\mathbf{H}}_i = \mathbf{J'M'} \tilde{\mathbf{G}}_i \mathbf{MJ} \qquad (23)$$

**Biography**

David Alleysson has a PhD in cognitive sciences from University Joseph Fourier at Grenoble France. He studied the processing of color signal in the retina as a basic model for human color perception. He is now a Post-Doc at the Audiovisual Communications Laboratory continuing his research on color-coding and color non-linearity.