On The Asymptotic Capacity of Gaussian Relay Networks

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Abstract — In this paper, we determine the asymptotic capacity of a Gaussian multi-relay channel as the number of relays tends to infinity. The upper bound is an application of the cut-set theorem, and the lower bound follows from an argument involving uncoded transmission. Hence, this paper gives one more example where the cut-set bound is achievable, and one more example where uncoded transmission achieves optimal performance. In the latter sense, the result is an extension to [1]. The arguments of this paper are also relevant to wireless networks, yielding an asymptotic capacity result [2].

I. RELAY NETWORK MODEL
Various relay network models have been studied in the literature, some of the most recent examples being [3, 4, 5]. However, capacity results are rare. The simple Gaussian relay network studied in this paper is obtained from the non-degraded Gaussian single-relay channel by adding $M - 1$ relay branches, as shown in Figure 1. Note that even for the case $M = 1$, capacity is not known for this channel. The $W_k$’s denote additive white Gaussian noise and are assumed to be independent. In this paper, we assume that all $W_k$’s are of the same variance $N$. The general case is studied in [6]. Not shown in the figure are the power constraints. For the scope of this paper, those are $EX^2 \leq P$ and $\sum_{k=1}^{M} X_k^2 \leq c(M)$. An interesting choice is for example $c(M) = MQ$ for some constant $Q$.

II. ASYMPTOTIC CAPACITY
To obtain an upper bound on the capacity of the network of Figure 1, we apply the cut-set bound [7, Thm. 14.10.1]; in particular, we separate the source node and connect the rest of the nodes ideally. The resulting system is a multi-antenna channel with one transmit and $M + 1$ receive antennas. Its capacity is an upper bound to the capacity of our network; we denote it by $C_{\text{upp}}$.

The lower bound is found by fixing the relay operation to be direct forwarding: The output at time $t$ is simply a scaled version of the input at time $t - 1$. Under this assumption, the relay network becomes a point-to-point channel whose capacity can be determined. This is a lower bound to the capacity of our network; we denote it by $C_{\text{low}}$.

Define the auxiliary functions

$$a(M) = \sum_{k=0}^{M} \alpha_k^2 \quad \text{and} \quad b(M) = \sum_{k=1}^{M} \alpha_k^2 \frac{2P + N}{\delta_k^2} \tag{1}$$

With this, our main result can be phrased as follows:

Theorem. If

$$\lim_{M \to \infty} \frac{1}{a(M)} = 0 \quad \text{and} \quad \lim_{M \to \infty} \frac{b(M)}{a(M)c(M)} = \delta < \infty \tag{2}$$

then

$$\lim_{M \to \infty} (C_{\text{upp}} - C_{\text{low}}) \leq \frac{1}{2} \log_2 \left( 1 + \frac{2P + N}{N} \right) \tag{3}$$

The proof is reported in [6].

III. ILLUSTRATIONS AND EXTENSIONS

Example: Suppose $\alpha_k = \delta_k = 1$, for all $k$, and $c(M) = MQ$. The conditions of the theorem are satisfied, and $\delta = 0$. Hence, the bounds are tight, and it can be shown that the asymptotic capacity is

$$C = \frac{1}{2} \log_2 \left( 1 + \frac{(M+1)P}{N} \right) \tag{4}$$

The result has extensions to wireless networks and to sensor networks, reported in part in [2], as well as to fading relay channels.

REFERENCES