The Multiple-relay Channel: Coding and Antenna-clustering Capacity

Michael Gastpar
Ecole Polytechnique Fédérale (EPFL)
Lausanne, Switzerland
Michael.Gastpar@epfl.ch

Gerhard Kramer
Bell Labs, Lucent Technologies
Murray Hill, NJ 07974, U.S.A.
gkr@research.bell-labs.com

Piyush Gupta
Bell Labs, Lucent Technologies
Murray Hill, NJ 07974, U.S.A.
pgupta@research.bell-labs.com

Abstract — **Two coding strategies are developed for a multiple-relay channel having one source terminal and one destination terminal. The first strategy mimics multiple-antenna transmission, while the second strategy mimics multiple-antenna reception. Both approaches are based on the concept of "antenna-pooling" and extend prior art for the single-relay channel. The strategies achieve capacity when the terminals form two closely-spaced clusters.**

I. Antenna-pooling

A multiple-relay channel [1] with $n-2$ relays is defined by a probability distribution $p(y_2, \dots, y_n | x_1, \dots, x_{n-1})$, where x_1 is transmitted by the source terminal, y_k and x_k are received and transmitted, respectively, by relay k for $k = 2, \dots, n-1$, and y_n is received by the destination terminal. The relays can use any causal coding functions, i.e., $x_k[t]$ can be any function of $(y_k[t-1], y_k[t-2], \dots)$.

Two coding strategies for the single-relay channel ($n = 3$) were introduced in [2, Thm. 1 and Thm. 6]. Our approach is to extend these techniques to *multiple* relays by using the concept of *antenna-pooling*. We describe three antenna-pooling methods for the two-relay channel ($n = 4$). Various approaches for more relays can be based on these techniques. While our terminology is inspired by wireless networks, our results apply to any relay networks that satisfy the above description.

Antenna-pooling at the Source: The paradigm is for the source to send a message to the relays first, and thereafter for the source and the relays to send cooperatively to the destination, thus mimicking a multiple-antenna transmitter. For the single-relay case, this strategy is the same as that given in [2, Thm. 1]. An extension of the strategy to multiple relays was presented in [3] where it was shown how one can pass the message to the relays. The approach of [3] involves solving a discrete optimization problem over all feed-forward graphs from the source via the relays to the destination.

Antenna-pooling at the Destination: The paradigm is for the relays to compress their received values and send these to the destination, thus mimicking a multiple-antenna receiver. For the single-relay case, this strategy is the same as that given in [2, Thm. 6]. We extend the strategy to multiple relays in the following theorem.

Theorem 1 *For the two-relay channel, antenna-pooling at the destination terminal achieves any rate less than*

$$R_{APD} = I(X_1; \hat{Y}_2 \hat{Y}_3 Y_4 | X_2 X_3), \tag{1}$$

where the joint probability distribution of the random variables factors as $p(x_1) \cdot p(u_2, x_2) \cdot p(u_3, x_3) \cdot p(y_2, y_3, y_4 | x_1, x_2, x_3) \cdot p(\hat{y}_2 | y_2, x_2) \cdot p(\hat{y}_3 | y_3, x_3)$ *and satisfies the bounds*

$$\begin{aligned} &I(\hat{Y}_2; Y_2 | X_2 U_3) - I(\hat{Y}_2; \hat{Y}_3 Y_4 | X_2 X_3) \\ &\leq \min\{I(U_2; Y_3 | X_3), I(U_2; Y_4 | U_3)\} + I(X_2; Y_4 | U_2 X_3), \end{aligned} \tag{2}$$

$$\begin{aligned} &I(\hat{Y}_3; Y_3 | U_2 X_3) - I(\hat{Y}_3; \hat{Y}_2 Y_4 | X_2 X_3) \\ &\leq \min\{I(U_3; Y_2 | X_2), I(U_3; Y_4 | U_2)\} + I(X_3; Y_4 | X_2 U_3), \end{aligned} \tag{3}$$

$$\begin{aligned} &I(\hat{Y}_2; Y_2 | X_2 U_3) + I(\hat{Y}_3; Y_3 | U_2 X_3) \\ &\quad - I(\hat{Y}_2; \hat{Y}_3 Y_4 | X_2 X_3) - I(\hat{Y}_3; \hat{Y}_2 Y_4 | X_2 X_3) \\ &\quad + I(\hat{Y}_2; \hat{Y}_3 | Y_4 X_2 X_3) \\ &\leq \min\{I(U_2; Y_3 | X_3) + I(U_3; Y_2 | X_2), I(U_2 U_3; Y_4)\} \\ &\quad + I(X_2 X_3; Y_4 | U_2 U_3). \end{aligned} \tag{4}$$

The auxiliary random variables U_2 and U_3 may be chosen from arbitrary alphabets. The rate (1) shows that one can realize a gain similar to 3-antenna diversity reception, but the gain is limited by the bounds (2)-(4).

Two Antenna Pools: For the single-relay channel, the relay can split its power and combine the above two strategies, as shown in [2, Thm. 7]. For the multiple-relay channel, many more degrees of freedom can be exploited. For instance, when one relay is close to the source and another relay close to the destination, a 2×2 multi-antenna system can be approximated. The strategy's performance is given by the following theorem.

Theorem 2 *For the two-relay channel, any rate R satisfying*

$$R < \min\{I(X_1 X_2; \hat{Y}_3 Y_4 | X_3), I(X_1; Y_2 | X_2)\} \tag{5}$$

is achievable, where the joint probability distribution of the random variables factors as $p(x_1, x_2) \cdot p(x_3) \cdot p(y_2, y_3, y_4 | x_1, x_2, x_3) \cdot p(\hat{y}_3 | y_3, x_3)$ *and satisfies*

$$I(\hat{Y}_3; Y_3 | Y_4 X_3) \leq I(X_3; Y_4). \tag{6}$$

II. Antenna-clustering Capacity

It can be shown that the rate of Thm. 1 is the capacity when the two relays form a *cluster* with the destination terminal, i.e., the two relays are very close to the destination terminal. In this sense, Thm. 1 gives the *antenna-clustering capacity* of such a relay network.

Similarly, one can show that Thm. 2 gives capacity when one of the relays is very close to the source terminal and the other relay is very close to the destination terminal. Thus, Thm. 2 gives the antenna-clustering capacity of that relay network.

References

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