A Convergence Theorem for Controlled Queues with Partial Information[1]

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Abstract — **We consider a queuing problem in which both the service rate of a finite-buffer queue and its rate of arrivals are functions of the same partially observed Markov chain. Basic performance indices of this device, such as long term throughput and loss rates, are expressed in terms of an invariant measure over a suitable finite-dimensional simplex. In this paper we prove the existence of that invariant measure.**

I. Introduction

We consider a queue with finite buffer and constant service rate, accessed by N sources that decide to turn between on/off transmitting state independently of each other. The injection rates (control actions) are modeled as Bernoulli random variables: an active source decides between transmitting a packet or staying idle with a certain probability. Thus the number of active sources x_k, where k is the time instant, forms a Markov chain. Sources are not able to communicate among them. The only information locally available is the feedback from the queue of whether the packet sent was accepted or not in the buffer. In [1] we derived the optimal controler u_k that a source should implement to maximize its throughput under some loss constraints. We showed that essentially the optimal controler is uniquely determined by the information state, which is a quantity that contains all the partial information about the hidden state of the system (number of active sources), when knowing the sequence of previous applied controls and observations. A good choice is the vector of conditional probability of the state, $\pi_k \in \Pi$, where $\Pi \subset \mathcal{R}_+{}^N$ is the simplex of probability vectors of size N.

II. Convergence in Distribution of the Information State

In order to evaluate the performance of the system (average throughput, average loss rate), we need to characterize the long term behavior of the information state. The main result of this paper is that the information state converges in distribution.

Theorem. The information state sequence π_k converges weakly (in distribution) $\pi_k \to \mu$ as $k \to \infty$ (where μ is the limit stationary distribution over the Π space).

We show that the information state process π_k forms a Markov chain over the simplex Π, for which jumps outside cells of size smaller than a fixed constant happen with non-zero probability, no matter what is the state. This helps constructing a family of discretizations for the chain, which induces finite state Markov chains shown to have recurrent classes that contain the cell to which the stationary distribution of the x_k process belongs. It follows that the corresponding recurrent restrictions have in the limit stationary distributions. Finally, using the framework in [5], we prove that when the size of the cell becomes infinitely small, there exist an invariant probability measure over Π that is positive on the recurrent subset of the simplex, and null outside, which is the limit probability distribution we are looking for.

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III. Related Work

There are some important results in the literature dealing with similar results on convergence in distribution for partially observed systems. Kaijser [3] proved convergence in distribution of the information state for finite state ergodic Markov chains under some mild conditions. Kaijser's results were used by Goldsmith and Varaiya [2] in the context of the finite state Markov channel formed by a set of discrete memoryless channels and where the partial observations are the input and output symbols of the active channel; that holds under the assumption of i.i.d. inputs. This assumption is removed by Sharma and Singh [4], where it is shown, using the theory of regenerative processes, that for convergence the pair [channel input, channel state] should be drawn from an irreducible, aperiodic and ergodic Markov chain. A seemingly feasible approach to establishing the sought convergence result for our system would have been to consider the control action u_k to play the role of a channel input symbol to the system in the formulations above, while the observation r_k could play the role of a channel output symbol (and consequently the control u and the observation r are the partial observations we have about the system). However, this approach does not yield the sought result, because in our case the control u is a function of the information state. In those previous papers, inputs are decoupled from this quantity. So it is this dependence due to feedback-control the main difference between our setup and all that previous work.

References

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