LOW-COMPLEXITY SUBSPACE METHODS FOR CHANNEL ESTIMATION AND SYNCHRONIZATION IN ULTRA-WIDEBAND SYSTEMS

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ABSTRACT

We consider the problem of low-complexity channel estimation in digital ultra-wideband receivers. We extend some of our recent sampling results for certain classes of parametric non-bandlimited signals and develop several methods that take advantage of transform techniques to estimate channel parameters from a low-dimensional subspace of a received signal, that is, by sampling the signal below the Nyquist rate. By lowering the sampling rate we reduce computational requirements compared to current digital solutions, allow for slower A/D converters and potentially significantly reduce power consumption of digital receivers. Our approach is particularly suitable for indoor wireless sensor networks, where low rates and low power consumption are required. One application of our framework to high-resolution acquisition in ultra-wideband localizers is also presented.

1. INTRODUCTION

Ultra-wideband (UWB) technology has received much attention due to benefits of exceptionally large fractional bandwidth [1], such as very fine time resolution for accurate ranging, imaging and multipath fading mitigation. One of the crucial tasks of an UWB system, which imposes serious restrictions on the system performance, is synchronization. There is a vast literature that has appeared recently, addressing both algorithmic and implementation issues of various synchronization techniques, with a clear trend to eliminate as much as possible the necessity for analog components and perform all processing digitally [4]. While high-performance schemes have already been proposed for analog systems [3], their application to digital-oriented solutions is still not feasible due to prohibitively high computational requirements. Furthermore, implementation of such techniques in digital systems would require very fast and expensive A/D converters (operating in the gigahertz range) and therefore high power consumption.

We propose several frequency domain methods for channel estimation and timing synchronization in UWB systems, which use low-rate uniform sampling and well developed computational procedures. We extend some of our recent sampling results for certain classes of parametric non-bandlimited signals [5] [8] to the problem of channel estimation in ultra-wideband channels, and develop an algorithm for estimating unknown time delays and propagation coefficients from the set of samples of a received signal taken below the Nyquist rate. Our approach leads to faster acquisition compared to currently proposed digital solutions, allows for slower A/D converters and potentially significantly reduces power consumption of digital receivers. Besides, the developed framework can be used for identification of more realistic UWB channel models, where different propagation paths undergo different frequency-selective mitigation, and is particularly suitable for applications such as precise position location or ranging.

2. SUBSPACE CHANNEL ESTIMATION

2.1. Problem Statement

A number of propagation studies for ultra-wideband signals have been done, which take into account temporal properties of a channel or characterize a spatio-temporal channel response [1]. A typical model for an impulse response of a multipath fading channel is given by

$$h(t) = \sum_{l=1}^{L} a_l \delta(t - t_l) \tag{1}$$

where t_l denotes a signal delay along the l-th path, while a_l is a complex propagation coefficient which includes a channel attenuation and a phase offset along the l-th path. While this model does not adequately reflect specific bandwidth-dependent effects, it provides a good characterization of the propagation channel used for diversity reception schemes (i.e. RAKE receivers). Equation (1) can be interpreted as saying that a received signal $y(t)$ is made up of a weighted sum of attenuated and delayed replicas of a transmitted signal $s(t)$, i.e.

$$y(t) = \sum_{l=1}^{L} a_l s(t - t_l) + \eta(t) \tag{2}$$

where $\eta(t)$ denotes receiver noise. Note that the received signal $y(t)$ has only $2L$ degrees of freedom, the time delays t_l and the propagation coefficients a_l. Although these parameters can be estimated using the time domain model (2), an efficient, closed-form solution is possible if we consider the problem in the frequency domain.

Let $Y(\omega)$ denote the Fourier transform of the received signal

$$Y(\omega) = \sum_{l=1}^{L} S(\omega) a_l e^{-j\omega t_l} + N(\omega) \tag{3}$$

where $S(\omega)$ and $N(\omega)$ are Fourier transforms of $s(t)$ and $\eta(t)$ respectively. Clearly, spectral components are given by a sum of complex exponentials, where the unknown time delays appear as complex frequencies while propagation coefficients appear as unknown weights. Thus the problem of estimating the channel parameters can be considered as a special case of harmonic retrieval problems that are well studied in spectral estimation literature. There is a particularly attractive class of subspace or SVD-based algorithms, called high-resolution methods [6] [7], which can resolve closely spaced sinusoids from a short record of noise-corrupted data. In the following, we will present a subspace method based on the State Space approach [7], which provides an elegant and numerically robust tool to estimate the channel parameters, and develop an algorithm that allows for parameter estimation from a signal subspace.

2.2. Algorithm outline

Suppose that the received signal $y(t)$ is filtered with an ideal lowpass filter $H_b = \text{rect}(-f_l, f_l)$ and sampled uniformly at a sub-Nyquist rate $R_s \geq 2f_l$. From the set of samples, we can compute DFT coefficients $Y[m]$ that correspond to the lowpass version of $y(t)$. Define next a $P \times Q$ data matrix $\mathbf{J}$ as

$$\mathbf{J} = \begin{pmatrix} Y_s[1] & Y_s[2] & \dots & Y_s[Q] \\ Y_s[2] & Y_s[3] & \dots & Y_s[Q+1] \\ \vdots & & & \\ Y_s[P] & Y_s[P+1] & \dots & Y_s[P+Q] \end{pmatrix} \tag{4}$$

Where $Y_s[m] = Y[m]/S[m]$, while $S[m]$ are DFT coefficients of the transmitted UWB pulse, assuming that in the considered frequency band this division is not ill-conditioned. If we let $P, Q \geq L$ and $z_k = e^{-j\omega_0 t_k}$, the data matrix $\mathbf{J}$ can be written as $\mathbf{J} = U \Lambda V^T$, with matrices U, Λ, and V defined as

$$U = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ z_1 & z_2 & z_3 & \dots & z_L \\ \vdots & & & & \\ z_1^{P-1} & z_2^{P-1} & z_3^{P-1} & \dots & z_L^{P-1} \end{pmatrix} \tag{5}$$

$$\Lambda = diag\,(\, a_1 \quad a_2 \quad a_3 \quad \dots \quad a_L \,) \tag{6}$$

$$V = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ z_1 & z_2 & z_3 & \dots & z_L \\ \vdots & & & & \\ z_1^{Q-1} & z_2^{Q-1} & z_3^{Q-1} & \dots & z_L^{Q-1} \end{pmatrix} \tag{7}$$

The State Space method is based on two properties of the data matrix $\mathbf{J}$. The first one is the fact that in the theoretical case of noiseless data, $\mathbf{J}$ has rank L. This will allow us to reduce the noise level by approximating the noisy data matrix with an optimal, rank L matrix. The second property is the Vandermonde structure of U and V, that is, they both satisfy the so-called shift-invariant subspace property,

$$\overline{U} = \underline{U} \cdot \Phi \quad \text{and} \quad \overline{V} = \underline{V} \cdot \Phi \tag{8}$$

where Φ is a diagonal matrix having z_i's along the main diagonal, while $\overline{(\cdot)}$ and $\underline{(\cdot)}$ denote the operations of omitting the first and the last row of $(\cdot)$ respectively. Our algorithm can be thus summarized as follows:

1. Compute the DFT coefficients $Y[m]$ from the set of samples

   $$y_n = < h_b(t - nT_s), y(t) >, \quad n = 1, \dots, L_1 \tag{9}$$

   where $T_s = 1/R_s$ and $L_1 \geq 2L$.

2. Define a $P \times Q$ matrix $\mathbf{J}$ as in (4), where P and Q satisfy the following relations: $P, Q \geq L$ and $P + Q \leq L_1$.

3. Compute the singular value decomposition of $\mathbf{J}$

   $$\mathbf{J} = U_s \Lambda_s V_s^H + U_n \Lambda_n V_n^H \tag{10}$$

4. Estimate the signal poles $z_l = e^{-j\omega_0 t_l}$ as generalized eigenvalues of the matrix pencil

   $$\overline{V_s} - z\underline{V_s} \tag{11}$$

5. Find the propagation coefficients a_l from the Vandermonde system

   $$Y_s[m] = \sum_{l=1}^{L} a_l e^{-jm\omega_0 t_l}, \quad m \in [1, L] \tag{12}$$

In the theoretical case of noiseless data, any subspace of a proper dimension can be used to estimate all the relevant parameters. For example, in the above algorithm, we estimated the channel by sampling the lowpass approximation of the signal. In practice, the best performance of our method is expected if we estimate the channel from a frequency band where a signal-to-noise ratio is highest. It is also worth noting that it suffices to use only a portion of the signal bandwidth and still obtain high-resolution estimates, due to the fact that the problem we consider belongs to the class of non-linear estimation problems.

2.3. Computational Complexity and Alternative Solutions

A major computational requirement of the developed algorithm is dominated by the singular value decomposition step in 3., which results in the overall computational order of $\mathcal{O}(M^3)$, where $M = max(P, Q)$. However, in certain applications, such as ranging and positioning, we are typically interested in estimating the time delay of a dominant component, thus computing the full SVD of the matrix $\mathbf{J}$ is not necessary. Alternatively, we can use some simpler methods from linear algebra [2], that have lower computational requirements and converge very fast to the desired solution. For example, an efficient way to compute the dominant eigenvector is by using the so-called *Power Method*, which we briefly summarize in the following.

Power Method

Consider a matrix $\mathbf{F} = \mathbf{J}\mathbf{J}^H$ of size $P \times P$, and suppose that $\mathbf{F}$ is diagonizable, that is, $\Lambda^{-1}\mathbf{F}\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_P)$ with $\Lambda = [\mathbf{x}_1, \ldots, \mathbf{x}_n]$ and $|\lambda_1| > |\lambda_2| \geq \ldots |\lambda_P|$. Given $\mathbf{y}^{(0)}$, the *power method* produces a sequence of vectors $\mathbf{y}^{(k)}$ as follows:

$$\mathbf{z}^{(k)} = \mathbf{F}\mathbf{y}^{(k-1)}$$

$$\mathbf{y}^{(k)} = \mathbf{z}^{(k)}/\|\mathbf{z}^{(k)}\|_2 \tag{13}$$

The above method converges[1] if λ_1 is dominant and if $\mathbf{y}^{(0)}$ has a component in the direction of the corresponding dominant eigenvector $\mathbf{x}_1$. Note that the power method involves only simple matrix multiplications and has a computational order of $\mathcal{O}(P^2)$.

2.4. More Realistic Channel Models

We will now extend our analysis to the more complex case of a channel that takes into account certain bandwidth-dependent properties. Namely, as a result of the very large bandwidth of UWB signals, components propagating along different propagation paths undergo different frequency selective distortion, so that a more realistic channel model for UWB systems is of the form

$$h(t) = \sum_{l=1}^{L} a_l p_l(t - t_l) \tag{14}$$

where $p_l(t)$ are different pulse shapes that correspond to different propagation paths. The DFT coefficients of the received signal are thus given by

$$Y[m] = S[m] \sum_{l=1}^{L} P_l[m] a_l e^{-jm\omega_0 t_l} + N[m] \tag{15}$$

where $P_l[m]$ are now unknown coefficients. Clearly, in order to completely characterize the channel, we need to estimate a_l's and t_l's, as well as the coefficients $P_l[m]$, which, in general, requires a non-linear estimation procedure. However, one possible way to obtain a closed form solution for the estimates of all the parameters, is to approximate the coefficients $P_l[m]$ with polynomials of degree $d \leq R - 1$, that is

$$P_l[m] = \sum_{r=0}^{R-1} p_{l,r} m^r \tag{16}$$

Equation (15) now becomes

$$Y[m] = S[m] \sum_{l=1}^{L} a_l \sum_{r=0}^{R-1} p_{l,r} m^r e^{-jm\omega_0 t_l} + N[m] \tag{17}$$

By denoting $c_{l,r} = a_l p_{l,r}$ and $Y_s[m] = Y[m]/S[m]$, we obtain

$$Y_s[m] = \sum_{l=1}^{L} \sum_{r=0}^{R-1} c_{l,r} m^r e^{-jm\omega_0 t_l} + N[m] \tag{18}$$

In this case, U and V from (4) no longer satisfy the shift-invariance property and the State Space approach cannot be used. We will thus present an alternative method, based on annihilating filters [8], where the main idea is to find the so-called annihilating filter $H(z) = \sum_{k=0}^{N} H[k]z^{-k}$ which satisfies

$$(H * Y_s)[m] = 0, \ \ \forall m \in \mathbb{Z} \tag{19}$$

A useful property of such a filter is that it has multiple roots at $z_l = e^{-j\omega_0 t_l}$, that is,

$$H(z) = \prod_{l=1}^{L} (1 - e^{-j\omega_0 t_l} z^{-1})^R = \sum_{k=0}^{RL} H[k] z^{-k} \tag{20}$$

In the following, we give an outline of the algorithm, while a more detailed discussion of the annihilating filters can be found in [8].

[1] It typically converges in less than 15 iterations.

Annihilating filter method

1. Find the coefficients $H[k]$ of the annihilating filter

$$H(z) = \prod_{l=1}^{L}(1 - e^{-j\omega_0 t_l} z^{-1})^R = \sum_{k=0}^{RL} H[k] z^{-k} \tag{21}$$

which satisfies

$$H[m] * Y_s[m] = \sum_{k=0}^{RL} H[k] Y_s[m-k] = 0, \quad \forall m \in \mathbb{Z} \tag{22}$$

Since there are $RL + 1$ unknown filter coefficients, we need at least $RL + 1$ equations.

2. Find the values of t_l by finding the roots of $H(z)$. Recall that $H(z)$, which satisfies (22), has multiple roots at $z_l = e^{-j\omega_0 t_l}$. While this is true in the noiseless case, in the presence of noise it is more desirable to estimate the time delays from L roots of $H(z)$ which are closest to the unit circle.

3. Solve for the coefficients $c_{l,r}$ by solving the linear system of at least RL equations in (18),

$$Y_s[m] = \sum_{l=1}^{L} \sum_{r=0}^{R-1} c_{l,r} m^r e^{-jm\omega_0 t_l} \tag{23}$$

3. APPLICATION: RAPID ACQUISITION IN UWB LOCALIZERS

One of the most interesting applications of the developed framework can be found in ultra-wideband transceivers intended for low-rate, low-power indoor wireless systems, for example, in systems used for precise position location. Such UWB transceivers, called localizers, have already been developed [3] and they use low duty-cycle episodic transmission of a coded sequence of impulses to ensure low-power operation and good performance in a multipath environment. Yet, the synchronization still presents a bottleneck in the transceiver design. The system developed by *Aetherwire* [3] is analog and uses a complex method for sequence acquisition, which resorts to exhaustive search through all possible code positions. A serious drawback of this approach is that it is inherently time consuming, since transmissions are spaced apart while the correlation window spans a small fraction of the sequence cycle time T_{cycle}.

We will show that our previous results can be directly extended to the problem of timing synchronization (acquisition) in such systems, by modeling the received signal $y(t)$ as a convolution of L delayed (possibly different) impulses with a known coding sequence $g(t)$, that is,

$$y(t) = \sum_{l=1}^{L} a_l p_l(t - t_l) * g(t) \tag{24}$$

while the DFT coefficients $Y[m]$ are given by

$$Y[m] = \sum_{l=1}^{L} a_l P_l[m] G[m] e^{-jm\omega_c t_l}, \quad \omega_c = 2\pi/T_{cycle} \tag{25}$$

where $G[m]$ denote DFT coefficients of $g(t)$. If we use the polynomial approximation (16) of the spectral coefficients $P_l[m]$, the total number of degrees of freedom per one cycle is $2RL$. Therefore, the signal parameters can be estimated using low-rate uniform sampling and the method we already described. This potentially leads to a few orders of magnitude faster acquisition, as will be shown later.

One of the most attractive features of our algorithm is that when the received waveform is unknown, the sequence timing can still be recovered independently of the pulse shape. Recall that solutions based on matched filters [3] [4] require prior knowledge of the received waveform or resort to complex algorithms for its estimation [1].

4. PERFORMANCE ANALYSIS

Simulation examples that illustrate the performance of the developed algorithms in a low duty-cycle regime are shown in Figure 1. We first considered the case where a coded sequence of 127 first-derivative Gaussian impulses is periodically transmitted, while the sequence spans approximately 20% of the cycle time T_{cycle}. Delay estimates for the case with eight propagation paths and one dominant (with 70% of the total power), were obtained for various values of signal-to-noise ratio (E_b/N_0) and the root-mean square errors (RMSE) of delay estimation for the dominant component is shown in Figure 1(a). We analyzed the performances of the SVD-based algorithm and the algorithm which uses the power method, and compare the results with the Cramer-Rao Bound (CRB). Both methods yield highly accurate estimates for a wide range of SNR's, for example, when the sampling rate is one tenth the Nyquist rate ($N_s = N_n/10$) and $E_b/N_0 = -10dB$, the time delay along the dominant path can be estimated with an RMSE of approximately one sample, and this within only 30 cycles[2]. Also note that the performance of both algorithms is very close to the CRB in the case of Nyquist-rate sampling.

We next considered the case of joint pulse shape and timing estimation, when the received UWB signal is made up of two pulses with different shapes. A received noiseless and noisy UWB signals (for SNR=0dB) are shown in Fig. 1(b). We used the annihilating filter method to estimate unknown time delays of the pulses, by sampling the received signal uniformly at one fifth the Nyquist rate and averaging samples over $N_c = 60$ cycles, while the corresponding

[2]Note that the main reason for using multiple cycles is to increase the effective SNR by averaging samples taken over N_c cycles.

pulse shapes are then obtained by polynomial approximation of the DFT coefficients. In this case, we used a polynomial of order $R = 20$, which clearly yields a very good approximation of the received waveforms, as illustrated in Figures 1(c)-(d).

5. CONCLUSION

We developed several frequency domain methods for channel estimation and synchronization in ultra-wideband systems operating over multipath fading channels. Our approach takes advantage of well-known spectral estimation techniques, requires lower sampling rate and, therefore, lower complexity and power consumption compared to existing methods. Besides, it uses standard computational procedures and allows for identification of more realistic channel models without resorting to complex algorithms.

6. REFERENCES

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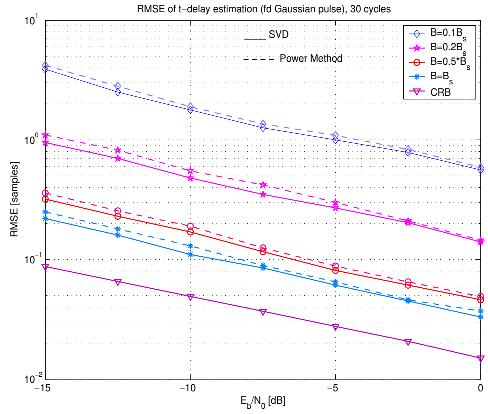
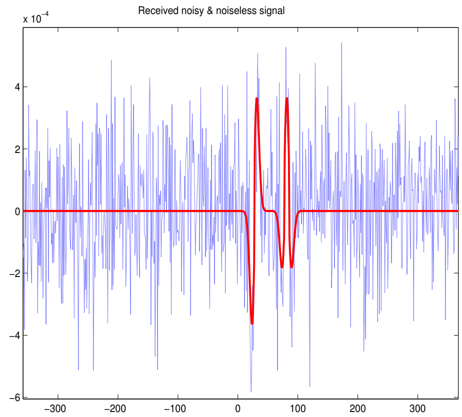


(a)



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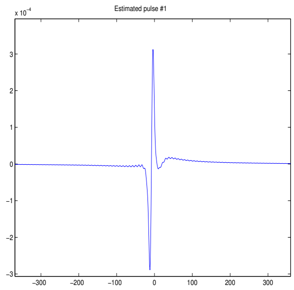

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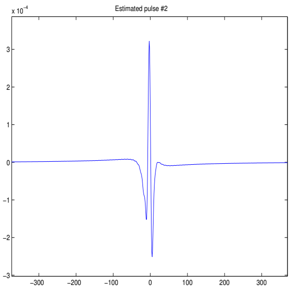

(d)

Fig. 1. **(a)** Root-mean square error (RMSE) of delay estimation vs. SNR (one dominant path) for different values of the sampling rate. We assumed that the samples of the received signal are averaged over $N_c = 30$ cycles and compared the performances of SVD-based algorithm and the power method with the Cramer-Rao bound. **(b)** Received UWB noisy signal (blue) and the noiseless signal (red) made up of two short pulses having different shapes. **(c)** Estimated shape of the first pulse. **(d)** Estimated shape of the second pulse. The received signal is sampled at one fifth the Nyquist rate. In both cases, we used a polynomial of order $R = 20$ to approximate the DFT coefficients of the received signal.