MULTIPLE DESCRIPTION SOURCE CODING AND DIVERSITY ROUTING: A JOINT SOURCE CHANNEL CODING APPROACH TO REAL-TIME SERVICES OVER DENSE NETWORKS.

Guillermo Barrenechea[1], *Baltasar Beferull-Lozano*[1], *Abhishek Verma*[2],
Pier Luigi Dragotti[3], Martin Vetterli[1,4]

[1] Laboratoire de Communications Audiovisuelles,
Swiss Federal Institute of Technology, CH-1015 Lausanne, Switzerland.
[2] Indian Institute of Technology Guwahati, Assam-781039, India.
[3] Department of Electrical and Electronic Engineering,
Imperial College of Science, Technology and Medicine, London SW7 2BT, UK.
[4] EECS Department, UC Berkeley CA 947020.

ABSTRACT

We investigate the interaction of the source coding mechanism and the transport mechanism for real time data packet transmission over dense networks. We show that single path routing does not make efficient use of network capacity whereas multi path routing techniques do (*Proposition 1 and 2*). We then consider the interaction of single versus multiple description source coding with single versus multi path routing. The most sophisticated scheme (multiple descriptions source coding and multi path routing) performs significantly better that the usual single path and single description scheme. This improvement is more remarkable in the case of low rates and shorter maximum allowed delay (about 18 dB in the best case).

1. INTRODUCTION

Ad Hoc networks have attracted many research efforts over the past few years [20, 11]. These networks present a number of challenges, some of which have still to be solved. The main features we deal with are the large number of nodes and the unreliability due to node and link failures. From any particular node, there are many possible paths to reach any other node. However, the probability of one of these paths to fail is not negligible. Multi path routing techniques have been found to be a good strategy under these conditions to increase robustness [2, 3, 23]. The principle of these algorithms consists of flowing data simultaneously along multiple routes.

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These particular networks strongly call for specific coding techniques capable of exploiting the path diversity present in the network. Multiple Description (MD) codes are conceived for communication over multi path networks. The idea is to encode the source information using several descriptions (packets) which are complementary and at the same time independently good. Each description can be sent along a different path. Some of these packets will be lost while others will reach the destination within a certain delay. Given a certain number of received packets, a reconstruction of the original data unit is obtained. This is illustrated in Fig. 1.

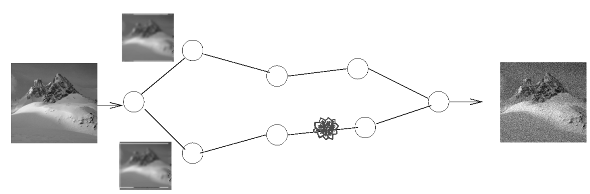

Fig. 1. Multi path routing and multiple description coding. Source information is encoded in two descriptions (packets) and each description is sent along a different path. Even if we lose some of these descriptions, we are able to reconstruct the original signal.

The reconstruction quality depends on the number of packets received by the source and on the method used to generate these descriptions. Therefore, if we want to maximize the quality of the reconstructed data, it is necessary to optimize jointly two elements: the number of packets reaching the destination and the reconstruction of the original data from the received packets.

We show that traditional single path routing algorithm does not make efficient use of network capacity whereas other multi path routing schemes do.

We concentrate on the interaction of the source coding mechanism and the transport mechanism. In practice, we consider the interaction of single versus multiple description source coding with single versus multi path routing. The most sophisticated scheme (using multiple descriptions source coding and multi path routing) performs significantly better (by about 18 dB in the best case) and is more robust over a wide range of networks behavior than the usual scheme, that is, single path routing and single description. This indicates the benefit of such methods in large networks where there are links and node failures.

In this paper, we concentrate on mesh networks [16, 13], because their simplicity permits some analytical results which would be difficult to obtain in general networks.

The rest of the paper is organized as follows. In Section 3, we analyze the influence of routing schemes in the end to end achieved distortion. We introduce a simple network model that allows us to derive some mathematical expressions for the multiuser capacity. We study the optimal routing strategy in terms of achieved capacity. In Section 4, we present multiple description coding as the natural technique to use in high connected networks. In Section 5, we formulate and solve analytically a "toy problem" that merges both communication aspects: routing and coding. In Section 6, we present some simulation results that help to build the intuition about the behavior in bigger networks. Conclusions are discussed in Section 7.

2. RELATED WORK

Multi path routing algorithms have already been considered in the context of mobile ad-hoc networks [19, 18]. Servetto and Barrenechea [23] presented a multi path routing algorithm based on constrained random walks. This algorithm is able to route messages along all possible routes between a source and a destination node. Chen and Nahrstedt proposed a parallel multiple route computation as a mechanism to provide QoS in ad-hoc networks [7].

Gupta and Kumar studied the transport capacity in wireless networks [10] and conclude that for a uniform traffic distribution the total end-to-end capacity is roughly $\mathcal{O}\left(\sqrt{n}\right)$ where n is the number of nodes. The capacity of regular grids has been investigated in the analysis of deflection routing algorithms [13, 6].

The first theoretical results in multiple description coding were provided by El Gamal, Cover and Ozarow [8, 17] for the case of Gaussian source, mean squared error distortion and two descriptions. An achievable region for the binary symmetric source with many descriptions was derived in [25]. Recently, achievable rate regions of the multiple description problem with more than two descriptions have been determined for the *symmetric* case by Pradhan, Puri and Ramchandran [21, 22]. Vaishampayan proposed in [24] a simple procedure for designing multiple descriptions scalar quantizers with remarkably good asymptotic properties. For an excellent tutorial on multiple description coding refer to [9].

Many network communication problems do not allow to use the separation theorem. Thus joint source channel coding can bring substantial improvements. McCanne et al considered the problem of JSCC in the context of multicast packet video [14, 15]. They proposed a receiver based approach where each receiver can dynamically adapt to local network capacity by adjusting the quality of the video it receives. M. Alasti et al [1] already studied the use of multiple description coding in networks with congestion problem. They investigated the problem of a simple network represented by a set of parallel queues with congestion problems and showed that double description coding significantly improves the overall average end-to-end distortion at high network loading compared to single description coding systems.

3. DIVERSITY ROUTING

We start by analyzing the influence of routing protocols in the end to end achieved distortion. The routing protocol determines the capacity of the network, in other words, the maximum number of data packets that can be transmitted (on average) simultaneously between any source destination pairs. First, we introduce a simple network model that allows us to derive some mathematical expressions for the multiuser capacity. We also analyze the optimal routing strategy for this particular network and the performance of different routing algorithms. We prove that multi path routing is in some cases optimal (*Proposition 2*) and that in more general scenarios, it outperforms single path routing.

3.1. Network Model

We study the problem of routing in a static graph with a very regular structure: the wrapped square grid. Fig. 2 shows an $n \times n$ wrapped square grid for $n = 4$. Vertices are used to represent communication devices with routing capabilities and edges to represent simplex communication channels between devices.

The length of a path is defined as the number of edges in the path. Moreover, $s(i, j)$ is the length of the shortest path between i and j. The longest of all the shortest path lengths over all pair of vertices is the network diameter d. The wrapped $n \times n$ square grid contains $N = n^2$ vertices and $M = 2n^2$ edges. It has diameter $d = n$ for n even, and $d = n - 1$ for n odd.

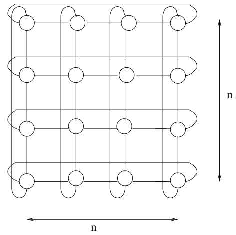

Fig. 2. Network model: $n \times n$ wrapped square grid or torus for $n = 4$.

The target of data routing protocols is to transport packets from any source i to any destination j using shortest paths in such a way that a maximum throughput per device is achieved. All our subsequent results are based on this model.

3.2. An Upper Bound on Sending Rate per source

We derive now the maximum traffic per source that can be carried by this network model under any circumstances. Consider the following set of assumptions:

- The network transports $\mathcal{R}n^2T$ data units over T time units.
- The average distance between source and destination of a data unit is $\overline{L}$.
- Transmissions are slotted into synchronized slots of length one time unit.
- Every link in the network has the same capacity C in data units per time unit.
- Links are simplex.
- Links are modeled by a first come first served single server queue with maximum length q.

Proposition 1: Under these assumptions, the maximum data unit sending rate per node $\mathcal{R}$ that can be transported by a N nodes M links (edges) network is bounded as follows:

$$\mathcal{R} \leq \frac{MC}{N\overline{L}} \tag{1}$$

Proof: Consider a data unit u, where $1 \leq u \leq \mathcal{R}NT$. Suppose it moves from the source i to the destination j following a shortest path of length $s(u)$. Then,

$$\sum_{u=1}^{\mathcal{R}NT} s(u) = \mathcal{R}NT\overline{L}. \tag{2}$$

Applying the condition that links are simplex:

$$\sum_{u=1}^{\mathcal{R}NT} \sum_{h=1}^{s(u)} 1_{\{\text{The } h_{th} \text{ hop of } u \text{ is over link } i \text{ in slot } s\}} \leq C.$$

Summing over the links and the slots gives:

$$\sum_{u=1}^{\mathcal{R}NT} s(u) \leq MTC. \tag{3}$$

Combining eqns. (2) and (3) yields the result. □

Substituting the values of N and M in eqn. (1), the rate per source in an $n \times n$ wrapped square lattice is upper bounded by $\mathcal{R}_{max} = \frac{2C}{\overline{L}}$. This upper bound depends on the traffic matrix that determines the value of $\overline{L}$. For instance, in the case of uniformly distributed traffic matrix, the maximum rate per source in an $n \times n$ wrapped square grid is bounded as follows:

$$\mathcal{R} \leq \begin{cases} \frac{4}{n}\left(1 - \frac{1}{n^2}\right)C & n \text{ even} \\ \frac{4}{n}C & n \text{ odd} \end{cases} \tag{4}$$

For a proof see [4].

3.3. Optimal routing policy

In previous sections we derived an upper bound for the maximum rate per device that can be accomplished in the considered scenario. We analyze now the achievability of this upper bound. We model links between devices as a FIFO queue with unitary service time ($1/\mu = 1$) and a maximum queue size of q data units. We assume that the processing time per data unit is negligible.

3.3.1. Infinite buffer queues

We first consider the case where links are modeled by an infinite waiting queue. For any pair of nodes in the grid (i, j), we can visualize the grid as a plane map and consider j to be displaced from i along Cartesian-like co-ordinates X,Y being x, y the actual displacements. We define for an $n \times n$ wrapped square grid the least displacements of i and j, $\delta(i, j) = [x_0, y_0]$ where $x_0 = min(|x|, (n - |x|))$ and $y_0 = min(|y|, (n - |y|))$.

Let (i, j) be a pair of nodes that want to communicate. The way packets flow from node i to node j through the network is determined by the routing policy $\Pi(i, j)$. We say that a routing policy Π is *space invariant* if Π is identical for nodes with the same least displacements. That is,

$$\forall\ (i, j), (k.l) : \delta(i, j) = \delta(k, l) \rightarrow \Pi(i, j) = \Pi(k, l).$$

This situation is shown in Fig. 3.

Proposition 2: Any routing algorithm achieves capacity under the infinite buffer queues hypothesis if and only if is *space invariant*.

Proof: First we prove that *space invariant* routing algorithms induce identical arrival rates per link, λ.

$$\lambda_i = \frac{\mathcal{R}n^2\overline{L}}{M} \quad \forall i \in [1..M]. \tag{6}$$

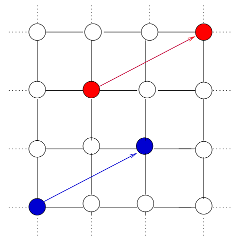

Fig. 3. Two identical least displacements pairs. A routing policy Π is time invariant if depends only on the least displacements between source and destination.

Consider a particular link of the wrapped square grid, l_k. We denote by $P^{\Pi}_{l_k}(i,j)$ the fraction of the traffic generated at node i with destination node j that travels through link l_k according to routing algorithm Π. With this notation, the arrival rate to l_k can be computed as follows:

$$\lambda^{\Pi}_{l_k} = \mathcal{R}\sum_{i=1}^{n^2}\sum_{j=1,j\neq i}^{n^2} A(i,j)P^{\Pi}_{l_k}(i,j). \qquad (7)$$

where A is the probability communication matrix between nodes.

Applying the fact that Π is *space invariant* and the structural homogeneity of the torus,

$$\sum_{k=1}^{M}\lambda^{\Pi}_{l_k} = \sum_{k=1}^{M}\lambda^{\Pi} = M\lambda^{\Pi}. \qquad (8)$$

Combining equations (7) and (8) and reordering summations:

$$M\lambda^{\Pi} = \mathcal{R}\sum_{i=1}^{n^2}\sum_{j=1,j\neq i}^{n^2} A(i,j)\sum_{k=1}^{k=M} P^{\Pi}_{l_k}(i,j). \qquad (9)$$

$\sum_{k=1}^{k=M} P^{\Pi}_{l_k}(i,j)$ is the fraction of the traffic generated at node i with destination node j that travels through any link of the network. Note that the set of links that communicate nodes at distances l and $l+1$ has to route the entire traffic fraction between i and j. Given that we have $s(i,j)$ of these sets,

$$\sum_{k=1}^{k=M} P^{\Pi}_{l_k}(i,j) = s(i,j) \quad \forall\Pi. \qquad (10)$$

Combining equations (9) and (10)

$$M\lambda^{\Pi} = \mathcal{R}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} A(i,j)s(i,j). \qquad (11)$$

Recalling the definition of the average distance between source and destination $\overline{L}$ and putting together equations (10) and (11),

$$\lambda^{\Pi} = \frac{\mathcal{R}}{M}\sum_{i=1}^{N}\overline{L} = \frac{\mathcal{R}N\overline{L}}{M}. \qquad (12)$$

Therefore, *space invariant* routing algorithms can accomplish the same maximum source rate $\mathcal{R}$, given that for stability conditions $\rho = \frac{\lambda}{\mu} \leq 1$.

If the routing algorithm is not *space invariant*, the load is not distributed uniformly among links, hence the arrival per node is not constant and capacity is limited by the most loaded link. $\square$

Many multi path routing schemes fulfill this condition. For instance, *spreading* routing [23] and Bernoulli routing algorithms [23] are *space invariant* and therefore achieve capacity in the infinity buffer case. On the contrary, a single path routing based on distributed Bellman-Ford [5] is not *space invariant*.

3.3.2. Finite buffer queues

In the infinite buffer case, every packet will eventually reach the destination within a big enough delay. Therefore, all routing algorithms assuring at least a "minimal condition" of load distribution will be able to achieve this capacity. However, we face now the problem of finite buffering. The desirable routing would be associated with a small mean and variance of packet delay at each queue. A convenient but somewhat imperfect alternative is to measure congestion at a link in terms of the average traffic carried by the link. We can formulate the optimal routing problem by the following minimization problem [5]. We call $F_{i,j}$ the flow of link (i,j). For each pair $w = (i,j)$ of distinct nodes i and j, the input traffic arrival process is assumed stationary with rate $\mathcal{R}$. The routing objective is to divide each $\mathcal{R}$ among the many paths from origin to destination in a way that the resulting total link flow pattern minimizes a monotonically increasing cost function $D_{i,j}(F_{i,j})$. If we denote by W the set of all origin destination pairs, by P_w the set of all paths connecting the origin and destination nodes $w = (i,j)$ and by x_p = Flow (data units/time unit) of path p, the problem can be written as:

$$\begin{aligned} &\text{minimize} && \textstyle\sum_{(i,j)} D_{ij}\left[\sum_{\text{all paths p containing } (i,j)} x_p\right] \\ &\text{subject to} && \textstyle\sum_{p\in P_w} x_p = \mathcal{R}, \quad \forall w \in W \\ & && x_p \geq 0, \quad \forall p \in P_w, w \in W. \end{aligned}$$

The underlying hypothesis here is that one achieves reasonably good routing by optimizing the average levels of link traffic without paying attention to other aspects of the traffic statistics. Therefore, the cost functions, D_{ij} is insensitive to undesirable behavior associated with high variance and with correlations of packet interarrivals times and transmission times. The embedded problem here is that delay on each link depends on second and higher moments of the arrival process, while the cost function reflects a dependence on just the first moment.

Therefore if we do not have any a priori knowledge of the communications patterns and the cost function D_{ij} depends only on the first moment of the traffic distribution, the best routing algorithm for a uniform traffic matrix would have the property that distributes the load as uniform as possible among all nodes.

The *spreading* routing algorithm presented in [23] accomplish this uniformly load distribution. On the contrary, other multi path routing algorithm like Bernoulli tends to generate uneven distributions[23]. This uneven distribution is more evident in the case of single path routing schemes.

Nevertheless, routing performance can be improved by using queue length information, although unfortunately, it is impractical to keep nodes informed of queue lengths in a large network [5].

3.4. Experimental Results

For illustration purposes, we compare the performance of four different routing algorithms. A single path routing algorithm, that for any given destination computes a single shortest path using Bellman-Ford. *Bernoulli* routing, consisting of flipping a fair coin to decide which of the two feasible neighbors in the shortest path toward a given destination on a next hop to pick at each node. *Spreading* routing, an algorithm that distributes the load evenly among nodes belonging to a shortest path, based on the schema proposed in [23]. And finally we consider a greedy suboptimal algorithm that out of the two neighbors on a next hop always picks the node with the shortest queue. We refer to this algorithm as *least filled* routing.

Nodes transmit following a Markov rule, switching between ON and OFF states with probabilities $P_{ON \to OFF}$ and $P_{OFF \to ON}$ respectively, independently from one another. The maximum rate per node is normalized by the stationary probability of any node to be transmitting. We assume that sender has instantaneous feedback and is requested to retransmit lost packets. We analyze the uniform traffic matrix case. Under this set of assumptions we compute the *goodput* achieved by a given routing algorithm as the number of packets successfully delivered to the destination divided by the total number of packets generated.

Fig. 4 shows the maximum goodput rate product against network size N for a very long buffer queue. Note that *Spreading* and *Bernoulli* routing schemes are both *space invariant*, achieving a maximum goodput rate product very close to the theoretical upper bound. *Least Filled* routing depends on cross traffic for each node, therefore, *space invariant* property can not be assured. Note also that all multi path routing schemes outperforms single path routing.

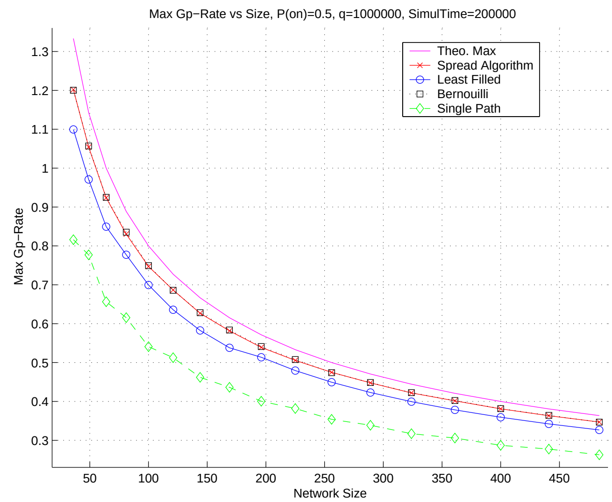

Fig. 4. Maximum Goodput Rate product vs Network size n for a buffer queue $q = 100000$ and a simulation time of 200000 time slots. The solid line shows the theoretical upper bound (eqn. 4).Dashed lines represent the performance of the different routing algorithms.

Fig. 5 shows the maximum goodput rate product versus queue size for a 100 nodes network. *Least Filled* routing outperforms all routing schemes since routing can be improved by using queue length information [5]. *Spreading* routing achieves better results than *Bernoulli* routing due to a more uniform distribution of the load. All multi path routing schemes largely outperform single path routing.

4. DIVERSITY CODING

In last section, we showed the benefit of using multi path routing schemes over single path routing. These multiple paths between source and destination can be used to increase the end to end connection reliability by considering specific coding techniques that exploit this diversity. Multiple description coding techniques fit very well in diversity networks, and consist of generating different complementary and at the same time independently good descriptions of each data unit that can be sent along different paths.

In order to compare practical and theoretical results, we turn our attention to the independent and identical distributed (iid) Gaussian source and two description case. Two different coding strategies are considered: multiple description scalar quantization and unequal error protection.

The achievable rate-distortion region for the symmetric case with $\frac{R}{2}$ bits per description is defined by the following equations [17]:

$$\begin{aligned} d_s &= 2^{-2\frac{R}{2}(1-\eta)} \\ d_c &= 2^{-2R}\frac{1}{1-(1-2d_s)^2}, \end{aligned} \tag{13}$$

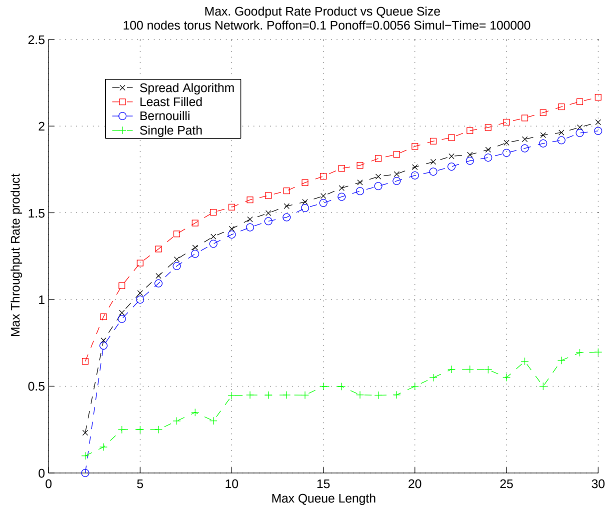

Fig. 5. Maximum Goodput Rate product vs Queue size in a 100 nodes square grid network. Simulation time: 100000 time slots. Dashed lines represent different routing schemes.

where d_s is the side distortion, that is, the distortion in the case only one description gets to the receiver, and d_c is the Central distortion, that is, the distortion achieved in the case both descriptions are received. The parameter η, $(0 \leq \eta \leq 1)$, represents the trade-off between the side and the central distortion.

A possible practical implementation of an MD coder is represented by the Multiple Description Scalar Quantizer (MDSQ) proposed by Vaishampayan [24]. An MD scalar quantizer can be seen as an ordinary quantizer plus an index assignment that generates two indices per quantized sample. This scheme has been proved to be asymptotically optimal [24].

An alternative way to implement MD coding is by using Unequal Error Protection (UEP) codes with progressive source coder [9]. For instance, one might use a progressive source coder to produce a representation at rate $(2-\zeta)R$, $\zeta \in [0,1]$, and them partition this representation in three parts. The initial (most important) ζR bits are repeated in each description; the second $(1-\zeta)R$ bits are put in description 1; and the final $(1-\zeta)R$ bits are put in description 2. This is summarized in the following table:

Description 1:	$R\zeta$	$(1-\zeta)R$
Description 2:	$R\zeta$	$(1-\zeta)R$

In the particular case of a memoryless Gaussian source with squared error distortion and two descriptions, the achievable rate-distortion region is given by the following formulas:

$$
\begin{aligned}
d_c &= 2^{-2(2-\zeta)\frac{R}{2}} \\
d_{s_1} &= 2^{-2\frac{R}{2}} \\
d_{s_2} &= 2^{-2\frac{R}{2}\zeta}
\end{aligned}
\qquad (14)
$$

The parameter ζ, $(0 \leq \zeta \leq 1)$, represents the trade-off between side and central distortions. Goyal [9] proposed a very simple method to implement practically this coding strategy using a uniform scalar quantizer followed by a double description generator.

Fig. 6 shows the side and central distortion trade off for both MDSQ and UEP coding techniques theoretic and experimentally using 8 bits per description. Note that MDSQ performs slightly better than UEP, however, this second approach is attractive for large numbers of descriptions given that good channel codes are available at a variety of rates. The difference between theoretical and experimental values is almost completely explained by the use of a non optimal quantizer.

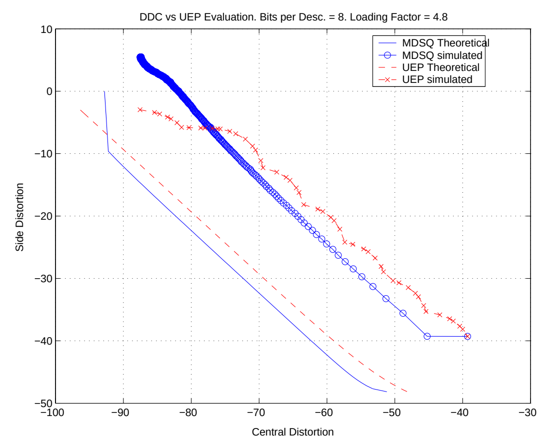

Fig. 6. Central and side distortion trade off for MDSQ and UEP using 8 bits per description. Solid lines represent MDSQ and dashed lines UEP

5. TOY PROBLEM ANALYSIS

We concentrate now on the interaction of the source coding mechanism and the transport mechanism in a simple scenario that allows us to derive analytical expressions and compare different approaches from a quantitative point of view.

5.1. Architecture and Assumptions

We start from the simplest yet non trivial instance of our original problem (Fig.7). We consider two devices acting as sources, S_1 and S_2, that want to send data to D_1 and

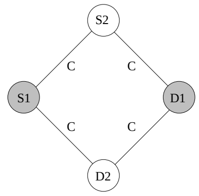

Fig. 7. The simplest yet non trivial instance of our problem.

D_2 respectively. Any device may serve as relay for ongoing communications.

Both sources generate packets of constant size B randomly, one every time slot of length T time units. The exact arrival time of the packet inside the interval is given by a uniformly distributed random variable. The arrival process of such a source can be characterized by the following formula:

$$A(i) = iT + a(i) \quad i = [0, 1, ...], \tag{15}$$

where a is a random variable uniformly distributed in the interval $[0, T]$.

To simplify the problem, we consider the following set of assumptions:

- The communication channels are noise-free.
- Packets will be forwarded to the receiver under all circumstances. The decoder drops out all packets that have gone through a delay exceeding Δ.
- Each path is modeled by a first come first served single-server queue.
- Sources generate traffic according to the process described in eqn.(15).
- Sources transmit zero mean unit variance Gaussian signals.

5.2. Single path routing and single description coding

In the network model considered previously, both packet flows share the same network and hence compete for a certain capacity. The results in terms of network usage and congestion depend on the routing scheme used by the transmitting sources. First, we consider single path routing in both flows, as depicted in Fig. 8.

Assuming that a packet is lost when the packet delay exceeds a fixed values Δ, the distortion for a zero mean unit variance Gaussian can be calculated as follows:

$$D = 2^{(-2B)} P(T_S \leq \Delta) + (1 - P(T_S \leq \Delta)). \tag{16}$$

where T_S is a random variable indicating the total delay that a packet experiences before reaching its destination. The

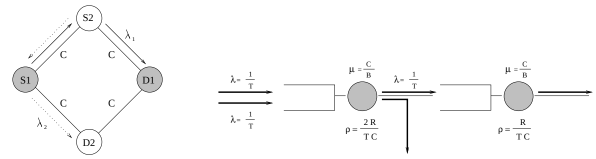

Fig. 8. Left: Single path routing and single description coding flow model. Right: queue model, where links are modeled by FIFO queues of service time $\frac{1}{\mu} = \frac{B}{C}$.

first transmission channel can be modeled as a G/D/1 queue with arrival rate $\lambda = \frac{2}{T}$. Since the time elapsed between two consecutive arrivals for the second queue has to be at least the service time of the first queue, this second queue can be modeled as a constant delay of $\frac{1}{\mu}$.

The system delay probability density function can be calculated as the product of these two queues delay probability density functions in the transform domain:

$$f_{T_S}(s) = f_{T_{S1}}(s) f_{T_{S2}}(s).$$

The first queue can be modeled as a G/D/1 with an arrival process given by the following inter arrival distribution [4],

$$A(t) = \begin{cases} \frac{t(t^3 - 8t^2T + 6tT^2 + 12T^3)}{12T^4} & \text{if } t \leq T \\ \frac{-4T^4 + 32tT^3 - 24t^2T^2 + 8t^3T - t^4}{12T^4} & \text{if } T \leq t \leq 2T. \end{cases}$$

We can make the following approximation to simplify the analysis. Under low to moderate load and for the arrival process given before, the system time cumulative distribution function can be approximated by the following linear expression:

$$\begin{aligned} &F_{T_{S_1}}(t) \approx \\ &\left[\beta + \mu(1-\beta)\left(t - \tfrac{1}{\mu}\right)\right]\left(u\left(t - \tfrac{1}{\mu}\right) - u\left(t - \tfrac{2}{\mu}\right)\right) \\ &+ u(t - \tfrac{2}{\mu}), \end{aligned} \tag{18}$$

where we defined $\beta = \sqrt{(1 - \rho)}$.

Under this approximation, the overall delay cumulative distribution function is given by:

$$F_{T_S}(t) \approx \begin{cases} 0 & \text{if } t \leq \frac{2}{\mu} \\ \beta + \mu(1-\beta)(t - \frac{2}{\mu}) & \text{if } \frac{2}{\mu} \leq t < \frac{3}{\mu} \\ 1 & \text{if } \frac{4}{\mu} \leq t \end{cases}$$

and the probability $P(T_S \leq \Delta) = F_{T_S}(\Delta)$.

5.3. Multi path routing and single description code

We turn our attention to the case of multi path routing algorithms that distributes load uniformly among the shortest paths from the source to the destination. This case is illustrated in Fig. 9. The system can be modeled as a two G/D/1 queues network with a total average arrival rate $\lambda = \frac{1}{T}$ and a service time $\frac{1}{\mu} = \frac{B}{C}$. Note that the load carried by the most loaded node has been reduced by a factor of two.

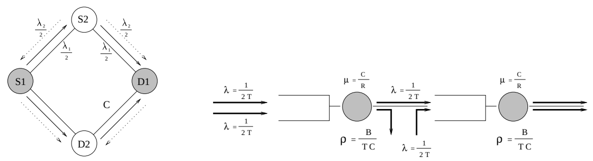

Fig. 9. Left: Multi path routing and single description coding flow model. Right: queue model, where links are modeled by FIFO queues of service time $\frac{1}{\mu} = \frac{B}{C}$.

To resolve this queues system, Kleinrock [12] suggested that merging several packet streams on a transmission line has an effect akin to restoring the independence of inter arrival times. Given that we have a new incoming flow in the second queue, we can assume that interval times and packet lengths are independent, and compute the system delay probability density function as the product of two identical probability density functions in the transform domain.

Operating on eqn. (18), the Laplace transform of the probability density function is given by:

$$f_{T_{S1}}(s) \approx \beta e^{-\frac{s}{\mu}} + \frac{(1-\beta)\mu}{s}\left(e^{-\frac{s}{\mu}} - e^{-\frac{2s}{\mu}}\right)$$

and the system delay probability density function,

$$\begin{aligned} f_{T_S}(t) &= \mathcal{L}^{-1}(f_{T_{S1}}(s) f_{T_{S2}}(s)) \\ &\approx \mathcal{L}^{-1}[\beta^2 e^{-\frac{2}{\mu}s} + \tfrac{(1-\beta)^2\mu^2}{s^2}\left(e^{-\frac{1}{\mu}s} - e^{-\frac{2}{\mu}s}\right)^2 \\ &+ \tfrac{2\beta(1-\beta)\mu}{s}\left(e^{-\frac{2}{\mu}s} - e^{-\frac{3}{\mu}s}\right)]. \end{aligned}$$

Computing the inverse Laplace transform and integrating we can derive the probability density function common to both descriptions

$$P(T_S \leq \Delta) \approx \begin{cases} 0 & \text{if } \Delta < \frac{2}{\mu} \\ \mathcal{F}_1 & \text{if } \frac{2}{\mu} \leq \Delta < \frac{3}{\mu} \\ \mathcal{F}_2 & \text{if } \frac{3}{\mu} \leq \Delta < \frac{4}{\mu} \\ 1 & \text{if } \frac{4}{\mu} \leq \Delta \end{cases},$$

where

$$\mathcal{F}_1 = \beta^2 + 2\beta(1-\beta)\mu\left(\Delta - \tfrac{2}{\mu}\right) + (1-\beta)^2\mu^2\frac{\left(\Delta-\frac{2}{\mu}\right)^2}{2}$$
$$\mathcal{F}_2 = \tfrac{1+2\beta-\beta^2}{2} + (1-\beta)^2\mu^2\left[\tfrac{1}{\mu}\left(\Delta - \tfrac{3}{\mu}\right) - \frac{\left(\Delta-\frac{3}{\mu}\right)^2}{2}\right].$$

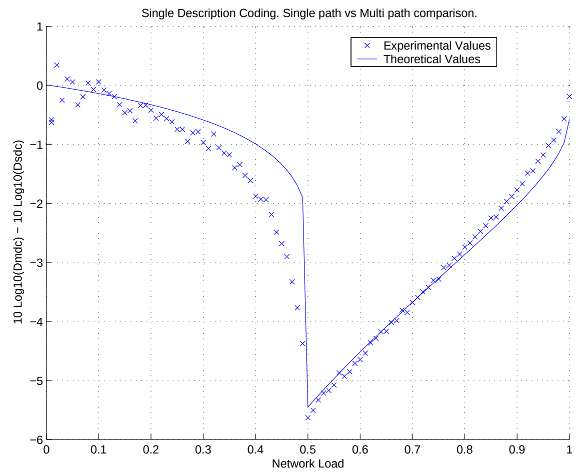

Fig. 10. Distortion improvement achieved when multi path routing is used. The y-axis represents the distortion gap in dB's and the x-axis indicates the network load. The solid line represents theoretical values while x-marks are results from simulations. The maximum delay is fixed at $\Delta = \frac{2.5}{\mu} = 2.5\frac{B}{C}$.

Fig. 10 shows the difference between the achieved distortion using single path and multi path routing in dB for different network load ρ. The distortion gap is maximum, about 5.5 dB, when the network load is $\rho = 0.5$. This performance gap is due to the load distribution achieved by the multi path routing resulting in a more efficient usage of the network capacity.

5.4. Multi path routing and Double description codes

In last section, we studied the advantage of using a routing scheme that uniformly distributes load. This is a scenario where MD fits very well. We analyze now the interaction between routing and source coding. Each source generates two descriptions of equal size, $\frac{B}{2}$ and, depending on the network conditions, possibly add some protection against packet losses. These two description are forwarded randomly using the shortest paths, however, two descriptions of the same sample are not routed through the same path.

The system can be modeled as two G/D/1 queues network with average arrival rate $\lambda = \frac{2}{T}$ and a service time $\frac{1}{\mu} = \frac{B}{2C}$. Under Kleinrock independence approximation, the total service delay distribution can be calculated as the product of the two queues system delay probability density function in the transform domain. The resulting function is equivalent to the result we derived in Section 5.3.

We jointly optimize two elements: given the source model and the coding scheme, we can calculate the loss probability of any individual description and once we have this network characterization, we compute the achievable distortion for the optimal coding scheme which depends on the network conditions.

Using MDSQ formulas (eqn. 13) the total Distortion can be computed as follows:

$$D_{MDSQ} = d_c P(T_S \leq \Delta)^2 + \\ 2d_s(1 - P(T_S \leq \Delta))P(T_S \leq \Delta) + (1 - P(T_S \leq \Delta))^2$$

and for the UEP case (eqn. 14)

$$D_{UEP} = d_c P(t \leq \Delta)^2 + \\ (d_{s_1} + d_{s_2})(1 - P(t \leq \Delta))P(t \leq \Delta) + (1 - P(t \leq \Delta))^2$$

Fig. 11 shows the benefits of using joint source channel coding coding. For each network load, data packets are coded using MDSQ and UEP for the central side distortion pair (d_c, d_s) that achieves the lowest distortion. Note that

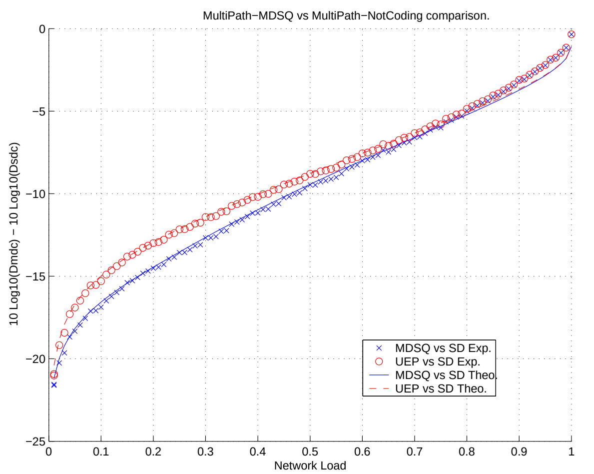

Fig. 11. Distortion improvement achieved when source coding is used. The y-axis represents the distortion gap in dB's and the x-axis indicates the network load. The solid line represents theoretical values, x-marks and circles are results from simulations. The maximum delay is fixed at $\Delta = \frac{2.5}{\mu}$. For any network load we code packets using MDSQ and UEP for the central side distortion pair (d_c, d_s) that achieves the lowest distortion.

JSCC scheme outperforms multi path no coding scheme for all networks loads. The maximum distortion improvement is almost 5 dB when $\rho = 0.25$. Note also that MDSQ outperforms UEP. Indeed, the maximum distortion improvement is of almost 1.6 dB for a network load $\rho = 0.125$.

The results presented in this section can be summarized in table 1.

6. SIMULATION RESULTS

For illustration purposes, we compare the average end-to-end distortion per source achieved by the most sophisticated

Routing	*Coding*	*Dist. Improvement* *ρ=0.25,0.5 and 0.75*
Single Path	Single des.	0 - 0 - 0 dB
Multi Path	Single des.	0.6 - 5.5 - 3.3 dB
Multi Path	UEP coding	12.1 - 8.8 - 5.7 dB
Multi Path	MDSQ	13.6 - 9.4 - 5.9 dB

Table 1. This table summarizes the different possible interactions between routing and coding. The first column is the routing scheme and the second column shows the coding technique applied to data packets. The third columns indicates the distortion improvement (in dB) with respect to the single path single description case for three different network load values, ρ= 0.25, 0.5 and 0.75, that is, for low, moderate and high traffic.

scheme, that is, multiple description source coding using scalar quantizers and multi path routing using *spreading* algorithm, with the distortion that would be achieved by the usual single path routing and single description in a bigger network. This comparison is presented in Fig. 12. Multi path - MDSQ scheme performs significantly better, being more remarkable in the case of low rates and shorter maximum allowed delay (about 18 dB in the best case).

7. CONCLUSIONS

In this paper we investigated the interaction of the source coding mechanism and the transport mechanism in dense network. These networks provide multiple paths between source and destination, so that both, the end to end reliability and the overall traffic handling capability can be increased. First, we studied the impact of routing algorithm in the achieved rate and showed that multi path routing algorithms that distribute the load uniformly among nodes exhibit a better performance. Then we introduced multiple description coding as the natural source coding to use over dense networks. We analyzed the interaction of routing and source coding in a simple scenario where we derived analytical results. Finally, we presented simulation results to build the intuition necessary for bigger networks. We showed that the most sophisticated scheme (multiple descriptions source coding and multi path routing) performs significantly better that the usual single path and single description scheme, being this improvement more remarkable in the case of low rates and shorter maximum allowed delay (about 18 dB in the best case). We are now interested in exploring the use of this method for transmission of real data such as video over a dense network.

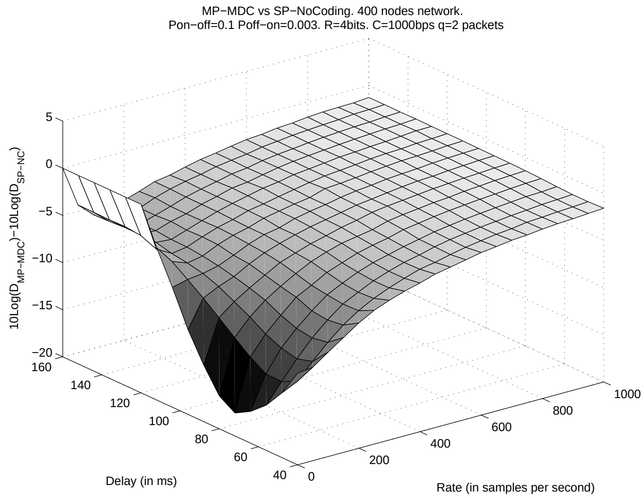

Fig. 12. Distortion improvement achieved when multi path routing and MDSQ is used instead of single path and single description in a 400 nodes torus. The z-axis represents the distortion gap in dB's, the y-axis the maximum delay and the x-axis the rate attempted per device. Nodes transmit following a Markov rule, switching between ON and OFF states over time with probabilities $P_{on\to off} = 0.1$ and $P_{off\to on} = 0.003$ independently from one another. Each sample is coded using 4 bits.

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