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GEOMETRICAL IMAGE DENOISING USING QUADTREE SEGMENTATION

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ABSTRACT

We propose a quadtree segmentation based denoising algorithm, which attempts to capture the underlying geometrical structure hidden in real images corrupted by random noise. The algorithm is based on the quadtree coding scheme proposed in our earlier work [12, 13] and on the key insight that the lossy compression of a noisy signal can provide the filtered/denoised signal. The key idea is to treat the denoising problem as the compression problem at low rates. The intuition is that, at low rates, the coding scheme captures the smooth features only, which basically belong to the original signal. We present simulation results for the proposed scheme and compare these results with the performance of wavelet based schemes. Our simulations show that the proposed denoising scheme is competitive with wavelet based schemes and achieves improved visual quality due to better representation for edges.

1. INTRODUCTION

Denoising is a classical problem in the estimation theory, which has received an extensive treatment in the literature [1, 3, 4, 5, 6, 7, 9, 14, 15]. In the denoising problem, the goal is to estimate the original signal x from an observed *noisy* signal y. More formally, consider that the signal x has been corrupted by the additive white noise z. Thus, the observed signal is

$$y = x + z.$$

Our goal is to obtain an estimate, $\widehat{x}$, of x from y such that the mean squared error $E\|x - \widehat{x}\|^2$ is minimized. Theoretically, the best estimate is

$$\widehat{x} = E\left[x|y\right], \tag{1}$$

which is quite difficult to solve in general. This is because Equation (1) is nonlinear, and the conditional probability density $p_{x|y}$ required for solving (1) is difficult to calculate. Therefore, one generally settles for the best linear estimate, which is obtained by the Wiener filtering approach [7]. However, Wiener filtering method requires the information about the spectra of the noise and the original signal and it works well only if the underlying signal is smooth. When the given signal is piecewise smooth, then the block based Wiener filtering can be used. But this approach generally fails to perform well as fixed segmentation simply cannot capture the real singularities of the underlying piecewise smooth signal. To overcome the weakness of the Wiener filtering, Donoho and Johnstone proposed the wavelet based denoising scheme in [4]. This wavelet based denoising scheme basically projects the noisy signal on the subspace, where the original signal is supposed to live. This scheme works as follows:

1. Apply the wavelet transform on the noisy signal.
2. Estimate noise strength σ from the high frequency subbands.
3. Compute the appropriate threshold t_0, e.g., $t_0 = 3\sigma$.
4. Threshold the wavelet coefficients via hard or soft thresholding scheme.
5. Reconstruct the signal from the thresholded wavelet coefficients. This reconstructed signal is our denoised signal.

This scheme generally works well but fails to perform the segmentation according to the real singularities of the given piecewise smooth signal. But it has been shown in [2, 10] that the wavelet based coders perform suboptimally due to their failure to precisely model singularities. This suggests that the above wavelet based denoising scheme might have some limitations for the denoising application. That means, a good denoising scheme should be capable of performing the segmentation according to the real singularities of the underlying signal. Since the quadtree algorithm proposed in [12, 13] accurately models both singularities and smooth pieces, we expect it to play an important role in denoising applications. Now, the key question is how to design a tree based denoising algorithm.

In [8], Natarajan has provided a crucial insight that the lossy compression of a noisy signal may lead to the filtered signal. The reason is as follows: At low rates, the coding

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scheme captures the smooth features only, which basically belong to the original signal. However, at high rates, coding scheme also tracks high frequency noise-like features. That means, as scheme codes the noisy signal from low rates to high rates, the scheme first tracks the signal up to a certain rate R_0. But above this rate R_0, the scheme starts tracking more noise than the signal. Moreover, the best denoising performance is obtained for the R-D operating point, which has the distortion equal to the noise strength. Figure 1 presents a typical R-D curve for a noisy signal. This graph clearly shows that the distortion reduces rapidly upto certain rate R_0. But beyond this rate, even a small reduction in distortion requires a large increase in rate as the coding scheme starts to track noise. This transition point R_0 is indeed the desired operating point for the coding scheme for denoising the given signal. This point is called the knee point of the R-D curve, i.e., the point at which its second derivative attains a maximum.

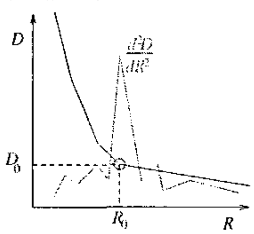
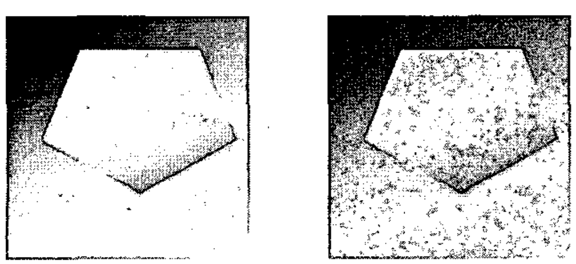


Fig. 1. R-D curve and its second derivative for a noisy signal. (R_0, D_0) represents the knee point of the R-D curve.

Natarajan's insight suggests us to select a coding scheme, which can represent the underlying original signal more efficiently. Therefore, the knowledge about the class of the original signal can be used to achieve better denoising performance. Suppose that the problem of our interest is as follows: Consider a piecewise polynomial image shown in Figure 2, which has been corrupted by the white Gaussian noise. What is a good way to recover the underlying piecewise polynomial image from the noisy image?



(a) Original image. (b) Noisy image.

Fig. 2. Original and noisy piecewise quadratic image. For noisy piecewise quadratic image, SNR= 20.32 dB.

Since a piecewise polynomial image can be precisely described by the segment boundaries and the polynomial models associated with segments, the key task is to correctly compute the segment boundaries and associated polynomial models. Since the quadtree algorithms proposed in [12, 13] can efficiently model both the smooth boundaries and smooth polynomials, we employ these schemes in our denoising algorithm. The rest of the paper is organized as follows: In Section 2, we first briefly describe the quadtree coding schemes proposed in [12, 13] and then outline the quadtree based denoising scheme. We then present some simulation results in Section 3. Finally, we conclude with a discussion of further research directions in Section 4.

2. QUADTREE BASED DENOISING ALGORITHM

Our goal is to develop a geometrical denoising algorithm based on the quadtree coding schemes proposed in [12, 13]. Before presenting our denoising algorithm, we briefly explain the prune and prune-join quadtree coding schemes. The prune quadtree coding algorithm can be summarized as follows:

Step 1: Initialization

1. Segmentation of the input image using the quadtree decomposition up to a tree depth J.
2. Approximation of each node by a geometrical model, which is composed of two 2-D polynomials separated by a linear edge, in the least square error sense.
3. Generation of the R-D curve for each node by approximating the node by the quantized version of geometrical model, which is obtained by scalar quantizing the associated 2-D polynomial coefficients.

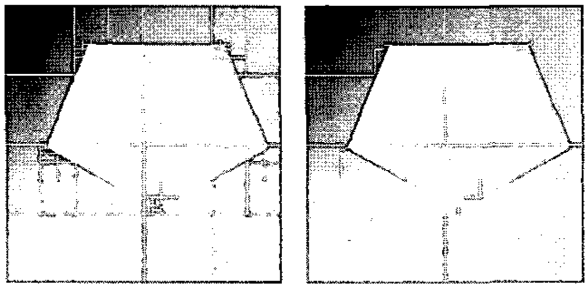
Step 2: The Lagrangian cost ($L(\lambda) = D + \lambda R$) based pruning

4. For the given operating slope λ, R-D optimal pruning criterion is as follows: Prune the children if the sum of the Lagrangian costs of the children is greater than or equal to the Lagrangian cost of the parent. This parent-children pruning criterion is used recursively to do fast pruning from the full tree depth towards the root to find the optimal subtree for a given λ [11]. This scheme provides a pruned tree.
5. Each leaf of the pruned tree for a given λ has an optimal rate choice and the corresponding distortion. Summing up the rates of all the tree leaves along with the tree segmentation cost will provide the overall bit-rate $R^*(\lambda)$. Similarly, summing up the associated distortions of all the tree leaves will give the net distortion $D^*(\lambda)$.

Step 3: Search for the desired R-D operating slope

6. The value for λ is determined iteratively until the bit-rate constraint R_0 is met as closely as possible. We employ the fast bisection search algorithm given in [11].

However, this independent pruning scheme fails to join neighboring blocks with similar information, if they have different parents. Thus, this coding scheme fails to exploit the complete dependency among neighbors. This drawback can be easily seen in Figure 3(a). For correcting the suboptimal behavior, we have proposed the prune-join quadtree algorithm, which performs the joint coding of similar neighboring leaves even if they have different parents. The prune-join coding scheme employs the prune quadtree scheme followed by the neighbor joint coding algorithm, which decides whether neighbors should be coded jointly or independently (see [13] for more details). It is evident from Figure 3(b) that, due to the neighbor joining, the prune-join scheme successfully joins similar neighbors as well as provides the correct segmentation.



(a) Prune quadtree. (b) Prune-join quadtree.

Fig. 3. Segmentation performed by the quadtree algorithms for a piecewise quadratic image.

Now, the quadtree based denoising scheme can be summarized as follows

Algorithm 1 ***The quadtree based denoising algorithm***

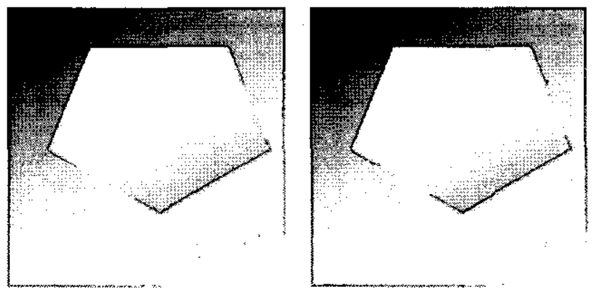
1. Compute the R-D curve using either the prune or prune-join quadtree coding scheme for the given noisy image.
2. Estimate the knee point of the R-D curve by calculating the point where second derivative of the R-D function achieves maximum. Note that we approximate the second derivative by the second order difference.
3. Compute the coded image representation for this R-D point.
4. This coded image is our filtered/denoised image.

Since the quadtree algorithms use the linear-edge model explicitly in the approximation tile, they are well suited to capture the geometry hidden in noisy images. In the next section, we perform denoising related experiments using the above described algorithmic framework.

3. SIMULATION RESULTS

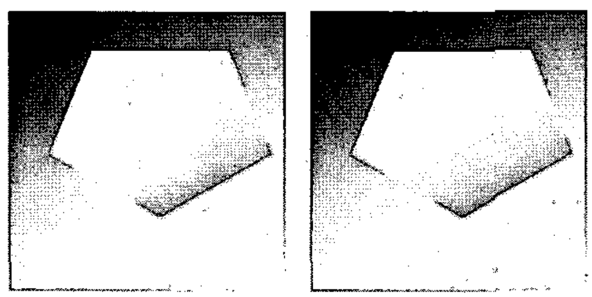
For denoising experimentation, we consider piecewise polynomial and real images. Figure 2 shows a piecewise quadratic image along with its noisy version. In Figure 4, we present the denoised images obtained by the prune and prune-join quadtree algorithms. On the other hand, Figure 5 displays the denoised images provided by the standard un-decimated wavelet transform based denoising method and the adaptive directional wavelet transform based denoising scheme [14]. These results clearly show that the quadtree based denoising schemes captures the underlying regular structure better than the wavelet based schemes.

In Figure 6, we show the cameraman image and its noisy version with SNR=17.72 dB. Figure 7 presents the denoised images obtained by the prune and prune-join quadtree schemes. Figure 8 shows the denoised images provided by the standard un-decimated wavelet transform and the adaptive directional wavelet transform based denoising schemes. One can easily see that the tree algorithms preserve the sharpness of edges while the wavelet based schemes suffer from the ringing artifacts around the edges. These simulation results clearly demonstrate that the proposed tree based schemes perform better than the wavelet based schemes if the original image exactly fits the piecewise polynomial model. Even for real images, the performance of the tree based denoising schemes is competitive with that of the state of the art wavelet based denoising schemes.



(a) Prune quadtree based denoising (SNR= 36.91 dB). (b) Prune-join quadtree based denoising (SNR= 37.96 dB).

Fig. 4. Denoised images obtained by the prune and prune-join quadtree schemes.



(a) Standard un-decimated wavelet transform (SNR= 34.66 dB). (b) Adaptive directional wavelet transform (SNR= 36.09 dB).

Fig. 5. Denoised images provided by the standard un-decimated wavelet transform and adaptive directional wavelet transform based schemes.

(a) Original cameraman. (b) Noisy cameraman.

Fig. 6. Original and noisy cameraman image. For noisy cameraman image, SNR= 17.72 dB.

(a) Prune quadtree based denoising (SNR= 24.74 dB). (b) Prune-join quadtree based denoising (SNR= 24.73 dB).

Fig. 7. Denoised images obtained by the prune and prune-join quadtree schemes.

4. CONCLUSIONS

We have presented a quadtree based denoising scheme, which can efficiently extract geometrical structures from noisy images. Simulation results shown in Section 3 clearly demonstrate that our scheme is competitive with state of the art wavelet based schemes. Moreover, it is also evident from Figures 4 and 7 that the proposed scheme captures geometrical features, like edges, of images more precisely compared to wavelet based schemes. Our on-going research effort is to extend the present algorithm such that it can also distinguish the texture present in images from random noise for further improving the visual quality of denoised images.

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(a) Standard un-decimated wavelet transform (SNR= 24.42 dB). (b) Adaptive directional wavelet transform (SNR= 24.82 dB).

Fig. 8. Denoised images provided by the standard un-decimated wavelet transform and adaptive directional wavelet transform based schemes.

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