Power Control is Not Required for Wireless Networks in the Linear Regime

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Abstract

We consider the design of optimal strategies for joint power adaptation, rate adaptation and scheduling in a multi-hop wireless network. Most existing strategies control either power and scheduling, or rates and scheduling, but not all three together as we do. We assume the underlying physical layer is in the linear regime (the rate of a link can be approximated by a linear function of the signal-to-interference-and-noise ratio), like in time hopping UWB (TH-UWB) and low gain CDMA systems, and that it allows fine-grained rate adaptation, like in 802.11a/g, HDR/CDMA, TH-UWB. The goal is to find properties of the power control in an optimal joint design. Our main finding is that optimal power control is simple $0 - P^{MAX}$ *power control, i.e. when a node is sending it uses the maximum transmitting power allowed. We consider both high rate networks where the goal is to maximize rates under power constraints and low power networks where the goal is to minimize average consumed power while meeting minimum rate constraints. We prove analytically that in both scenarios the optimal can always be attained with* $0-P^{MAX}$ *power allocation. Moreover, we prove that, when maximizing rates, and if power constraints are on peak and not average,* $0 - P^{MAX}$ *is the only optimal power control strategy, and any other is strictly suboptimal.*

1 Introduction

1.1 Power Control and Optimal Wireless MAC Design

The first wireless MAC protocols for multi-hop networks were designed to control only medium-access. A typical example is the original 802.11 MAC. It always uses maximum power for transmitting a packet and aims to establish communication on a fixed, predefined link rate. Then several improvements to the initial approach were proposed. According to the type of improvement, the MAC protocols can be divided globally in two groups. The former group of protocols [19, 8, 11] is focused on rate adaptation: the transmission power is still kept fixed, but the rate is adapted to the actual channel conditions and the amount of interference. The latter group of protocols [10, 12, 14, 13] considers power adaptation while keeping the rates fixed. However, there are no MAC protocols that adapt both rate and power at the same time, and the fundamental issues in this joint adaptation problem are not well understood. In this paper we make a first step by showing that, perhaps contrary to intuition, there is a whole class of networks (those operating in the linear rate function regime, see paragraph 1.3) for which power control is not required, or may even be suboptimal.

We consider a wireless network with arbitrary scheduling, rate adaptation and routing strategies, and we are interested in characterizing the properties of the optimal power allocation strategy in this setting.

1.2 Rate Adaptation and Rate Function

The physical layer of a wireless link defines communication parameters such as bandwidth, modulation and coding, which can be used to establish communication with some level of bit or packet errors. One of the most important parameters of the physical layer is signal-to-interference-and-noise ratio (SINR) at the receiver. The higher the SINR is, the higher communication rates can be attained, and one of the goals of networking design is to efficiently track and adapt SINRs and/or rates on links.

Some of the existing wireless systems use fixed communication rates. A typical example is a cellular voice network, where one voice channel has a fixed rate. There, a goal of the system is to maintain the SINR of each user above a threshold, such that there are no outages. Initially, the first version of 802.11 used the same approach. In contrast, most of the recently proposed wireless physical layers allow rates to vary with SINR. Typical examples are

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802.11a/g [1], CDMA/HDR [15], and TH-UWB [23]. The physical layers use adaptive modulation [16, 1] and/or adaptive coding [23] to adjust the rate to the SINR at the receiver while maintaining a constant, guaranteed bit-error rate. The function that gives the maximum achievable rate for a given SINR is called the *rate function*. Examples of protocols that use rate adaptation can be found in [19, 8, 11].

1.3 Linear Regime

The rate function of an efficiently designed system is a concave function of SINR. Furthermore, in many cases, especially when bandwidth is large or the target SINR is low, it is a linear function. Some examples of physical layers, where rate function is linear, are TH-UWB [23] and low or moderate-gain CDMA [4]. These physical layers are in linear regime in the whole operational SINR range due to a very large bandwidth, and they can operate on high as well as low data rates. Also, physical layers with non-linear rate functions, like 802.11a/g, may operate in the linear regime if the received power is low (e.g. distances between nodes are large). Our findings in this paper are for networks whose physical layers operates in the linear regime.

1.4 Rate Maximization and Power Minimization

There are two typical deployment scenarios for wireless networks: high bit-rate networks and low power consumption networks. The first one considers real-time video and audio communication, web surfing, data transfer, and the like. The primary design focus here is to maximize available rates, subject to power constraints. Typical examples of this type of networks are 802.11 and 802.15.3a wireless LANs and CDMA-HDR cellular systems. We call this case *rate maximization scenario*; here we are interested in the set of feasible rates.

The second scenario is focused on low power networks like sensor networks or networks of computer peripherals. The main goal is to maximize network lifetime, or equivalently, to minimize average consumed power. At the same time, end-to-end flow rates are lower bounded by application requests, and each sender typically has a minimum amount of information to send to a destination in a given time. Here we are interested in minimizing power consumption, subject to minimum long term rate constraints. Long-term average power consumption is defined in Section 2.4. We call this case *power minimization scenario*; here we are interested in the set of feasible power allocations.

Different performance objectives for comparing the feasible sets in both scenarios are presented in detail in Section 2.5.

1.5 Power Control in Existing Systems

The goal of power control is to determine which power a transmitter should use when transmitting a packet. The optimal transmitted power of a packet depends on a large number of parameters, such as the distance from the destination, the background noise, the amount of interference incurred by concurrent transmissions, etc. In an ad-hoc network, the optimal power also depends on transmitting powers of other concurrently scheduled links. Since power control is tightly coupled with scheduling, it is typically implemented within the MAC protocol.

Perhaps the simplest way to choose the transmitted power is not to do any power control. In other words, whenever a packet is sent, it is sent with maximum allowed power. We call this $0 - P^{MAX}$ power control. The $0 - P^{MAX}$ power control was widely used in the design of the first wireless MAC protocols, such as 802.11, due to its simplicity, and due to the fact that the optimal power control was not well understood.

Much of the research on power control is focused on voice cellular systems. Those systems typically use quasi-orthogonal channels for different users (e.g. CDMA spreading) in order to decrease multi-user interference, i.e. interference between competing users in the same network. However, the orthogonality of channels is not complete, and some amount of interference between users cannot be avoided (this is captured by the orthogonality factor in Section 3.1). Classically, the physical layer of CDMA systems is designed to operate when multi-user interference is small; otherwise (this is is known as the *near-far problem*), signal acquisition and decoding do not work. This is why such systems must employ some form of power control; for example, on the CDMA-HDR uplink, the near-far problem is avoided by equalizing all received powers at the base station. Some pioneering work in this area can be found in [7, 2, 24, 13]. These papers propose iterative algorithms that converge to a power allocation where all nodes' SINRs are above thresholds, should such allocation be possible. These ideas have been extended to multi-hop wireless networks in [6].

An attempt to design an optimal power control protocol for 802.11 networks has been made in [10, 12, 14]. They consider the 802.11b physical layer with a fixed rate, and the common conclusion is that the power should be adjusted to the minimal value required to be successfully decoded at the destination. The above power control protocols are optimal only when the physical layer offers a fixed rate, regardless of the signal-to-noise level at the receiver. Not too much work has been done on power control for networks with variable link rates. An adaptive power control mechanism for cellular networks with variable link rates is presented in [16]. However, this mechanism is adapted to voice traffic. It does not consider scheduling and thus leaves out an important design parameter of data wireless networks.

Several power adaptation protocols have been proposed for power minimization scenarios. A typical example is

given in [18] where the power of a link is adjusted to a minimum necessary to reach a destination, and the routing is chosen to minimize the overall power dissipation.

In most of these existing systems, the benefits of power control derive from assumptions on the physical layer (such as fixed rate coding, or the need to avoid near-far problems). It is however possible to do without such assumptions: some examples are the CDMA-HDR downlink (which does rate adaptation), or TH-UWB systems with interference mitigation [11]. This motivates us to pose the problem of optimal MAC design in general terms, assuming power control is an option but not a requirement. Protocols that consider rate adaptation, power adaptation and scheduling in this general setting have been proposed in [4, 5]: they focus on low processing gain CDMA or UWB networks (thus linear regime) and show that $0 - P^{MAX}$ power control is optimal when the objective is to maximize the total sum of rates. However this objective is known to be defective [17], as it requires that the most expensive links be shut down completely. We go beyond these results and establish the optimality of $0 - P^{MAX}$ for any performance objective, and the non-optimality of any non $0 - P^{MAX}$ power control for rate maximization scenarios.

1.6 Performance Comparison

For different power control strategies, we are interested in comparing the resulting rate allocations. By using different scheduling strategies with one power control strategy however, one can obtain different rate allocations. The set of all possible rate allocations that can be obtained with a given power control strategy, and with different schedules, is called the feasible rates set. In this paper we use the most general way to compare performances of the two power control strategies, which consists in comparing the sets of their Pareto efficient feasible allocations. Precise definitions of these concepts are given in Section 2.5.

1.7 Modeling of Wireless Networks

We are interested in the fundamental principles in a design of a wireless MAC, and not in designing a specific protocol. Therefore, we assume an ideal, zero overhead MAC protocol, which comprises ideal scheduling and rate adaptation strategies. And we are interested in characterizing properties of an optimal power control strategy.

General models of wireless networks that incorporate various physical layers, MAC and routing protocols, are discussed in [4, 21, 9, 17]. These models represent the most general assumptions on physical layers (including variable rate 802.11, UWB or CDMA) and MAC protocols. Note however that they exclude the possibility of cooperative coding and decoding at the physical layer across multiple links, as this requires synchronization assumptions that are not realistic today.

We use a model similar to these; we assume arbitrary routing (single-hop or multi-hop) and we assume point-to-point links whose conditions change randomly over time due to fading or mobility. For a given network topology and traffic demand, we characterize the set of feasible average end-to-end rate allocations under given maximum average power constraints, and equivalently the set of feasible average power constraints under minimal average end-to-end rate constraints. We use the model to prove our findings by theoretical analysis and numerical simulations. More detailed assumptions on the network model are given in Section 2.

1.8 Our Contribution

We consider a general multi-hop wireless network with random channels due to fading or mobility, where link rates, transmission powers and medium access can be varied, and we focus on physical layers that operate in the linear regime. For such systems, one can find rate control, power control and theoretical MAC protocols that maximize the performance. This is a joint optimization problem and a change in any of the three components influences the choice of the other two. We consider different power control strategies, for each of them we assume the optimal MAC and rate adaptation, and we compare their performances. The goal is to characterize the optimal power control.

We first consider the rate maximization scenario and we mathematically prove that every feasible rate allocation can be achieved without power control (power adaptation is not needed beyond $0 - P^{MAX}$), and that, if there are no average power constraints (i.e. only peak power constraints), any power control that does not use $0 - P^{MAX}$ power control is not Pareto efficient (power adaptation is suboptimal).

We further consider the power minimization scenario. We prove that any feasible average power allocation is achievable without power adaptation. In other words, any feasible average power allocation is achievable with $0 - P^{MAX}$ power control and an appropriate schedule, and power adaptation is not needed.

Our findings are based on the assumption that, for every power control protocol of choice, we design an optimal scheduling and rate adaptation protocol, which is not necessarily simple to implement. However, our results do suggest that, for multihop networks operating in the linear regime and that can live with arbitrary levels of rate and power, power control beyond $0 - P^{MAX}$ can be avoided, and thus, the MAC layer should concentrate on scheduling (by means of a protocol) and rate adaptation, using full power whenever a transmission is allowed by the protocol.

1.9 Organization of The Paper

In the next section we describe system assumption. In Section 3 we give a mathematical formulation of the model

of a network. In section Section 4 we present our main findings. In the last section we give conclusions and directions for further work. Proofs of the results are in the appendix.

2 System Assumptions

We analyze an arbitrary multi-hop wireless network that consists of a set of nodes, and every two nodes that directly exchange information are called a link. For each pair of nodes we define a signal attenuation, i.e. a level of signal received at the receiver, assuming the sender is sending with unit power. This attenuation is usually a decreasing function of a link size due to power spreading in all directions, but here we assume it can be an arbitrary number defined for each pair of nodes. We assume the network is located on a finite surface and that all attenuations are always strictly positive, hence every node can be heard by any other node in the network and there is no clustering. Signal attenuation also changes in time due to mobility and different variations of characteristics of paths the signal takes, thus we will model it as a random process. Next, we give properties of the physical model of communications on links.

2.1 Physical Model Properties

All physical links are point-to-point, this means each link has a single source and a single destination. A node can either send to one next hop or receive from one at a time.

We model rate as a function $r(\mathrm{SINR})$ of the signal-to-interference-and-noise ratio at the receiver, which is the ratio of received power by the total interference perceived by the receiver including the ambient noise and the transmissions of other links that occur at the same time. In case of systems with spreading, such as CDMA, frequency-hopping OFDM or TH-UWB, a receiver does not capture the full power of an interferer, but just a fraction that depends on the correlation of the spreading sequences of the sender and the interferer. The total noise at a receiver can thus be modeled as the sum of the ambient noise and the total interference multiplied by the *orthogonality factor*. The more efficient the spreading is, the smaller the orthogonality factor is.

This model corresponds to a large class of physical layer models, for example:

- Shannon capacity of Gaussian channels [3]: $r(\mathrm{SINR}) = 1/2 \log_2(1 + \mathrm{SINR})$.
- Low-power and/or wide-band Gaussian channels [20]: $r(\mathrm{SINR}) \approx K \times \mathrm{SINR}$
- Time-hopping ultra-wide band [23]: $r(\mathrm{SINR}) = K \times \mathrm{SINR}$.
- Moderate processing gain CDMA [4]: $r(\mathrm{SINR}) = K \times \mathrm{SINR}$.
- Fixed rate 802.11b [standard]: $r(\mathrm{SINR})$ is a step function of SINR
- Variable rate 802.11a/g [standard]: $r(\mathrm{SINR})$ is a stair function of SINR.
- CDMA HDR [15]: $r(\mathrm{SINR})$ is a stair function of SINR.

In all the examples except 802.11b, the rate is variable and is a function of signal-to-noise ratio at a receiver. This is achieved by adaptive modulation, like in [16, 19, 8], or adaptive coding [11]. Rate as a function of SINR is a concave function. For an efficiently designed system, it usually approaches the Shannon capacity of the system [3], which is a log-like function. However, for low-power (e.g. sensor networks) or high-bandwidth systems (e.g. UWB [23] or CDMA systems with moderate processing gain [4]), the total noise is much larger than received powers, and the capacity can be approximated with a linear function of SINR [20]. Also, physical layers with a non-linear rate function operate in a linear regime when the SINR at the receiver is low. In this paper we focus on physical layers with a linear rate function.

2.2 MAC Protocol

The model of the MAC protocol is similar to the one from [17]. We assume a slotted system. In each slot a node can either send data, receive or stay idle, according to the rules defined in Section 2.1. Each slot has a power allocation vector associated with it, which denotes what power is used for transmitting by the source of each link. If a link is not active in a given slot, its transmitting power is 0. A schedule consists of an arbitrary number of slots of arbitrary lengths.

The first part of a MAC is a power control strategy. The power control strategy is defined by a set of possible powers that can be allocated to links in any slot. An example of power control strategy is $0 - P^{MAX}$ power control where any link in any slot can send with power P^{MAX} or stay idle. This is the simplest strategy where powers are fixed and there is no power adaptation.

The second part of a MAC is the rate adaptation and scheduling. Having chosen a power control strategy, a MAC chooses a schedule and assigns powers that belong to the set of possible powers to links in each slot. Finally, the rate on each link in each slot is adapted to the SINRs at receivers.

We assume that for a given power control strategy we have an optimal MAC protocol that calculates the optimal transmission power of each link out of the set of possible powers defined by power control, and in each slot in a ideal manner and according to a predefined metric. This is equivalent to a network where nodes dispose of an ideal control plane with zero delay and infinite throughput to negotiate schedule and power allocation.

A more realistic MAC protocol would introduce some errors and delays, but a good approximation should be close to the ideal case. Also, by considering an ideal protocol,

we focus our analysis on properties of performance metrics, and not artifacts of leaks in protocol design. Our assumption corresponds to neglecting the overhead (in rate and power) of the actual MAC protocol.

We also assume random fading. Since we have an ideal MAC protocol, it can instantly adapt the schedule and the power and rate allocation to any state of the random fading of links. For a precise mathematical model of MAC protocol, see Section 3.2.

2.3 Routing Protocol and Traffic Flows

We assume an arbitrary but constant routing protocol (routes and flow demands do not change in time). Flows between sources and destinations are mapped to paths, according to some rules specific to the routing protocol. At one end of the spectrum, nodes do not relay and only one-hop direct paths are possible. At the other end, nodes are willing to relay data for others and multi-hop paths are possible. There can be several parallel paths. All these cases correspond to different constraint sets in our model, as defined in Section 3.2. Sources can send to several destinations (multicast) or to one (unicast).

2.4 Power and Rate Constraints

There are four types of power and rate constraints in a wireless network: peak power constraint, short-term average power constraint, long-term average power constraint and average rate constraint. Here we describe them in detail:

Peak power constraint: Given a noise level on a receiver, a sender can decide which codebook it will use to send data over the link during one time slot. Different symbols in the codebook have different powers. The maximum power of a symbol in a codebook is then called *peak power*. In our model, the peak power constraint is integrated in a rate function, given as an input.

Short-term average power constraint: We assume a slotted system. In each slot a node chooses a codebook and its average power, and it sends data using this codebook within the duration of the slot. We call *transmitted power* the average power of a symbol in the codebook. This is a short-term average power within a slot, since a codebook is fixed during one slot. We assume that this transmission power is upper-bounded by P^{MAX}. This power limit is implied by technical characteristics of a sender and by regulations, and is not necessarily the same for all nodes. For example, this is the only power constraint that can be set by users on 802.11 equipment.

Long-term average power constraint: While transmitting a burst of data (made of a large number of bits), a node uses several slots, and possibly several different codebooks. Each of these codebooks has its transmission power. We call the *consumed power* the average of transmission powers during a burst, and we assume it is limited by $\overline{P}^{MAX}$. Consumed power $\overline{P}^{MAX}$ is set by a node to control its lifetime and it can vary from node to node.

Average rate constraint: In networks such as sensor or peripheral networks, the goal is to minimize power consumption and to maximize the lifetime of nodes rather than maximize the rates of links. Still, there is a lower bound on the rate a node has to transmit. For example, a temperature sensor on a car engine or a computer mouse have a well-defined rate of information they need to communicate to a central system. This is what we call the *average rate constraint* and we define it as the average amount of bits a node has to transmit over the network in one second. We assume this average limit is the same on both long and short timescales.

We incorporate explicitly in our model the transmission power constraints, the average consumed power constraints and the average rate constraints. The peak power is incorporated implicitly through the choice of the rate function.

2.5 Performance Objectives

Design criteria in wireless networks can be divided into two groups: rate maximization and power minimization. We first consider rate maximization. Given a network topology and a family of MAC protocols, one can define a set of feasible rate allocations as the set of all rate allocations that can be achieved on the network with some MAC protocol from the given family. An interesting subset of the feasible rate set is the set of Pareto efficient rate allocations. A rate allocation is *Pareto efficient* if no rate can be increased without decreasing some other rate. When maximizing rates, we are clearly interested only in the Pareto efficient rate allocations.

The most general way to compare two families of network protocols on a same network is to compare their Pareto efficient rates' sets. If all Pareto efficient rates of one family of protocols are feasible under the other family of protocol, then one can undoubtedly say that the second family is as good as the first. Furthermore, if neither of the Pareto efficient rates of the second family is achievable under the first family of MAC protocols, then we can say that the second family is strictly better than the first. We will use this criterion to compare different power control strategies throughout the paper. We use the analog approach to compare different power minimization scenarios: in this case a power allocation is Pareto efficient if no average power can be decreased without increasing some other power. Mathematical definitions of terms are given in Section 3.2.

3 Mathematical Model

3.1 Notations

We model the wireless network as a set of I flows, L links and O nodes. Flows are unicast or multicast. We as-

sume the network is in a random state S belonging to set $\mathcal{S}$. For each state $s \in \mathcal{S}$ we define the attenuations among nodes in the network and the power of background noise at every receiver. Since we analyze a theoretical MAC, we assume for each system state $s \in \mathcal{S}$ that there is a separate instance of the MAC. We give here a list of notations used in this section to describe the model. The precise definitions are given in subsequent subsections:

– $h_{l_1 l_2}(s)$ is the attenuation of a signal from the source of link l_1 to the destination of link l_2 when the system is in state s. We assume no clustering, hence $h_{l_1 l_2}(s) > 0$.

– β is the orthogonality factor that defines how much power of interfering signals is captured by a receiver.

– $\mathbf{f} \in \mathbb{R}^I$ is the vector of average rates achieved by flows.

– $\bar{\mathbf{x}} \in \mathbb{R}^L$ is the vector of average rates achieved on links.

– $\{1, \cdots, N(s)\} \in \mathbb{R}$ is the number of slots in the schedule corresponding to the system state s.

– for every $n \in \{1, \cdots, N(s)\}$, $\mathbf{x}^n(s) \in \mathbb{R}^L$ is the vector of rates achieved on links in time slot n when the system is in state s.

– for every $n \in \{1, \cdots, N(s)\}$, $\mathbf{p}^n(s) \in \mathbb{R}^L$, $\mathbf{p}_{\text{rcv}}{}^n(s) \in \mathbb{R}^L$ are the vectors of transmitted and received powers allocated on links in time slot n, respectively, when the system is in state s.

– $\overline{\mathbf{F}}^{MIN} \in \mathbb{R}^I$ is the vector of minimum average rates achieved by end-to-end flows (every flow may have a different minimum average rate).

– $\mathbf{P}^{MAX} \in \mathbb{R}^L$ is the vector of maximum allowed transmission powers on links, which are assumed constant in time (every link may have a different maximum power).

– $\overline{\mathbf{P}}^{MAX} \in \mathbb{R}^L$ is the vector of maximum allowed average transmission powers on links (every link may have a different maximum power).

– $\eta_l(s) \in \mathbb{R}$ is the white noise at the receiver of link l when the system is in state s.

– for every $n \in \{1, \cdots, N(s)\}$, $\mathbf{SINR}^n(s) \in \mathbb{R}^L$ is the vector of signal-to-interference-and-noise ratios at the links' receivers in time slot n, when the system is in state s.

– for every $n \in \{1, \cdots, N(s)\}$, $\alpha^n(s) \in [0, 1]$ is the relative frequency of time slot n (fraction of the total schedule duration occupied by slot n) in the schedule assigned to the system when in state s.

– R (routing matrix) is such that $R_{l,i} = 1$ if flow i uses link l. We have $R\mathbf{f} \leq \bar{\mathbf{x}}$. The matrix R is defined by the routing algorithm.

3.2 Mathematical Formulation

We assume that for every state s there is a schedule consisting of time slots $n = 1...N(s)$ of frequency $\alpha_n(s)$. This is an abstract view of the MAC protocol, without overhead. We normalize these lengths such that $\sum_{n=1}^{N(s)} \alpha_n(s) = 1$. Let us call $\mathbf{p}^n(s)$ the vector of transmission powers assigned to links in slot n and state s, and let $\mathbf{SINR}^n(s)$ be the vector of signal-to-interference-and-noise ratios at receivers of the links, induced by $\mathbf{p}^n(s)$. The rate achievable on link l in slot n and state s is $x_l^n(s) = K\,\text{SINR}_l^n(s)$. The vector of average rates on the links is thus $\bar{\mathbf{x}} = \mathbb{E}\left[\sum_{n=1}^{N(S)} \alpha_n(S)\mathbf{x}^n(S)\right]$, averaged over the distribution of states. Since $\mathbf{x}^n(s)$ has dimension L (where L is a number of links), by virtue of Carathéodory theorem, when in state s, it is enough to consider $N(s) \leq N = L + 1$ time slots of arbitrary lengths $\boldsymbol{\alpha}(s)$ in order to achieve any point in the convex closure of points $\mathbf{x}^n(s)$.

Feasible rate and power allocations: Given a network topology and a routing matrix R, we define the *set of feasible average powers, link rates and end-to-end rates* $\mathcal{T}$ (without average power or rate constraints). It is the set of $\mathbf{f} \in \mathbb{R}^I$, $\bar{\mathbf{x}} \in \mathbb{R}^L$ and $\bar{\mathbf{p}} \in \mathbb{R}^L$ such that there exist schedules $\boldsymbol{\alpha}(s)$, sets of power allocations $\mathbf{p}^n(s)$ and corresponding sets of rate allocations $\mathbf{x}^n(s)$ for all $n = 1 \cdots N$ and all states $s \in \mathcal{S}$, such that the following set of equalities and inequalities are satisfied for all $n = 1 \cdots N, i = 1 \cdots I, l = 1 \cdots L, o = 1 \cdots O$:

$$\begin{aligned}
R\mathbf{f} &\leq \bar{\mathbf{x}} \\
\bar{\mathbf{p}} &= \mathbb{E}\left[\textstyle\sum_{n=1}^{L+1} \alpha_n(S)\mathbf{p}^n(S)\right] \\
\bar{\mathbf{x}} &= \mathbb{E}\left[\textstyle\sum_{n=1}^{L+1} \alpha_n(S)\mathbf{x}^n(S)\right] \\
\mathbf{x}_l^n(s) &= K\,\text{SINR}_l(\mathbf{p}^n(s)) \\
\text{SINR}_l(\mathbf{p}^n(s)) &= \tfrac{p_l^n(s)h_{ll}(s)}{\eta_l(s)+\beta\sum_{k\neq l} p_k^n(s)h_{kl}(s)}
\end{aligned} \tag{1}$$

$$\begin{aligned}
1 &= \textstyle\sum_{n=1}^{L+1} \alpha_n(s) \\
1 &\geq \textstyle\sum_{l\,:\,l.\text{src}=o} 1_{\{p_l^n(s)>0\}} + \\
&\quad\ \textstyle\sum_{l\,:\,l.\text{dst}=o} 1_{\{p_l^n(s)>0\}} \\
p_l^n(s) &\leq P_l^{MAX}
\end{aligned}$$

where $l.\text{src} = o$ and $l.\text{dst} = o$ are true if node o is the source or the destination of link l, respectively.

We are interested in comparing average rates and power consumptions with $0 - P^{MAX}$ and with arbitrary control. With $0 - P^{MAX}$ power control, a node sends with maximum power when sending. More formally this means that in any slot n, power allocation vector $\mathbf{p}^n$ has to belong to the set of extreme power allocations $\mathcal{P}^E = \{\mathbf{p} \,|\, (\forall l = 1 \cdots L)\, p_l \in \{0, P_l^{MAX}\}\}$. In contrast, with an arbitrary power control, any power from the set of all possible power allocations $\mathcal{P}$ is possible. The set $\mathcal{P}$ is defined as $\mathcal{P} = \{\mathbf{p} \,|\, (\forall l = 1 \cdots L)\, p_l \in [0, P_l^{MAX}]\}$.

We say that an average rate allocation $\mathbf{f}$ and average power consumption $\bar{\mathbf{p}}$ is *achievable* with a set of power allocations belonging to $\mathcal{P}$ if for all $n = 1 \cdots N, i = 1 \cdots I, l = 1 \cdots L, o = 1 \cdots O$, it satisfies constraints (1), and for all $n = 1 \cdots N, s \in \mathcal{S}, \mathbf{p}^n(s) \in \mathcal{P}$.

We can similarly define the set of average end-to-end rates, link rates and power allocations $\mathcal{T} = \mathcal{T}(\mathcal{P})$ that is achievable with power allocations belonging to $\mathcal{P}$, as the set of all $(\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}})$ that are achievable using power allocation $\mathcal{P}$. Thus, sets $\mathcal{T}$ and $\mathcal{T}(\mathcal{P}^E)$ represent the sets of all possible average end-to-end rates, link rates and power consumptions with an arbitrary and with $0 - P^{MAX}$ power control, respectively.

When we consider rate maximization under constraints on average consumed power, we are interested only in the set of feasible rates. If the average consumed power is limited by $\overline{\mathbf{P}}^{MAX}$, then the set of feasible rates is $\mathcal{F} = \{\mathbf{f} \,|\, (\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{T}, \bar{\mathbf{p}} \leq \overline{\mathbf{P}}^{MAX}\}$. Similarly, with $0 - P^{MAX}$ power control, the set of feasible rate is $\mathcal{F}^E = \{\mathbf{f} \,|\, (\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{T}(\mathcal{P}^E), \bar{\mathbf{p}} \leq \overline{\mathbf{P}}^{MAX}\}$. For notational conveience, we analogly define $\bar{\mathcal{X}} = \{\bar{\mathbf{x}} \,|\, (\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{T}, \bar{\mathbf{p}} \leq \overline{\mathbf{P}}^{MAX}\}$ and $\bar{\mathcal{X}}^E = \{\bar{\mathbf{x}} \,|\, (\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{T}(\mathcal{P}^E), \bar{\mathbf{p}} \leq \overline{\mathbf{P}}^{MAX}\}$.

Similarly, when considering power minimization, we focus on the set of feasible average consumed powers. If the average end-to-end flow rate is lower-bounded by $\overline{\mathbf{F}}^{MIN}$, then the set of feasible average consumed powers, under arbitrary power control, is $\overline{\mathcal{P}} = \{\bar{\mathbf{p}} \,|\, (\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{T}, \mathbf{f} \geq \overline{\mathbf{F}}^{MIN}\}$. Similarly, with $0 - P^{MAX}$ power control, the set of feasible rate is $\overline{\mathcal{P}}^E = \{\bar{\mathbf{p}} \,|\, (\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{T}(\mathcal{P}^E), \mathbf{f} \geq \overline{\mathbf{F}}^{MIN}\}$.

Performance Objectives: Finally, we formally define notion of Pareto efficiency that was introduced in Section 2.5. Rate vector $\mathbf{f} \in \mathcal{F}$ is Pareto efficient on $\mathcal{F}$ if there exist no other vector $\mathbf{f}' \in \mathcal{F}$ such that for all $i, \mathbf{f}'_i \geq \mathbf{f}_i$ and for some $j, \mathbf{f}'_j > \mathbf{f}_j$. Average power dissipation vector $\mathbf{p} \in \overline{\mathcal{P}}$ is Pareto efficient on $\overline{\mathcal{P}}$ if there exists no other vector $\mathbf{p}' \in \overline{\mathcal{P}}$ such that for all $i, \mathbf{p}'_i \leq \mathbf{p}_i$ and for some $j, \mathbf{p}'_j < \mathbf{p}_j$.

4 Main Findings

4.1 Rate Maximization

In this section we show that any rate allocation that is feasible with an arbitrary power control and under some average power constraint, is also achievable with $0 - P^{MAX}$ power control. Moreover, if we consider a scenario without average power constraints, then $0 - P^{MAX}$ is the only optimal power control.

We clearly have $\mathcal{F}^E \subseteq \mathcal{F}$, and we want to show that every feasible flow rate allocation can be achieved by a set of extreme power allocation from $\mathcal{P}^E$, that is $\mathcal{F} \subseteq \mathcal{F}^E$. In other words that every feasible flow rate allocation can be achieved only with an appropriate scheduling, and without power control.

Theorem 1 *If the rate is a linear function of the SINR, then for arbitrary values of parameters of constraint set (1), we have that $\mathcal{F}^E = \mathcal{F}$.*

(Proof in appendix) The theorem says that every feasible rate allocation, thus including the Pareto efficient ones, can be achieved with $0 - P^{MAX}$ power control, and with an appropriate scheduling. Hence $0 - P^{MAX}$ is at least as good as any other power control, and power adaptation is not needed.

To interpret this finding, consider a UWB MAC protocol presented in [5] where both power adaptation and scheduling is used. Any rate achieved by this MAC protocol could be achieved with another protocol that would not adapt power and would use an appropriate scheduling.

Comment. One could be tempted to interpret Theorem 1 by saying that in the case of a linear rate function, adapting power can be directly mapped into scheduling time shares. This is not correct. Indeed, in the linear regime, the rate of a link is a linear function of the SINR, but it is *not* a linear function of the transmission powers of interfering nodes. Therefore, it is not obvious that the gain from power adaptation can be completely achieved by making linear combinations through scheduling. The intuitive explanation lies in the fact that, since the SINR is non-linear in interference, when increasing the transmitted power of a node, this has a more important effect on increasing the signal than on increasing interference (which yields a strictly positive second derivative, as shown in the proof). We conjecture that there are non-linear rate functions that yield the same conclusion.

We next consider a scenario where there are no constraints on average consumed power (or equivalently $\overline{\mathbf{P}}^{MAX} \geq \mathbf{P}^{MAX}$), and we prove that power adaptation is strictly suboptimal. In other words, if any node at any time uses less than maximum power for a transmission, then there exists an alternative schedule with $0 - P^{MAX}$ power assignment which yield higher rate for at least one flow, and higher or equal rates for other flows. The finding is more precisely formulated in the following theorem:

Theorem 2 *Consider an arbitrary network where the rate is a linear function of the SINR, and an arbitrary schedule $\boldsymbol{\alpha}$ and a set of power allocations $\mathbf{p}^n$ for that network. If for some n, $\alpha^n > 0$ and power allocation $\mathbf{p}^n \notin \mathcal{P}^E$ then the resulting average rate allocation $\mathbf{f}$ is not Pareto efficient on $\mathcal{F}$.*

(Proof in appendix) The theorem says that a Pareto efficient allocation cannot be achieved if in any time slot a power allocation different from $0 - P^{MAX}$ is used.

Applying the finding on the framework of [5] we can conclude that there exists a different schedule that does not use power control and that improves the performance of a network.

The main use of Theorem 1 and Theorem 2 is the following corollary:

Corollary 1 *When the rate is a linear function of the SINR, $0 - P^{MAX}$ power control is actually the single optimal power control strategy, and any other power adaptation is strictly suboptimal.*

4.2 Power Minimization

We next analyze the effect of power adaptation on minimizing dissipated power of a network. By decreasing transmitting power, one decreases the dissipated power and also the destructive effect of interference on others, hence, intuitively, power control should minimize power consumption. However, as we show here, in the case of linear rate function, power control does not bring any benefit.

Theorem 3 *If the rate is a linear function of the SINR, then for arbitrary values of parameters of the constraint set (1), we have that $\overline{\mathcal{P}}^E = \overline{\mathcal{P}}$.*

(Proof in appendix) The theorem says that any average power consumption that is feasible under some average rate constraints is achievable with $0 - P^{MAX}$ power control. Intuitively, by using maximum power for every transmission we can increase the transmission rate, and use the channel for a shorter time. It can be shown that this compensates the effects of power control.

All feasible average power dissipations, hence all Pareto efficient ones, can be achieved with $0 - P^{MAX}$ power control, hence it is at least as good as any other power control. Again here, power adaptation is not needed. We note here that for power minimization there is no statement analog to Theorem 2. Theorem 2 assumes that there are no average power constraints. In the framework of power minimization, this corresponds to a setting with no average rate constraints, which leads to the trivial solution of having the network silent all the time.

4.3 Numerical Example

In order to illustrate the above findings we give a simple example. Consider a network of two links presented on the left of Figure 1. This network is known as the near-far scenario as an interferer is closer to a receiver than the corresponding transmitter. Node S_1 transmits to D_1 and node S_2 transmits to D_2. We introduce two simple MAC protocols. The first MAC protocol assumes $0 - P^{MAX}$ power control and arbitrary scheduling. The second assumes no scheduling (constant power allocations through time, like in some cellular systems), and arbitrary power control strategy. The corresponding sets of feasible rates and powers are given on the right of Figure 1.

We see that when maximizing rates, only $0 - P^{MAX}$ power control can achieve all feasible rates, including the Pareto efficient ones. On the contrary, the second MAC protocol that does not use scheduling but uses power adaptation achieves only a fraction of feasible rates. Furthermore, only in cases when power allocation is $0 - P^{MAX}$, the achieved rates are Pareto efficient.

However, when there is an average power limit, there might exist a schedule and a power control strategy, different from $0 - P^{MAX}$, that can achieve Pareto efficient allocations, as discussed in Section 4.2. To see this, consider an even simpler example of a single link. Let P^{MAX} be the maximum transmitting power, $\overline{P}^{MAX} < P^{MAX}$ the maximum average consumed power, h be the fading from the source and η be the power of background white noise. There exist only one Pareto efficient rate allocation which is $R = \overline{P}^{MAX} h/\eta$. It can be achieved by sending $\alpha = \overline{P}^{MAX}/P^{MAX}$ fraction of the time using full power, or by sending all the time using $\overline{P}^{MAX}$ as the transmitting power. The second strategy thus does not have the form of $0 - P^{MAX}$ power control, yet it achieves the Pareto efficient allocation. An analogous construction can be made to show that a non-$0 - P^{MAX}$ power control can achieve Pareto efficient average power allocation.

On the right of Figure 1 we depict the feasible average dissipated powers allocations for an arbitrary power control, and for $0 - P^{MAX}$ power control. We see that the two sets coincide.

5 Conclusion

We have considered multi-hop wireless networks in the linear regime and have shown that $0 - P^{MAX}$ power control, is always optimal, both for power minimization and rate maximization. We have also shown that for rate maximization, and when there are no average power limitations, any other power control strategy yields non-Pareto optimal rate allocations, hence power adaptation is strictly suboptimal.

Since power control is a difficult task in a distributed, ad-hoc system, and is not needed beyond $0 - P^{MAX}$ is not needed, our findings suggest that the complexity of a protocol should be invested in optimizing scheduling and rate adaptation, and not the power adaptation. If the number of possible physical link rates is small, one should use power adaptation and scheduling (as for example in [13]), but if the number of possible link rates is large, which is usually the case with adaptive modulation and/or coding, one should adapt rates, use $0 - P^{MAX}$ power control and scheduling.

Another conclusion that stems from our work is that, unlike common belief that in CDMA or similar data networks with almost-orthogonal links' transmissions, and for rate maximization, it is better to solve near-far problems by scheduling and rate adaptation and to use $0 - P^{MAX}$ power control, instead of using power adaptation that tend to equalize received powers.

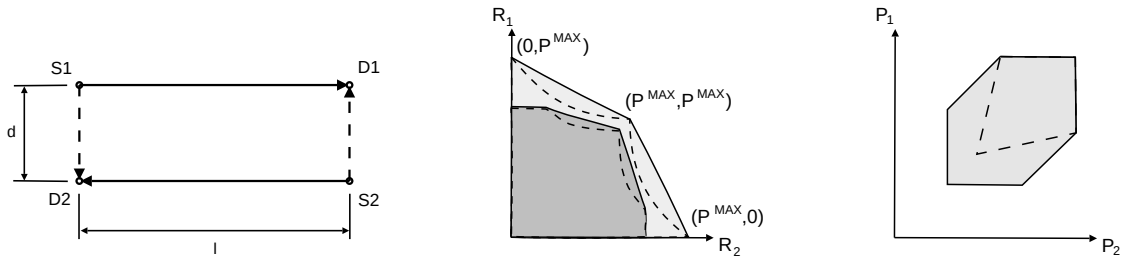

Figure 1. A simple example of a network with 2 links. The topology of the network is given on the left. Node S_1 sends to node D_1 while node S_2 sends to D_2. The feasible rate set for this network is given in the middle. The lighter region in dashed lines represents the set of feasible rates that can be achieved without scheduling, only with power adaptation. The lighter region in full lines represents an increase that is achieved by scheduling and without power adaptation ($0 - P^{MAX}$ power control). The darker region in dashed lines is the same example without scheduling and with power control, but this time with additional average power constraints. Again the darker region in full lines represents an improvement introduced by scheduling. We see that the second protocol cannot achieve Pareto efficient rates of the feasible rate set, except for the three rate allocations. But these three rate allocations are achieved with power allocations $(0, P^{MAX})$, $(P^{MAX}, 0)$ and (P^{MAX}, P^{MAX}) which belong to $0 - P^{MAX}$ power strategy. In the figure on the left, the feasible set of average consumed power under minimum rate constraints is depicted in gray. The region in full lines represents average power consumption achievable with scheduling and without power adaptation, and the region in dashed lines represents average power consumptions achievable without scheduling and with power adaptation. All average powers belonging to this set can be achieved without power adaptation.

It remains as a future work to further investigate the trade-off between scheduling and power adaptation by incorporating costs of different power control and scheduling protocols.

6 Appendix

6.1 Proof of Theorem 1

The outline of the proofs is as follows: we first take an arbitrary linear objective function of the form $U(\bar{\mathbf{x}}, \bar{\mathbf{p}}) = \sum_i \mu_i \bar{x}_i - \sum_i \lambda_i \bar{p}_i$, we maximize it on $\mathcal{T}$, and we show that the maximization point has to be generated with powers from $\mathcal{P}^E$. This is showed in Lemma 1. Next, we use the property of convex sets saying that for every Paretto efficien point there exists a plane touching the set in that point. Since this Pareto efficient point is also the maximizer of a linear objective function that corresponds to the plane, it follows that it has to be generated with $0 - P^{MAX}$.

We start by proving some properties of the optimal point with respect to a linear objective function:

Lemma 1 *Consider a function $U(\bar{\mathbf{x}}, \bar{\mathbf{p}}) = \sum_i \mu_i \bar{x}_i - \sum_i \lambda_i \bar{p}_i$ for some arbitrary vectors $\boldsymbol{\mu}, \boldsymbol{\lambda}$. Then:*

1. *There is a unique maximum $U^* = U(\bar{\mathbf{x}}^*, \bar{\mathbf{p}}^*)$ on set $\mathcal{T}$,*
2. *The maximum $(\mathbf{f}^*, \bar{\mathbf{x}}^*, \bar{\mathbf{p}}^*) \in \mathcal{T}^E$,*
3. *If some i, $|\mu_i| > 0$ and for all j, $\lambda_j = 0$, then for arbitrary $\boldsymbol{\alpha}$ and $\{\mathbf{p}^n(s)\}_{1,\cdots,N}$, such that for some n, $\boldsymbol{\alpha}_n > 0$ and $0 < p_i^n < P_i^{MAX}$, and the resulting $(\bar{\mathbf{x}}, \bar{\mathbf{p}})$ we have $U(\bar{\mathbf{x}}, \bar{\mathbf{p}}) < U(\bar{\mathbf{x}}^*, \bar{\mathbf{p}}^*)$.*

Proof of 1): Both function $U(\bar{\mathbf{x}}, \bar{\mathbf{p}})$ and set $\mathcal{T}$ are convex, hence the maximum is attained in some $(\mathbf{f}^*, \bar{\mathbf{x}}^*, \bar{\mathbf{p}}^*) \in \mathcal{T}$. We also know there exist $\boldsymbol{\alpha}^*, \{\mathbf{p}^{n*}(s)\}_{1,\cdots,N}$ that satisfy (1).

Proof of 2): Let us first assume there is a single system state $\mathcal{S} = \{s\}$ hence there is no randomness in the system. We use an approach similar to [4, 5]. Without loss of generality, we fix all $\boldsymbol{\alpha}(s), \{\mathbf{p}^n(s)\}_{1,\cdots,N}$ to arbitrary values, except $p_1^1(s)$, and we consider a function $p_1^1(s) \xrightarrow{V} \sum_i \mu_i \bar{x}_i - \sum_i \lambda_i \bar{p}_i$ as a function of a single free variable $p_1^1(s)$.

We then have the following derivatives:

$$\frac{\partial V}{\partial p_1^1(s)} = \frac{\mu_1 \alpha_1(s)\, h_{11}(s)}{\eta_1(s) + \beta \sum_{k \neq 1} p_k^n(s) h_{k1}(s)} - \lambda_1 \alpha_1 \quad (2)$$

$$- \sum_{i=2\cdots I} \frac{\mu_i \alpha_i(s)\, p_i^1(s) h_{ii}(s) h_{1i}(s)}{\left(\eta_i(s) + \beta \sum_{k \neq i} p_k^1(s) h_{ki}(s)\right)^2} \quad (3)$$

$$\frac{\partial^2 V}{(\partial p_1^1(s))^2} = \sum_{i=2\cdots I} \frac{\mu_i \alpha_i(s)\, 2 p_i^1(s) h_{ii}(s) h_{1i}^2(s)}{\left(\eta_i(s) + \beta \sum_{k \neq i} p_k^1(s) h_{ki}(s)\right)^3} \quad (4)$$

We first suppose that for all i, $\mu_i > 0$. It is easy to see from (4) that regardless of the values of other variables, the second derivative is always positive, $V(p_1^1(s))$ is always convex, hence the maximum is attained for $p_1^1(s) \in \{0, P^{MAX}\}$.

Next we suppose, without loss of generality, that for some m we have $\mu_1 \leq 0, \cdots, \mu_m \leq 0$. Then clearly the optimal is to have $\bar{x}_1 = 0, \cdots, \bar{x}_m = 0$. Then by setting $\mu_1 = 0, \cdots, \mu_m = 0$, the new optimization problem has the same maximum as the old one, and we again have that the optimal values belong to $\{\mathbf{p}^{n*}(s)\}_{1,\cdots,N} \in \mathcal{P}^E$, and $\bar{\mathbf{x}} \in \bar{\mathcal{X}}^E$.

At this point we proved the second claim under assumption that there is no randomness in the system. We next relax this assumption. From the above we know that for every state $s \in \mathcal{S}$ there is a power allocation from $\mathcal{P}^E$ that maximizes the utility. Since averaging over S is a linear op-

eration, the average over S is also going to be maximized, which concludes the proof of the second claim.

Proof of 3): In the previous point we proved that maximum of function $V(p_1^1)$ is reached at one end of the interval $[0, P^{MAX}]$. Here we want to prove that it is reached only at one end of the interval and that any point in between yields smaller V. Suppose the maximum is reached at $V(0)$. Intuitively, due to convexity of V we have that, if there is another p such that $V(0) = V(p)$, then we have $V(0) = V(p') = V(p)$ for every $0 \leq p' \leq p$. Furthermore, this means that $V'(p') = 0$ for every $0 \leq p' \leq p$. We want to show that this is not possible.

More formally, consider the case when $|\mu_1| > 0$ and $\lambda_j = 0$ for all j. We again suppose no randomness ($\mathcal{S} = \{s\}$), and we suppose that $\alpha_1(s) > 0$ and $0 < p_1^1(s) < P_1^{MAX}$. It is easy to verify from (3) that equation $\frac{\partial V}{\partial p_1^1(s)} = 0$ can be transformed into $Q(p_1^1(s)) = 0$ where Q is some polynomial of degree n_Q. Furthermore, one can verify that the coefficient of the polynomial of degree n_Q is strictly positive, hence Q is not identical to 0. Therefore there is only a finite number of values of $p_1^1(s)$ that solves $Q(p_1^1(s)) = 0$, and thus also $\frac{\partial V}{\partial p_1^1(s)} = 0$.

We know from above that the maximum V^* is achieved at one of the extremal points, say P_1^{MAX} without loss of generality. By assumptions, we have $V(p_1^1(s)) = V^*$. Now for some γ we have that $p_1^1(s) = \gamma P_1^{MAX}$ and $V^* = V(p_1^1(s)) \leq (1-\gamma)V(0) + \gamma V(P_j^{MAX}) \leq V^*$. We thus have $V^* = V(p)$ for all $p \in [0, P_j^{MAX}]$. Now this is impossible since $V'(p)$ has only a finite number of zeros, hence $\{\mathbf{p}^n(s)\}_{1,\cdots,N}$ cannot maximize V.

Now we introduce randomness. Again, due to linearity of averaging it is easy to see that if for any state s with positive probability ($P[S = s] > 0$) we have $\{\mathbf{p}^n(s)\}_{1,\cdots,N} \notin \mathcal{P}^E$, then the utility in that slot is going to be strictly smaller than the maximum achievable, hence the overall utility will be strictly smaller than the maximum, which proves the last claim. *q.e.d.*

Proof of Theorem 1: We will first show that $\bar{\mathcal{X}} = \bar{\mathcal{X}}^E$, and then that $\mathcal{F} = \mathcal{F}^E$. We clearly have $\bar{\mathcal{X}}^E \subseteq \bar{\mathcal{X}}$, and it remains to be shown that $\bar{\mathcal{X}} \subseteq \bar{\mathcal{X}}^E$. First, consider the optimization problem $\max \sum_i \mu_i \bar{x}_i$ such that $\bar{\mathbf{p}} \leq \overline{\mathbf{P}}^{MAX}, (\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{T}$. This is a convex optimization since set $\mathcal{T}$ is convex, hence if the constraint set is not empty there is a unique maximum $(\mathbf{f}^*, \bar{\mathbf{x}}^*, \boldsymbol{\mu}^*)$. The dual problem is $\min_{\boldsymbol{\lambda} \geq 0} g(\boldsymbol{\lambda}) + \sum_i \lambda_i \overline{P}_i^{MAX}$ where $g(\boldsymbol{\lambda}) = \max_{(\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{T}} \sum_i \mu_i \bar{x}_i - \sum_i \lambda_i \bar{p}_i$. According to lemma 1, point $(\mathbf{f}^*, \bar{\mathbf{x}}^*, \bar{\mathbf{p}}^*)$ that maximizes $g(\boldsymbol{\lambda}^*)$, thus also maximizes the above maximization problem, belongs to $\mathcal{T}(\mathcal{P}^E)$.

We now prove the theorem by contradiction. Suppose there exists a point $\bar{\mathbf{x}} \in \bar{\mathcal{X}}$ that is not in $\bar{\mathcal{X}}^E$. Then by the separating hyperplane theorem [22] there exists a hyperplane defined by $(\mathbf{c}, b)$ that separates $\bar{\mathbf{x}}$ and $\bar{\mathcal{X}}^E$, that is $\mathbf{c}^T\mathbf{f} > b$ and for all $\mathbf{g} \in \bar{\mathcal{X}}^E$, $\mathbf{c}^T\mathbf{g} < b$. This on the other hand means that $\bar{\mathbf{x}} \notin \bar{\mathcal{X}}^E$ maximizes the above maximization problem, which leads to contradiction, and we prooved $\bar{\mathcal{X}} = \bar{\mathcal{X}}^E$.

Finally, set $\mathcal{F}$ is completely determined by set $\bar{\mathcal{X}}$ and the routing matrice R. Since we have $\bar{\mathcal{X}} = \bar{\mathcal{X}}^E$, it follows that $\mathcal{F} = \mathcal{F}^E$. *q.e.d.*

6.2 Proof of Theorem 2

Next we proove Theorem 2. The proof consists of two parts. In the first part we get rid of routing by showing that if $\mathbf{f}$ is Pareto efficient on $\mathcal{F}$ then $\bar{\mathbf{x}}$ is Pareto efficient on $\bar{\mathcal{X}}$ regardless of the network topology or routing strategy. In the second part we use the last statement from Lemma 1 to show that if in any slot the power allocation does not belong to $\mathcal{P}^E$, then the resulting average link rate allocation does not belong to $\bar{\mathcal{X}}^E$.

Lemma 2 *Let $(\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{T}$. If $\mathbf{f}$ is Pareto efficient on $\mathcal{F}$ then the corresponding average link rate $\bar{\mathbf{x}}$ has to be Pareto efficient on $\bar{\mathcal{X}}$.*

*Proof:*We proceed by contradiction. Suppose that $\mathbf{f}$ is Pareto efficient on $\mathcal{F}$ but $\bar{\mathbf{x}}$ is not Pareto efficient on $\bar{\mathcal{X}}$, and we can increase $\bar{x}_i$ for some i without decreasing other rates. In other words, we have $(\mathbf{f}, \bar{\mathbf{x}}', \bar{\mathbf{p}}) \in \mathcal{T}$ such that $\bar{x}_i' = \bar{x}_i + \epsilon$ and $\bar{x}_k' = \bar{x}_k$ for all $k \neq i$. If there is flow f_j such that link i is its bottleneck, than we can increase the rate of f_j since $\bar{x}_i' > \bar{x}_i$, hence $\mathbf{f}$ is not Pareto efficient. Hence we conclude no flow has a bottleneck on link i.

We next choose an arbitrary flow k, we start from link rate allocation $\bar{\mathbf{x}}'$ and we show how to construct a schedule that will increase the rate of flow k. Let us denote with j the bottleneck link of flow k (if k has no bottleneck $\mathbf{f}$ is obviously not Pareto efficient). We first try to find a slot in which both j and i are active. If this slot exists (say it is slot n) than we can decrease the power $p_i^n(s)$ (in some state s with positive probability) by some $\epsilon_i > 0$ such that the resulting $\bar{x}_i''$ has the property $\bar{x}_i' > \bar{x}_i'' > \bar{x}_i$. Since link j is also active in slot n and we have decrease the interference (due to the assumption, $h_{ij}(s) > 0$), we have also increased $\bar{x}_j'' > \bar{x}_j$, and we can in turn increase f_k which violates the Pareto efficient property of $\mathbf{f}$.

Finally we have the case when links i and j are not active in the same slot. We pick slot n_i and n_j, in which i and j are active, respectively, such that $\alpha_i^n(s) > 0$ and $\alpha_j^n(s) > 0$ (for some states s with positive probability). We then decrease the power $p_i^{n_i}$ by some $\epsilon_i > 0$ such that the resulting $\bar{x}_i''$ has the property $\bar{x}_i' > \bar{x}_i'' > \bar{x}_i$. At the same time, we increased the average rates of all links scheduled during the slot $\alpha_i^n(s)$. Now we can decrease the duration of the slot $\alpha_i^n(s)$ such that in the new allocation all those links will have at least the same rates as in the initial configuration $\bar{x}_l'' \geq \bar{x}_l$. However, since we decreased $\alpha_i^n(s)$, we can now increase $\alpha_j^n(s)$, hence also increase $\bar{x}_j$. Now

flow k looses its bottleneck, hence we can increase f_k again violating Pareto efficient assumption. *q.e.d.*

Proof of Theorem 2: We proceed by contradiction, and assume there exist a schedule $\boldsymbol{\alpha}$ and a set of power allocations $\{\mathbf{p}^n(s)\}_{1,\cdots,N}$ such that the resulting average rate allocation $\mathbf{f}$ is Pareto efficient (and thus on the boundary of set $\mathcal{F}$), and for some n, i, $0 < \mathbf{p}_i^n < P_i^{MAX}$. From Lemma 2 we have that since $\mathbf{f}$ is Pareto efficient on $\mathcal{F}$ then is so $\bar{\mathbf{x}}$ on $\bar{\mathcal{X}}$, and we focus further on contradicting that $\bar{\mathbf{x}}$ is Pareto efficient.

Since $\bar{\mathcal{X}}$ is convex and $\bar{\mathbf{x}}$ is on the boundary, there exists a supporting hyperplane [22] $(\boldsymbol{\mu}, b)$ which contains $\bar{\mathbf{x}}$ (that is $\boldsymbol{\mu}^T\bar{\mathbf{x}} = b$) and contains $\bar{\mathcal{X}}$ in one of the half-spaces (that is for all $\bar{\mathbf{x}}' \in \bar{\mathcal{X}}, \boldsymbol{\mu}^T\bar{\mathbf{x}}' \leq b$).

Let us first suppose $|\mu_i| > 0$. Then, according to lemma 1 there exists $(\bar{\mathbf{x}}^*, \bar{\mathbf{p}}^*) \in \bar{\mathcal{X}}^E$ such that $\boldsymbol{\mu}^T\bar{\mathbf{x}}^* > \boldsymbol{\mu}^T\mathbf{f} = b$, which leads to contradiction. Therefore, we have that $\mu_i = 0$, and $\sum_{j\neq i}\mu_j\bar{x}_j' \leq b$ for all $\bar{\mathbf{x}}' \in \bar{\mathcal{X}}$. However, it is then easy to construct find a counter example, starting from $\bar{\mathbf{x}}$. If there exist another $j \neq i$ such that $p_j^n > 0$, then by setting $p_i^n = 0$ we increase $\bar{x}_j$, thus now $\sum_{j\neq i}\mu_j\bar{x}_j' > b$, hence the contradiction. On the contrary, if for all $j \neq i$, $p_j^n = 0$, we then set $\alpha_n = 0$ and increase some other α^m such that for some j, $p_j^m > 0$. Again, this way we increase $\bar{x}_j$, thus $\sum_{j\neq i}\mu_j\bar{x}_j > b$, that also leads us to a contradiction. *q.e.d.*

6.3 Proof of Theorem 3

Consider the optimization problem $\min\sum_i \mu_i\bar{\mathbf{p}}_i$ such that $\mathbf{f} \geq \overline{\mathbf{F}}^{MIN}, (\mathbf{f}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{X}$. Since we have a constraint $R\mathbf{f} \leq \bar{\mathbf{x}}$, we can also express the minimum constraint as $\bar{\mathbf{x}} \geq \overline{\mathbf{X}}^{MIN} = R\overline{\mathbf{F}}^{MIN}$.

This is a convex optimization since set $\mathcal{X}$ is convex, hence if the constraint set is not empty there is a unique minimum $(\mathbf{f}^*, \bar{\mathbf{x}}^*, \boldsymbol{\mu}^*)$. The dual problem is $\max_{\boldsymbol{\lambda}\geq 0} g(\boldsymbol{\lambda}) + \sum_i \lambda_i\overline{\mathbf{X}}_i^{MIN}$ where $g(\lambda) = \min_{(\mathbf{f},\bar{\mathbf{x}},\bar{\mathbf{p}})\in\mathcal{X}}\sum_i \mu_i\bar{p}_i - \sum_i \lambda_i\bar{x}_i$. According to lemma 1, point $(\mathbf{f}^*, \bar{\mathbf{x}}^*, \boldsymbol{\mu}^*)$ that maximizes $-g(\boldsymbol{\lambda}^*)$, thus also minimizes the above minimization problem, belongs to $\mathcal{X}(\mathcal{P}^E)$. The rest of the proof is the same as in Theorem 1. *q.e.d.*

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