DIMENSIONAL ANALYSIS AND SIMILITUDE
IN MICROSYSTEM DESIGN AND ASSEMBLY

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Abstract

The effects miniaturization has on microsystems and related fields are the central subject of this work. Physical phenomena we are used to in our conventionally sized world seldom work at smaller scales. Before attempting to design a microsystem or try to perform any operation on or with it, the outcome of size reduction has to be considered. When well-known phenomena are involved it remains relatively simple to predict the outcome as a function of size. Unfortunately the behavior of microsystems is, in many cases, governed by phenomena that have been systematically and correctly neglected. In consequence our knowledge about them is limited to the point that we can't establish a relation between them and the size of the system.

Dimensional analysis and theory of similitude are two engineering tools that have been long used to solve problems where analytical solutions were not available. While the first one gives us the possibility of obtaining prediction equations for our phenomena based on our initial hypothesis, the second allows us to test the validity of our findings through the use of models of convenient size. Combined they allow information and data obtained from the study of different cases where similar phenomena are involved to be extrapolated to the case under consideration. Ship engineering has benefited from those tools: the flow around a ship's hull is difficult to describe analytically and full-sized models are uneconomical to build for testing purposes alone.

Since microsystems find themselves in a similar situation (some effects can't be described analytically and full-size models are difficult to work with) the use of dimensional analysis and similitude to solve different problems encountered in this domain is proposed.

The subject of forces at the microsystem level is treated in detail to show how dimensional analysis can be used both qualitatively and quantitatively to help understand how forces behave at this scale.
Version Abrégée

Le sujet central de ce travail porte sur les effets que la miniaturisation entraîne sur les microsystèmes et les techniques conjointes. Les phénomènes physiques que nous avons l'habitude d'observer dans notre environnement habituel ne se rencontrent que rarement dans un monde à échelle réduite. Avant de tenter de concevoir un microsystème ou de faire une quelconque opération sur ou avec lui, il faut prendre en compte les conséquences de la réduction de la taille. Si le phénomène impliqué est bien connu, il est relativement aisé de prédire l'effet du changement de taille sur son évolution. Malheureusement, le comportement des microsystèmes est dans de nombreux cas régis par des phénomènes qui ont été systématiquement négligés. En conséquence, nos connaissances à leur sujet sont si limitées que nous ne pouvons établir de relations entre elles et la taille du système.

L'analyse dimensionnelle et la théorie des similitudes sont deux outils de l'ingénierie qui ont été longuement utilisées pour résoudre des problèmes quand les solutions analytiques n'étaient pas disponibles. Alors que la première nous donne la possibilité d'obtenir des équations prédictives pour notre phénomène, la seconde nous permet de contrôler la validité de nos trouvailles, par l'utilisation de modèles construits à une échelle idoine. Combinées sur la base d'informations et de valeurs numériques obtenues par l'étude de plusieurs cas de phénomènes similaires, elles permettent d'extrapoler les résultats à obtenir pour d'autres cas. L'ingénierie navale a bénéficié de tels outils: les flux autour d'une coque de bateau sont difficiles à décrire analytiquement et la construction de modèles à pleine échelle uniquement pour ces essais est économiquement injustifiable.

Puisque les microsystèmes se trouvent dans une situation similaire, (certains effets ne peuvent être décrits analytiquement et il est difficile de travailler avec les modèles à pleine échelle), l'utilisation de l'analyse dimensionnelle et les similitudes résolvent différents problèmes rencontrés dans ce domaine.

Le sujet des forces dans le domaine des microsystèmes est traité en détail pour montrer comment l'analyse dimensionnelle peut être utilisée à la fois pour obtenir des valeurs qualitatives et quantitatives afin d'aider à comprendre comment les forces se comportent à cette échelle.
...how facts which at first seem improbable will, even on scant explanation, drop the cloak which has hidden them and stand forth in naked and simple beauty. Who does not know that a horse falling from a height of three or four braccia will break his bones, while a dog falling from the same height or a cat from eight or ten, or even more, will suffer no injury? Equally harmless would be the fall of a grasshopper from a tower or the fall of an ant from the distance of the Moon. Do not children fall with impunity from heights which would cost their elders a broken leg or perhaps a fractured skull? And just as smaller animals are proportionately stronger and more robust than the larger, so also smaller plants are able to stand up better than larger. I am certain you both know that an oak two hundred feet high would not be able to sustain its own branches if they were distributed as in a tree of ordinary size; and that

Nature cannot produce a horse as large as twenty ordinary horses, or a giant ten times taller than an ordinary man, unless by miracle or by greatly altering the proportions of his limbs and especially of his bones, which would have to be considerably enlarged over the ordinary.

Likewise the current belief that, in the case of artificial machines the very large and the small are equally feasible and lasting is a manifest error.

I have sketched a bone whose natural length has been increased three times and whose thickness has been multiplied until, for a correspondingly large animal, it would perform the same function which the small bone performs for its small animal. From the figures here shown you can see how out of proportion the enlarged bone appears. Clearly then if one wishes to maintain in a great giant the same proportion of limbs as that found in an ordinary man he must either find a harder and stronger material for making the bones, or he must admit a diminution of strength in comparison with men of medium stature; for if his height be increased inordinately he will fall and be crushed under his own weight. Whereas, if the size of a body be diminished, the strength of that body is not diminished in the same proportion; indeed the smaller the body the greater its relative strength. Thus a small dog could probably carry on his back two or three dogs of his own size; but I believe that a horse could not carry even one horse of his own size.

Galileo's comparative bone drawing. [614]

[Galilei, Galileo. Dialogue Concerning Two New Sciences, 1638]

*When Galileo wrote "Dialogue Concerning Two New Sciences" physics was not as developed as it is today. For example, Newton's laws of motion were published 49 years later in 1687 long after Galileo's death. This seems a valid excuse for his lack of precision when using terms such as "strength" or "the function which it performs" in reference to the mechanical properties of the dog's bone.
I would like to thank my advisor, Professor Jacques Jacot, for having accepted me into the IPM and giving me the opportunity to work on this doctoral thesis. I have very much appreciated the liberty that I was given in doing my work. Moreover I wish to thank the members of my doctoral committee, Professor Olivier Besson of the Universite de Neuchatel, Professor Biran of Technion (Israel Institute of Technology) from whom I heard for the first time about dimensional analysis, Doctor Yves-Olivier Perriard of EPFL and Professor Philippe Renaud the president of the committee who, to my pleasure, showed a great interest in my work. Their comments, opinions and suggestions have been very helpful throughout the whole process of producing this work.

Furthermore I would like to thank my officemates Mrs. Sandra Koelemeijer (Dr. Koelemeijer in a couple of months), Mr. Jean-Marc Uehlinger, Mr. Olivier Demont and lately Dr. Yuri Lopez de Meneses for their invaluable help and discussions from which many ideas for this thesis were born.

My gratitude goes to all the staff of the IPM that has always been willing to help at one stage or another.

Finally I wish to thank my parents, brothers, grandfather and, of course, Britta, for their continuous encouragement throughout this work.
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Chapter 1: Introduction

1.1 Overview

Microsystems, micro-electro-mechanical systems (MEMS), mechatronics, etc. are terms that were born when the need for more efficient and inexpensive systems started a few years ago. The call for this new generation of systems is explained on one hand by the increasing demand for domestic appliances, vehicles and other micro-system equipped goods, and additionally, by the fact that also their sophistication is increasing making it necessary to have more micro-systems in each item produced. This does not only affect the number of micro-systems produced, it also affects their technical characteristics. They must be less expensive and more reliable. Less expensive because it is becoming more and more difficult to justify an increase in price by adding a few extra features to the product, and more reliable because if the overall reliability of the product must be kept the same or improved while adding extra sub-systems to it, then the individual reliability of those sub-systems must be increased.

Microsystems are small as a consequence of having to improve their performance and reduce their price. Microsystems with direct size constraints are limited basically to those employed in medicine and those integrated into portable devices. In such cases cost and performance are frequently sacrificed in order to achieve miniaturization.

A fundamental process a system must undergo when being miniaturized is that of integration; that is, a minimal number of parts carry out the functions the system must perform. Integration has reached levels where mechanical parts, electromechanical devices and electronic circuits are merged into a single element. The progress made in the microelectronics industry in the past decades has made the combination of mechanical and electronic functions on the same silicon chip possible. However, the technology used to produce integrated circuits (ICs) does not adapt entirely to the requirements of mechanical parts. Consequently new technologies that derive from the IC technology are being developed. This has led to a situation where more classical production techniques have been left behind.

Micro-engineering is a multidisciplinary field in engineering: the nature of microsystems implies a combination of electronics, electromagnetism, mechanics, fluid mechanics, heat transfer, etc. Not only must the microsystem designer be aware of the relation the product bears with the different domains but he or she must also take into consideration the fact that when miniaturizing the different physical phenomena that take place within the system do not scale down at the same rate.
In recent years miniaturization has been pushed to levels never explored before. At this scale experience and engineering intuition developed after years of practice begin to fail if precautions are not taken. In some cases an analytical analysis of the behavior of a system will suffice to get a picture of what will happen at a smaller scale. The performance of a system can be altered, by a simple reduction of size, to a point where the only alternative left to obtain a functional product is to change its working principle.

Similar scaling problems are encountered in other fields of engineering such as naval architecture, civil engineering and aeronautics. The behavior of ships, buildings, bridges or planes can't be deduced from empirical formulas and at the same time a bridge or ship can't be built just to test if it will work properly or not. Scaled-down models are used to reproduce the behavior of the system under different circumstances. The relations that allow extrapolating the behavior of the model to that of the real size system are known in those domains as “similitude laws”. The same “similitude laws” can be applied to understand how the performance of a system will evolve with size, even if empirical formulas aren't available.

From all the aspects of microsystem production affected by size, assembly is among those most concerned. Not only from the technical point of view is microassembly different from conventional assembly, but also at the economic level the repercussion of assembly costs on the overall cost of the product bears no resemblance with those of conventional-size products.

Finally, some of the problems encountered in the microsystem domain are fairly new to engineers. The reason is that because of scale effects, the inconveniences introduced by problems of the same nature in conventional size systems is so small that they are systematically neglected. Therefore research in these fields is very recent. The use of dimensional analysis can prove very helpful in situations like this one because combining the limited knowledge available with the information supplied by this analysis can lead, with the help of some experimental work, to an understanding of the phenomena under consideration.

### 1.2 Main Contribution of this work

The first direct contact we had with microsystems before starting this work consisted in producing a 1 mm long pin with a diameter of 97 μm from a small receptacle containing several hundreds of them. The first obstacle we encountered was the difficulty we had to keep our eyes focused on such small parts and if our intention were to be working with them for more than just a few minutes it would be better to have some kind of visual aid. Next we realized that we would need a tool other than our fingers if we were to get only one of the pins. With the appropriate equipment and some practice we managed to grasp, with the tweezers, one of the pins, but to our surprise, while pulling it out of the container an additional four or five pins were clinging on to our pin and tweezers.
This experience gave us a lot of things to think about. The idea of discussing the relation between the different forces and the size of an object was born at that point. The strategies used to grasp and release parts automatically would certainly be different to what we are used to in conventional robotics. Vision systems with higher resolution, or smaller vision field would be necessary. From there we went on to think about the shape and dimensions of parts. When they are so small planes are no longer flat since they are affected by surface roughness. When planes are no longer planes, the definition of the dimensions of a part automatically becomes a problem because there are no reference points. When the tolerances of mating parts within a microsystem have to be established the problem becomes even more complex. Since measurement at this scale is critical we underline the importance of defining the relevant dimensions that will influence the performance of the part or the system. The concept of functional dimensions in microsystems is of utmost importance. Putting all those factors together we realize that microassembly must be substantially different from conventional assembly and consequently it must be examined with great care.

With our ideas more or less clear we started our bibliographic research to see what has already been done in this domain. A significant amount of literature was found and our impressions of the work done up to that point can be summarized in the following points:

- A systematic use of models and laws without verifying their domain of validity is observed.
- The urge to create new products makes micro-systems engineers oversee fundamental steps required in the design process of any product. The first thing that must be done is to understand the functioning principle of the system being designed.
- Many micro-systems are simply reduced copies of more conventionally sized ones, even when simple straightforward calculations show that they will not work.
- Shielded behind the comfort of modern technologies, simplifications that can yield accurate results are discarded in favor of sophisticated measurement and simulation systems where, as mentioned further up in this list, the validity of models is not even checked.

The second and third chapters in this report try to make micro-engineers aware of the effect size has on the micro-system. The structure, material properties, performance, production, assembly techniques, etc. are related to the system's dimensions. This part is not meant to solve all the problems that come along with size reduction. There is no general formula to do that. The purpose is to try to make designers aware of where the origin of their troubles may be and give the outlines on how to tackle them. As we will see, very simple cases are treated here mainly because of our lack of knowledge in some of the fields mentioned; but it may hint experts in the different domains on how to proceed.
The use of dimensional analysis and similitude, which are described in chapter 4 can be seen as an aid to designers. The first can help find relations between parameters that are known to influence a given process and at the same time it helps reduce laboratory work considerably. Similitude, on its hand, can help the engineer by allowing to reproduce the microscopic phenomena present in micro-systems at a larger, more tangible scale. Not only will the phenomena be more understandable to the engineer, he will also be able to perform more accurate measurements. Imagine that the flow in a channel measuring 50x50 \( \mu \)m can be up-scaled to one measuring a few centimeters while being sure of how its behavior would map into the smaller one. Valuable data such as velocity profiles and pressure gradients would be determined in a much easier way in the larger channel and a simple mathematical calculation would give us information on what happens in the microscopic channel.

The subject of forces is brought up in the fifth chapter. Forces are used as an example to show how the ideas mentioned in the previous chapters can be put to use. The problem of surface forces is addressed. The classic friction force model loses its validity in the micro-system domain; experimental procedures to determine friction and surface forces are proposed, but once again, not having enough knowledge in the field of interfacial engineering our approach to the problem may seem incomplete. Finally micro-force sensors are analyzed. The most appropriate working principle is discussed as well as the influence micro-part production technologies may have on its performance.

Before conclusions are drawn in the final chapter further examples are illustrated in chapter 6. Special attention is given to the bonding effect number (Be) since it appears in the dimensional analysis of all the problems where surface forces can't be neglected.
Chapter 2: What is a Microsystem?

2.1 Introduction

Being a fairly recent field of engineering it is necessary to define the objectives microsystems aim at before attempting to overcome the many problems encountered. Once defined, the different design strategies that can be used to realize them can be applied. In occasions traditional design approaches are not applicable and alternative strategies must be followed.

In this chapter we show how different technical properties and characteristics of systems and their components are related to size. Mechanical properties of materials, manufacturing techniques and dimensional and geometric tolerances are just a few of the fields related to system design and manufacturing that are deeply affected by size.

Many of the problems encountered in the micro-system world are originated because those relationships are ignored or are not fully understood. The working principles of conventional-sized systems are assumed to be constant regardless of size. With such an assumption in mind many microsystems are simply scaled-down versions of conventional-sized ones. Models used systematically at the human usual scale do not necessarily hold when size is drastically reduced. Their domain of validity has to be defined and the designer has to make sure that his or her design falls within the limits of the model.

2.2 Definition of a micro-system

Micro-systems are relatively small sized integrated systems capable of performing different tasks. The high level of integration found in this family of systems responds mainly to the need of making them as small as possible. For this reason micro-systems are generally composed of few, multipurpose elements.

The term "micro-system" denotes a system of small dimensions. We should not understand the particle "micro" as $10^{-6}$, but rather as it appears in the word "microscope". In the same way as we use a microscope to look at things that can't be seen with the naked eye, micro-systems can be described as systems that can't be designed, manufactured and assembled with bare hands or even with traditional industrial technology.

The size of the objects that can be observed with a microscope range over several orders of magnitude, in the same way as the size of a micro-system can vary from a few
hundred microns to a few centimeters. Size alone is, consequently, not a good parameter to use if we want to define a micro-system.

The majority of existing micro-systems falls in the category of sensors. In such case the system acquires an input consisting of the variable we want to measure or monitor and generates an output which is the signal required by an external element to make it react to the changes detected on the monitored variable. There exist other micro-systems that can’t be classified as sensors. Such is the case of micro-pumps, micro-valves, ink-jet printer heads, etc. Nevertheless they still receive an input, process it internally and produce the desired output.

A well-known example of a mass produced micro-system is the accelerometer used to trigger the airbags present in all modern cars. The vehicle’s acceleration is the sensor’s input, which is internally converted to an electric signal, for processing purposes. If the acceleration exceeds a predefined value, an electric signal that triggers the airbag is output.

This is only an example and there exist many different micro-systems. The Figure below shows some of the most typical inputs and outputs of existing micro-systems.

---

**Figure 2-1 Possible I/O's for different Micro-Systems**

The typical structure of a micro-system is shown in Fig. 2-2. There are two main elements within the system. The first one is in charge of converting a physical signal into an electric one before being transferred to the second element for processing and generation of the appropriate output signals. The housing that protects the delicate core of the system from the environment together with the connectors are the parts that complete the micro-system.
The use of modern silicon processing techniques allows, in many cases, to produce modules that contain, in a single part, the main elements. As we will see later on, this single part structure has some advantages when it comes to assembling the system since the number of parts is minimal. However, due to manufacturing tolerances and other functional characteristics of the structure, the assembly operations can be very complex and time consuming.

Micro-systems can also be seen as miniaturized versions of larger systems. It is important to mention that the idea of miniaturizing larger systems is one of the main sources of the problems encountered in the domain if it is not done carefully. How to do this is one of the messages this work intends to pass on to micro-system designers.
Micro-systems can be found in a variety of places. Cars are equipped with a few of them: accelerometers, pressure and temperature sensors, speed sensors, transponders, remote control, etc. Domestic appliances, medical instruments, fire and gas chemical detection systems in buildings, etc. are equipped with micro-systems. [FEL95] and [FRA95] illustrate many examples of the use of micro-systems.

We have concluded that it is rather a combination of various factors that allow defining a micro-system. Among those factors we can include the following:

- Whether a system is considered a micro-system or not may depend on the time it has been on the market; a classical example would be the watch industry where both mechanical and electronic watches could be considered from the engineering point of view as micro-systems, but in reality, and due to historical reasons, form a completely detached industry.
- In addition micro-systems can’t be repaired and in case of failure must be replaced.
- We also find systems that at a point are re-classified as micro-systems because their production rate has increased. Such is the case of ink-jet printer heads.
- The techniques used to produce the parts that will later constitute the micro-system can also influence the definition of a micro-system. Parts produced using technologies derived from the micro-electronics industry will automatically make part of a micro-system, whereas more classical techniques such as milling capable of producing parts of the same size and characteristics are not easily associated to the micro-system domain.

From the point of view of this work a microsystem can be understood as a small system whose working principles are (or should be) based on phenomena different from those we are used to at a human scale.

### 2.3 The problem of size reduction

System miniaturization is desired either to reduce manufacturing costs or to improve the system's performance. In this work we concentrate mainly on the technical aspects of microsystems. For a thorough analysis of microsystem production costs the reader is referred to [KOE00]. The system’s performance may be related to different aspects of a system as the examples in the following table show:

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<tr>
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<tr>
<td>Endoscope</td>
<td>Minimize pain and tissue damage</td>
</tr>
<tr>
<td>Accelerometer</td>
<td>Filter out low frequency noise</td>
</tr>
<tr>
<td>Portable electronics: laptop, cell-phone, etc</td>
<td>Increase portability, reduce power consumption</td>
</tr>
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From the constructive point of view micro-systems tend to integrate as many functions as possible in the smallest number of parts. Electronic and mechanical functions are integrated whenever possible on the same chip. In general micro-systems consist of the parts that carry out the functions it is designed for (the micro-chip in many cases), the connections that interface it with the surroundings and the packaging that protects the system from the environment.

Micro-system miniaturization is often jeopardized by the necessary connections between the system and the environment. A straightforward example is illustrated in Figure 2-4 where the size of the user interface cannot be reduced since constraints imposed by human anatomy and physiology cannot be avoided. Another example is found in gas-pressure sensors.

Figure 2-4 While the size of computer chips has been reduced significantly in recent years, that of the interface with the user (keyboard and monitor) has remained the same.

The tube connecting the sensor to the zone where the pressure is to be measured has to have a sufficiently large diameter since pressure derives from the average velocity of the molecules of the gas molecules. Molecular behavior has to be considered if the mean free path (average distance between molecular collisions) is larger than one hundredth of the tube's diameter. Figure 2-5 shows the evolution of an automotive pressure sensor. The size of the micro-system (left column) has been considerably reduced while the casing that includes both the electrical and fluid connections has remained of the same size.

---

1 When molecular movement is restricted so severely the zero-velocity assumption at the walls of the tube is no longer valid.
2.3.1 Influence of size (ex. valves/filters)

Another interesting aspect of micro-systems that is not sufficiently well exploited at present is that because of their reduced size and with a sufficient understanding of the processes involved, the design can be completely modified to the point that it keeps no resemblance whatsoever with an equivalent system of bigger size. Not considering this often leads to complex micro-systems that are consequently more expensive and less reliable.
2.3.1.1 Example of well-exploited small-scale phenomena: Micro-filter

Laminar flow, that occurs at low values of Reynolds number, and the effect of diffusion can be combined to produce simple filters where no porous material requiring cleaning and replacing is needed [Bro96]. Figure 2-7 represents a micro-filter. It consists merely of an I-shaped channel. The fluid that has to be filtered enters at the lower left end of the channel. At the opposite side of the lower branch a dilutant that will be used as carrier for the small particles is introduced.

Figure 2-7 The concept of diffusion-based filtration.
Both fluids are forced up the channel, but since the flow is laminar they will not mix; they flow side by side until they split again when they reach the top of the channel. During the time they are flowing in parallel, diffusion takes place according to the following law:

\[ \Delta = \frac{k_B T}{6 \pi \eta R_0} \]  

where:

- \( \Delta \) = Diffusion coefficient,
- \( R_0 \) = size of particle,
- \( \eta \) = viscosity of fluid,
- \( T \) = temperature,
- \( k_B \) = Boltzmann's Constant.

The distance \( l \) a particle will diffuse depends both on the diffusion coefficient \( D \), and time \( t \). Its value is given by:

\[ l = \sqrt{\Delta t_d} \]  

where:

- \( t_d \) = diffusion time.

The smaller the particles, the greater the distance that they will travel, so the fluid that outputs to the right is enriched with smaller particles.

[BRO96] has designed some micro-filters. The central channel has a length of about 300 \( \mu m \), a width ranging between 20 and 50 \( \mu m \), and a depth of 40-50 \( \mu m \).

With a pressure difference of 2 cm of water the flow velocity at the center of the channel is approximately 100 \( \mu m/sec \). With those parameters the concentration ratio at the output ranges from 5 to 1000 depending on the diffusion factor of the input solution (eq. 2-1).

Micro-filter designers have many parameters to play with to obtain an output with the desired concentration. Not only can they vary the central channel's length and width. Temperature, flow rate of each of the input fluids and their particle concentration levels can also be used in the designers benefit. Furthermore, since size and
manufacturing costs allow for it, an array of those filters can be connected in series (to obtain specific concentration levels) and in parallel (to obtain the desired output flow rate).

![Diagram of two micro-filters connected in series](image)

Figure 2-8 Two micro-filters connected in series. The output of the first filter is fed to the input port of the second one. The final filter output rate remains the same but the concentration of small particles is higher and that of big particles smaller.

![Diagram of two micro-filters connected in parallel](image)

Figure 2-9 Two micro-filters connected in parallel. Filter output rate is doubled without affecting flow conditions in diffusion zone. The dotted line indicates that waste flow can eventually be reutilised.

2.3.1.2 Example of well-exploited small-scale phenomena: Micro-valve

The flow through one or another port of the "T" shaped channel can be directed by controlling the pressure at each end of the vertical segment of the channel shown in Fig. 2-10. Apart from being easy to produce, micro-valves are capable of very short switching times. Table 2-1 shows the switching times achieved by micro-valves of different sizes.
The tests carried out by Brody et al. [BRO96] were run with water flowing through 10 μm deep channels and a pressure difference of 1 PSI.

<table>
<thead>
<tr>
<th>Width of channel</th>
<th>Switching time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 μm</td>
<td>100 μs</td>
</tr>
<tr>
<td>100 μm</td>
<td>10 ms</td>
</tr>
<tr>
<td>1 mm</td>
<td>1 s</td>
</tr>
</tbody>
</table>

Table 2-1 Approximate fastest switching times of a micro-valve [BRO96].

The relation between inertial and surface tension forces (known as the Weber number We) is so small that surface tension alone is sufficient to stop a flow.

2.3.2 Boundary between micro- and ordinary size systems

A good way to differentiate between conventional and micro-systems is to look at the behavior of the system or its parts and search for the limiting conditions that make it behave "normally". This limit can be taken as the border between micro- and conventional systems, which in reality is a transition zone. The graph below shows the magnitude of the relevant forces in a pick and place operation for a part as a function of its size. Normally when a gripper that is grasping a part is opened it will fall because at the scales we are used to the forces due to gravity are considerably bigger than the forces induced at the contact surfaces. However as the size of the part is reduced the gravity forces decrease much faster than the surface forces till a point where the latter becomes larger than the former. At that point we can say that we are dealing with micro-systems.
Figure 2-11 The limit (actually a transition zone) between Micro and Macro-Systems is not well defined. Furthermore it depends on the aspect or characteristics of the system we are looking at.

Obviously this limiting point is never well defined. Later on we will see that it can be in a range of several orders of magnitude. For further inconvenience, the size of micro-systems falls in the transition zone where it is difficult to ascertain which phenomenon predominates over the others (Fig. 2-11).

However this approach to defining micro-systems highlights the importance of the relation between the size of the system and the phenomena that occur within them. The moment the behavior of a system starts to "defy" the "classical" laws of nature can give us a hint on when to start considering the system as a micro-system.

To illustrate this we can use the example of a micro-turbine. Micro-turbines are being produced experimentally in different laboratories [MAT94]. The working principle of a micro-turbine is the same as that of a conventional turbine: kinetic energy supplied by a fluid is transformed into mechanical energy. The fluid going through a conventional turbine is always in the turbulent regime; the same thing can't be said about a micro-turbine.
In laminar flows the viscous forces exerted by the fluid become more important with respect to inertia forces, or in other words, Reynolds number decreases. This is important because the effects laminar and turbulent flows have on the turbine blades are completely different and strongly influence the efficiency of the turbine (which is one of its most important parameters). After all this has been said, it seems adequate to use the type of flow to classify them as micro or macro-turbines. Simple as it seems the type of flow present does not only depend on the size of the turbine, it also depends on the viscosity of the fluid, the velocity at which it flows, and even on the efficiency we consider acceptable. This could make a turbine "micro" under certain operating conditions and become a conventional one if the conditions change slightly. For this reason we say that a limit can be defined but it is not very precisely located.

This thesis is meant to show designers why and where to look for size-related problems, and how to solve them both when the phenomena can be fully identified and described by a formula and when the phenomena are only understood partially.

2.3.3 Mechanical/electrical structure

From the constructive point of view micro-systems tend to integrate as many functions as possible in the smallest number of parts. Electronic and mechanical functions are integrated whenever possible on the same chip.

Micro-systems are seldom composed of moving parts. If, however, movement is necessary it is achieved by means of local elastic deformations of a conveniently designed part as can be seen in Fig. 2-14, since manufacturing tolerances do not permit to build so small bearings or linear guides. Furthermore energy losses due to friction forces in micro-bearings are greater than energy dissipation due to hysteresis phenomena in a small flexure hinge for example.
Overall bearing friction torque is composed of skidding and rolling friction between the different elements of the bearing: roller, raceways and cages. Due to its complexity, empirical formulas are used to estimate the torque friction coefficient.

Under certain conditions (low charge, good lubrication and normal working temperature) it can be approximated by the following relation [SKF89].

\[ M = M_0 + M_1 \]  
\[ 2-3 \]

where \( M_0 \) is the friction torque independent from the load on the bearing and \( M_1 \) the friction torque, which depends on the load. \( M_0 \) is proportional to the cube of the bearing's diameter while \( M_1 \) is proportional to the diameter.

The expressions for \( M_0 \) and \( M_1 \) are:

\[ M_0 = 160 \times 10^{-7} f_0 d_m^3 \]  
\[ 2-4 \]

\[ M_1 = f_1 P_1 d_m \]  
\[ 2-5 \]

The factors \( f_0 \) and \( f_1 \) depend on the type of bearing while \( P_1 \) depends on the load on the bearing.

What matters to us is that \( M_1 \) is always larger than \( M_0 \) (even for small bearings with light loads).

Consequently we can consider that the friction torque in a bearing is roughly proportional to its diameter, or more generally speaking, to a linear dimension:

\[ M \propto [L] \]  
\[ 2-6 \]

The work required to turn the bearing an angle \( \alpha \) is given by

\[ W = Ma \]  
\[ 2-7 \]

And from eq. 2-6 it is apparent that

\[ W \propto [L] \]  
\[ 2-8 \]

From the analysis of energy losses, \( U_L \), due to hysteresis phenomena we obtain that they are proportional to the volume of the flexure hinge. This can be done by looking at the hysteresis loop of the material of the hinge (Fig. 2-13).
Figure 2-13 Typical hysteresis loop of low carbon steel.

The work required to load the hinge is greater than the work it gives back when unloaded. The relation between the necessary work and the volume of the hinge is given by simple strength of material's theory:

\[ U = \int_V \frac{\sigma_x^2}{2E} \, dV = \int_V \frac{1}{2E} \left( -\frac{My}{I} \right)^2 \, dx \, dA \]  \hspace{1cm} 2-9

Hence,

\[ U_L \propto L^3 \]  \hspace{1cm} 2-10

Figure 2-14 Detail of a flexure hinge in an electrostatic micro-actuator. The height of the hinge is 100 μm and has a minimum thickness of 1 μm (from [FAH99]).
Many micro-systems must be designed to work in harsh environments under extreme temperature, humidity, vibration, and other conditions. The core of the system is usually very delicate and must be well protected from its environment. At the same time the packaging in charge of protecting the system must allow the different signals to cross this protective layer. When only electrical signal must be exchanged the problem remains fairly simple to solve thanks to the microelectronics industry. However, when fluidic, temperature, light and other non-electric connections are necessary techniques not found in the microelectronics industry must be developed to make them reliable and economically feasible.

As far as strength of materials is concerned, micro-parts have a great advantage over larger parts. Micro-machining techniques reduce considerably the number of cracks and other flaws on the surface of parts. Stress concentration is avoided resulting in structures that can be loaded to stresses higher than larger structures built with high-performance steels. This unexpected behavior holds as long as the overall dimensions of the part are small compared to the average distance between surface imperfections.

For similar reasons, fatigue in micro-systems is far less critical than in conventional-sized ones. This is, together with the energy dissipation mentioned above, another good reason to employ flexure hinges in micro-systems.

2.3.4 Functional dimensions & Tolerances

In conventional mechanics, the outer dimensions of parts usually coincide with those of its active elements (ex. a pin). Those active elements must be dimensioned with the appropriate tolerances that will allow the assembled system to operate correctly. This "inherent" property of conventional-size mechanical parts facilitates part grasping, manipulation and assembly, since the position uncertainty of the elements that play a role during assembly coincides usually with the active or functional elements and is consequently within their tolerances.

When moving to the micro-system world things are different. The position tolerance of the active elements with respect to the outer surfaces (that can be used for grasping purposes) exceeds the maximum positioning error that will assure a correct operation of the finished product. The combination on one single part of electronic and mechanical elements together with limitations and interfacing problems are at the origin of such inconveniences. Silicon chips are the central element of most micro-systems and their characteristics deserve special attention.
2.3.4.1 Silicon Parts

Many of the parts used in today's micro-systems are made out of silicon mainly because they can be manufactured using techniques derived from the microelectronics industry, saving manufacturers research and development expenditures.

One of the main problems encountered when manipulating micro-parts is that their functional dimensions are unrelated to the outer dimensions. By functional dimensions we mean the dimensions of the features on the part that play an active role. It is with respect to those dimensions that the parts must be positioned. However most of the grippers grasp the part by its outer dimensions that in general have very loose tolerances and have no position tolerance with respect to the functional dimensions.

When assembling an all-electric chip, the relative position between the chip and the frame is not very important because the electric wire is flexible\(^2\). But when other connections are involved, the positioning of the chip relative to the housing becomes critical if a reliable connection is desired, and in order to obtain a quality product the correct assembly strategies must be used.

A clear example can be found in silicon components similar to computer chips. It is common to find optical micro-sensors where the signal-processing unit (chip) has a built-in photo-detector. The pattern on the chip has very accurate dimensions, with tolerances in the order of 0.1 microns, whereas the outer dimensions of the chip can have tolerances in the order of 100 microns. In such a case we can’t rely on the external borders of the part if we want to position precisely the photo-detector with respect to a focusing lens. To overcome this difficulty we need a system that can measure the position of the detector with respect to the outer dimensions of the chip and transmit this information to the manipulator’s controller so it can take corrective measures. This type of system, commonly known as feedback systems, is analyzed in the following chapters.

Other restrictions (some of which are not a direct consequence of size reduction) are encountered when manipulating silicon micro-parts:

\(^2\) Although the wire is flexible, the position tolerance of the chip must be tight enough to be sure that the bonding pad falls within the scope of the bonding machine’s vision system.
• Extremely delicate surfaces that cannot be touched without the risk of scratching the part,
• Internal stresses due to external manipulation forces or other related stresses,
• Electrostatic discharges may cause permanent damage, especially to semiconductor parts,
• Risk of contaminating the surface of very clean parts,
• Phenomena that become very important due to size reduction (surface forces, grasping pressure, friction forces, etc.)

2.4 Micro-part manufacturing techniques and materials

Most of the manufacturing technologies and materials used for micro-system component fabrication have been adapted from similar technologies applied in other fields. This is the case of micro-milling, micro-drilling, laser processes and ion beam processes. LIGA (X-ray micro-machining) may be one of the exceptions since it was developed especially for micro-system manufacturing. The techniques that derive from the microelectronics field have the disadvantage of not being capable of producing parts with high aspect ratios. The latter is importance since the efficiency of many micro-systems depends on it. The torque delivered by electrostatic and magnetic motors depends on the aspect ratio of the gap between the rotor and stator. Micro-turbines and mechanical gears benefit too from high aspect ratios. Flow and power output of turbines increase as the aspect ratio. Similarly the maximum torque a gear can transmit depends linearly on the aspect ratio.

2.4.1 Manufacturing technologies

Micro-machining with diamond tools allows us to fabricate shafts 25 \( \mu \text{m} \) in diameter, 125 \( \mu \text{m} \) in length and a surface roughness of 20-30 nm (rms). One of the problems of micro-machining is that parts tend to deflect during machining. In the case of the shaft mentioned a taper of 1 part in 200 over the entire length has been measured [FRA95].

Parts can also be produced using micro-molding technologies. Batch fabricated metallic micro-structures use these processes. Micro-molds can be formed in thin films of polymer materials on planar substrates by means of photolithographic techniques. An electroplating process is used to fill the molds, which can be later removed to leave free standing micro-structures. Those structures can be released from the substrate by etching away an eventual layer of sacrificial material.

3 The ratio between the height of an object and its width or depth. The aspect ratio of a pin would be its length divided by its diameter.
Microstereolithography consists in projecting an "active" mask that induces a space-resolved photo-polymerization of a liquid resin. Successive masks can be used to polymerize the resin layer by layer (more than 1000 layers can be superimposed) until the desired structure is obtained [REN99]. Additional structures can be added to existing parts using this technology (right-hand side photograph in Fig.2-15).

![Microstereolithography Diagram]

*Figure 2-15* Relatively complex structures can be produced with the microstereolithography process (from [REN99]).

In the LIGA\(^4\) technique the source material is a polymeric plastic layer of a few hundred-micron thickness, which is applied onto a substrate. The pattern of a mask is transferred into the thick resist layer by means of a highly parallel, high intensity and high-energy synchrotron radiation. The long chains of molecules that form the polymeric plastic break due to the irradiation causing a change in their chemical stability. This allows removing, through the use of a solvent, the resist that has been irradiated. Micro-electro-deposition techniques are applied successively to build a complementary structure of metal. Once the structure is built, almost any number of plastic copies can be produced accurately and inexpensively using injection molding techniques or vacuum embossing techniques. The microturbine shown in Fig. 2-12 is built using the LIGA technique.

To end this very brief description of micro-manufacturing technologies we will mention the technologies that have been developed specifically for silicon. Two main groups of technologies can be made: bulk silicon processes and surface micro-machined silicon processes.

Among the first group we have wet and dry etching processes for machining into and through silicon substrates either isotropically or anisotropically. Thin diaphragms, suspended and overhanging structures can be produced out of bulk silicon by a process known as the etch-stop process. Those processes are carried out in batch on a silicon wafer. Once the parts have been machined they need to be sawed to detach them and form single parts. The sawing tolerances are usually several orders of magnitude bigger

\(^{4}\) LIGA is a German acronym for RöntgenLithographie (X-ray lithography) Galvanik (electrodeposition) Abformung (molding).
than the dimensional tolerances achieved with the etching processes. During assembly sawed parts introduce a series of inconveniences that are treated in the next chapter.

Surface micro machining includes the processes that produce MEMS\(^5\) structures on the surface of planar substrates. They involve deposition of different materials on the surface of the substrate that are then patterned and finally released to form micro machined components.

<table>
<thead>
<tr>
<th>Feature Height</th>
<th>Complementary micro-manufacturing processes</th>
<th>X-ray Micromachining</th>
<th>Bulk Si processing techniques</th>
<th>Surface processing techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td>1mm cm's</td>
<td>1mm cm's</td>
<td>500μm</td>
<td>10-20 μm</td>
<td></td>
</tr>
<tr>
<td>Cross-sectional shape</td>
<td>Very good</td>
<td>Very good</td>
<td>Fixed by crystal</td>
<td>Very good</td>
</tr>
<tr>
<td>Cross-sectional variation</td>
<td>Good</td>
<td>Limited</td>
<td>Limited</td>
<td>Very limited</td>
</tr>
<tr>
<td>with depth</td>
<td>Wide range possible</td>
<td>New materials being developed continuously</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
<tr>
<td>IC Compatibility</td>
<td>Good</td>
<td>Good</td>
<td>Fair</td>
<td>Excellent</td>
</tr>
<tr>
<td>Process Maturity</td>
<td>Developing</td>
<td>First products</td>
<td>Fairly mature</td>
<td>First products</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>Large</td>
<td>Large</td>
<td>Small</td>
<td>Medium</td>
</tr>
<tr>
<td>Low volume production</td>
<td>Excellent</td>
<td>Poor</td>
<td>Fair</td>
<td>Poor</td>
</tr>
<tr>
<td>High volume production</td>
<td>Possible</td>
<td>Good</td>
<td>Excellent</td>
<td>Excellent</td>
</tr>
</tbody>
</table>

Table 2-2 Comparison of basic micro-system processing technologies (from [FRA95]. The complementary processes include micro-milling, micro-drilling, laser processes and ion beam processes.

### 2.4.2 Materials for micro-components

The most common materials used in the micro-system field include single crystal silicon, polysilicon, silicon dioxide, optical material compounds such as III-V compounds (gallium arsenide), piezoelectric materials such s zinc oxide, shape memory alloys, electroplated and vapor deposited metals, polymer materials and materials for complementary processes like steel. The material out of which a micro-component is made must fulfill two requisites: first it has to be compatible with the manufacturing process used to manufacture it and second, as for any mechanical or electrical part in conventional size systems it must have the appropriate properties. For example parts that are in

\(^5\) MEMS: Micro-electro-mechanical systems.
permanent contact with other elements and must be wear resistant can be made out of materials such as nickel.

Materials compatible with such technologies are limited since most of them derive from the microelectronic field where silicon is the main element employed. The more conventional manufacturing processes allow for the use of almost any material: metals, plastics, etc.

2.5 Conclusions

Many microsystems have been analyzed in the frame of this work and we have realized that in a great number of them it is fairly easy to find the relation between the system's functions and its size. When the relations have been established it is relatively simple to fix the domain of validity of the phenomena involved and consequently if they will remain valid at the scale of the microsystem being studied. If they do they can be considered as conventional systems.

The degree of miniaturization aimed should be in concordance with the requirements of the final product and not simply to benefit from the possibilities offered by micro-parts manufacturing technologies. Miniaturization is a design strategy used to improve the performance of the system or to reduce manufacturing costs.

The need to interface a system with its environment is often an impediment for further miniaturization. In the case of autonomous micro-systems, energy supply is severely restricted.

In general more efficient microsystem designs are obtained when its working principle is such that measurements are taken from surface related phenomena and disturbing factors are associated with volume related effects.

Micro-parts are manufactured using infinity of new technologies which are compatible with an increasing number of materials. Nevertheless many of those new technologies can be replaced by more conventional techniques where higher quality parts can be produced at much lower costs.
Chapter 3: Description of Microassembly

3.1 Introduction

Micro-assembly, because of the different effects caused by miniaturisation on micro-systems, plays a very important role in the chain of processes that lead to the completion of a sound micro-system. In addition to physically piecing together the micro-system, micro-assembly comprises control, calibration and testing tasks. Consequently micro-assembly equipment does not consist of part manipulators alone; sensors of different nature are necessary to determine the position of parts, check their dimensions and perform quality control and calibration tests. Part feeders are also required to provide the assembly station with an acceptable degree of autonomy. However, as we will see, they don’t necessarily have to consist of complex and expensive bowl feeders; simple pallets can perform much better in the micro-system domain. The way parts are handled differs completely from what is done in conventional assembly, and the reason for this is found when the way micro-parts are produced is analysed in detail. The reliability and efficiency of a micro-assembly procedure does not depend entirely on the available equipment. The way a micro-system is designed is crucial for the success of the future assembly operations it will have to undergo. Design for micro-assembly (DfμA), a tool especially conceived to guarantee that micro-assembly operations are feasible in first place, and sufficiently reliable, is positioning itself in a prominent place, especially in this field where in addition to the long list of technical limitations, the economic side is of great importance.

3.2 Definition of Micro-Assembly

Micro-assembly is the last stage of production a micro-system must undergo before it is marketed. It obviously includes – but is not limited to – the process of putting together the parts that will constitute the finished product. Unlike in conventional assembly where control and calibration operations are performed after the system has been put together, micro-systems must be assembled, calibrated and controlled simultaneously¹.

¹ Exceptions can be found where the calibration is done once the system is completely assembled, provided the design of the system allows for post-assembly interventions. A good example is the quartz oscillator found in electronic watches. The assembly operations are basically encapsulating and wiring the quartz crystal to the connector. Once this is done, an opening in the casing allows the passage of a laser beam that trims the crystal till the desired resonance frequency is achieved.
The use of non-reversible processes to produce a micro-system is one of the main reasons why piecing together, calibration, tuning and quality control are combined, becoming a single stage in the production chain.

3.3 Comparison to "conventional" assembly

The origin of all the differences between "conventional" and micro-assembly lies, obviously, on the size of the product. The physical laws that describe the phenomena that take place during an assembly sequence are without doubt the same whether the product is big or small. However size does influence the domain of "technical validity" of each law. For this reason an assembly strategy can be effective and reliable for a product of conventional size, but completely futile if applied to a micro-system.

As we will see later on some of those domains of validity are automatically identified while others are much more difficult to perceive and may delude highly trained and experienced engineers.

From the economic standpoint figuring out where those limits stand can lead to important cost reductions. The work done by Mrs. Koelemeijer [KOE99] on this subject includes a detailed analysis on the possibilities of cost reduction in micro-assembly by comparing it to conventional assembly.

The use of robots for conventional assembly purposes is not very common mainly because of the complexity of the tasks involved. In Germany, for example, automated assembly is the third most frequent application for the approximately 11,000 robots in service. Although we have found no data that covers a reasonably large group of industries, we can state that robots present in the micro-system industry are mainly used for assembly purposes. This information reflects that probably micro-system assembly operations are more apt for automation.

3.4 Manual and Automatic Assembly

As for conventional assembly, the decision on whether to assemble a micro-system manually or automatically will depend on both economical and technical factors. Those factors must be analysed bearing in mind that micro-systems and not ordinary sized systems are being assembled. As a general rule the diagram in Fig.1 can be used to give an idea of the degree of automation required for our product depending on production volumes, batch size, number of variants, etc.
Figure 3-1. Relation between different production parameters and degree of automation (modified from [Et99]).

There is no unanimous explanation on whether automatic assembly or manual assembly is more suitable from the technical point of view. Some authors ([REI97] and [ZUH97]) state that because of the high quality and accuracy requirements on the assembly of miniaturised products manual methods still prevail although they may put a lot of stress on the operator, and others say that the required precision can only be attained by means of a manipulator.

The claims sustained in both perspectives are valid but so centred on precision and accuracy issues that they cannot be generalised to micro-assembly as a whole since aspects other than those mentioned are as influential when deciding upon the degree of automation of the assembly operations. In addition to the parameters that appear in Figure 3-1 the following must be considered when the manual versus automatic assembly debate comes up:

- Complexity of assembly operation
- Fragility of parts
- Dimensions of components
- Assembly accuracy
- Identification of assembly reference features

### 3.5 Requirements for micro-assembly equipment

Equipment found in a micro-system assembly plant differs considerably from that found in a conventional-sized system assembly plant. Some of those differences are obvious; nobody would conceive a micro-assembly plant where a batch of silicon chips weighing a few grams was transported around in a forklift.
On the opposite side there are many requirements for micro-assembly equipment that are systematically overseen, or ignored simply because the unfolding of well-known physical laws and principles as size is reduced is not considered adequately. As an example the reader is referred to [Koe99] where the author elegantly proofs why part feeders, other than a simple pallet, are not necessary in the discipline of micro-assembly. The proof is based on simple geometric characteristics of both the parts being assembled and the manipulator, and on the amount of time spent by an operator working on the mentioned machine.

Microsystem manufacturers repeatedly look at the positioning precision and repeatability of a manipulator while ignoring other parameters that may be equally important such as robustness, reliability, characteristics of peripheral equipment, etc. Section 3.6.2 describes the dependence of a manipulator's precision on the different elements that compose it (control strategy, measurement system, type of gripper, etc.).

Throughout this thesis the reader will find different ways of establishing the applicability of working principles depending on system size. In turn, this will prevent designers from overlooking important design aspects in their projects.

### 3.5.1 Bibliographic research

An extensive analysis of existing literature in the field of micro-assembly equipment has been made and, indeed, a lot of useful information has been gathered. However it seems as though efforts are mainly directed at reducing the size of assembly tools alone. Important elements that relate directly to the effectiveness of an assembly strategy (cycle time, robustness of operations, movement accuracy and repeatability, etc.) are not analysed rigorously enough. The validity of the information collected has to be checked thoroughly; micro-systems and related processes and technologies are not merely smaller versions of their conventional sized equivalents. Downscaling an existing system or any process related to it involves much more than that. Manufacturing processes must be understood, physical laws applied correctly, design criteria has to be adapted, etc.

Gunther Reinhart et al. [REi97] wrote a very good article that summarizes most of the problems found in micro-system automated assembly together with useful hints on how they can be overcome.

A lot has been written about robot precision and accuracy:

"The required assembly accuracies for a micro-assembly robot are in the range of 0.1 to 20 μm" [Zuh97], "sub-micron precision is often required" [NEL98].

Russel [Rus93] states that "the assembly robot itself need not be micron-sized only capable of adroit, delicate and precise movements at the scale of the components it is required to manipulate".
Another important characteristic of micro-assembly robots is that they must be capable of performing accurate and repeatable movements to allow computer control [Rus93].

Koelemeijer [KOE00] has analysed how the different parts of an automatic micro-assembly stations influence its overall positioning accuracy. Information on this analysis can be found in section 3.6.2.

The use of micro-robots for micro-assembly purposes is gaining momentum, at least at the experimental level. There is no doubt that small robots have some advantages over bigger ones. They consume less energy. They can work in smaller volumes saving expensive clean-room space, etc. A vast number of subjects related to micro-assembly applied micro-robotics are treated in the literature.

Sulzmann et al. [SUL96] describes the use of micro-robots for microsystem manipulation. The robot working on the stick-slip principle is controlled by means of a sophisticated vision system. The combination of 2D pattern recognition and passive auto-focus algorithms allows for a control of the robot in the three dimensions of the workspace. While this paper clearly brings new ideas to the microassembly domain industrial applications cannot be derived from them. More conventional manipulators are much faster and despite being less precise match many of the present microsystem assembly requirements. An analysis of how different features of the robot scale down with its size shows that micro-robots have some advantages with respect to conventional robots. On the other hand other important properties are not favored by miniaturization.

The design and construction of microgrippers is of capital importance in this domain. In the beginning attempts to produce universal micro-grippers yielded no positive results. The main problem when manipulating small parts comes from the effect of surface forces and is described in chapters 2 and 5. Different explanations on how to overcome such a problem can be found in the literature, but reliable models capable of predicting accurately enough the magnitude of the so-called surface forces do not exist due to the enormous complexity surrounding their origins. Despite those limitations several microgrippers based on different working principles appear in the literature. The use of shape memory alloys [BEL98] and piezoelectric materials [GAL89] combined with different flexible structures [BRE98] [XU96] that permit to amplify, and therefore render useful for gripping purposes\(^2\), the small deformation achieved by the materials mentioned.

\(^2\) The same references use the same structures for different purposes related to microassembly such as translation stages, force sensors and motion elements for microrobots.
3.5.2 Characteristics of micro-system parts

The specifications for micro-assembly equipment are dictated by the characteristics of the parts involved, which differ considerably from those of conventional-size parts. The most significant differences are listed below:

- Micro-parts are designed to perform several tasks. Electronic and mechanical functions may have to be carried out simultaneously.
- Functional dimensions are unrelated to outer dimensions, or the position tolerance of the active elements with respect to the outer surfaces exceed the maximum positioning error that will guarantee a successful operation.
- Internal stresses due to external manipulation forces or other related stresses may cause permanent damage to the part and are difficult to avoid if part has to be manipulated.
- Semiconductor parts are very sensitive to electrostatic discharges.
- The surface of some parts has to be very clean and can be easily contaminated or scratched during assembly.
- Physical phenomena that are heavily influenced by size: surface forces, grasping pressure, friction forces, etc.

Another important aspect of micro-systems in terms of assembly is the number of degrees of freedom that have to be controlled. Conventional sized products consist of parts that fit together or that slide with respect to each other to ensure a mechanical function. The geometry of the components is used to define all the degrees of freedom except for one or two (see Fig. 3-2).

![A typical assembly task: mechanical constraints help position the part.](image1)

![A typical microassembly task: no mechanical constraints to help position the part.](image2)

Figure 3-2 Many microassembly tasks are characterised by lack of mechanical constraints that help positioning the parts.
Micro-systems, on the other hand, can't benefit from their geometry to make micro-assembly operations simpler: the tolerances obtained with present fabrication technologies don't allow for it. All the degrees of freedom must therefore be controlled during the assembly process. To make things worse the use of attachment techniques such as gluing or welding require the part to be kept precisely in place till it holds. Valuable resource time is "wasted" while the glue polymerizes or the welding material solidifies.

There are a few cases where parts are inserted as for example when assembling a micro-gearbox. Problems related to friction forces appear in this case. More details can be found in chapter 5.

3.5.3 Manual micro-assembly equipment

An operator, with the appropriate visual aid, is capable of performing very precise manipulations, such as assembling a wristwatch or other small mechanical systems. It makes no sense to talk about precision or resolution in a manual assembly operation due to the complexity of the resources employed by a human being to perform such tasks.

3.5.4 Automatic micro-assembly equipment

Any micro-assembly process requires picking a part from its original location and placing it in its target location. Additionally the part may be checked or adjusted during the displacement or once it has reached its final position. A third task may have to be performed in order to attach the part: it may take place before the part reaches its final destination (gluing), or it may have to be done once the part is in place (welding). The generalised use of permanent attachment methods such as gluing or welding implies that any adjustment necessary has to be done before binding the component in place. Evidently the part must be designed in a way that will render the adjustment possible.

In order to analyze the effects that size reduction has on the assembly operations the equipment and strategies used are divided into the following four groups:

- Position feedback: Position control & Target location,
- Part Feeding,
- Grippers,
- Auxiliary operations.

The fact that the magnitude of surface and perimeter forces become relevant when compared to weight and other inertia forces (see the chapter dedicated to surface forces) and the circumstance that the functional elements of the components have no geometric relation with its external features, which are normally used to reference them, are the two main sources of problems encountered in any of the categories mentioned above.
The requirements for position control and reference location in micro-assembly equipment are dictated mainly by the lack of relation between functional and external dimensions of the parts involved. More details can be found in section 3.6 below.

Both part feeding and gripping are influenced by the relative magnitude of the different forces involved and the dimensional independence mentioned above. References to those issues appear in the present chapter and in chapter 5.

Assembly equipment has other characteristics that influence directly the overall performance. Those characteristics are not directly related to the technical feasibility of the assembly operation itself, but involve the amount of time an operator must spend assisting the machine. Set-up time and autonomy of the machine are two of the most relevant characteristics that fall in this category. Although less evident than in other fields the size of the parts influences heavily both set-up time and autonomy [KOE99].

3.6 Position Feedback: Position Control & Target Location

The term position feedback is used when describing the position control loop of a robotic arm and the measuring systems that provide the robot's controller with information on the location of their target destination.

Feedback control is used to obtain accurate control of a process while it is being performed. The variable that is being monitored is constantly being measured and compared to its reference or pre-programmed value. Process parameters are then varied according to a control strategy in a way that the difference between measured and pre-programmed value is minimised [MCK91].

In a pick & place task the position of the end-effector is the measured variable (as a function of joint position and robot's geometry) and the programmed path or trajectory is used as reference. Indeed the path is programmed only after the target position has been established. In the micro-system domain it is hardly possible to define the position of the parts and then let the robot arm manipulate them "blindly". Some sort of measuring device that furnishes the robot with information on the real position of the part is necessary. Although this is also based on position feedback, it is used for an entirely different purpose than trajectory control.

Very few robots are equipped with stepper motors or other actuators that can be controlled in an open loop fashion [CRA86]. Most manipulators mount motors or actuators that output a force or torque, which is applied to the joints. Since external factors such as friction, variable load on gripper, geometric tolerances and other disturbances cannot be eliminated the use of some kind of position feedback control reveals necessary.
At present the number of industrial processes carried out by robots is limited because of their lack of sensing capabilities. Many of these robots can only measure joint position, and a few interlocks and timing signals. The exact location of the object that has to be manipulated is programmed into the robot by manually moving the gripper to the object's location and recording the angular position of each joint. If, for any reason, the object is not exactly where it was when the robot was programmed the operation will fail.

In addition in some industrial and most micro-assembly applications the reference cannot be determined precisely enough a priori. In consequence measurement of environment parameters becomes crucial to the successful application of robots.

To guarantee that parts are correctly positioned and oriented parts-handling equipment or manual positioning is necessary, unless the robot can be equipped with sensors of some kind that allow it to determine the exact position of the parts.

3.6.1 Target Location in Micro-Assembly

Passive compliance\(^3\) provides a simple means of absorbing position errors during pick & place operations as well as adapting trajectories during motion. This can make up for position errors to a certain extent.

Micro-parts are usually fed to robots on pallets. For the robot to have an acceptable autonomy the pallets must accommodate a reasonable number of parts. If to this we add the necessity of providing enough clearance between the parts to allow gripper access to the parts we end up with quite a large pallet. Manufacturing such pallets becomes expensive; not only must the parts be nested tightly enough\(^4\) to reduce play, but in addition the position tolerance between nests must be small enough. Furthermore the advantages of passive compliance can only be exploited as long as the assembly accuracy requirements are based on the external physical features of the part. When this is not the case (like in many micro-assembly cases) a method that allows establishing a geometric relation between the "active" element of the part and the gripping surfaces must be used.

\(^3\) Passive compliance is the tendency of a body to move due to the internal effects of the forces applied to it, e.g. when placing an object on a surface it is almost impossible to program the trajectory so that the object is just touching the surface when it is released. The object may be dropped from a short distance or forced against the surface. In such case passive compliance prevents damage to part, surface and manipulator.

\(^4\) Custom-made pallets are necessary if parts must be nested tightly, making them even more expensive.
Silicon chips are the clearest example of parts whose functional features bear no relation with its external surfaces. A brief glance at the fabrication process of a chip reveals that while features on the chip can be etched with sub-micron precision, its outer edges after sawing may have tolerances of about a tenth of a millimetre. Although sub-micron precision is used to manufacture the active elements of the part, it does not mean that a positioning with a similar precision is necessary. However light detectors, emitters and other optical devices do require positioning accuracy of a few microns with respect to other elements within the system such as focusing lenses or mirrors. [AND96] shows an example where a lens must be precisely positioned with respect to a source. Since the lenses optical axis cannot be determined from its external dimensions a feedback system consisting of a light emitter and a sensor at the other side of the lens has been integrated to the assembly station. The manipulator places the lens in its approximate position while the feedback system starts measuring the light received by its sensor. According to a predefined algorithm the manipulator shifts the lens slightly till an optimal signal is received. Then by means of an ingenious support the lens is welded in its exact position.

The examples above illustrate the two strategies used to position precisely parts during a micro-assembly operation.

The first one is based on identifying the functional elements of the part before grasping it. With this information the manipulator "knows" the position of those functional elements with respect to the gripper's coordinate frame and can proceed as if the part had been mechanically referenced. A video camera with the appropriate image processing algorithms is often used for this purpose.

The second approach consists in measuring the function the part being assembled must accomplish. This measurement can be used to determine the desired position of the part. Vision systems can also be used for this purpose but customized measuring systems that work directly with the part's function can be much more reliable, accurate and fast. This modus operandi allows relaxing a bit the precision and repeatability requirements for the gripping system. In micro-assembly, due to the characteristics of the parts and the phenomena involved this can be greatly advantageous.

3.6.1.1 Camera-based vision systems

The advantage of using computer vision lies on the fact that since it can be used for many different applications, a ready-to-use vision package can be purchased and can be operational after a short start-up period when compared to other measuring systems.
Still a vision system is quite complex and environment sensitive, especially to variations in illumination. The importance of the system's light source should not be underestimated. The correct choice can lead to a dramatic increase of the system's overall performance. Not only the nature of the light\textsuperscript{6} has to be considered, but also its direction and any interaction it may have with the illuminated object. The most common types of light source include: diffuse (coaxial, annular, retro-illumination), directional, polarised, stroboscopic, etc.

Due to the nature of light and the size of micro-systems with respect to conventional-sizes ones the behaviour of light when it is reflected on a micro-system is the same as when it is reflected on a larger system.

Differences may appear at the lens level where excessive magnification can increase the distortion of the image.

Sub-pixeling can be used when the resolution given by whole pixels does not suffice [THE98]. This technique allows us to define with greater accuracy where the edge of a part is located, even if it does not coincide with the border between two pixels.

### 3.6.1.2 Other measuring systems

Contactless measuring devices are preferred in the micro-system domain. Among them, optical sensors are widely used since they provide acceptable measuring ranges and resolutions.

Unlike camera based vision systems that are able to perform different measurements (surface, centre of gravity, pattern recognition, and even 3-D measurements), optical sensors measure mainly distances in one or at most two directions. More complex sensors can measure diameters and even distance between simply shaped parts [KEY94]. For specific applications optical sensors can be especially designed [LOO00]. They have the advantages and disadvantages of any custom made product.

When aiming at micron resolutions with optical sensors the interaction between the surface of the object being measured and the light used to do so has to be analysed since it may be the source of considerable errors [BEN96]. There are measuring stations equipped with optical sensors able of delivering sub-micron resolution even when measuring parts with surface roughness of the order of one micron. How this is achieved is unknown by the author, however it points out the importance of knowing what really needs to be measured. It is the functional dimensions of a part that have to be controlled and not merely those that are easy to measure. Functional dimensions are discussed in chapter 2, but there is still another aspect to be treated: the appropriate

\textsuperscript{6} Intensity, wavelength, polarisation.
sensor (according to its measuring principle) must be chosen in accordance with the functional dimension that is to be controlled. An example concerning a sensor to measure the deflection of a beam is given in section 5.7 in chapter 5.

3.6.2 Remarks on the precision of microassembly equipment

The concept of precision of a microassembly manipulator is not well defined. A piece of equipment with axis resolution of, say, 1 μm does not guarantee an assembly precision of 1 μm. The assembly precision depends, in addition to the manipulators resolution and precision, on:

- the dimension and geometric tolerances of the components and their functional elements,
- the way parts are presented and fed to the manipulator,
- the type and characteristics of the gripper,
- the measuring systems,
- the way the resources are operated.

In many practical situations an assembly precision is demanded without evaluating the components and their function within the system. This is in part due to the fact that it is not clearly defined. The required assembly precision can be defined as the required tolerance on the position of the functional dimensions of one component with respect to a second one. Similarly we define the obtained assembly precision as the difference between the nominal and the real position of the two parts in a specific case [KOE00]. It is important to note that both precisions relate solely to the relative position of the functional dimensions of the components.

The obtained assembly precision results from the accumulation of several errors that originate in the components, grippers, manipulator, etc. This stack of errors is treated statistically rather than additively since it is highly improbable that they will all have a large deviation and on the same side at the same time. We consider that the obtained real positions $P_{\text{obt}}$, are distributed following a normal distribution with variance $\sigma$ around $P_{\text{nom}}$ (the desired targeted position). The tolerances for a well-mastered process are within $3\sigma$, which means that 99.73% of the obtained positions are within the tolerance limits.

The obtained assembly position that results from the accumulation of the different elements involved in the operation is distributed normally around $P_{\text{nom}}$ with a resulting variance $\sigma_{\text{res}}$ equal to:

$$\sigma_{\text{res}} = \sqrt{\sum_i \sigma_i^2}$$

3-1
where:

\( i \) = index for each element involved in the operation.

The precision of each element is set to:

\[
P_i = 3\sigma_i
\]  
\(3-2\)

Similarly the obtained assembly precision is:

\[
P_{\text{obt}} = 3\sigma_{\text{assembly}}
\]  
\(3-3\)

where:

\( \sigma_{\text{assembly}} \) = variance of the distribution of the distance between the functional dimensions of the two parts.

The factors that play a role in the obtained assembly precisions are:

- The position tolerances of the functional elements, \( t_{\text{edge}} \), of a micro-component with respect to the elements used for gripping\(^7\) (for a silicon component a typical tolerance is \( +/- 50 \mu m \)).
- The position tolerance (or play) \( t_{\text{feed}} \), with which the parts feeding system positions the part for the manipulator (when parts are fed in pallets a typical value is \( +/- 200 \mu m \)).
- The gripper used. The error introduced by the gripper depends on its working principle: with or without mechanical reference, gripper that does not modify part position (except for vertical component), compliant gripper, etc.
- The type of manipulator control used: open or closed loop, with or without servoed axes, cam driven, with or without measuring system, etc.

Table 3.1 has been obtained by considering the factors and applying the method mentioned above.

\(^7\) Those elements vary with the type of gripper used but are normally the edges of the component.
<table>
<thead>
<tr>
<th>Type of manipulator</th>
<th>Type of gripper</th>
<th>Obtained Assembly Precision, $\sigma_P$</th>
<th>Precision [\mu m]</th>
<th>Technological time [sec]</th>
<th>Investment cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open loop</td>
<td>Without part position modification</td>
<td>$3\sqrt{2(\sigma_{feed}^2 + \sigma_{edge}^2 + 2\sigma_{manip}^2 + \sigma_{trans}^2)}$</td>
<td>263</td>
<td>3</td>
<td>$I_{base}$</td>
</tr>
<tr>
<td></td>
<td>Without mechanical reference</td>
<td>$3\sqrt{2(\sigma_{grip}^2 + \sigma_{edge}^2 + 2\sigma_{manip}^2 + \sigma_{trans}^2)}$</td>
<td>244</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>With mechanical reference</td>
<td>$3\sqrt{2(\sigma_{edge}^2 + 2\sigma_{manip}^2 + \sigma_{trans}^2)}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closed loop</td>
<td>Without part position modification</td>
<td>$3\sqrt{2(\sigma_{edge}^2 + 2\sigma_{manip}^2 + \sigma_{feed}^2)}$</td>
<td>169</td>
<td>3</td>
<td>$10\times I_{base}$</td>
</tr>
<tr>
<td>and servoed axes</td>
<td>Without mechanical reference</td>
<td>$3\sqrt{2(\sigma_{grip}^2 + \sigma_{edge}^2 + \sigma_{manip}^2)}$</td>
<td>118</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>With mechanical reference</td>
<td>$3\sqrt{2(\sigma_{edge}^2 + 2\sigma_{manip}^2)}$</td>
<td>94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closed loop</td>
<td>Without part position modification</td>
<td>$3\sqrt{2(\sigma_{res-manip}^2 + \sigma_{manip}^2 + \sigma_{camera}^2)}$</td>
<td>22</td>
<td>5</td>
<td>$15\times I_{base}$</td>
</tr>
<tr>
<td>and measuring system</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Servo-controlled</td>
<td>Conventional robot</td>
<td>$3\sqrt{\sigma_{manip}^2 + \sigma_{camera}^2}$</td>
<td>1.5</td>
<td>10</td>
<td>$25\times I_{base}$</td>
</tr>
<tr>
<td>external dimension</td>
<td>Micro-robot</td>
<td>$3\sqrt{\sigma_{manip}^2 + \sigma_{camera}^2}$</td>
<td>1.5</td>
<td>30</td>
<td>$20\times I_{base}$</td>
</tr>
</tbody>
</table>

Table 3-1 Obtained assembly precision for different micro-assembly manipulator configurations (from [600]).

Typical values for the different variances used in Table 3-1 are presented in Table 3-2. The last two columns show the technological time and investment cost (normalized with respect to the cost of an open-loop assembly system) of the different configurations.
<table>
<thead>
<tr>
<th>Type of manipulator</th>
<th>Type of gripper</th>
<th>$\sigma_{\text{feed}}$</th>
<th>$\sigma_{\text{manip}}$</th>
<th>$\sigma_{\text{edge}}$</th>
<th>$\sigma_{\text{trans}}$</th>
<th>$\sigma_{\text{grip}}$</th>
<th>$\sigma_{\text{res-manip}}$</th>
<th>$\sigma_{\text{camera}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open loop</td>
<td>Without part position modification</td>
<td>33</td>
<td>33</td>
<td>17</td>
<td>33</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Without mechanical reference</td>
<td>-</td>
<td>33</td>
<td>17</td>
<td>33</td>
<td>17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>With mechanical reference</td>
<td>-</td>
<td>33</td>
<td>17</td>
<td>33</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Closed loop and servoed axes</td>
<td>Without part position modification</td>
<td>33</td>
<td>10</td>
<td>17</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Without mechanical reference</td>
<td>-</td>
<td>10</td>
<td>17</td>
<td>-</td>
<td>17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>With mechanical reference</td>
<td>-</td>
<td>10</td>
<td>17</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Closed loop and measuring system</td>
<td>Without part position modification</td>
<td>-</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>Servo-controlled external dimension</td>
<td>Conventional robot</td>
<td>-</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Micro-robot</td>
<td>-</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 3-2 Typical values for the variances of the different elements of an assembly system. The values are given in microns.

The following list briefly describes the meaning of the variances that appear in the preceding tables. For more details on how the expressions for $P_{\text{opt}}$ are obtained consult [KOE00]:
σ_{feed} : positioning precision of part feeder when it delivers a part to the manipulator.

σ_{manip} : positioning precision of the manipulator (fixed automation manipulators).

σ_{edge} : position of functional elements of part with respect to its edges.

σ_{trans} : positioning of working pallet in manipulator working space (only used with fixed automation manipulators).

σ_{grip} : positioning of part with respect to gripper (for grippers without mechanical reference).

σ_{res-manip} : influence of the manipulators resolution (when external measuring system is used).

σ_{camera} : positioning of part by means of camera.

### 3.7 Design for Micro-Assembly (DfμA)

The efficiency and reliability of a micro-assembly line do not depend solely on the equipment used; an appropriate design that simplifies the assembly process proves essential. Commercial success of micro-systems is measured in cents. If to this we add that the assembly costs represent between 70 and 80% of the total cost of the product the interest in having an efficient, simple, fast, reliable and consequently economic assembly process is obvious.

From experience production costs of an existing product can be cut down by 10-15% by updating assembly lines and modifying assembly procedures. Not much more can be done when about 75% of such costs are set at the design stage (see Fig. 3-3).
Figure 3-3 Factors that determine the cost of a micro-system, and the influence they have on it. (from KDOO).

Besides defining the mechanical, electrical and other characteristics of a system, the design stages will also determine the number of components it will have, the equipment required to assemble it, the technological time to do so, the rate of successful operations and the number and duration of production interruptions.

As far as assembly is concerned a good design results in a simple assembly process, a short technological time, non-sophisticated assembly equipment, and operations with a high rate of success.

**DESIGN FOR FUNCTIONALITY: Before Design For Assembly (DFA)**

With some exceptions, size is not among the essential attributes a micro-system must possess, since they are usually integrated in larger machines or appliances and are therefore out of the user's sight.

Way more important are the reliability and accuracy with which they carry out the functions they are designed for.

Of comparable concern for micro-system manufacturers are the economic aspects of their products. Not only production costs have to be considered; also factors such as
time to market, commercial life of the product, availability of raw materials and access to necessary technology must be considered.

Size plays a role in most, if not all, of the points just mentioned. Reducing the size may enhance the functional properties of a given micro-system: A small accelerometer tends to have a higher resonance frequency than a larger one. Should this be desirable, miniaturization gains importance, but within certain limits; a very small accelerometer may involve high assembly costs that would prevent it from competing in the market.

The equation for a product's success includes all of the terms acknowledged above and many more that have not been addressed, since they are more product-specific.

The following figure schematizes the steps that lead to a sound micro-system design. The dashed arrows indicate that once a design is obtained the steps that involve some kind of decision by the manufacturer have to be revised since they are inter-related and they influence each other. The assemblage of a micro-system is held responsible for great part of its cost and of the resources needed to manufacture it. Assembly aspects touch all the design steps mentioned in the figure below although it is not explicitly indicated.
3.7.1 Design for Assembly

A significant amount of work has been done in the domain of "Design for Assembly" for ordinary size products. "Design for Manual Assembly" and "Design for High Speed Automatic Assembly" are two methods developed by Boothroyd in [BOO91] and [BOO92].

Those are simple and transparent methods consisting of sets of rules that have to be followed in order to obtain a product that can be assembled easily. Although in principle they can be applied to micro-system assembly, in practice some changes have to be made. They are summarized in the following seven points:

- Component nature: Size, fragility and clean environment conditions make manipulation by an operator a task unfit for mass production. Introduction of complex automatic equipment becomes necessary.
- Nature of micro-assembly operations: Basically they are pick & place cycles as in conventional assembly. However part position detection differs radically as well as the mechanical constraints encountered when releasing the part (see chapter 2).
- Micro-assembly specific operations: A number of assembly operations such as bonding, fluid deposition, etc. are specific to the micro-system domain and are not considered in the DFA rules.
- Problems introduced by adhesion forces: When small parts are manipulated care must be taken to ensure that the part remains in its initial position until it is firmly grasped by the gripper. The same requirements are valid when releasing the part. The surface forces to volumetric forces ratio is at the origin of those disturbances (see chapter 5).
- Parts feeding: Micro-parts cannot, in general, be handled in bulk. For this and other reasons, bowl feeders are rarely used in micro-assembly. Micro-part feeding is far more complex than in conventional assembly, but if it is analyzed correctly it can be rational and inexpensive. This added complexity is not reflected in existing DFA rules.
- Required precision: Manual assembly cannot provide either the necessary precision or the required production rate. High precision positioning equipment is

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8 See insert on bowl feeders.
therefore mandatory. In general a position feedback loop involving direct measurements on the concerned parts is necessary. Not all the assembly operations require the same degree of precision. Less complex equipment is required to assemble elements such as the casing. This difference in complexity at present ignored by the DFA methods analyzed has to be accounted for in DfμA.

- Reduction of number of components: The rules concerning the reduction of number of parts found in DFA continue to be valid for micro-systems. However the integration of an excessive number of functions in a single element can make the fabrication process too complex, resulting in a process with an unacceptably low yield.

### 3.8 Conclusions

Assembly of microsystems differs in many ways from conventional assembly. In this domain it is not limited to putting together the different components of the system. Due to the use of permanent attachment techniques control and calibration procedures must be carried out during the assembly stages.

The assembly process has to be taken into account when designing a microsystem if a reliable, fast and inexpensive assembly cycle is desired. At present assembly costs represent about 70% of the total system cost, and at the same time the design of a system can alter by 75% its cost. Therefore all effort invested in improving either the design of a system from the assembly point of view or developing new assembly strategies or equipment will yield significant earnings in terms of system cost reduction.

The adequateness of a microassembly station is not measured in terms of precision alone. Although microsystem components are very small their characteristics are such that loose assembly tolerances are often possible, making pointless the use of ultra high-accuracy positioning devices. Furthermore the effective positioning precision or repeatability of an assembly station may be degraded significantly due to inaccuracies in gripper, measuring system, part feeder, etc. Other important factors to consider when deciding on the assembly equipment are the effective working time of the machine and setup and calibration time since they have a direct impact on the machine’s yield.
Chapter 4: Dimensional Analysis and Similitude for Microsystems

4.1 Introduction

In modern engineering practice no machine, structure or system is built before the design has been verified, tested, redesigned and checked over again. In many cases the process is repeated several times until a final trustworthy design is achieved. In order to design it some initial knowledge of how the system works and on what parameters it depends is necessary. Once an initial design has been established it needs to be tested.

Dimensional analysis and theory of similitude are very useful when it comes to designing and testing new systems.

When confronted with a new design, an engineer makes use of the experience obtained in the different branches of engineering. This experience comprises theories, models and experimental data. Theories and models are obtained by simplifying real processes and validating them by making measurements on them. Unfortunately those models are ideal and do not match exactly the requirements and restrictions imposed on the real design. Experimental data may hint the engineer on the reasons why there is a deviation between his models and the real behavior of a system.

Thus the engineer has part of the information he needs to obtain a prediction equation that will describe the behavior of his system. The extra bit of information he requires can be obtained thanks to the use of dimensional analysis\textsuperscript{1}.

The reasons why prediction equations usually have to be adapted to the case under study can be synthesized in two points:

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\textsuperscript{1} Dimensional analysis is a powerful analytical tool developed from a consideration of the dimensions in which each of the pertinent quantities involved in a phenomenon is expressed [MUR50].
a) Prediction equations are often obtained from a simplified model that neglects phenomena that have a small impact on the process. This simplification is required if an analytical solution to the phenomena is desired; it is not done merely to simplify calculations.

b) Prediction equations may be accurate enough but fall out of the domain of validity for which they were established.

In the micro-system domain there is a lack of analytical solutions for phenomena that are neglected in conventionally sized systems. Due to size considerations the influence of surface forces and geometric tolerances among others can no longer be ignored, forcing the micro-engineer to study the long ignored phenomena.

Testing is carried out to validate the prediction equations and therefore guarantee that the final product will meet the specifications it is intended to fulfill before the product is mass produced or, if it is a large and expensive product, before it is built. Tests are run with models of the system being developed. Common sense indicates that if the model is real size the information obtained from the experiments can be applied straightforward to the final product. If for economical or technical reasons a full size model cannot be used a scaled-down (or enlarged) version of the prototype will have to be used. Here the domain of validity of the laws we may use to analyze the model must be thoroughly checked as well as the relative importance of the different phenomena involved. Theory of similitude\(^2\) allows extrapolating the information obtained via the model to the definitive design.

The degree of success achieved thanks to the use of dimensional analysis depends both on the type of results anticipated and on the complexity of the system under analysis. If a rough estimate of the behavior of a simple system is desired, results will be easily obtained (such is the case of the simple pendulum where the period of oscillation is searched neglecting air friction and assuming small angles). If the angle is not assumed to be small, a valid prediction equation for the period will be found but a considerably greater amount of investigation will be necessary. When air resistance is considered the analysis becomes more cumbersome and it is possible that different effects will have to be studied separately; in this case the effect of air resistance in function of speed can be analyzed separately and its effects added to the previous results. In general

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\(^2\) Murphy describes this theory as "the theory that comprises the principles which underlie the proper design and construction, operation and interpretation of test results of the models. In other words the theory of similitude includes a consideration of the conditions under which the behaviour of two separate entities or systems will be similar, and the techniques of accurately predicting results on the one from observations on the other." [MUR50].
problems that have analytical solutions are the best candidates for dimensional analysis, but in such cases it is not necessary. However the reason why they are good candidates is important: analytical solutions are found when the relevant parameters are known, and this is exactly the requirement for the success of dimensional analysis. The more sure we are about which parameters intervene and which don't, the better results we will obtain.

Having partial knowledge on the behavior of a system, even if it is a complex one is of great help when applying dimensional analysis. This is the case of surface forces where we have a general idea of the origin of the forces but difficulty is found to predict their magnitude. Even though there are many parameters involved in surface force effects dimensional analysis proofs useful because the purpose of microengineers is not to understand why those forces appear but rather they are interested in knowing how to reduce (or increase) their effects.

A highly complex problem where accurate results concerning detailed parts of the system is desired can also be attacked with dimensional analysis either by splitting the problem into smaller parts or creating prediction equations that can be used in more "sophisticated" analysis methods such as finite element analysis.

This chapter describes by means of several examples how to apply dimensional analysis to obtain the prediction equations that describe the behavior of a system. The use of models where for the first time (to our knowledge) up-scaled models are needed is also analyzed in a simplified way due to lack of space and time.

### 4.2 Dimensional Analysis and Prediction Equations

Dimensional analysis is based on Fourier's principle of dimensional homogeneity, which states that an equation describing a physical relationship between quantities must be dimensionally homogeneous. This principle can be used to determine the dimensions of a physical quantity and is very useful to check engineering calculations. But what really interests us about dimensional analysis is that the form of a physical equation can be obtained if the relevant parameters and their dimensions are known. Dimensional analysis will directly provide us with qualitative solutions to our problems. In order to obtain quantitative answers experiments must be run.

The example of the simple pendulum in appendix A shows how the equation for its period is obtained. The only information provided by the engineer in this case is restricted to the parameters that are thought to have an influence on the pendulums period, namely the pendulum's length and mass and the acceleration due to gravity. Even if part of the information was incorrect (the mass plays no role in this case) the method
worked\(^3\). A simple laboratory experiment leads to a complete solution to the problem of determining the pendulums period.

![Graph showing the relationship between the number of parameters, qualitative prediction equation, quantitative prediction equation, and reduce number of variables to analyze against the amount of experimental work.]

Representative problems
- □ Simple pendulum.
- Sphere in fluid.
- Micro-dispenser.
- Microsystem manufacturing costs.
- □ Analysis of surface and friction forces.

Figure 4-1 Applicability of Dimensional Analysis. The number of parameters involved and the difficulty with which they can be identified influence the amount of experimental work required to obtain a qualitative or quantitative prediction equation.

Appendix A gives a detailed description and examples of dimensional analysis.

### 4.2.1 Prediction Equations

Prediction (or operating) equations are the equations that describe the behavior of a system as a function of the significant variables. There are two general methods to obtain those equations:

- **analytical method**: relations between the significant variables are obtained by applying the natural laws that are pertinent to the problem.

- **experimental method**: the effect of the pertinent variables on the quantity to be predicted is established by observation and measurement.

The necessary knowledge to develop prediction equations for micro-systems analytically is not solid enough. The recentness of this branch of engineering is possibly the main

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\(^3\) This is not always the case. In a real situation we would have checked experimentally which variables do influence the period. Overlooking a parameter usually has a more negative effect although in some cases the omission can be detected.
cause of this lack of comprehension. However, this handicap should not be seen as an obstacle that cannot be cleared. Thanks to the bits of reliable knowledge we have at our disposal, and the simultaneous use of dimensional analysis and models we can experimentally obtain trustworthy prediction equations that describe the behavior of micro-systems and the techniques associated to them.

4.2.1.1 The Buckingham \( \Pi \) Theorem

When the number of variables is greater than in the case of the pendulum obtaining a prediction equation is not straightforward. More experimental work is necessary, but its amount is limited thanks to Buckingham's theorem (see appendix A for more details).

The theorem states that the number of dimensionless and independent quantities (or \( \Pi \) terms) required to express a relationship among the variables in any phenomenon is equal to the number of quantities involved, minus the number of dimensions in which those quantities are measured, or in equation form:

\[
s = n - b
\]

where:

\( s \) = the number of \( \Pi \) terms;

\( n \) = the total number of quantities involved;

\( b \) = the number of basic dimensions involved.

The \( \Pi \) terms have a single requisite: they must be dimensionless and independent.

Now that the \( \Pi \) term has been defined we can add that prediction equations have an additional advantage: for a group of similar systems the prediction equations obtained for each one of them may contain similar \( \Pi \) terms. This enables, under certain conditions and with certain limitations, to extrapolate knowledge or experience from one system to another.

4.2.2 Determination of Functions

At this point dimensional analysis gains importance again since it allows us to obtain a maximum of information with a minimum of experimental work.

By using dimensional analysis and the \( \Pi \) theorem we obtain a prediction equation that is a function of the different \( \Pi \) terms. In order to formulate a useful prediction equation the function that relates those terms must be established. The only way to do this is by means of experimental data. Occasionally data may be extrapolated from similar previous experiments, reducing further the testing phase. Different series of
experiments have to be run each time keeping all but one \( \Pi \) term fixed. At this point we can clearly see the advantage of simultaneous use of dimensional analysis and the Buckingham \( \Pi \) Theorem: the reduction in the number of variables that must be investigated.

Unfortunately there are still experiments to be carried out, and when designing large, expensive structures such as planes, ships, bridges, dams, etc. it is evident that the system itself is not available. This is where the role of models becomes important.

## 4.3 Similitude and Models

Dimensional analysis complemented by experimental results can lead to the formulation of an expression that describes the phenomenon under consideration. If the number of influential parameters is large, an expression with many \( \Pi \) terms will be obtained, consequently making data collection and interpretation too time consuming to be viable. In many cases, however, a general expression is not required; all that the engineer requires for the design is an indication of the relationship of the variables for a specific design, or within a narrow range of variation of the significant variables. In such cases a model can produce the desired result quickly and inexpensively.

### 4.3.1 Models

If it is possible to construct a full size model, then the data obtained can be used straightforward to make improvements and other modifications. However many models are scaled down versions of the final system and certain rules must be followed to "transform" the data from the real system into equivalent data for the model. Those rules are grouped under the similitude theory.

Murphy [MUR50] defines a model as "a device which is so related to a physical system that observations on the model might be used to predict accurately the performance of the physical system in the desired respect. The physical system for which the predictions are to be made is called the prototype."

As we mentioned in the preceding paragraph, models are required because a prototype is not available to run the experiments we need to determine the prediction equations. When analyzing big, expensive structures such as airplanes, dams or ships, it is not economical or even possible to build a full-scale model for investigation purposes only. This is why scaled down models are built. The inconvenience of scaled-down models is that a certain amount of restrictions or design conditions must be imposed to the model so that it behaves like the prototype. Many of the restrictions can be defined thanks to the laws of similitude (see following paragraph). However, due to the difference in size between model and prototype, it may occur that it is not possible to reproduce the same phenomenon on both of them. Once again, if the phenomenon is sufficiently mastered we
may be able to relax some of the similitude constraints while being sure of the influence it will have on the experiments.

In the micro-systems domain we encounter the same problems but originated by different causes. When designing a micro-system, efficiency, reliability and cost, among other factors, must be considered for the same reasons as they are considered in more conventional systems. The problem here is that it is very hard or even impossible to make measurements on the micro-systems themselves due to their size. If an up-scaled model is built, measurements will be possible and we will be able to obtain prediction equations that are accurate enough. Many problems appear here, but before confronting them lets understand the way similitude laws work.

4.3.2 Similitude

Although similitude theory\textsuperscript{4} is quite simple it is rarely possible to design and run tests on a model based on theory alone. The feeling of the engineer together with experience, judgment, ingenuity and patience play a vital role if useful results are to be obtained, correctly interpreted, and prototype performance predicted therefrom [Ven75].

4.3.2.1 "Traditional" uses of Similitude

Similitude has been used in many fields of engineering for a long time, to the point that it has become an engineering specialty in itself. The development of similitude laws requires not only a good scientific knowledge; a lot of imagination and improvisation skills are required if relevant data is desired. A clear example of the skills required to develop reliable similitude laws can be found in [LAT89]. Ship engineering is a very appropriate domain for the development of similitude laws because the design of a ship involves very complex fluid mechanics. Since analytical equations and computer simulations are not reliable enough, downscaled models must be used.

4.4 Application to micro-systems

To our knowledge the use of similitude laws is limited to downscaled models. In the micro-system domain we are interested in up-scaled models, that is, we want to build a model that is bigger than our final system. This is done in order to make data acquisition simple and reliable, as well as to allow the micro-engineer to get a picture of what happens at smaller scales.

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\textsuperscript{4} Can be briefly described as the basic theory for the interpretation of model tests.
The following sections illustrate a few examples of dimensional analysis and
development of prediction equations in the micro-system world. Although the technique
is described from beginning to end, experimental work required to obtain ready-to-use
formulae has not be carried out completely.

4.4.1 Development of Prediction Equations and use of Models
for a Micro-Dispenser

Gluing is a common attachment technique used in the micro-system world. The quantity
of glue required is very small, but because of the properties of most glues (viscosity,
surface tension, etc.) it is very difficult, in practice, to produce so small drops. Micro-
dispensers are currently being developed for this purpose. In order to produce small
drops the formation process must be known.

Since there is no previous experience in the design of micro-dispensers, we are
interested in finding the relation that exists between the adjustable parameters of the
dispenser, the fluid characteristics, and the size and homogeneity (or other qualitative
characteristics) of the drops. It is clear that an analytical solution could be attempted;
even a simulation program could be used, but by doing so it is sure that critical elements
typical of the micro-world would be inadvertently left out. Dimensional analysis alone
suffers the same inconvenience. If we don’t include all the relevant parameters in the
initial list they will not appear in our prediction equation. Among other advantages such
as considering factors that may have been omitted, performing measurements on a
model allows the engineer to identify the critical parts of the system that require a
deeper analysis.

The working principle of one type of micro-dispenser we are working with at our
laboratory is quite simple. A chamber with a small hole at the bottom is filled with the
fluid we want to dispense. A mechanism of some sort transmits a certain amount of
energy to the fluid, creating a pressure wave that propagates through the volume of
the chamber. When it reaches the hole, fluid is forced out the whole. If the energy
suffices a drop is formed and expelled.
The first step is to identify the different parameters that may influence the system.

The properties of the fluid: density ($\rho$), viscosity ($\mu$) and surface tension ($\sigma$) will surely influence the process of drop formation. Common engineering sense leads to affirm the latter. However, in case of doubt, a set of simple experiments will suffice to confirm that they are influent parameters.

There are cases where the influence of the parameters cannot be tested experimentally due to the lack of a running model. In such cases the engineer may resort to cases of similar nature to determine whether a certain parameter must be, or not, taken into account. For instance the influence of density and viscosity could be corroborated by analogy to flow in a pipe and that of surface tension by analogy to the capillary effect.

Even if an extrapolation as described above is not reassuring there is one last option as we have seen in the simple example of the pendulum (appendix A). The engineer has a margin of error: if he erroneously introduces a parameter that has no influence on the system there is a possibility$^5$ for it to be eliminated by the method.

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$^5$ This possibility is related to the dimensions of the parameter. If it includes a basic dimension that appears only due to it then it will be eliminated, otherwise it will not and the final result might be altered considerably.
A chemical engineer would probably point out that the size of the molecular chains within the glue or the size of aggregate particles could influence the size of the drops. In such case an extra property of the fluid should be added. For sake of simplicity we will not add it here. In order to simplify the analysis at the first stages of the research we could decide that the size of the chains is the same for all the glues we want to work with. By doing this, the influence of their size would be reflected in some other parameter with the same dimensions, i.e. the diameter of the hole (as we will see later on).

Compressibility is not an issue in this case since the speed at which the membrane of the reservoir hits the fluid is considerably lower than the wave-propagation speed. Should it be an issue, the compressibility modulus of the fluid and the wave propagation speed would have to be introduced in the analysis.

At this point it is not necessary, nor recommended, to discard parameters because their influence is thought to be negligible; they will be identified and removed at a later stage.

The rest of parameters we think may have an influence on the behavior of the system are:

- $P_L$: Power transmitted to the liquid;
- $D_h$: Diameter of hole;
- $L_h$: Length of hole;
- $D_d$: Diameter of drop;
- $V_d$: Velocity of drop.

A hammer symbolizes the mechanism transmitting the energy to the liquid since in this example we are interested in the formation of the drops. Evidently the amount of energy available to form the drop will also depend on the geometry of the reservoir and the amount of time over which the energy is delivered (power).

It is important to note that our interest here is to determine the relationship that exists between the drop characteristics, the fluid, the energy we transmit to it, and the geometry of the hole through which the fluid exits the dispenser. A similar approach could be used to study the most appropriate shape for the dispenser’s reservoir, or to analyze the energy transmission mechanism. For this reason parameters

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6 Do not confuse a parameter with negligible influence with one with no influence at all.
related to the geometry of the reservoir and to the system symbolized by the hammer are not included in our list.

### 4.4.1.1 Development of the Prediction Equation

The next step consists in developing a prediction equation for the parameter we are interested in, the diameter of the drop.

We will use the following set of independent basic quantities (one cannot be expressed as a combination of the two others):


Next we express the parameters of our list as a function of those dimensions:

- $\rho_L = [M][L]^{-3}$
- $\mu_L = [M][L]^{-1}[T]^{-1}$
- $\sigma_L = [M][T]^{-2}$
- $P_L = [M][L]^2[T]^{-3}$
- $D_H = [L]$  
- $L_H = [L]$  
- $D_D = [L]$  
- $V_D = [L][T]^{-1}$

We have 8 parameters that can be expressed with an independent set of 3 dimensions. According to the Buckingham $\Pi$ Theorem we will have:

$$8 - 3 = 5 \text{ $\Pi$ terms.}$$

The diameter of the drop will be a function of the other seven parameters:

$$D_D = F(E_L, \rho_L, \mu_L, \sigma_L, D_H, L_H, V_D) \quad (4-1)$$

Since this is a function that describes a physical phenomenon it has to be dimensionally homogeneous and therefore can be rewritten in the following way:

$$C_\alpha D_D^{c_1} \rho_L^{c_2} \mu_L^{c_3} \sigma_L^{c_4} D_H^{c_5} L_H^{c_6} V_D^{c_8} = 1 \quad (4-2)$$

where $C_\alpha$ is a dimensionless coefficient that may depend on the other parameters.
In order to use the preceding equation as a prediction equation we should obtain the values for the 9 unknowns. This would be a very cumbersome process and the results cannot be guaranteed beforehand. However we can reduce the number of unknowns by pushing further the dimensional analysis.

Substituting each parameter by its dimensions we obtain the dimensional equation:

\[ L^{c_1} (ML^2T^{-3})^{c_2} (ML^{-3})^{c_3} (ML^{-1}T^{-1})^{c_4} \]
\[ (MT^{-2})^{c_5} L^{c_6} L^{c_7} (LT^{-1})^{c_8} = 0 \]

Equating each basic dimension to zero leads to the following three equations:

For \( M \):
\[ c_2 + c_3 + c_4 + c_5 = 0 \]

For \( L \):
\[ c_1 + 2c_2 - 3c_3 - c_4 + c_6 + c_7 + c_8 = 0 \]

For \( T \):
\[ -3c_2 - c_4 - 2c_5 - c_8 = 0 \]

This is a system of three equations in 8 unknowns. In order to solve it we need to assign arbitrary values to 5 of the unknowns and solve for the remaining 3.

We choose \( c_2 \), \( c_6 \) and \( c_8 \) as independent variables. To make sure that the resulting equations are independent and the selection, therefore, valid, the determinant of the coefficients of the independent variables must be different from 0. So our determinant is:

\[
\begin{vmatrix}
1 & 0 & 0 \\
2 & 1 & 1 \\
-3 & 0 & -1 \\
\end{vmatrix} = -1 \neq 0
\]

Therefore the selection is valid. Now we have to assign 5 sets of values to the remaining 5 unknowns \( c_1, c_3, c_4, c_5 \) and \( c_7 \). Evidently the five sets must be independent.
The first set of values is:

\[ c_1 = 1 \]
\[ c_3 = c_4 = c_5 = c_7 = 0 \]

Substituting those values in our original set of equations we obtain:

\[
\begin{align*}
    c_2 &= 0 \\
    1 + 2c_2 + c_6 + c_8 &= 0 \\
    -3c_2 - c_8 &= 0
\end{align*}
\]

Solving this system we get:

\[ c_2 = 0, \ c_6 = -1 \text{ and } c_8 = 0 \]

So, for this set of values we obtain our first \( \Pi \) term, which is equal to:

\[ \Pi_1 = \frac{D_B}{D_H} \]

We proceed in a similar way to obtain the remaining 4 \( \Pi \) terms. The following values are assigned to the independent variables:

For \( \Pi_2 \):

\[
\begin{align*}
    c_3 &= 1 \\
    c_1 &= c_4 = c_5 = c_7 = 0
\end{align*}
\]

\Rightarrow \Pi_2 = \frac{\rho_L D_H V_B^3}{P_L}

For \( \Pi_3 \):

\[
\begin{align*}
    c_3 &= 1 \\
    c_4 &= -1 \\
    c_1 &= c_5 = c_7 = 0
\end{align*}
\]

\Rightarrow \Pi_3 = \rho_L \frac{D_H V_B}{\mu_L}

For $\Pi_4$:

\[
\begin{align*}
  c_3 &= \frac{1}{2} \\
  c_5 &= -\frac{1}{2} \\
  c_1 = c_4 = c_7 &= 0
\end{align*}
\]

\[\Rightarrow \Pi_4 = \frac{V_D}{\sqrt{\frac{\sigma_L}{\rho_L D_H}}}
\]

For $\Pi_5$:

\[
\begin{align*}
  c_7 &= 1 \\
  c_1 = c_3 = c_4 = c_5 &= 0
\end{align*}
\]

\[\Rightarrow \Pi_5 = \frac{L_H}{D_H}
\]

Our equation then becomes:

\[
\begin{align*}
  \frac{D_D}{D_H} &= C_p \left( \frac{\rho_L D_H^2 V_D^3}{\rho_L^2} \right)^{d_1} \left( \frac{\rho_L D_H V_D}{\mu_L} \right)^{d_2} \left( \frac{V_D}{\sqrt{\frac{\sigma_L}{\rho_L D_H}}} \right)^{d_3} \left( \frac{L_H}{D_H} \right)^{d_4}
\end{align*}
\]

or

\[
\begin{align*}
  \frac{D_D}{D_H} &= F \left( \frac{\rho_L D_H^2 V_D^3}{\rho_L^2}, \frac{\rho_L D_H V_D}{\mu_L}, \frac{V_D}{\sqrt{\frac{\sigma_L}{\rho_L D_H}}}, \frac{L_H}{D_H} \right)
\end{align*}
\]

By using the equation above the laboratory work required to obtain the values of the exponents is greatly reduced, since there are 5 unknowns instead of 9.

It is not surprising that some of the $\Pi$ terms are found in other domains where this method is used. This "coincidence" has been achieved by choosing the appropriate values for $c_1$, $c_3$, $c_4$, $c_5$ and $c_7$.

The idea behind looking for this "coincidence" is that the behavior of fluids as a function of such $\Pi$ terms has been studied in other engineering fields (naval
architecture for example) and existing data can be used to predict the behavior of the dispenser without having to run additional tests.

The term \( \rho_L D_H V_D / \mu_L \) is known as Reynolds's number (Re). It gives an indication of the ratio of inertial forces to viscous forces. The lower this value, the more important viscous forces are with respect to inertial forces, and, in consequence, the flow is more laminar than turbulent.

Another important term is \( \frac{V_D}{\sqrt{\frac{\sigma_L}{\rho_L D_H}}} \), known as Weber's number (We). It relates the forces due to inertia to those induced by the surface tension of the fluid. Without a deep knowledge in fluid mechanics we can see that as both Re and We become bigger the less influence viscous and surface tension related forces have on the formation of drops since they will be easily overcome by inertial forces. So, even before any experiments are run and without writing any fluid mechanics equation we realize that dense fluids at high speeds will form drops more easily. Furthermore, low viscosity and surface tension will also help in the creation of drops. Just compare honey falling from a spoon with water coming out of a hose.

In addition, the bigger the hole, the easier it will be to form drops. This can be verified by observing capillary tubes. However there is a relation between the diameter of the hole and the diameter of the drops formed, and it appears in the term \( \Pi_1 \).

The conclusions reached up to now were fairly easy to obtain since both Re and We appear in many fluid mechanics problems and their meaning is sufficiently mastered. Let's focus our attention on the remaining terms.

Since the different terms are independent any combination of them will still be independent. This property will be used to create a new term that will substitute \( \Pi_2 \) which is a rather complex term.

\[
\Pi_6 = \frac{\Pi_2}{(\Pi_4)^2} = \frac{\sigma_L D_H V_D}{\rho_L} \tag{4-7}
\]

Our prediction equation now looks like this:

\[
\frac{D_D}{D_H} = F\left(\frac{\sigma_L D_H V_D}{\rho_L}, Re, We, \frac{L_H}{D_H}\right) \tag{4-8}
\]
The first term of eq. 4-8 can be explained as the ratio of energy necessary to create a drop to the energy given to the system, or in other words, the efficiency of drop formation.

It is not our intention to fully develop a prediction equation for the glue dispenser. At this point experiments must be run where one of the parameters is changed while the others are kept constant. Good engineering judgment and experience helps reducing the parameters that must be changed and their ranges. In this particular case the only parameters we must be able to measure are the diameter of the drop, its velocity at the exit and the energy delivered to the system. The first two measurements can be done by means of optical sensors or a high-speed camera. Measuring the height from which a steel ball is dropped can easily control the energy delivered. Its diameter and elastic properties are needed to determine the duration of the impact and therefore the power transmitted. We assume that the properties of the fluid are known.

However as soon as we start changing the values of the different parameters incompatibilities start to appear. Imagine we want to change the ratio $L_H/D_H$, to obtain a relation between the diameter of the hole and the diameter of the drop for a given configuration of the system. We can change $L_H$ or $D_H$ indistinctly. If the diameter is changed the value of all the other terms will also change, and additional parameters will have to be changed. However since $L_H$ appears only in one place it is wiser to change it rather than the diameter. Next imagine we want to change $Re$ without modifying any other term. The only parameter that appears only in $Re$ is $\mu$. It could occur that we had at our disposal different fluids having the same surface tension and density but a different viscosity. In that case we would be able to modify $Re$ without influencing the other terms, but unfortunately such fluids are not generally available. In conclusion keeping all the terms but one constant by changing some of the parameters is in most cases physically impossible.

In this example we have considered the energy per unit of time (power) transmitted to the fluid rather than the difference of pressure created by the pressure wave. The same technique can be carried out taking into consideration the pressure within the liquid and the velocity of the wave within the fluid. The analysis is carried out in detail in chapter 6.

4.4.1.2 Parallelism between Reynolds number and $\frac{\sigma L D_H V_D}{P_L}$

Reynolds number can be seen as the ratio of inertia forces to viscous forces. The bigger the number, the bigger the inertia forces with respect to the viscous ones. This implies that turbulent flows are associated with large Reynolds numbers while laminar ones are described by smaller $Re$. While this tendency is valid for every flow configuration (from flow in a pipe to a whale swimming in the ocean), numerical values of $Re$ are closely related to each flow configuration.
The first flow configuration for which a relation between Re and type of flow (laminar or turbulent) was established was done by O. Reynolds. He studied the flow through a pipe of diameter d. He found that the upper limit for laminar flow was \(12000 < \text{Re} < 14000\). Unfortunately this upper limit depends on factors such as shape of pipe entrance and pipe roughness. In the engineering world the limit is established at \(2700 < \text{Re} < 4000\). The lower limit of turbulent flow at about \(\text{Re} = 2100\) is of great importance for engineers since it defines a condition below which all turbulence entering the flow will be dampened out by viscous forces [VEN75]. The following table shows Re values for turbulent and laminar flows in different configurations:

<table>
<thead>
<tr>
<th>Flow Configuration</th>
<th>Upper limit for laminar flow</th>
<th>Lower limit for turbulent flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Re} = \frac{\rho V D}{\mu})</td>
<td>(2700 &lt; \text{Re} &lt; 4000)</td>
<td>(\text{Re} = 2100)</td>
</tr>
<tr>
<td>Flow in a pipe of diameter D.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\text{Re} = \frac{\rho V X}{\mu})</td>
<td>(\text{Re} &lt; 500000)</td>
<td>(\text{Re} &gt; 500000)</td>
</tr>
<tr>
<td>Flow over a semi-infinite plate (in boundary layer).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\text{Re} = \frac{\rho V D}{\mu})</td>
<td>(\text{Re} = 0.5)</td>
<td>(30 &lt; \text{Re} &lt; 1000)</td>
</tr>
<tr>
<td>Flow over a 2-D cylinder</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-1. Limits for laminar and turbulent flows for different configurations.

Whenever a prediction equation containing Re is obtained, data such as that presented in the previous table can be useful in order to understand the relevance of the different parameters involved in the equation. Drag coefficients, heat-convection coefficients, etc. are strongly related to the type of flow present so having a way to estimate what type of flow will exist in our system can be very convenient.

A similar data recollection could be envisaged for the term \(\alpha L_{Dv} V_{Dv}/P_{L}\) where we could determine, for a specific configuration, when drops are formed and when they aren't.
The first tests could be run on a system similar to the one described in the following figure:

![Diagram showing experimental setup for studying the $\sigma LDH/V_B/P_L$ term.]

Figure 4-3. Experimental set-up for the study of the $\sigma LD_H/V_B/P_L$ term.

By dropping the steel ball from different heights we would vary the amount of energy transmitted to the fluid. Given a sufficient height a drop will be formed each time we drop the ball. The value of $\sigma LD_H/V_B/P_L$ at which drops are formed is registered. The same procedure is followed again but changing the diameter of the hole. Again, after a certain diameter drops will always be formed. The experiment can be continued by using fluids with different surface tension values. If the values obtained in the different procedures coincide then we have a base that may allow us to, qualitatively, say what conditions are required for the formation of drops in any drop formation process. If it doesn't work it would mean that an important parameter has been omitted, and has to be found and incorporated to the analysis.

### 4.4.2 Theory of Similitude

Throughout the development of our equation we have not mentioned the size of the system. This means that it can be used indistinctly for a big or a small system.

If we want to analyze the system at a larger scale we just need to create a bigger model in a way that all the terms remain with the same value.

In equation form:

$$\left( \frac{D_B}{D_H} \right)_m = \left( \frac{D_B}{D_H} \right)_p$$
if

\[
\left( \frac{\sigma_L D_H V_D}{P_L} \right)_m = \left( \frac{\sigma_L D_H V_D}{P_L} \right)_p ; Re_m = Re_p ; We_m = We_p \ldots \tag{4-10}
\]

where the indexes m and p stand for model and prototype respectively.

In this specific case building a larger model is not justified since the measurements we are interested in can be taken from a full-size model and there are no economical reasons that justify making the model of a bigger size. However it is possible that in a more advanced analysis we were interested in understanding the flow pattern around the nozzle. In such case it would be difficult to obtain useful data from a full-scale model. A larger model would allow us to obtain the velocity profile of the fluid, the pressure changes, etc.

The first thing that has to be done is establish the design parameters for the up-scaled model. They are obtained by combining eq. 4-10 with the scale factor n we want to use (a geometrically similar model is used here since the design parameters are easy to obtain. See appendix A for other types of models).

The first condition is obtained from the first equality:

\[
\left( \frac{\sigma_L L D_H V_D}{P_L} \right)_p = \left( \frac{\sigma_L L D_H V_D}{P_L} \right)_m \tag{4-11}
\]

Considering geometric similarity:

\[
D_H = nD_H \tag{4-12}
\]

Furthermore if we use the same fluid for prototype and model our design condition becomes:

\[
n V_{D_p} = V_D \tag{4-13}
\]

Proceeding similarly for the other \(\Pi\) terms we obtain the remaining design conditions:

\[
Re_p = Re_m \Rightarrow nV_{D_p} = V_D \tag{4-14}
\]

\[
We_p = We_m \Rightarrow \sqrt{n}V_{D_p} = V_D \tag{4-15}
\]
The use of a geometrically similar model is reflected in the last $\Pi$ term. Since it is a ratio of two characteristic lengths of the prototype it can be understood as being the condition that requires geometric similarity between model and prototype.

Our model can't be built and run following all the design conditions since they are contradictory. This does not mean that there is an error. It simply means that for $n \neq 1$ and using the same fluid for prototype and model complete similarity is not possible. There may exist a fluid with properties such that for a given scaling factor all design conditions are met. In general a fluid with the necessary characteristics is not found. To overcome this inconvenience two possibilities exist:

4.4.2.1 The use of a distorted model

The model may be distorted in a way that the conditions above can be met. A fluid's surface tension coefficient can be altered without changing considerably its other properties by adding chemical substances. In our example something could be done to change the design conditions where the surface tension coefficient appears.

The surface tension coefficient appears only in $We$ and in the first $\Pi$ term. By changing $P_{Lm}$ in eq. 4-13 we can absorb the fluctuations of the equality introduced by $\sigma_{Lm}$ so there is no need to worry about the first design condition.

Rewriting the equality for $We$ in detail we have:

\[
We_p = We_m \Rightarrow \frac{V_{Dp}}{\sqrt{\sigma_{Lp}}} = \frac{V_{Dm}}{\sqrt{\sigma_{Lm}}} \]

\[
\sqrt{\rho_{Lp}D_H} = \sqrt{\rho_{Lm}D_{Hm}} \]

Considering that $\rho_{Lp} = \rho_{Lm}$ and geometric similarity we obtain:

\[
\frac{V_{Dp}}{\sqrt{\sigma_{Lp}}} = \frac{V_{Dm}}{\sqrt{n\sigma_{Lm}}} \Rightarrow V_{Dm} = \sqrt{\frac{n\sigma_{Lm}}{\sigma_{Lp}}} V_{Dp} \]

To be consistent with the design condition imposed by eq. 4-14 the following must hold:

\[
n = \sqrt{\frac{n\sigma_{Lm}}{\sigma_{Lp}}} \]

From this expression we obtain the design condition that indicates how the surface tension coefficient must be changed:
\[ \sigma_{Lm} = n \sigma_{Lp} \]

It is now obvious that if an appropriate scaling factor is chosen it will be possible to fulfill this design condition along with all the others. A model distorted with respect to the surface tension parameter has been obtained.

4.4.2.2 Neglecting the less influential term

In some cases previous experience involving similar systems and phenomena may allow us to ignore certain \( \Pi \) terms from eq. 4-8 because:

- Their influence is negligible with respect to the other terms,
- Their influence on the system's behavior is well known and may be combined at a later stage with the results obtained from the simplified model.

In our example the flow through the nozzle has not been analyzed in detail because extensive research has been carried out for fluid flows in closed conduits. Therefore important parameters such as the pressure drop within the nozzle or its wall's surface roughness have not been considered. Chapter 6 shows how the problem of flow in conduits is treated.

4.5 Conclusions

In the micro-system domain analytical solutions are generally not available. This is due to the fact that many phenomena present are neglected in larger systems. Since micro-systems are fairly recent there has not yet been time for researchers to analyze completely the phenomena at this level. This situation is ideal for the use of dimensional analysis and similitude.

Those methods should not be underestimated because of their simplicity. At worst they can provide the researcher with information concerning the dimensions of the variables involved, but if some experience and imagination is brought in, the results can be very gratifying. The \( \Pi \) terms can be understood as indicators that hint us on the behavior of a system with respect to a given variable.

In the engineering world, where in many cases the objective is to know under which conditions a system will work instead of why it will work, similitude and dimensional analysis can be even more useful, since the behavior of a system under different conditions may be extrapolated from a relatively small amount laboratory work.

The advantage of being able to use existing data from previous similar system analyses is also important because R&D time and expenditure, critical in the micro-system industry, can be reduced significantly.
Chapter 5: Forces at the microsystem scale

5.1 Introduction

Forces condition in many ways the design, appearance, life expectancy, reliability, etc of mechanical systems. The optimal material, shape, and dimensions of a part will depend on the forces that it will withstand. Strength, fatigue, deformation, friction, wear, etc. are all important aspects that must be considered when a part is being designed, and they are all somehow related to the forces that act on it.

The techniques used to assemble the system are also influenced heavily by the mechanical solicitation it is subject to during operation. The attachment techniques used to put the system together (bolts, welding, glue, etc.), type of bearings used for rotary joints, etc.

From the assembly point of view forces are also of great importance; the force a part may have to withstand during assembly can be greater than the maximum allowable force, or even worse it may act in a different direction, causing permanent damage. Forces present when two parts are in contact during an assembly operation can also give us information on the position of one part relative to the other.

As long as we deal with systems of conventional size (that is not too big nor too small) we can quite accurately calculate the forces mentioned above. However when it comes to very large or very small systems our intuition starts to fail. Force related phenomena of importance for systems of, say, large size may not be so for systems a couple of orders of magnitude smaller. Although this may seem a mistake that engineers elude easily, we repeatedly come across with this type of error.

In this chapter the importance of the different forces present in the micro-world and related assembly operations is assessed. The differences between conventional and micro-assembly regarding the forces at play are also described.

5.2 Relation between size and forces

As the size of an object is reduced, the ratio surface/volume increases (see Fig. 5-1). In consequence the effects that are proportional to its surface will increase with respect to those proportional to the volume.
\[ P = 12L \]
\[ S = 6L^2 \]
\[ V = L^3 \]

\[ S/V = 6/L \]

\[ P = 12(L/n) \]
\[ S = 6(L/n)^2 \]
\[ V = (L/n)^3 \]

\[ S/V = n(6/L) \]

Figure 5-1 Relation between P (length of edge), S (surface of sides), V (volume) and size reduction (n=reduction factor). The ratio S/V increases linearly with size reduction.

Many mechanisms, systems and processes are limited (often much more severely that we can imagine prior to an analysis) by this relation between surface and volume forces. The size of airplanes, for example, cannot grow indefinitely because the lift they can produce depends on the area of the wings whereas the lift required for flying depends on the mass of the plane (or its volume). So if we double the size of an existing plane the weight would increase eight fold while the area of the wings would only be four times bigger. In order to make it fly we would either need wings that produce more lift or make the wings twice as big. While the first point is continuously being improved for fuel efficiency considerations the second is limited by the strength of the materials used.

A very small airplane cannot be built either because the weight becomes small compared to the surface of the wings. This does not allow for a lot of fuel to be carried on board. For such a small plane the drag (which depends mainly on the surface of the aircraft) would be much higher than the lift required for flight. To overcome this opposing force a great amount of fuel would be required thus reducing notably the range of the plane, till a point where it would not be possible even for it to take off. Small airplanes are being developed [MRA98]. Requirements for such vehicles are:
Longest dimension < 150 mm

Mass < 115 grams

Range = 10 km

Maximum speed = 50 km/h

Flight time > 20 minutes

Using internal combustion engines they achieved autonomies of up to 5 minutes, falling quite short of the requirements imposed. Battery powered motors performed better: a 9-mile flight that took 16 minutes was accomplished. High-density batteries costing around $200 dollars were used. As expected autonomy is the major problem for small aircraft.

Micro-assembly is substantially affected by this relation too. Since weight and other inertia forces depend on the volume of the object, it is evident that their magnitude will decrease with respect to forces that depend on its surface\(^1\) such as electrostatic, humidity and van der Waals\(^2\) forces. Objects of the size found in micro-systems (micro-parts) have a very small weight with relation to the surface forces that act on them (Fig. 1-3) [FEB99], [ARA96], [FUK95], [ARA98], [FEA95], [HEC90]. This presents a number of inconveniences that are not found in conventional assembly; parts will not fall when the gripper opens to release them, bowl feeders will not work or will have a very bad efficiency, positioning of parts will be difficult, etc.

---

**Bowl Feeders in Micro-Assembly**

Bowl feeders are intended to feed parts to a manipulator. Parts are supplied to the bowl feeder in bulk and randomly oriented, who in turn separates them, and orients them

---

\(^1\) While it is clear that weight and other inertia forces are directly proportional to the volume of the part, it is not accurate to say that the surface forces mentioned depend linearly on the surface. In any case they are proportional to \(L^p\) to some power smaller than 3.

\(^2\) Van der Waals forces are normally several orders of magnitude smaller than other surface forces. From the engineering point of view they can be neglected. We don't understand why this force is constantly mentioned in micro-assembly literature.
before presenting them to the manipulator. In addition a bowl feeder may have to
discard defective parts.

As it names indicates a bowl feeder consists of a bowl with a ramp that winds up around
its inner wall. Parts climb up this ramp thanks to the combined upper and rotational
movement of the bowl, which is achieved thanks to the configuration of its suspension
rods. The bowl is supported by a number (usually three) of inclined rods. An
electromagnet or other actuator applies a vertical force to the bowl in a harmonic
pattern.

An exit opening is provided at the upper end of the ramp. Towards the top of the bowl
there are a number of chicanes and other mechanisms that discard parts that are not
properly oriented or defective.

Figure 5-2 represents a part on the ramp of a bowl feeder with all the forces that act
on it just as the bowl starts its downward movement. For analysis simplicity the frame
of reference used here is fixed to the bowl.

\( \Theta \) - angle of ramp, in practice has a value of 3° or 4°,

\( \Psi \) - angle between direction of motion and ramp, about 30°,

\( a_b \) - acceleration of bowl,

\( F_f \) - friction force,

\( F_N \) - normal force,

\( m \) - mass of part.

Figure 5-2. Force diagram of part on bowl feeder ramp at the instant
when the bowl starts its downward motion.
The bowl performs the downward-upward movement cycle several times per second (reason for which it is referred to as "vibrating bowl" in other languages). On its upward movement the normal force acting on the part is greater than when it descends. The friction force, which is considered as proportional to the normal force, is big enough to prevent the part from sliding downwards on the upward displacement of the bowl, and if properly dimensioned will be sufficiently small to allow the part to slide upward as the bowl moves downwards.

The condition for the part to advance is given by the following expression:

\[ ma_b \cos \Psi > mg \sin \theta + F_f \]  

5-1

If the conventional model for friction is used \((F_f=\mu F_N)\) then \(F_f\) can be expressed as:

\[ F_f = \mu (mg \cos \theta - ma_b \sin \Psi) \]  

5-2

since

\[ F_N = mg \cos \theta - ma_b \sin \Psi \]  

5-3

Eq. 5-1 combined with eq. 5-2 gives the condition for the part to slide upwards as a function of the friction coefficient, the acceleration of the bowl and its geometry:

\[ \frac{a_b}{g} > \frac{\mu \cos \theta + \sin \theta}{\cos \Psi + \mu \sin \Psi} \]  

5-4

However, as it is mentioned frequently in this thesis, the friction model used here is not valid for parts this small. No model exists for friction forces when small surfaces are in contact. To show that bowl feeders are not suitable for micro-parts data from the graph in Figure 5-7 is used. The tangential force\(^3\) required to make the part slide along the ramp can be assumed to be the same as the force necessary to lift it\(^4\) (which is the data reflected on the graph). Average values for the bowl parameters\(^5\) and

---

3 Note that the term "tangential force" is used instead of "friction force" since in the micro-world adhesion forces can be several orders of magnitude bigger than a the friction force.

4 This assumption can be made because of the nature of the forces (mainly humidity and electrostatic) that attract the part to the ramp.

5 \(\Theta=4^\circ; \Psi=30^\circ; a_b=100m/s^2\)
information from Fig. 5-7 (for simplicity only the adhesion forces due to grease after a fine cleaning is used) together with eq. 5-1 yield the results presented in the following table:

<table>
<thead>
<tr>
<th>Object weight (N)</th>
<th>Surface force/Weight</th>
<th>Surface force (N)</th>
<th>ma&lt;sub&gt;b&lt;/sub&gt;cosΨ - mgsinθ &gt; F&lt;sub&gt;s&lt;/sub&gt; ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel cube 40mm side</td>
<td>4.8</td>
<td>0.5</td>
<td>2.4</td>
</tr>
<tr>
<td>Steel cube 10mm side</td>
<td>0.075</td>
<td>1</td>
<td>0.075</td>
</tr>
<tr>
<td>Steel gear Ø4mm, t=0.03mm</td>
<td>1.13x10&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>300</td>
<td>0.34</td>
</tr>
<tr>
<td>Steel sphere Ø10µm</td>
<td>3.14x10&lt;sup&gt;-10&lt;/sup&gt;</td>
<td>10000</td>
<td>3.14x10&lt;sup&gt;-6&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Table 5-1 Calculations to determine which parts are suitable for use in bowl feeders.

As predicted the smaller parts don’t have enough mass to produce an inertial force that will overcome the opposing surface force and therefore will not make it up the ramp.

Equation 5-1 can be rearranged as follows:

\[
\frac{m(a_b \cos \Psi - g \sin \theta)}{F_s} > 1 \tag{5-5}
\]

The left hand side of this inequality resembles the "Bonding effect number" that appears in the example of the vacuum gripper. The term in brackets is nothing else than the sum of all the accelerations that the mass m undergoes. The sine and cosine terms are specific to the case being studied; it can be expressed in broader terms as \( C_B A_T \). Eq. 5-5 can be rewritten in a more general form as:

\[
\frac{pL A_T}{\gamma} > \frac{1}{C_B} \tag{5-6}
\]

\(^6 C_B\) will be called the “bowl constant” for clarity.
where \( m \) has been replaced by \( pL^3 \) and \( F_s \) by \( yL^2 \) since we assume that the surface forces are proportional to the area of contact between part and ramp.

Now the left hand side of eq. 5-6 is the "Bonding Effect" number (chapter 6). The question that arises next is: what is this good for? Eq. 5-6 is the working condition for a bowl feeder. The "Bonding Effect" number describes the part that has to be fed and \( 1/C \) the characteristics of the bowl. The "bowl constant" can be found experimentally by observing any geometrically similar bowl feeder. In order to know if a part of a given size will work on a bowl with constant \( C_b \) its parameters must be introduced in the "Bonding Effect" number and eq. 5-6 verified.

**Remarks:** The problem of bowl feeders has been treated in a very simple way. Throughout the discussion the acceleration has been assumed to be small enough to ensure that the part is always in contact with the ramp. Other cases that have not been addressed here include parts that intermittently lose contact with the ramp and parts that can slide backwards as well as forward. Another simplification has been made when \( F_s \) has been replaced by \( yL^2 \) since surface forces depend not only on the area of contact but also on its perimeter and other factors that still remain unknown. If data on how the surface force between the part and the ramp changes as the size of the part is reduced we could use it instead of making such a simplification.

At the same time the fact that surface forces gain relevance with respect to inertia forces can be advantageous for the micro-assembly domain. A typical case is when small ball bearings are assembled. At a point when the balls are already in place the upper half of the cage must be inserted and to do that the bearing must be turned round. Obviously this can't be done without dropping the balls if it weren't for the fact that grease is added before it is turned around. The surface forces induced by the grease are bigger than the gravity forces thus preventing the balls from falling.

Although it is true that the mechanisms that drive those forces are not yet understood there is excessive concern about such forces. Presumably this is due to the fact that those forces work in a way that opposes our ordinary-size biased intuition. Nevertheless, advantage can be taken from the existence of such forces to make micro-assembly operations simpler. A bit of imagination and avoiding comparison with "conventional" assembly strategies is at present much more efficient than trying to eliminate those forces.

From the engineering point of view the ultimate interest is not to understand the mechanisms that originate the forces present in a micro-assembly operation. The engineers are interested in knowing when those forces will appear and what their domain of validity is. If the parameters that influence the forces are known and in
addition can be controlled the engineer will find its way towards the solution of many surface force related problems including micro-assembly using force feedback. In other words a plot of surface forces as a function of parameters such as contact surface, humidity, electric conductivity of material, etc. can suffice for an engineer to overcome those problems (see paragraph 3.1).

This is exactly what dimension analysis and similitude theory are good for: find a physically reasonable relation among the different parameters involved in a process. Additionally a relatively small amount of experimental work will lead to quantitative prediction equations.

In this chapter we combine existing research and information on the different surface forces and combine them with dimensional analysis and similitude theory to obtain useable prediction equations. We show how the methods described in previous chapters and in appendix "Similitude" are applied in this particular domain of micro-forces to obtain functional relations between the different parameters involved. This is done with a substantially reduced amount of work both at the experimental and theoretical levels. Furthermore, the behavior of similar processes studied in other fields is extrapolated to this domain further reducing research time.

5.2.1 Analysis of Surface Forces in the world of Micro-Systems

The term "surface forces" encompasses forces of different nature. Performing a dimensional analysis with all the variables that influence any of the surface forces would be extremely slow and would require an immense amount of laboratory work, especially since it is pretty sure that additive terms would appear and discerning those would require an even greater quantity of experimental data.

Throughout the literature (see appendix B) surface forces are basically divided in three groups according to their nature: humidity related forces, electric charge originated forces and van der Waals forces.

From the appendix we find a series of parameters that play a role in generating those forces. Different authors using different models arrive at different equations. There are cases where the constants they introduce are not even well defined, or values for them do not exist. In any case the fundamental thing here is that they all use the same parameters and this is what we do too. We can use the same ones as a departure point for our analysis.

Our initial analysis does not take into consideration the specific geometry of a problem. The contact zone of two micro-parts can't be defined by a simple geometric model such as a plane and a sphere, since a surface as an entity can't be defined in the range of dimensions we are concerned with. Surface roughness can't be scaled down in the same proportion as linear dimensions (see Fig. 5-3).
Figure 5-3 Photographs of two pins with equal surface roughness $R_A$ 0.05-0.10 µm. The pin on the left has a diameter of 0.5 mm, while the one on the right has a diameter of 0.1 mm. The magnifications are such that on the image they present a similar size. This enhances the fact that surface roughness doesn’t scale down in the same proportion as linear dimensions.

The use of simple geometric models leads to relatively simple formulas since the distance between any point in the sphere and in the plane is easily calculated. Furthermore electric charges and fluid films can be assumed to be distributed homogeneously throughout the surfaces. Simple laboratory experiences show that those distributions are far from being homogeneous in most common micro-parts surfaces. An appropriate statistical description of the surfaces would allow estimating a charge and fluid film distribution that would in turn yield more accurate surface force estimations. Chang et al. [CHA87] use a similar approach to assess friction forces due to the asperities on the surfaces. In their work asperities are assumed to have spherical tips while their height and radius is given by statistically by a distribution curve. The distance between asperities is also defined statistically.

An alternative approach involving the use of dimensional analysis and experimentation to quantify the effects of the geometric configuration while making use of the equations that describe accurately the base phenomena could also be used. For example when analyzing the surface forces due to electrostatic charges we come across equations such as

$$F_{ei} = \frac{\pi}{4\varepsilon_0} \frac{\varepsilon - \varepsilon_0}{\varepsilon + \varepsilon_0} d^2 \sigma^2$$  and  $$F_e = \frac{\pi \sigma_1 \sigma_2 d^2}{\varepsilon_0}$$
would reduce to one equation of the form:

\[ F_{ei} = G\sigma_1 \sigma_2 d^2 \]  

where:

\( G \) is a function of geometry, surface finish and dielectric properties of the bodies involved,

\( \sigma_1, \sigma_2 \) surface charge density,

\( d \) characteristic distance between bodies.

The values for \( G \) are obtained by measuring the forces for different geometric configurations, surface roughnesses and dielectric coefficients.

Dimensional analysis is applied here to reduce the number of experiments so that for example the effect of surface roughness can be established testing a single configuration and extrapolated to the rest of configurations.

Evidently the same approach can be applied to humidity and van der Waals forces, which can be also decomposed into two factors: one that depends on the geometry and one that quantifies the corresponding phenomenon that originates the force.

While our approach seems quite promising we should proceed cautiously; we should not forget that the parameters we have used are those that are used in conventional size physics, where the models have been obtained through observation of infinity of conventionally sized systems and processes.

A prudent micro-engineer researching surface forces should point out the following facts before approving his general equation that would be of the form:

\[ F_{tens} = G l \sigma_L \]  

where \( G \) would be, as before, a function of geometry, \( l \) a linear dimension to which the perimeter of the wetted surface would be proportional to\(^7\) and \( \sigma_L \) the surface tension coefficient.

\(^7\) The proportionality constant would be included in \( G \).
• Since surface tension forces depend on the perimeter of the wetted surface, its shape as well as the number of separated wet surfaces play an important role in determining the magnitude of the force (Fig. 5-5).

• Surface tension is a result of intramolecular forces in the liquid. Imagine two molecules, one on the surface of the liquid and one deeper into the liquid. The molecule beneath the surface experiences forces in all directions due to its neighbors while the one on the surface only experiences forces from below and from the sides. This imbalance tends to pull the "surface'' molecules back into the liquid. Consequently it is legitimate to ask what the minimum amount of liquid is necessary to obtain this surface tension effect, and also what is the relation between the surface tension coefficient $\gamma$, and the quantity of fluid.

• The surface tension coefficient is considered to be constant for a given fluid at a given temperature. When two surfaces with a drop of fluid between them are pulled apart the effect depicted in Figure 5-4 is observed. The force due to the fluid that opposes separation can be obtained by multiplying the surface tension coefficient by the perimeter of the wetted surface.

![Diagram of surface tension forces](image)

**Figure 5-4** The meniscus effect. As the overall size is reduced the difference between $D$ and $d$ becomes more significant.

Suppose we apply a force equal to $\pi D\gamma$. This is the maximum force we can apply before the fluid "breaks" apart. If we analyze the forces present at the section $A-A$ we observe that the maximum force the fluid can withstand is reduced to $\pi d\gamma$. 
Since the angle $\alpha$ depends on the properties of the fluid and the surface with which it is in contact alone\(^8\) the ratio $D/d$ will be bigger for smaller surfaces. For this reason if measurements are made with large surfaces the difference between $D$ and $d$ are so small that variations in the measured force can be attributed to instrument precision or uncertainty in any of the parameters of the experiment ($D$, $d$ or $\gamma$).

The following questions arise: Does the maximum allowed pulling force decrease as the distance between the surfaces increases and the diameter at the center decreases? Is $\gamma_1$ really constant, or does its value change as the amount of fluid present decreases? Are other factors such as fluid viscosity relevant? If so, why? Does it affect the ratio $D/d$ and consequently the maximum force? Does the velocity $dc/dt$ at which the surfaces are pulled apart influence the force?

A series of tests are being prepared to find an answer to all those questions.

\[
\begin{align*}
D & \quad \quad \quad \quad \quad 4 \\
S &= \frac{\pi D^2}{4} \\
S &= \frac{\pi D^2}{16}
\end{align*}
\]

Figure 5-5 Depending on the number of "wet surfaces" the perimeter may be kept constant while reducing considerably the overall wet surface or the total amount of fluid present.

Parameters that describe the environmental conditions have been omitted on purpose. We will assume that conditions are constant for the first stages of the analysis. At a

\(^8\) The angle $\alpha$ can be obtained from the following relation $\gamma_{\alpha} = \gamma_{s1} + \gamma_{v} \cos \alpha$, where $\gamma$ is half the work of cohesion between the different interfaces ($S$-solid; L-liquid and V-vapor). The work of cohesion corresponds to the work required to pull apart a volume of unit cross-sectional area from the bulk, creating two interfaces (liquid-vapor, liquid-solid or vapor-solid) of unit area [STO97].
later stage it would be interesting to study the relation between the forces and temperature or humidity conditions.

The domain of validity of the different models used to produce the equations in the appendix has to include the phenomenon we are studying at the scale we are studying it. The problem of the surface tension coefficient when the quantity of fluid is so small is described above and although there is evidence that its value changes as the quantity of fluid diminishes the models cited in the appendix use the surface tension coefficient found by using the "wire-frame technique" as described below. From the configuration of the wire frame it is assumed that the surface tension force depends only on the length or perimeter of the wetted surface. Maybe at a large scale (as compared to micro-systems) and with this experimental set-up the surface covered by the fluid is small compared to the perimeter it encloses so any possible effects related to the surface are diminished by the "oversized" perimeter. In other words, the question of whether a certain parameter depends on size or not should be posed before deliberately applying it to a domain where linear, surface and volumetric dimensions are several orders of magnitude smaller.

**Surface Tension**

*Definition:* Work/unit area  
*Symbol:* $\sigma$ (sigma)  
*Dimensions:* $FL^{-1}$ or $MT^{-2}$  
*Units:* U.S.: lbf/ft SI: N/m

\[
\sigma = \frac{\text{work}}{\text{unit area}} = \frac{dW}{ds} = \frac{2F_\sigma}{2Ldx} = \frac{F_\sigma}{L}
\]

or,
\[
F_\sigma = \sigma L
\]

Figure 5-6 Elements and notation for the wire-frame set-up used to calculate the surface tension coefficient of fluids at a conventional size. Would the coefficient found by using this method be the same as if the wire frame was 100 or 1000 times smaller? (from [MUR93]).
5.3 Friction forces in microassembly

In conventional size engineering the friction forces created at the interface of two parts are traditionally estimated by using the "friction coefficient" model, which states:

\[ F_F = \mu F_N \]  \hspace{1cm} 5-9

where:

- \( F_F \): friction force;
- \( \mu \): friction coefficient, and
- \( F_N \): force acting perpendicularly to the contact surface.

In this model the friction force is independent of the contact area, or analogously, of the contact pressure.

The reason why this model is acceptable is that for ordinary size parts in common applications the relation between material properties, surface roughness and normal force is such that conditions at the interface remain pretty constant. This allows obtaining empirical values for \( \mu \), and extrapolating them for common engineering situations.

Chang et al. present a more accurate model\(^9\) [CHA87]. The friction coefficient, \( \mu \), is defined as:

\[ \mu = \frac{Q}{F} = \frac{Q}{P_c - F_s} \]  \hspace{1cm} 5-10

where \( Q \) is the tangential force necessary to shear the junctions between the contacting asperities, and \( F \) is the external normal force, which is equal to the contact load \( P_c \), minus the adhesion force \( F_s \) acting between the surfaces in contact.

\(^9\) Their interest in modelling the friction coefficient arose from their need to calculate friction forces between the surface of hard disks and the reading head. In this scenario surfaces are very smooth and contact forces (hence pressure) are very small, making the traditional friction coefficient values very inaccurate.
This model is more accurate than the previous one, and although in principle it can be used indistinctly for big or small surfaces it does not work for small area contacts because in order to obtain a value for $Q$ statistical assumptions on asperity size and distribution are made. When the number of asperities is small it is evident that statistical values can no longer be used.

In addition, as we have seen in the previous section, the adhesion force $F_a$ is not easy to quantify.

Figure 5-7 Surface Forces vs. Weight of Part under different conditions. The values shown are mere indicators. The forces can vary over several orders of magnitude due to the influence of parameters that have not been fully identified. (Adapted from [KOE96]).

Highly variable surface forces due to humidity and electrostatic effects can significantly vary the normal force acting between the part and the contact surface. Figure 5-7 shows the relation between the surface force acting on a part due to different environment conditions and the weight; note that the vertical scale is exponential. Therefore even if the friction coefficient model could be used and the value of $\mu$ known, the resulting friction force could not be calculated in the traditional way ($F_F = \mu F_N$).

Because of all the uncertainties it is more effective to measure the friction forces between different materials and surface roughness in different environments and use the data to assess the friction force we can expect in our particular case. Beerschwitzinger, Yang, Reuben, et al. [BEE94] present an actuator capable of measuring the friction force between a micro-object and a surface, and the results obtained for copper and nickel parts sliding on a 450nm sputtered silver on chrome layer or on a
naked $\text{Al}_2\text{O}_3$ ceramic substrate. The friction coefficients obtained vary over a large range. This shows that for small objects (characteristic dimensions ranging from a few hundred microns to a couple of millimeters) the friction forces may bear no relation whatsoever with the normal force acting between the surfaces in contact.

5.3.1 Conclusions

Two groups of forces have been discussed in this section: surface forces and friction forces. The reasons that brought us to talk about them are quite different. The fact that surface forces become important with respect to the others as size is reduced is the reason why they are important in the micro-system world. On the other hand the model used to describe friction forces in the macro-domain is no longer valid as size is reduced; requiring a specific model for this range of sizes.

Although at first sight this may seem a trivial comment we are aware of many industrialized products and processes where those basic considerations have not been taken into consideration.

Excessive measures are not necessary either. For the sizes we deal with the introduction of the van der Waals forces produces no significant changes next to the others mentioned and are obviously left out. For similar reasons modeling the asperities by means of a simple geometric figure requiring one or at most two parameters to be completely defined will largely cover our needs; there is no need to simulate complex shapes that will only yield minor changes to our over all values while increasing in a non-quantifiable way the uncertainty of our analysis.

5.4 Force Measurement in Assembly

The term force-feedback is frequently used to denote a robot that performs force measurements of some kind. The reader should be aware of the difference between using force as feedback signal for control and processing force information to make corrections on the robots position feedback loop. We are not aware of any robotic system working with force as feedback signal. On the other hand, many examples can be found where force is successfully used to perform assembly operations. As the reader will discover in the remainder of this chapter, force measurement and subsequent processing is not a simple, straightforward task.

Damage to parts or equipment may be prevented with the use of force sensors. Even though this looks like a simple, straightforward application for force sensors and for the processing of the information acquired, reality is quite different. The high mechanical stiffness of the manipulator, which is required to achieve the necessary positioning accuracy, is the main cause for the high, but of short duration, forces that appear when contact between two parts is made. Youcef-Toumi et al. [YOU94] developed an analytical model for impact loads. Forces due to initial contact between manipulator
and object are several times higher than the expected one, which is attained after the initial impact and consequent vibrations are damped out (Fig. 5-8).

![Graphs showing force vs. time for three experiments.](image)

**Figure 5-8** Dimensionless force vs. dimensionless time plots of three impact tests (from [Yoo94]).

In consequence a maximum allowable force can't be used as a safety feature. Instead the energy invested during the contact phase is calculated and used as a triggering event to halt the process. This example shows that even for apparently simple applications a deep knowledge of the process and the systems involved is necessary.

For more complex operations where force sensors can be useful to determine the position of mating parts such as pin-in-hole insertion or to assess the amount of deformation in the assembly of very elastic elements such as O-rings, acquisition and subsequent processing of the information are far from being mastered. In many such cases the use of passive compliance greatly facilitates the assembly process by allowing the relaxation of several constraints [DRA77].
Force sensing can be used not only to avoid damage or to indirectly determine the relative position of two parts; a force-position plot can provide information on the quality of the assembled components. Force measurement must be performed continuously and synchronized with the position of the manipulator. This will allow us to construct a force-displacement curve from which the necessary information can be obtained. The number of parameters that influence a force-position plot is very large and their nature includes: dimensional tolerances, displacement velocities, surface finish, positioning accuracy, etc. Restrictions on the availability of force information (since it will only be present when there is contact between the parts) must also be considered.

Force-displacement data was obtained from an automatic press-fit pin-inserting machine where the vertical force is monitored. The force-displacement graph obtained can be used to determine if the operation has been correct as well as to identify several defects. The data is quite repetitive from test to test to the point that an envelope limiting the maximum and minimum forces can be defined. Figure 5-9 shows a typical force-displacement plot with its corresponding envelope. Valuable information can be obtained from this plot. First of all the initial peak corresponding to the initial contact between the pin and the borders of the hole; due to the difficulty of assessing the magnitude of the initial shock no limit is imposed to it. However if the offset between pin and hole is so high the insertion will not be possible. This problem is identified by monitoring the work expended during the shock, or by means of the graph, the area covered by the peak. An excess of work would mean an unacceptable offset and the operation would be aborted.

![Diagram of force-displacement plot for a press-fit pin insertion](image-url)
After the initial shock, the force starts increasing steadily as the pin penetrates the hole. "Noise" due to surface irregularities and materials elasticity can be observed at this stage. An interesting relation between the magnitude of the noise and the insertion velocity can be observed. If a certain velocity is exceeded the ripples become notably larger. They are thought to be caused by micro-welds that form due to local overheating and subsequent fracture. Additionally the slope of the straight line gives an indication of the interference between pin and hole. The sudden increase of the force at the end of the process indicates that the pin has made contact with the bottom of the hole. Once the pin is in its place the gripper releases it. During this phase a negative force appears. The shape of the plot corresponding to this last phase will depend on the type of gripper used.

5.5 Force measurement in microassembly

Position and angular inaccuracies between parts during a micro-assembly operation can’t be detected efficiently by optical means, especially when the parts are partially assembled.

The problem of parts tolerances is of great importance in micro-assembly (see chapter 3). Expensive vision and other optics based measurement systems that have to be recalibrated often, are not as reliable as they should, and the price we have to pay is high: in most cases an additional control station is required, or, if implemented on the assembly cell, the cycle time is increased considerably.

The forces generated during an assembly process are, in general, very sensitive both to tolerance changes and to position inaccuracies (Fig. 5-10). Moreover forces are measured while the assembly operation takes place so no significant increase of cycle time is generated.
Figure 5-10 Vertical force vs. insertion depth for two different offsets between hole and pin centers. The pin is made out of stainless steel and is 98 μm in diameter. The lower and upper plots correspond to an offset of 30 μm and 40 μm respectively. The abrupt change of slope indicates when the one the second contact point between the pin and the hole is established.

The chances of damaging parts during a micro-assembly operation are greater since the size relation between the parts and the equipment used to assemble them is defined by criteria other than the forces that intervene (e.g. axis resolution, working space, etc.).

A robot used to perform the pin in hole insertion depicted in Figure 5-10 has to be capable of withstanding the lateral forces given by:

$$ F = \frac{3EI}{l^3} \delta $$  \hspace{1cm} 5-11

where:

E = Young's modulus of elasticity,

I = Moment of inertia of cross-section (for cylindrical pin $I=\pi d^4/64$ with $d$=pin diameter),

$\delta$ = Offset between pin and hole axes,

l = Length of pin.
If the same robot is used to insert two different pins\(^\text{10}\) one having half the diameter and half the length (that is they are geometrically similar) of the other the force \(F\) varies by a factor of 2, instead of 4 that would be obtained without leaving the offset unchanged. This means that when we assemble the small pins we are either loading the pin closer to its maximum admissible elastic stress, or we are exceeding it causing a permanent deformation to it.

Using a robot capable of withstanding the forces of the large pin will certainly permit the assembly of the smaller pins (and also risk permanent deformation). Proceeding the other way round, that is, using a robot capable of resisting the forces imposed by the small pin would not allow for the assembly of the larger pins since the maximum allowable force would by exceeded and the regulator would disengage more often than not.

This example based on the elastic properties of the elements being assembled is quite trivial. In our laboratories we have observed that as the size of the pin is geometrically decreased the possibility of buckling increases. Once again we believe that surface forces are at the origin of this phenomenon. The ratio “insertion force/buckling force” for a given pin increases as the diameter decreases, which shows that insertion forces do not decrease proportionally to \([L]^2\) but somewhat slower. There are many possible causes for this, but among them the one that most interests us for our future research is that surface forces are not really proportional to the surface but to something between surface and length.

### 5.5.1 Position Feedback vs. Force Feedback

Robots usually move from one point in space to another following a determined path. In the case of assembly robots this must be done as fast and as accurately as possible. The knowledge of parameters such as actual position, actual velocity, mass, moment of inertia, required accuracy of final position, friction on guides, etc. and the use of well-known laws (Newton’s Laws of Motion especially) allow the controller to act in advance of the robots movements and perform the corrections that will permit the robot to attain the target position in the most efficient way.

Force feedback follows the same principle: we want to attain a certain force on the end-effector and to do this we measure the forces acting on it. With this information the controller can perform the required force and torque corrections till the desired force is obtained. There are some important technical problems when trying to implement such a system. Forces act on the end-effector only when it (or the part attached to it) physically contacts another object (inertia forces are left aside for the time being). As opposed to the robot’s position that can be obtained at any point of the

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\(^{\text{10}}\) By using the same robot we assume that the offset \(\delta\) remains constant.
working volume, force detection is limited to a small number of zones (the size of those zones depends on the elasticity of the combined robot-gripper-part system). An additional problem encountered in micro-assembly is that the magnitude of surface forces approaches that of the admissible contact forces that are present while executing an assembly operation and can jeopardize its success. Furthermore such forces appear before contact between part and gripper is made.

The force exerted on the end-effector has to be measured somehow. In conventional robotics the force sensing elements can be located in different positions:

- Sensors at robot joints,
- Sensor between last link and gripper,
- Sensors on robot platform/working table,
- Sensors on gripper.

In micro-assembly the force cannot be derived from knowledge of the forces or torques applied at each joint because there are too many disturbing factors that lead to inaccurate output.

The problem can be partially overcome by adding a force sensor in series with the end-effector. By doing this the "quality factor"\textsuperscript{11} of the robot will decrease considerably if the stiffness coefficient of the sensor is lower than that of the robot\textsuperscript{12}. There exists the possibility of placing the force sensor on the working table of the robot and attach to it the part onto which the robot will assemble the second part, or using force sensing grippers.

Let's assume we are now able to measure the force acting on the gripper in those limited zones. We next need a relation between the actual force, the target force and the relative position of the parts since it is the position of one of the parts that the robot can act on. This relation will depend on the elasticity of the robot at the given configuration, the elasticity of the parts, the stick & slip present at the joints of the robot, the axis controllers, etc. In order to obtain a robot with a force feedback control loop the relation between the magnitude and sense of those forces and the distance between the parts should be known. As long as this relation remains unknown or unpredictable, as is the case in the micro-world, the advantages of force feedback will be limited to control of static forces. Many articles on the subject are being published continually, but as we have seen in previous sections we are still far from being able to

\textsuperscript{11} The "quality factor" Q is defined by Demaurex [DEM79] as the product of the lowest resonance frequency of the robot arm and its maximum reach.

\textsuperscript{12} Unfortunately this is always the case.
predict accurately enough those surface forces [FED99] [ARA96] [FUK95] [ARA98] [FEA95] [HEC90].

The importance of force feedback in automated assembly should not be exaggerated; successful experiments have been made using force sensors to help in assembly tasks. However close analysis of the methods used led to the conclusion that success was not due to the use of a force feedback control loop but rather to the passive compliance introduced by the elasticity of the robot’s joints and force sensor.

**Gripping Forces**

The function of a gripper is to guarantee a fixed position and orientation of the part with respect to the last link of the robot. Inertia forces due to the acceleration of the robot and contact forces between the part and other objects present in the robot's working domain are the main disturbing factors. In micro-assembly the inertia forces are not as severe as in conventional assembly because the weight of the parts is much smaller in relation to the surface available for gripping. The gripper must therefore grasp the part firmly enough to prevent it from shifting while at the same time it must not deform it or damage its surface. This is where gripping force sensors come onto the scene.

The magnitude of the gripping force required depends on the following factors:

- the geometry of part and gripper (Fig. 5-11),
- the friction forces between gripper jaws and part.

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Positioning of a sphere by means of a vacuum gripper: The required suction force in the first case will be smaller because upon contact a force acting along the axis of the gripper will appear. On the second case, a shear force is applied to the suction cup and therefore a higher force is necessary to prevent shifting of the sphere.

Pin-in-hole insertion by means of a two-finger gripper: The gripping force of the left hand side gripper must be bigger to prevent vertical sliding of the pin because it is not equipped with a mechanical limit as is the case of the gripper on the right.

**Figure 5-11** Effects of contact point position and orientation on gripping force for two types of gripper.
In the micro-assembly domain the issue of force sensing grippers is to be reviewed thoroughly since the problem in many cases is the opposite of what has just been described: parts tend to stick to the gripper once the gripping action is cancelled. Pick & Place operations are not only troubled by the releasing problem. This force interaction occurs even before contact between part and gripper is made, so when the gripper is approaching a part to pick it up, the latter occasionally springs towards the former. Positioning problems appear at this point since any positioning reference established prior to the gripper’s approach is lost during the brief “flight” of the part.

5.6 Contact Forces

When two parts are being assembled together the position and orientation of one with respect to the other must be known. Measurement errors, robot inaccuracies and other disturbing factors originate an uncertainty of this relative position that can’t be measured by means of a vision system or other traditional position measurement devices. However the contact forces between the parts can vary drastically due to these inaccuracies. This can result in a defective assembly operation or can even damage the parts or the assembly equipment, but at the same time provide us with a mean of measuring those inaccuracies.

By measuring the forces acting on one of the parts not only damage can be prevented, but also valuable information about the actual state of the assembly process can be obtained. Knowledge of the forces gives some information about the relative position of the parts. The accuracy, and use we can make, of these forces depends on how well we know the elastic properties of both the robot and the parts being assembled.

The application of force sensors to automatic assembly equipment has not yet been developed sufficiently. At present force sensors can trigger an alarm or message if a certain force is exceeded, not only for safety reasons but also in peg-in-hole applications to indicate that there is a positioning problem of some sort [QIA93]. They are also used to exert a desired static, or quasi-static, force like for example in the case of a press. There exist some assembly operations that only require the direction and sense of the force acting on the part. However systems working with force as a feedback signal are not developed enough, and their effectiveness is still to be proved.

5.6.1 Working principle of a Force Sensor.

The principle for the force measuring devices mentioned in the literature is always the same: the force to be detected is applied to a flexible element whose displacement we measure. There exist different configurations for the flexible elements as well as different techniques to measure the displacement. Tanigawa’s force sensor [TAN89] consists of a flexible piezoresistive silicon membrane to which the force is applied (by
forces at the microsystem scale

means of an intermediate element). The displacement is measured thanks to the piezoelectric characteristics of the material. An electronic circuit will then convert the displacement to force. Ramakrishnan [RAM91] is original in the way he has to measure the deformation of the elastic element. The sensing element is a vibrating metallic glass element under tension. The external force changes the tension along the element, causing its resonance frequency to change. Modulated light excites the element by photovoltaic conversion and the resonant oscillations are detected by light guided through a fiber.

![Diagram of Ramakrishnan's force sensor](image)

Figure 5-12 Ramakrishnan's force sensor. This measuring strategy yields a range to sensitivity ratio of $10^5$. A force of up to 20 Newtons with a resolution of 200 milli-Newton can be detected (Adapted from [RAM91]).

Others use a simple cantilever beam as flexible element and an optical displacement meter to measure the displacement [NEL98]. Whenever designing a micro-force sensor a way of calibrating it should be available. The reason for this is that for the case of the cantilever beam, for example, "simple beam theory" is used to convert the measured displacement into force. Dimensional tolerances of the beam, which become more important as the size becomes smaller, play an important role in determining the rigidity constant of the beam, and therefore a means of checking the theoretical values should be provided. Arai, Nonoda et al. [ARA96b] present a cantilever beam but instead of measuring the displacement of the tip, measures the stress on certain parts of the beam using a laser Raman spectrophotometer. In theory when the structure is miniaturized, the stress per unit force is a more reliable measurement than displacement per unit force. A micro-force measuring device capable of measuring forces in two perpendicular directions consisting of two elastic parallelograms exists [BEN98], where the force is determined from the displacement of the parallelograms. Although measuring the stress instead of the displacement has advantages that are
related to the size of the beam, this sensor measures the displacement for the reasons that are described in the following paragraph.

There are force sensors that are deformed under a given force and are equipped with a system that allows them to create a force in the opposite direction until the deformation is cancelled. At this point the force done by the actuator equals the external force being applied to the sensor. This method is widely used in weighing scales. However this system allows measuring forces that do not depend on the position of the part that is applying the force. In the case of a weighing scale the weight of the part (force due to gravity) does not vary if the part is shifted slightly. The force exerted by an elastic element cannot be measured since when the force sensor applies the opposing force it will compress the elastic element and in turn will exert a larger force. Consequently we can say that in order to measure a force we must deal with the fact that the sensor has to deform.

5.6.2 Stiffness of a Micro-Force Sensor

As it has already been mentioned forces on the gripper can only be detected when the part is in contact with an element of the environment. In order for the robot to have time to measure the force as it increases towards the desired value and perform the required corrective tasks, the forces must be detectable before the targeted force appears. Modifying the stiffness coefficient of the force sensor can do this.

![Figure 5-13 Robot with force sensor on wrist.](image)

Figure 5-13 shows a robot equipped with a force sensor on its wrist, during an assembly operation. X₀ is the length of the elastic element of the force sensor when no force is being applied to it. Xₜ is the length of the elastic element at which the targeted force is reached. The distance X₀-Xₜ obviously depends on the stiffness of the force sensor. The importance of this value is explained in the following paragraph.
The force sensor detects no force as long as there is no contact between the part grasped by the robot and the environment. The moment there is contact the elastic element deforms and a force is measured. If the robot continues moving the force will increase. If the desired force is attained after a relatively large deformation of the sensor it means that "long" before the target force is reached the sensor was already measuring a force and therefore had time to perform the corrective measures. This leads to the conclusion that the stiffness of the sensor must be such that the magnitude of the desired force will be attained after a deformation that is big enough to give time and space for the robot to react is reached.

What really matters when measuring forces is the magnitude of the stiffness of the force sensors with respect to the robot's and the assembled part's stiffness. If the sensor has a considerably lower stiffness coefficient than the rest of elements involved those can be considered as being infinitely rigid as can be seen from the model described in Fig. 5-14 (springs in series). This can, in general, be achieved when assembling relatively large parts that can be firmly gripped and when the assembly involves only parts with high Young modulus. On the other hand it is difficult to achieve when there are very elastic elements involved in the process (when parts are glued or O-rings must be placed between two parts the term Kp decreases.), when parts have delicate surfaces and can't be grasped firmly enough and when working in the micro-assembly domain.

![Diagram](image)

**Figure 5-14** Simple model of a robot equipped with a force sensor on the wrist. Kr, Ks and Kp stand for the "stiffness" of the robot arm, the sensor and the parts being assembled respectively.
When Kr, Ks and Kp have similar values it will be difficult to know what distance we have to move the gripper to obtain the desired increase in force since the whole system will deform and as we will see in the next paragraph the stiffness of some elements are not known (only the stiffness of the force sensor is accurately known).

Force sensors on a robot can be installed on the robot's joints, wrist or working platform. Apart from the effects they have on the "quality factor" of the robot, the rest of problems introduced due to the use of a force sensor are common. The stiffness is composed of three main elements. The first two (elasticity and damping) have been thoroughly analyzed. The box with an interrogation mark inside stands for the third element. It is related to friction forces, mechanical inaccuracies, surface forces, control strategy, etc. The value of this third element is not known since the factors that influence it are not all identified, and those that have been identified will be difficult to model because in many cases they are time dependant or random. In addition we have evidence that its importance increases as the size and the resolution of the robot become smaller. Based on dimensional analysis and similitude laws used in other fields of engineering such as naval architecture we are developing methods that will allow us to quantify the magnitude of this element (see chapter 4).

The following conclusions regarding the use of a force sensor are obtained:

- Integration of a force sensor on an assembly robot can modify the "quality" factor\textsuperscript{13} of the robot depending on where it is placed.
- If a relation between the force measured at the wrist of the robot and its displacement in the directions of interest is not available, the advantages of using a force feedback reduce to the advantages of having a compliant end effector, or at best to having a damage prevention system capable of signaling when a given force is exceeded.
- The stiffness of the sensor must be small enough to allow the robot to receive information of the existing forces sufficiently in advance so as to allow for the corrective measures to be executed.
- If the stiffness of the robot, sensor and parts are similar control problems will appear.

5.7 Design of Micro-Force Sensors

As any other micro-system related element, micro-force sensors must be designed carefully because of the effects size reduction has on the processes involved. In this section we illustrate the procedure that allows identifying the most suitable working principle of a micro-force sensor. In the analysis parameters such as displacement

\textsuperscript{13} The degradation of the "quality" factor can be compensated by choosing a smoother velocity profile [DEM79]. This inevitably leads to longer cycle times.
measuring principle, maximum force, resolution and manufacturing tolerances are considered.

5.7.1 Dimensional Analysis as a Tool to Decide on the correct Measuring System

Two force sensors using the flexure of a cantilever beam as elastic element are discussed in this section. The measuring strategies followed are different; while one measures the displacement of the tip [NEL98] the second measures the maximum stress on the beam [ARA96b]. Both measurements are indirect measurements of force. They are related to the applied force by the mechanical properties and geometry of the cantilever beam in the following way:

For displacement measurement: \[ F = \frac{Ew^3}{4l^3} \delta \]  

Strain, \( \varepsilon \), rather than stress, \( \sigma \), is measured but are related by the equation \( \sigma = \varepsilon E \), where \( E \) is Young's modulus of elasticity and is a property of the material. In practice this strain is easiest measured on the most external fibers of the beam where it reaches its highest value. A look at the bending moment diagram of a cantilever beam like the one in Fig. 5-15 shows that the maximum stress, and therefore strain, occurs at the attachment point between the beam and the wall.

For stress measurement: \[ F = \frac{wl^2}{6l} \sigma_{\text{max}} \quad \text{or} \quad F = \frac{Ew^2}{6l} \varepsilon_{\text{max}} \]  

Both eqs. 5-12 and 5-13 have been obtained assuming the following:

- Plane sections through a beam, taken normally to its axis, remain plane after the beam is subjected to bending;
- Hooke's law is applicable to the individual fibers, i.e., stress is proportional to strain. The same elastic modulus \( E \) is assumed to apply to the material in tension as well as in compression;
- The forces applied to the beam are steady and delivered without shock or impact (energy methods can be used if shocks and impacts exist);
- The beam is stable under the applied forces.
Since we are designing a force sensor we are interested in the ratios $\frac{\delta}{F}$ and $\frac{\sigma_{\text{max}}}{F}$, which will give us an indication of the amount of displacement or stress we must be able to detect per unit of force applied to the sensor. Rearranging eqs. 5-12 and 5-13 we obtain:

$$\frac{\delta}{F} = \frac{4l^3}{Ewt^3} \tag{5-14}$$

$$\frac{\sigma_{\text{max}}}{F} = \frac{6l}{wt^2} \quad \text{or} \quad \frac{\epsilon_{\text{max}}}{F} = \frac{6l}{Ewt^2} \tag{5-15}$$

A look at the dimensions of the right hand side of eqs. 5-14 and 5-15 shows that $\frac{\delta}{F}$ is proportional to $[L]^{-1}$ while $\frac{\sigma_{\text{max}}}{F}$ is proportional to $[L]^{-2}$ ($[L]$ is a characteristic length of the cantilever beam). This means that if we make a cantilever beam $n$ times smaller $\frac{\delta}{F}$ will be $n$ times bigger and $\frac{\sigma_{\text{max}}}{F}$ will be $n^2$ times bigger. If we were capable of measuring with equal accuracy and resolution displacement and stress, there is no doubt that we would obtain a better ratio by measuring the stress, especially if trying to miniaturize the sensor further. In addition if by measuring say, stress we would obtain a higher resolution than required we could increase the stiffness of the cantilever and while maintaining the required resolution we would be able to measure a larger range of forces or measure the same forces but with a smaller displacement of the elastic element. At this point it becomes clear that it is of great importance to know the characteristics of the measuring systems available. An interesting example application of this method where the design of a system must be optimized in different ways can be found in [BEN99].
We developed a force sensor \cite{BEN98} where we measured displacement although in principle better measurements should have been possible by measuring stress. We tried to measure stress indirectly by means of strain gauges that we stuck to the surfaces of the elastic elements at the required places. The results did not quite agree with the theory and we later found out that during the machining of the elastic elements (wire EDM) a layer of metal results, which is not firmly attached to the bulk of the material. During flexure of the element this outer layer slides over the bulk metal giving an incorrect reading of the actual strain on the surface. A solution to this is first to avoid the formation of the outer layer by using other machining processes and by integrating the strain gauges directly on the elastic element \cite{HUB97}. Further miniaturized elastic elements offer the possibility of machining the strain gauge directly on its surface if it is made of silicon \cite{TAN89}.

Putting aside the characteristics of the displacement or stress sensors there is another factor that can be crucial when choosing what we are going to measure: the influence of manufacturing tolerances. As we have said before the elastic element, through its geometry and material properties converts a force into a displacement or a deformation. Since the relation between the input force and the displacement of the element differs from the relation between the input force and the stress at a point of the element, the influence of the manufacturing tolerances will be different and therefore the most favorable case should be identified. In our example of the cantilever beam we find two groups of tolerances: geometric tolerances and material properties tolerances. If $\delta$ and $\varepsilon_{\text{max}}$ are measured, the influence of the variation of $E$ will be the same for both cases and there is no advantage in using one or the other. However if we had a system capable of measuring $\sigma_{\text{max}}$ directly we would have a force sensor insensitive to variations in the material's properties.

Geometric tolerances must be analyzed in detail. Lets assume that all the dimensions have a tolerance equally proportional to their value. Eqs. 5-14 and 5-15 would become:

\[
\frac{\delta}{F} = \frac{4(l \pm al)^3}{E(w \pm aw)(t \pm at)^3}
\]

\[
\frac{\varepsilon_{\text{max}}}{F} = \frac{6(l \pm al)}{E(w \pm aw)(t \pm at)^2}
\]

Depending on the fabrication process some tolerances may be tighter than others, especially if chemical methods are used, but for clarity in the example we will assume that an "isotropic" fabrication process is involved.

The maximum values for $\delta/F$ and $\varepsilon_{\text{max}}/F$ will be:
\[
\frac{\delta}{F_{\text{max}}} = \frac{\delta}{F} \frac{1 + a}{(1 - a)^4} \quad \text{and} \quad \frac{\delta}{F_{\text{min}}} = \frac{\delta}{F} \frac{1 - a}{(1 + a)^4}
\]

\[
\frac{\varepsilon_{\text{max}}}{F_{\text{max}}} = \frac{\varepsilon_{\text{max}}}{F} \frac{(1 + a)}{(1 - a)^3} \quad \text{and} \quad \frac{\varepsilon_{\text{max}}}{F_{\text{min}}} = \frac{\varepsilon_{\text{max}}}{F} \frac{(1 - a)}{(1 + a)^3}
\]

For small systems tolerances of the order of 1% are common. If we substitute a by 0.01 we observe that \(\delta/F\) can vary, due to geometric tolerances, between 95% and 105% of the estimated value whereas \(\varepsilon_{\text{max}}/F\) varies between 96% and 104% of the calculated value. For mass produced force sensors where a control of the dimensions of each sensor is not economical the use of strain measurement will minimize the effect of parts tolerances. Evidently if the sensors go through a calibration stage the interest in this approach loses some of its strength but still it would allow to produce equally effective sensors using production techniques that achieve less tight tolerances and are more economical.
6.1 Dimensional Analysis: Glue Dispenser

The problem of the glue dispenser presented in chapter 4 has been developed using the power delivered to the fluid. Here we develop a new prediction equation for the diameter of the drop where we consider the velocity at which the membrane moves when it is hit by the hammer.

![Diagram of glue dispenser](image)

Figure 6-1 The problem of the glue dispenser. Pressure propagation due to velocity of membrane is considered instead of the power applied to the system.

Introducing the membrane velocity requires to introduce additional parameters. The primary quantities that will influence the drop will be:
P-Pressure within the fluid $[M][L]^{-1}[T]^{-2}$  

$V_m$-Velocity of membrane $[L][T]^{-1}$  

$t$- Time $[T]$  

$C$- Propagation velocity of wave in fluid $[L][T]^{-1}$  

$\rho$-Fluid density $[M][L]^{-3}$  

$\mu$-Dynamic viscosity of fluid $[M][L]^{-1}[T]^{-1}$  

$\sigma$-Surface tension of fluid $[M][T]^{-2}$  

$D_H$-Hole diameter $[L]$  

$L_H$-Hole length $[L]$  

$D_D$-Drop diameter $[L]$  

$V_D$-Drop velocity $[L][T]^{-1}$  

Apart from adding the velocity at which the membrane moves we have added the increase in pressure created by the shock wave propagating through the fluid. The time the membrane is moving is also included in the list for obvious reasons. Finally the velocity of wave propagation in the fluid is introduced.

The general prediction equation will be of the form:

$$C_\alpha D_D^{c_1} \rho^{c_2} V_m^{c_3} \mu^{c_5} \sigma^{c_8} D_H^{c_9} L^{c_{10}} = 1 \tag{6-1}$$

The dimensional equation corresponding to eq. 6-1 is, after rearranging terms:

$$[M]^{c_2 + c_6 + c_7 + c_8} [L]^{c_1 - c_2 + c_3 + c_5 - 3c_6 - c_7 + c_9 + c_{10}} [T]^{-2c_2 - c_3 + c_4 - c_5 - c_7 - 2c_8} = 1 \tag{6-2}$$

which leads to the system of three equations in 10 unknowns for $M$, $L$ and $T$ respectively:
\[ \begin{align*}
  c_2 + c_6 + c_7 + c_8 &= 0 \\
  c_1 - c_2 + c_3 + c_5 - 3c_6 - c_7 + c_9 + c_{10} &= 0 \\
  -2c_2 - c_3 + c_4 - c_5 - c_7 - 2c_8 &= 0
\end{align*} \]

The system is solved for \( c_2, c_3 \) and \( c_9 \). Arbitrary values have been assigned to the remaining seven unknowns. The following prediction equation is obtained:

\[ \frac{D_d}{D_H} = F\left( \frac{\sigma}{PD_H}, \frac{V_m}{C}, \frac{V_m^+}{D_H}, Re, We, \frac{L_H}{D_H} \right) \]

If we compare eq. 6-4 above with eq. 4-8 in chapter 4 we find some similarities. The last three TI terms are exactly the same. Obviously the effects the fluid, due to its motion and the geometry of the nozzle, has on the size of the drop does not depend on the way the energy is delivered to the system.

The term \( V_m/C \), known as the Mach number \( M \), is a measure of the tendency of flow to compress at a boundary or wherever the flow may be restricted. If fluids such as glue are being investigated, this term can be dropped out since the fluid is considered to be incompressible.

The first term \( \sigma/\pi D_H \) is similar in form to the first term in eq. 4-8 in chapter 4. If we consider Figure 6-2, the product \( \sigma \pi D_H \) denotes the force originated by the surface tension phenomenon at the interface between the drop and the nozzle. Note that the perimeter of the nozzle is equal to \( \pi D_H \). The cross-sectional area of the nozzle is \( \pi D_H^2/4 \). The pressure within the fluid \( P \) multiplied by the cross-sectional area yields the force exerted on the drop. The ratio of the surface tension forces to the pressure induced forces is \( 4\sigma/\pi D_H \) which, except for the constant, is identical to the first term in eq. 6-4.
Figure 6-2 Top: Picture of a drop as it leaves the nozzle. Bottom: Main parameters for the analysis and free-body diagram of the drop.

The diameter of the hole is small compared to its length on the prototypes tested in our laboratories. A pressure drop along the nozzle will certainly occur and dimensional analysis can be used to quantify it. If the fluid was ideal\(^1\) and the inner walls of the tube ideally smooth there would be no need for this analysis.

Many authors have done this analysis previously. The pressure drop is expressed as:

\[
\frac{\Delta P}{\rho v^2} = f \left( \frac{D_H}{L_H}, r, \frac{\rho v D_H}{\mu} \right)
\]

6-5

where:

\(\Delta P\) = Pressure drop \([ML^{-1}T^{-2}]\)

\(L_H\) = Length of hole \([L]\)

\(D_H\) = Diameter of hole \([L]\)

\(r\) = Roughness of pipe \([-]\)

\(^{1}\) If a fluid has no viscosity and does not flow in a turbulent manner, the fluid is said to be an ideal fluid, or more correctly the flow is said to be ideal. An ideal flow then has no internal friction and hence no internal dissipation or losses.
\( \rho = \text{Density of fluid} \ [\text{ML}^{-3}] \)

\( \mu = \text{Viscosity of fluid} \ [\text{ML}^{-1}\text{T}^{-1}] \)

\( v = \text{Velocity of fluid} \ [\text{LT}^{-1}] \)

In this analysis the acceleration due to gravity, \( g \), has been omitted since the influence of weight in this case is negligible. Within the nozzle no fluid surface or interface is involved. Therefore the surface tension of the fluid is not introduced in the calculations.

Extensive data has been recollected for different values of the Pi terms shown in eq. 6-5. Figure 6-3 is an example where \( \Delta P / p v^2 \) is plotted against Reynolds number \( (p v D_{in}/\mu) \). The aspect ratio of the nozzle, \( D_{in}/L_{in} \) is fixed to 1/2 and two pipes of different roughness have been used.

![Figure 6-3 Plot of \( \Delta P / p v^2 \) against Reynolds number for two tubes of different surface roughness and \( D_{in}/L_{in}=1/2 \) [MUR50].](image)

This chart allows to determine the pressure drop within a tube provided that the viscosity and density of the fluid are known as well as the geometry involved.

The roughness is expressed in eq. 6-5 by a dimensionless parameter. It is obvious that a single parameter does not suffice to express the effects surface irregularities have on the fluid flow. Figure 6-4 shows the different parameters that describe the characteristics of the wall roughness.
Figure 6-4 Surface roughness descriptors.

Considering the parameters in Figure 6-4 the prediction equation for the surface roughness, \( r \), would be of the form:

\[
  r = \Phi \left( \frac{a}{l}, \frac{b}{l}, \frac{c}{l}, \frac{d}{l}, \frac{e}{l}, \frac{g}{l}, S_f \right)
\]

where:

- \( s_f \)=Dimensionless shape factor\(^2\)
- \( g \)=Spacing between asperities in the direction perpendicular to the paper
- \( l \)=Length of pipe

The quantities in eq. 6-6 are usually evaluated by using statistically mean values of the individual dimensions (an example of using statistical methods to describe surface roughness can be found in [CHA87]).

In order to determine the influence surface roughness has on flow experiments were carried out and the results expressed in the form of a graph (Fig. 6-5).

---

\(^2\) This dimensionless shape factor evaluates the effect of a projection of the type at A on Figure 6-4 relative to that of the projection at B.
Figure 6-5 Overall effect of roughness on pressure drop [MUR50].

The friction factor, $f$ that appears on the vertical scale of Figure 6-5 is defined as:

$$f = \frac{2\Delta P d}{\rho v^2 l}$$

From the graph we see that the effects of roughness on flow decrease as the Reynolds number decreases and has virtually no effect for $Re<2000$.

### 6.2 The Bonding Effect Numbers (Be)

In this section we present an interesting dimensionless parameter that appears in the analysis of different micro-assembly related situations. Consider a vacuum gripper like the one in Fig. 6-6. The coefficient $\gamma$ denotes the force per unit area exerted at the interface between the part and the gripper. $\lambda$ similarly denotes the force per unit distance exerted at the perimeter of the same contacting area. The adhesion forces are represented by two coefficients instead of one because previously we have seen that surface tension forces are neither proportional to the contact area nor its perimeter. Furthermore using two parameters such as $\gamma$ and $\lambda$ seems reasonable since the interface between two bodies usually consists of one or more surfaces whose main characteristics are their surface and perimeter.

Since a gripper is normally attached to a robot and therefore is subject to accelerations in directions other than that of gravity, the parameter $\alpha$ is used to denote the overall acceleration the part is exposed to.
Figure 6-6 Schematic view of a vacuum gripper with the different parameters that play a role on the gripping force required to grasp the part.

In this case we are interested in the conditions for which we will be able to pick and place the part. The parameters that may influence the gripping force are:

\(\alpha\)-Acceleration of part \([LT^{-2}]\)

\(\rho\)-Density of part \([ML^{-3}]\)

\(L\)-Characteristic length of part \([L]\)

\(\lambda\)-Linear force coefficient \([MT^{-2}]\)

\(\gamma\)-Surface force coefficient \([ML^{-1}T^{-2}]\)

\(P\)-Under pressure generated by gripper \([ML^{-1}T^{-2}]\)

\(D\) or \(d\)-Characteristic diameter of gripper \([L]\)

\(F\)-Gripping force \([MLT^{-2}]\)

The generic form of the prediction equation is of the form:

\[ F = f(\alpha, \rho, L, \lambda, \gamma, P, d) \]

The same equation can be expressed by means of Pi terms as:
\[
\frac{F}{\rho a L^3} = f\left(\frac{ad}{\gamma}, \frac{ad^2}{\lambda}, \frac{ad}{P}, \frac{L}{d}\right)
\]

When conventional size objects have to be manipulated by means of a vacuum gripper (in normal size manipulation they are mainly suction cups) equation 6-9 is used with the particularity that the first two terms on the right-hand side are neglected.

In effect, eq. 6-9 would become:

\[
\frac{F}{\rho a L^3} = f\left(\frac{ad}{P}, \frac{L}{d}\right)
\]

which includes all the design parameters we would require to build (or in the case of a suction cup, choose) the appropriate suction cup and vacuum pump.

The first two terms in eq. 6-9 are referred to as the "Bonding Effect" numbers for microassembly (Be) since at the micro-system scale they become preponderant in the design of grippers and other situations where interfacial adhesion phenomena intervene. The first one of them, \(ad/\gamma\), has a great similarity with the Reynolds number. If numerator and denominator are multiplied by \(d^2\) we obtain

\[
\frac{\rho a d^3}{\gamma d^2}
\]

which represents the ratio of inertia forces to surface phenomena induced forces. Similarly the Reynolds number represents the ratio of inertial forces to viscous forces.

There is a significant difference in the parameters used in both dimensionless numbers. The viscosity of a fluid can be measured from a sample of the fluid and if environmental conditions remain pretty constant so will the viscosity. This is not the case for the surface force coefficient \(\gamma\). It varies so much that statistical values have to be taken. [ARA98] statistically measures \(\gamma\) by placing small particles on the external surface of a cylinder. Spinning the cylinder creates a centrifugal force on the spheres, which ejects part of them. By controlling the angular velocity of the cylinder and keeping a count of the remaining particles a distribution of the magnitude of the adhesion forces can be obtained. Some of the data obtained with this method is plotted in Fig. 6-7 where four different types of surface were used. SiO\(_2\) spheres with a diameter of 30\(\mu\)m were adhered to all the surfaces. Before running the experiments precautions were taken to reduce to a minimum the influence of surface tension forces and hydrogen bonding forces.
Figure 6-7 Experimental data that allows estimating the value of $\gamma$.

Figure 6-7 shows that $\gamma$, for a specific pair of surfaces, can vary by a factor of three or even four, which leads us to wonder if a similar thing would occur with the viscosity of a fluid when small amounts are involved. To our knowledge measurements to determine the viscosity of a fluid are done using relatively large amounts of it. What would happen if the volume of fluid used to determine its viscosity is drastically reduced? Whether the fluid’s viscosity $\mu$ would suffer a variation or not is of great interest for the micro-system domain as we may now see if we look back at the example of the micro-dispenser.

Returning to the gripping numbers, the second term in eq. 6-9 is similar in form to the first one but it takes into consideration forces related to linear dimensions. The presence of surface tension induced forces would be reflected in this Pi term.

Figure 5-7 in chapter 5 can be seen as a table from which the “combined” Bonding effect number can be obtained for different objects under different conditions. The “surface” forces mentioned in the graph include both linear and surface dependent forces. An advantage of using the dimensional analysis method is that data concerning Be can be collected when studying the problems of micro-parts handling and later on this same data can be directly applied to see, for example, if such parts are compatible with a bowl feeder$^3$.

---

$^3$ The “bowl feeder example presented in chapter 5 could be approached using the gripping numbers.
6.3 Assembly of a high sensitivity Hall sensor

Magnetic sensors based on the Hall effect are widely used in industry. They can be found in home appliances, computers, speed meters, electric windows, ABS braking systems, etc. The need for smaller, more sensitive and less expensive sensors is hence justified. As its name indicates a Hall sensor measures the magnitude of a magnetic field by means of the Hall effect, which is described in the following section.

6.3.1 The Hall Effect

In a Hall effect meter there is a current flowing in a thin conducting bar as shown in Fig. 6-8. Suppose that the current I flowing from left to right, corresponds to the net motion of electrons from right to left with mean drift velocity \( v \). The existence of a magnetic flux density \( \vec{B} \) in the direction shown induces a force \( \vec{F} \) on the electrons equal to \(-ev \times \vec{B}\), which tends to move them towards face 2 of the bar. This tendency will continue until the resulting charge distribution generates an electric field \( \vec{E}_L \) that prevents more electrons from shifting towards face 2. At that point the electric force \(-e\vec{E}_L\) on the electrons is equal and of opposite direction to that due to the magnetic field:

\[
-e\vec{E}_L = ev \times \vec{B}
\]  

6-12
Figure 6-8 When a current $I$ flows in a conducting bar placed in a magnetic flux $B$, charges are produced on faces 1 and 2. This charge distribution induces a uniform transverse electric field.

The appearance of the transverse electric field is known as the Hall effect. The magnitude of the electric field in the bar is constant and equal to $vB$. Its direction is from face 1 to face 2. If faces 1 and 2 are separated by a distance $l$ the potential difference between them is:

$$V_{12} = E_l l = vBl$$  \hspace{1cm} 6-13

The current $I$ is given by:

$$I = NevA$$  \hspace{1cm} 6-14

where:

- $N=$ Number of conduction electrons per unit volume,
- $A=$ Cross-sectional area of the bar.

Solving eq. 6-13 for $v$ and substituting into eq. 6-14 gives:

$$V_{12} = \frac{IL}{NeA} B$$  \hspace{1cm} 6-15

Therefore a measurement of the potential difference across the bar gives the magnitude of the magnetic flux density $B$ as long as the other quantities in eq. 6-15 are known.

### 6.3.2 The High Sensitivity Hall Sensor

As we have seen, a simple piece of conducting material with a suitable number of conduction electrons per unit volume, a current generator and a voltmeter are the only elements necessary to build a Hall sensor. Proceeding like this would result in an extremely low sensitivity sensor. A way to increase the sensitivity is to use magnetic flux concentrators. As the name indicates, their purpose is to concentrate the magnetic flux towards the Hall sensor that is located between them (Fig. 6-9). The problem of the concentrators in existing sensors is that they are big and expensive (often more expensive than the sensor itself).
Figure 6-9 Hall sensor with flux concentrators used at present in industry.

The use of low cost integrated flux concentrators that are compatible with microelectronic integration processes allows reducing the cost of the concentrators to a quarter of the cost of the sensor. In addition the sensor's detectivity is increased considerably [BLA96]. The advantages of such concentrators are being studied at the Institute for Microsystems (IMS) of the EPFL.

Figure 6-10 Micro Hall sensor with integrated flow concentrators. On the right, a schematic view of the sensor where the circular active region can be seen.

6.3.3 Part Positioning

The main problem here is that the concentrators have not got a regular outline as can be seen in Fig. 6-12. Should the outline be perfect it would be easy to identify the short border and work out the position of the part from it.
Figure 6-11 Main dimensions of flux concentrator (a) and assembly tolerances (b). Since the ferromagnetic material the concentrator is made of has a much better magnetic permeability than air it can be assumed that the flux is concentrated about six times (1400/230) as long as saturation is not reached.

Since the outline of the part is not regular enough we should not define a reference mark with respect to it. However since the concentrator has not got any relevant feature we are forced to use the outline to define its position. Several strategies that allow averaging out the edge imperfections can be used. The center of gravity of the part or its main axes of inertia (which are easily and rapidly determined by a vision system) are not very sensitive to outline imperfections.

Figure 6-12 Close up image of a concentrator. Observe the irregularities at the edges and the lack of parallelism between the upper and lower edges. All the information required to position the part must be taken from this image.
Before a choice is made it is useful to understand the functionality of this part within the sensor. The Hall sensor is designed to work within a certain range of flux density. Additionally flux homogeneity must be kept within certain limits to avoid magnetic flux saturation. Should this occur the flow going through the sensor would remain constant while the excess flow would "leak" from the concentrators and bypass the sensor, resulting in false readings.

The reasons that may lead to magnetic flux saturation are:

- Misalignment of concentrators, especially in the z direction (Fig. 6-11b).
- Lack of parallelism between short edges of opposing concentrators.
- Incorrect design of concentrator (ratio of length of long edge to that of the short edge too big).

Considering the difference in magnetic permeability between air or silicon and the ferromagnetic material of the concentrator, the shape of the sides of the concentrator is irrelevant (it could be a simple trapeze).

The distance between the edge of the concentrator and the Hall sensor\(^4\) will also influence the magnetic flow through it. All those considerations have led the designers to impose the assembly tolerances shown in Fig. 6-11.

---

\(^4\) The task of positioning the concentrators with respect to the Hall sensor is simpler because clearly identifiable marks that can be accurately identified and positioned by the vision system are available.
From the remarks made above it is clear that the functional features of the concentrator are two: the short edge and the ratio of lengths of both edges. Therefore, although we are restricted by the part's irregular outline, it has to be positioned with respect to the short edge. Since its position is determined by averaging the irregularities it is evident that the longer the edge the more precisely we will be able to determine an average line. As we have said before, the main axes of inertia are rapidly determined by a vision system and can be used to check the validity of the averaging strategy by comparison\(^5\).

### 6.3.4 Part Manipulation

The concentrators have large, flat and resistant surfaces that make use of a simple vacuum or electro-magnetic gripper possible. The chip containing the Hall probe can be manipulated with the same tools as long as contact with the gripper is made where no damage will be done; that is the place where the concentrators will be positioned. The size of the concentrators does not allow stating that gravitational forces will always exceed the surface bonding forces (the Bonding effect numbers aren't large enough). Attention will have to be paid when gripping the parts to avoid a shift in the relative position between part and gripper. Releasing the part should pose no problem since the presence of the glue will assure that the bonding forces at the glue-part interface are larger than at the part-gripper interface. Nevertheless, the position accuracy demanded in the z direction requires a good understanding of the polymerization process of the glue.

### 6.3.5 Design for Manufacturing

An analysis of the list of operations of the assembly procedure for this product [UEH99] reveals that the major difficulty and most time consuming step in the assembly process is the positioning and alignment of the concentrators. A high positioning precision is demanded whereas the concentrator's geometry (surface roughness and parallelism of edges) does not allow for it.

---

\(^5\) One of the main axes of inertia will always be parallel to the straight-line edges.
In order to overcome the alignment problem and simultaneously reduce the cycle time the concentrators could be produced and assembled in a single part as seen in Fig. 6-14. Once in place the temporary positioning rods can be removed. Additional positioning rods attached to the narrow ends of the concentrators could be added to avoid bending if, due to glue polymerization effects, the tolerances in the z direction are difficult to respect. This idea was born while analyzing the technology used to produce the concentrators. Since they are produced by a photolithographic method a perfect alignment and positioning are assured. Evidently the geometry in Fig. 6-14 should be revised to avoid damaging sensitive parts of the silicon chip.
Chapter 7: Conclusions

Two main objectives have been followed in this dissertation. The first one has consisted in making the concerned people aware of the effects size reduction has on microsystems at the different levels. The second one is a proposition of a method based on dimensional analysis and similitude that helps researchers in their search for prediction equations that describe phenomena occurring at the microsystem.

Size influences both positively and negatively the characteristics and behavior of microsystems and related fields such as microassembly. Usually when the effects are negative the cause is that a conventional size system has been scaled down without taking into consideration the dependence of the different phenomena on size. The most common mistake is that phenomena that are important at conventional scale and completely irrelevant at the microsystem level are not neglected; the opposite is also true: in many cases important phenomena at the micro level are neglected simply because they are neglected too at larger scales.

One of the areas that is heavily influenced by miniaturization is that of assembly. Properties inherent to microsystems result in an extension of the functions grouped under assembly. In the microsystem domain assembly is not limited to putting together the different elements; control, calibration and testing routines must be incorporated to the assembly process. In addition the constructive characteristics of many microsystem components, where functional and physical dimensions are unrelated, requires a completely new approach to part detection, manipulation and positioning. Special attention is paid to micro-part manipulation due to the fact that forces present at the common contact surfaces are of the same order of magnitude (if not bigger) as inertia forces.

The dependence of the behavior of phenomena (that are found frequently in conventional size life and therefore can be predicted easily) can be obtained straightforward. However this does not suffice to obtain an effective design for a microsystem. Many effects that appear only at small scale can't be predicted as accurately as those mentioned in the preceding paragraph. Frequently even the origin of such effects is unknown. The methods of dimensional analysis and similitude presented here allow prediction equations for those effects to be obtained. By means of several examples we show that dimensional analysis and similitude can be used to obtain prediction equations that give us a relation between the relevant parameters of the system. A great advantage of dimensional analysis is that we can concentrate on the study of the parameters we are interested in. The remaining "secondary" parameters have as well an influence on the system's performance and it is reflected when experiments are carried out.
The methods have limitations: in the worst case, for a situation that does not have all the necessary requisites for the application of the methods, the researcher will obtain extra information that comes from the physical dimensions of the parameters involved (principle of homogeneity). Nonetheless, the real advantage of those methods is that with some imagination and experience from the side of the engineer, more or less useful results can be attained. Furthermore results or information from previous studies where similar\(^1\) effects take place can be very useful since they can considerably reduce the amount of experimental work and give hints on the behavioral tendencies of the effect under consideration.

7.1 Future Work

Several examples concerning the use of dimensional analysis and similitude have been presented throughout the report. The purpose has been to show how the techniques are applied and how information coming from previous work can be incorporated to a current research project.

The example of the micro-drop dispenser will be continued in the immediate future due to the importance of being able to apply small quantities of fluid (grease, oil, glue, etc.) in very precise locations. Experimental setups of dispensers working under different principles have already been built [RAN99] in the scope of MINAST project No. 6.04 and will be used to obtain the necessary information required for the dimensional and similitude analysis.

The field of application of those methods is not restricted to strictly physical phenomena. Economic aspects of microsystems can be analyzed too. This was done by Mrs. Koelmeijer [KOE99] when the assembly costs of two pressure sensors (a conventional one and a microsystem based one) were compared. In the future cost analysis will be extended to microassembly equipment, assembly plant organization, etc.

The study of the complex domain of surface and friction forces will also be pursued. Knowing some of the phenomena that originate those forces (van der Wals forces, Coulomb's law and surface tension) allows us to have a base from which influent parameters can be obtained. The methods presented here can help establish the relations that exist between those forces and the different geometry, material and environment influent parameters.

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\(^1\) The term "similar" is not restricted to the dictionary meaning. See Appendix A for an explanation of the word "similarity".
In the longer term we expect that the methods of dimensional analysis and similitude will be used by a larger number of microsystem researchers. This will lead to the establishment of several Pi terms that will be used as indicators for the phenomena that are important at this scale (in the same way as the numbers of Froude, Reynolds, etc. are used in other branches of engineering). Once settled in, more and more experimental data relating the Pi terms between them will become available. At that point it will be easy for microsystem designers to extrapolate data from other systems to their own. All aspects of microsystems, design, part manufacturing, assembly, calibration, etc. will be improved while expensive and time-consuming experimental research will be reduced to a strict minimum.
Appendix A: Dimensional Analysis and Similitude

A.1 Dimensional Analysis

Dimensional analysis is a powerful analytical tool developed from consideration of the dimensions in which each of the pertinent quantities involved in a phenomenon is expressed. It is based on Fourier’s principle of dimensional homogeneity (1822), which states that an equation that expresses a physical relationship between quantities must be dimensionally homogeneous, or, in other words, the dimensions of each side of the equation must be the same.

A.1.1 Basis of Dimensional Analysis

Considering our methods of measurement and evaluation of quantities two axioms can be derived from Fourier’s principle:

Axiom 1: Absolute numerical equality of quantities may exist only when the quantities are similar qualitatively.

Axiom 2: The ratio of the magnitudes of two like quantities is independent of the units used in their measurement, provided that the same units are used for evaluating each.

A.1.2 Applications of Dimensional Analysis

The most important uses dimensional analysis has in engineering are:

i) Classifying equations and indicating their generality.
ii) Converting equations or data from one system of units to another.
iii) Developing equations.
iv) Systematizing the collection of data in an experimental program and reducing the number of variables that must be investigated.
v) Establishing the principles of model design, operation and interpretation.
A.2 Determination of exponents by dimensional analysis

The equation for a given phenomenon may be expressed as:

$$\alpha = C_{\alpha} a_1^{c_1} a_2^{c_2} \ldots a_n^{c_n}$$  \hspace{1cm} A-1

where the $a$ terms are referred to as the primary quantities and $\alpha$ as the primary quantity. $C_{\alpha}$ is a dimensionless term that must be determined experimentally. At the same time both the primary and secondary quantities can be expressed in terms of basic quantities as the basic quantities raised to appropriate powers. Any purely mechanical quantity may be expressed dimensionally as:

$$[A] = F^{d_1} L^{d_2} T^{d_3} \quad \text{or} \quad [A] = M^{d_1} L^{d_2} T^{d_3}$$  \hspace{1cm} A-2

where:

$F$=force; $L$=length; $T$=time and $M$=Mass. (Square brackets are used to indicate that the dimensions of the quantity and not their magnitude is considered.)

Note that $F$, $L$ and $T$ in the first equation and $M$, $L$ and $T$ in the second are two bases\(^1\) that span the whole "space" of primary and secondary quantities we can find in a purely mechanical phenomenon. For problems involving heat or electricity additional basic quantities must be used. Those could be temperature ($\Theta$), and electric charge ($Q$). Throughout this report we stick to the $M$, $L$, $T$ base.

Knowing the general form of the equation for any phenomena and the factors that will influence the secondary quantity (that is the primary quantities) dimensional analysis can be applied, as described in the following example, to determine the values of the exponents.

A.2.1 Example: The Simple Pendulum

Suppose we want to find an equation that allows us to calculate the period of a pendulum (in vacuum) as a function of the variables we think can influence it.

\(^1\) None of the elements of the base can be obtained by combination of the other two.
In this example it is obvious that the only parameters that can affect the period are the mass $m$ of the pendulum, its length $l$, and the gravity $g$.

Therefore the equation for the period, $T$, will be of the form:

$$T = C_\alpha m^{c_1} l^{c_2} g^{c_3} \quad \text{(A-3)}$$

According to axiom 1 the equation has to be dimensionally homogeneous.

The primary and secondary quantities can be expressed in terms of their basic quantities:

$$T = [T]$$
$$m = [M]$$
$$l = [L]$$
$$g = [L][T]^{-2} \quad \text{(A-4)}$$

The dimensional form of eq. A-3 is:

$$[T] = [M]^{c_1} [L]^{c_2} [L]^{c_3} [T]^{-2c_3} \quad \text{(A-5)}$$
And since the dimensions on the left hand side must be equal to those on the right hand side we can write the following set of equations:

for [T]: \[ 1 = -2c_3 \]
for [M]: \[ 0 = c_1 \]
for [L]: \[ 0 = c_2 + c_3 \]

from which we obtain the values for the exponents:

\[
\begin{align*}
  c_1 &= 0 \\
  c_2 &= \frac{1}{2} \\
  c_3 &= -\frac{1}{2}
\end{align*}
\]

Equation A-3 can be rewritten as:

\[ T = C_\alpha \sqrt{\frac{1}{g}} \] \hspace{1cm} \text{(A-6)}

and only one simple experiment\(^2\) is required to obtain the value of \(C_\alpha\) and have a complete equation.

The example of the pendulum given in the previous section is relatively simple and straightforward because the unknowns \(c_1\), \(c_2\) and \(c_3\) can be determined from the three

\(^2\) Actually this is not so simple because it may occur that \(C_\alpha\) be a function of the other variables.
equations we obtained thanks to the homogeneity principle (one for each basic dimension involved). But what happens when the number of exponents is bigger than the number of basic dimensions?

### A.3 Form of Dimensional Equations

In previous sections we have assumed that any quantity can be expressed dimensionally in terms of appropriate primary quantities in the following way:

\[
\alpha = C_\alpha a_1^{c_1} a_2^{c_2} \ldots a_n^{c_n} \tag{A-7}
\]

In a more general form any secondary quantity can be expresses as a function of the primary quantities:

\[
\alpha = f(a_1, a_2, ..., a_n) \tag{A-8}
\]

If we continue with the pendulum \( \alpha \) is the period of oscillation while \( a_1, a_2 \) and \( a_3 \) are the pendulum’s mass, length and gravity acceleration.

Our purpose is to establish the nature of the function.

Let the equation

\[
\beta = f(b_1, b_2, ..., b_n) \tag{A-9}
\]

describe the same phenomenon as eq. A-8. Both eq. A-8 and eq. A-9 are of the same nature; only the numerical values differ.

If the primary quantities in eqs. A-8 and A-9 are expressed in different units, the secondary quantities will change respectively to \( \alpha' \) and \( \beta' \). In equation form it is expressed as:

\[
\alpha' = f(x_1 a_1, x_2 a_2, ..., x_n a_n) \tag{A-10}
\]

and

\[
\beta' = f(x_1 b_1, x_2 b_2, ..., x_n b_n) \tag{A-11}
\]

in which \( x_1, x_2, ..., x_n \) represent the ratios of the set of units used for \( a \) and \( b \) and the set of units used for \( \alpha' \) and \( \beta' \). For example if \( a_1 \) were the length of the pendulum measured in meters and \( x_1 a_1 \) the length measured in centimeters, \( x_1 \) would equal 1/100.
From axiom 2,

\[ \frac{\alpha'}{\beta'} = \frac{\beta}{\alpha} \quad \text{or} \quad \frac{\alpha'}{\beta'} = \frac{\alpha}{\beta} \quad \text{A-12} \]

Substituting eqs. A-8 through A-11 into eq. A-12 we obtain:

\[
f(x_1a_1, x_2a_2, ..., x_na_n) =
= \frac{f(a_1, a_2, ..., a_n)}{f(b_1, b_2, ..., b_n)} f(x_1b_1, x_2b_2, ..., x_nb_n) \quad \text{A-13}
\]

Equation A-13 can be differentiated partially with respect to each \( x_i \) resulting in a series of equations of the form

\[
a_i \frac{\partial f(x_1a_1, x_2a_2, ..., x_na_n)}{\partial (a_ix_i)} =
= \frac{f(a_1, a_2, ..., a_n)}{f(b_1, b_2, ..., b_n)} b_i \frac{\partial f(x_1b_1, x_2b_2, ..., x_nb_n)}{\partial (b_ix_i)} \quad \text{A-14}
\]

Setting the \( x_i \) terms in eq. A-14 to 1 and rearranging terms yields:

\[
a_i \frac{\partial f(a_1, a_2, ..., a_n)}{\partial a_i} = b_i \frac{\partial f(b_1, b_2, ..., b_n)}{\partial b_i} \quad \text{f(a_1, a_2, ..., a_n)} = \frac{f(b_1, b_2, ..., b_n)}{f(a_1, a_2, ..., a_n)} \quad \text{A-15}
\]

This equation must hold for all values of \( a_i \) and \( b_i \). For any given value of \( b_i \), the right-hand side of the equation is constant and can be designated as \( c_i \). It can then be rewritten as:

\[
\frac{\partial f(a_1, a_2, ..., a_n)}{\partial a_i} \quad \frac{1}{f(a_1, a_2, ..., a_n)} = \frac{c_i}{a_i} \quad \text{A-16}
\]

An equation of the same sort as eq. A-16 exists for each value of \( b_i \) and each value of \( a_i \). Each equation is a differential equation of the general relationship between
\( f(a_1, a_2, ..., a_n) \) and the particular \( a_i \) involved. Since \( a_1, a_2, ..., a_n \) are independent (the mass of the pendulum can be changed without altering its length or the gravity constant) eq. A-16 can be expressed as

\[
\frac{df(a_1, a_2, ..., a_n)}{f(a_1, a_2, ..., a_n)} = c_i \frac{da_i}{a_i} \tag{A-17}
\]

which can be integrated straightforward to give

\[
\ln f(a_1, a_2, ..., a_n) = c_i \ln a_i + \text{const.} \tag{A-18}
\]

Repeating the same procedure for each \( a_i \) term the general solution becomes:

\[
\ln f(a_1, a_2, ..., a_n) = c_1 \ln a_1 + c_2 \ln a_2 + ... + c_n \ln a_n + \ln C_\alpha =
\]

\[
= \ln a_1^{c_1} a_2^{c_2} ... a_n^{c_n} + \ln C_\alpha =
\]

\[
= \ln C_\alpha a_1^{c_1} a_2^{c_2} ... a_n^{c_n} \tag{A-19}
\]

or

\[
f(a_1, a_2, ..., a_n) = C_\alpha a_1^{c_1} a_2^{c_2} ... a_n^{c_n} \tag{A-20}
\]

from which

\[
\alpha = C_\alpha a_1^{c_1} a_2^{c_2} ... a_n^{c_n} \tag{A-21}
\]

that agrees with eq. A-7.

### A.4 Development of Prediction Equations

Prediction equations can be developed by following one of the two following methods:

i) Experimental method

ii) Analytical method

The first one consists in establishing, by careful observation and measurement, the effect of the pertinent variables upon the quantity to be predicted. The other method
consists in applying those "natural laws\(^3\) which are pertinent to the problem in order to develop relationships among the significant variables. The experimental method will be discussed here.

Three general experimental procedures may be followed, and they will be described by applying them to the problem of a falling body: find an expression for the distance \(s\) a body will fall in time \(t\).

1) The first method consists in simply expressing the secondary quantity as a function of the primary quantities:

\[
s = f(v, g, t, m)
\]

Experimental data has to be recorded and values of \(s\) compared with or plotted against the values of the controllable primary quantities. Proceeding this way may lead to false conclusions and is time consuming. Besides if more than three independent variables are involved, establishing a relation among them can become quite difficult.

2) Dimensional analysis can be used to establish a relationship among the variables. Equation A-22 would become

\[
s = C_{a} v^{c_{1}} g^{c_{2}} t^{c_{3}} m^{c_{4}}
\]

There are five unknowns in eq. A-23, \(C_{a}, c_{1}, c_{2}, c_{3}\) and \(c_{4}\). Five sets of observations can be introduced into eq. A-23 producing five equations in five unknowns. \(C_{a}\) may be eliminated by division resulting in a system of four equations in four unknowns. Since the remaining unknowns are the exponents which are constant they may be determined from these four equations. Since \(C_{a}\) is not necessarily a constant, it is not necessarily determinable from any of the four original equations, but must be evaluated by multiple substitutions or by constructing a graph. The latter procedure involves some difficulties, especially when \(C_{a}\) is a function of all the primary quantities. One improvement consists in systematizing the data by holding all the primary quantities but one constant, and varying it to study its effect on the secondary quantity. The same procedure is repeated for the remaining primary variables.

3) The last approach consists in pushing further the use of dimensional analysis. It consists in equating the dimensions of the quantities present in eq. A-23 as was done for the pendulum. This leads to:

\[^{3}\text{The natural laws used in this method are simply generalizations of reliable information, which has been assembled through observation and measurement.}\]
\[ s = C_\alpha v t \left( \frac{g t}{v} \right)^{c_1} \quad \text{A-24} \]

The body's mass which, we know from other sources, does not influence its period no longer appears in our equation. This automatically reduces the number of primary quantities to investigate. Furthermore the relation between the dimensions of the different quantities involved has reduced the number of unknowns to just two, \( C_\alpha \) and \( c_1 \). If it were a constant only two sets of observations would be necessary. Generally it is not the case so a series of readings must be taken. In this case it is convenient to rearrange eq. A-24 as follows:

\[ \frac{s}{v t} = C_\alpha \left( \frac{g t}{v} \right)^{c_1} \quad \text{A-25} \]

The values of \( s/v t \) can be plotted against the values of \( g t/v \). The resulting curve can be used to obtain the values for both \( C_\alpha \) and \( c_1 \):

If the experimental data appears as a straight line (as is the case of this example) the unknowns are determined straight away.

If the data plots as a curve, values of \( C_\alpha \) and \( c_1 \) can be determined by substituting \( s/v t \) and \( g t/v \) for two representative points from the graph into eq. A-25. Solving the resulting two equations in two unknowns yields values for \( C_\alpha \) and \( c_1 \). The accuracy of these values may be checked by means of a third point of the graph.

When a large number of primary quantities is involved in a phenomenon it will not be possible to describe it with an equation as simple as eq. A-25 since there are too many unknown exponents and not enough dimensional relations to reduce them to a single exponent. However the process above can be applied, with some adaptations, to more complex problems. The procedure will be described by means of a sophisticated version of the problem of the falling body as shown in Fig. A-1.
Determine the form of an equation for the distance $s$ that a sphere will fall in a time $t$ if it starts with an initial velocity $v_o$ and the resistance of the fluid through which it falls is considered.

Figure A-1 The problem of a sphere falling through a fluid.

The parameters that influence the distance $s$ are, in addition to $g$, $v_o$, $t$ and $m$:

- $d$-sphere diameter, $[L]$
- $\rho$-fluid density, $[M][L]^{-3}$
- $\mu$-fluid viscosity, $[M][L]^{-1}[T]^{-1}$

The general expression for $s$ is then:

$$s = f(g, v, t, m, d, \rho, \mu)$$  \hspace{1cm} A-26

or

$$s = C_\alpha g^{c_1} v_0^{c_2} t^{c_3} m^{c_4} d^{c_5} \rho^{c_6} \mu^{c_7}$$  \hspace{1cm} A-27

Rewriting eq. A-27 in dimensional form gives:

$$L = (LT^{-2})^{c_1} (LT^{-1})^{c_2} T^{c_3} M^{c_4} L^{c_5} (ML^{-3})^{c_6} (ML^{-1}T^{-1})^{c_7}$$  \hspace{1cm} A-28

Homogeneity of a physical equation leads to the three component auxiliary equations:

$$M \rightarrow 0 = c_4 + c_6 + c_7$$  \hspace{1cm} A-29

$$L \rightarrow 1 = c_1 + c_2 + c_5 - 3c_6 - c_7$$

$$T \rightarrow 0 = -2c_1 - c_2 + c_3 - c_7$$
We have three equations and seven unknowns. Three of the unknowns can be expressed in terms of the other four. A possible combination would be to express \( c_1, c_2, \) and \( c_7 \) as a function of the remaining unknowns. Those can be substituted back in eq. A-27 that, after rearranging gives:

\[
\frac{sg}{v_0^2} = C_\alpha \left( \frac{gt}{v_0} \right)^{c_1} \left( \frac{g^2m}{v_0^3\mu} \right)^{c_3} \left( \frac{gd}{v_0^2} \right)^{c_4} \left( \frac{v_0^3\rho}{g\mu} \right)^{c_7}
\]

Equation A-27 containing eight unknowns has been simplified to an equation with five unknowns, which in case of having to be solved experimentally would mean a substantial reduction of laboratory work. Eq. A-30 is just one of the several that are possible. Another combination would result in:

\[
\frac{s}{v_0^t} = C_\beta \left( \frac{gt}{v_0} \right)^{c_1} \left( \frac{d}{v_0^t} \right)^{c_5} \left( \frac{\rho v_0^3t^3}{m} \right)^{c_6} \left( \frac{\mu t^2v_0}{m} \right)^{c_7}
\]

### A.5 Meaning of \( C_\alpha \)

The equations for a falling body in vacuum (eq. A-25) and eq. A-31 are similar in form. If the experiments required to obtain a value for \( C_\alpha \) in eq. A-25 were carried out with a sphere falling in air, \( C_\alpha \) would automatically take its effect into consideration. That is

\[
C_\alpha = \phi \left( \frac{gt}{v_0}, \frac{d}{v_0^t}, \frac{\rho v_0^3t^3}{m}, \frac{\mu t^2v_0}{m} \right)
\]

In consequence eq. A-31 can be rewritten as

\[
\frac{s}{v_0^t} = \Phi \left( \frac{gt}{v_0}, \frac{d}{v_0^t}, \frac{\rho v_0^3t^3}{m}, \frac{\mu t^2v_0}{m} \right)^4
\]

The term \( C_\alpha \) is a function of the dimensionless groups of the variables influencing the phenomenon. Those dimensionless groups are referred to as Pi terms and are designated as \( \pi_i \). Using this notation, eq. A-33 becomes in general terms

\[
\pi_1 = \Phi(\pi_2, \pi_3, ..., \pi_d)
\]

---

*Note that all the terms in this equation are dimensionless.*
where \( s \) denotes the total number of dimensionless groups influencing the phenomenon.

The number of \( \Pi \) terms required to describe a phenomenon can be found by applying the Buckingham \( \Pi \) theorem.

**A.6 The Buckingham \( \Pi \) Theorem.**

The theorem states that the number of dimensionless and independent quantities required to express a relationship among the variables in any phenomenon is equal to the number of quantities involved, minus the number of dimensions in which those quantities are measured, or in equation form:

\[
 s = n - b \tag{A-35}
\]

where:

\( s \) = the number of \( \Pi \) terms;

\( n \) = the total number of quantities involved;

\( b \) = the number of basic dimensions involved.

The previous example involved eight quantities and three dimensions. Thus five \( \Pi \) terms are necessary.

The general form of a prediction equation (eq. A-21) can be rearranged as

\[
 C_a q_1^{c_1} q_2^{c_2} \ldots q_n^{c_n} = 1 \tag{A-36}
\]

Its corresponding dimensional equation is:

\[
 (d_1^{x_{11}}, d_2^{x_{21}}, \ldots, d_b^{x_{b1}})^{c_1} (d_1^{x_{12}}, d_2^{x_{22}}, \ldots, d_b^{x_{b2}})^{c_2} \ldots \]

\[
 \ldots (d_1^{x_{1n}}, d_2^{x_{2n}}, \ldots, d_b^{x_{bn}})^{c_n} = 0 \tag{A-37}
\]

where the \( d_i \) terms designate the \( b \) basic dimensions involved.

Eq. A-37 can be rewritten as a set of auxiliary equations one for each basic dimension):
\[ x_{11}c_1 + x_{12}c_2 + \ldots + x_{1n}c_n = 0 \]
\[ x_{21}c_1 + x_{22}c_2 + \ldots + x_{2n}c_n = 0 \]
\[ \ldots \]
\[ x_{b1}c_1 + x_{b2}c_2 + \ldots + x_{bn}c_n = 0 \]

This set of equations contains \( n \) unknowns. Any \( b \) of the unknowns can be expressed in terms of the remaining \( n-b \) unknowns as long as the determinant of the coefficients of the \( b \) unknowns selected does not equal zero, i.e. they must be linearly independent.

Consequently \( b \) of the exponents in eq. A-36 can be replaced by their equivalents from eq. A-38. Thus the number of exponents in eq. A-26 will be reduced to \( n-b \). Terms with the same exponent can be grouped leading to dimensionless groups since each exponent satisfies the simultaneous equations based on dimensional homogeneity.

The \( \pi \) terms have a single requisite: they must be dimensionless and independent.

### A.7 Establishment of Pi terms

Several procedures can be used for the determination of a suitable set of Pi terms. The following procedure is used throughout this thesis:

i) write the auxiliary dimensional equations

ii) assign arbitrary numerical values to \( s \) of the unknown exponents

iii) solve the resulting set of simultaneous equations

iv) combine the result to form one Pi term

v) repeat steps ii) to iv) until \( s \) Pi terms are determined

vi) combine the results in the form indicated by eq. A-34.

To illustrate the procedure we will solve the problem of the falling sphere:

The prediction equation in its general form

\[ C_\alpha s^{c_1} g^{c_2} v_0^{c_3} + c_4 m^{c_5} d^{c_6} \rho^{c_7} \mu^{c_8} = 1 \]

is written in dimensional form:

\[ L^{c_1} \left( LT^{-2}\right)^{c_2} \left( LT^{-1}\right)^{c_3} T^{c_4} M^{c_5} \ldots \]

\[ \ldots L^{c_6} \left( ML^{-3}\right)^{c_7} \left( ML^{-1}T^{-1}\right)^{c_8} = 0 \]
from which the following system of equations derives:

\[
\begin{align*}
M & \rightarrow c_5 + c_7 + c_8 = 0 \\
L & \rightarrow c_1 + c_2 + c_3 + c_6 - 3c_7 - c_8 = 0 \\
T & \rightarrow -2c_2 - c_3 + c_4 - c_8 = 0
\end{align*}
\]  

A-41

The system consists of three equations and eight unknowns. Therefore arbitrary values must be assigned to 8-3=5 of the unknowns. From all the possible combinations we choose to give values to \( c_1, c_2, c_6, c_7 \) and \( c_8 \). The determinant of the three remaining coefficients in eqs. A-41 is

\[
\begin{vmatrix}
0 & 0 & 1 \\
1 & 0 & 0 \\
-1 & 1 & 0
\end{vmatrix} = 1
\]  

A-42

The selection we have made is valid since the resulting equations are independent.

The next step consists in assigning arbitrary values to the variables. For convenience simple values are assigned as follows:

\[
c_1 = 1 \\
c_2 = c_6 = c_7 = c_8 = 0
\]  

A-43

Substituting those values in eqs. A-41 and solving for \( c_3, c_4 \) and \( c_5 \) gives:

\[
c_3 = -1; c_4 = -1; c_5 = 0
\]

Substituting those values into eq. A-39 and dropping \( C_a \) yields

\[
\pi_1 = \frac{s}{v_0^f}
\]  

A-44

The same procedure is repeated four more times with the following choices:

For \( \pi_2 \):

\[
\begin{align*}
c_2 & = 1 \\
c_1 = c_6 = c_7 = c_8 & = 0 \\
\Rightarrow \pi_2 & = \frac{g^f}{v_0}
\end{align*}
\]
For $\pi_3$:
\[ c_6 = 1 \quad c_1 = c_2 = c_7 = c_8 = 0 \quad \Rightarrow \pi_3 = \frac{d}{v_0^t} \]

For $\pi_4$:
\[ c_7 = 1 \quad c_1 = c_2 = c_6 = c_8 = 0 \quad \Rightarrow \pi_4 = \frac{\rho v_0^3 t^3}{m} \]

For $\pi_5$:
\[ c_8 = 1 \quad c_1 = c_2 = c_6 = c_7 = 0 \quad \Rightarrow \pi_5 = \frac{\mu v_0 t^2}{m} \]

A general solution to the problem may be written as:
\[
\frac{s}{v_0^t} = F\left( \frac{gt \cdot d \cdot \rho v_0^3 t^3 \cdot \mu v_0 t^2}{v_0 \cdot v_0^t \cdot m \cdot m} \right)
\]

An infinite number of correct solutions exist, but just a few will be relatively simple and will reflect the physical phenomena that hide behind each of the terms.

To obtain simpler Pi terms it is not necessary to go over the whole process with a different set of values. Since the Pi terms are independent and dimensionless they can be combined between them to produce new terms that will still be independent and dimensionless.

For example the following new terms can be obtained:

\[ \pi_6 = \frac{\pi_4}{\pi_5} = \frac{\rho v_0^2 t}{\mu} \]
\[ \pi_7 = \pi_3 \pi_6 = \frac{\rho v_0 d}{\mu} \]
\[ \pi_8 = \pi_3^2 \pi_4 = \frac{\rho d^3}{m} \]
\[ \pi_9 = \pi_2 \pi_3 = \frac{gd}{v_0^2} \]

Eq. A-45 would then become:
\[
\frac{s}{v_0^t} = F_2\left( \frac{gt \cdot gd \cdot \rho d^3 \cdot \rho v_0 d}{v_0 \cdot v_0^t \cdot m \cdot \mu} \right)
\]
The Pi terms are now presented in a way that help understand several of the phenomena that take place while the sphere falls in the fluid. As an example, the third term on the right-hand side of eq. A-47 is nothing else but the ratio between the force due to gravity and the buoyancy force. Archimedes principle appears without having even talked about it in our analysis. This is important when laboratory work is to be carried out since it gives us an idea of what the importance of each term is. If we were dealing with a steel sphere falling through the air we could surely neglect this term because we know its influence is minimal. Being able to neglect some of the terms may prove very useful when models are built.

### A.7.1 Common Pi Terms

Some of the most useful Pi terms are listed in Table A-1 together with the problems in which they are relevant. Since dimensional analysis has been mostly used to solve situations involving the flow of fluids, most of the terms are related to it. However at the end of the table we find the Bonding Effect Numbers that appear when working in the micro-system domain.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Definition</th>
<th>Physical Significance</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary Geometry</td>
<td>Slenderness=$\frac{L}{D}$</td>
<td>Scale boundary geometry</td>
<td>General analysis.</td>
</tr>
<tr>
<td></td>
<td>Roughness=$\frac{\varepsilon}{D}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Aspect Ratio=$\frac{S}{D}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euler Number</td>
<td>$\frac{\Delta P}{\frac{1}{2}\rho U^2}$</td>
<td>Pressure Force _ Inertia Force</td>
<td>General fluid dynamic analysis.</td>
</tr>
<tr>
<td>Reynolds Number</td>
<td>$Re = \frac{\rho UD}{\mu}$</td>
<td>Inertia Force _ Viscous Force</td>
<td>General viscous fluid dynamic analysis.</td>
</tr>
<tr>
<td>Mach Number</td>
<td>$M = \frac{U}{c}$</td>
<td>Fluid Velocity _ Speed of Sound</td>
<td>Employed in flows with $M&gt;0.3$. At this point the flow tends to compress at the boundaries. Example: supersonic aircraft.</td>
</tr>
<tr>
<td>Number</td>
<td>Formula</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------------------</td>
<td>--------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Froude</td>
<td>Fr = ( \frac{U}{\sqrt{gD}} )</td>
<td>Free surface fluid dynamics. Example: effect of ocean waves on vessels.</td>
<td></td>
</tr>
<tr>
<td>Weber</td>
<td>We = ( \frac{U}{\sqrt{\sigma/\rho D}} )</td>
<td>Fluid dynamics of small free surface flows. Example: microdispenser.</td>
<td></td>
</tr>
<tr>
<td>Cavitation</td>
<td>( C = \frac{2(p_v - p)}{\rho U^2} )</td>
<td>Liquid flows in which static pressure in fluid may fall below vapor pressure of fluid.</td>
<td></td>
</tr>
<tr>
<td>Richardson</td>
<td>( Ri = \frac{-g \frac{\partial p}{\partial z}}{\frac{\rho}{\partial z} \frac{\partial U}{\partial z}} )</td>
<td>Stability of fluid field with density stratification. Example: thermal-induced density stratification in oceans.</td>
<td></td>
</tr>
<tr>
<td>Rayleigh</td>
<td>( Ra = \frac{gD^3 \left[ \rho(T_1) - \rho(T_2) \right]}{\alpha u \rho} )</td>
<td>Free convective flows impelled by thermal gradients.</td>
<td></td>
</tr>
<tr>
<td>Strouhal</td>
<td>( St = \frac{U}{fD} )</td>
<td>Fluid flows with periodic oscillations.</td>
<td></td>
</tr>
<tr>
<td>Bonding Effect</td>
<td>For linear related forces: ( Be = \rho \frac{aD^2}{\lambda} )</td>
<td>Problems involving contact between small parts or between small parts and a surface. Example: micro-grippers, micro-bowl feeders.</td>
<td></td>
</tr>
<tr>
<td>Numbers</td>
<td>For surface related forces: ( Be = \rho \frac{aD}{\gamma} )</td>
<td>( Surface_Forces ) ( Inertia_Forces )</td>
<td></td>
</tr>
</tbody>
</table>

Table A-1 List of most common PI terms.
The notation for table A-1 follows:

$c = speed of sound,$
$D, S, L = characteristic lengths,$
$a, g = acceleration (due to robot movement or gravity),$
$p = fluid static pressure,$
$p_a = ambient pressure,$
$T = temperature,$
$U = flow velocity,$
$z = vertical distance,$
$\alpha = thermal diffusivity,$
$\gamma = surface force coefficient,$
$\varepsilon = asperity height (in Boundary geometry parameter),$
$\nu = kinematic viscosity,$
$\lambda = linear force coefficient,$
$\rho = fluid density,$
$\sigma = surface tension.$

A.8 Determination of Functions

By using dimensional analysis and Buckingham's Pi theorem we obtain a prediction equation that is a function of the different \( \pi \) terms. In order to formulate a useful prediction equation the function that relates those terms must be established. The only way to do this is by means of experimental data. Different series of experiments have to be run each time keeping all but one \( \pi \) terms fixed. At this point we can clearly see the advantage of simultaneous use of dimensional analysis and the Buckingham \( \pi \) Theorem: the reduction in the number of variables that must be investigated.
The function $F$ in the general equation (eq. A-34) may denote any combination of the $\Pi$ terms. However it is interesting to investigate the conditions that are necessary and/or sufficient for certain simple combinations to exist.

### A.8.1 Conditions for $F$ to be a product

Consider a phenomenon where three $\Pi$ terms are involved:

$$\pi_1 = F(\pi_2, \pi_3)$$  \hspace{1cm} (A-48)

Experiments where $\pi_2$ is varied while keeping $\pi_3$ constant would be carried out. The relationship

$$(\pi_1)\bar{3} = f_1(\pi_2, \pi_3)$$  \hspace{1cm} (A-49)

where the bar denotes constant values can be obtained by plotting $\pi_1$ against $\pi_2$.

From a second set of experiments the following relation can be established too:

$$(\pi_1)\bar{2} = f_2(\pi_2, \pi_3)$$  \hspace{1cm} (A-50)

Eqs. A-49 and A-50, called component equations, can be combined under certain conditions to form the general prediction equation by multiplication

$$\pi_1 = C(\pi_1)\bar{3} (\pi_1)\bar{2}$$  \hspace{1cm} (A-51)

In order to determine those conditions $C$ has to be determined. This can be done by assuming that the component equations multiply themselves to form the general equation

$$F(\pi_2, \pi_3) = f_1(\pi_2, \pi_3)f_2(\pi_2, \pi_3)$$  \hspace{1cm} (A-52)

If this is true, the first set of tests (where $\pi_3$ is kept constant) will give:

$$F(\pi_2, \bar{\pi}_3) = f_1(\pi_2, \bar{\pi}_3)f_2(\pi_2, \bar{\pi}_3)$$  \hspace{1cm} (A-53)

from which

$$f_1(\pi_2, \bar{\pi}_3) = \frac{F(\pi_2, \bar{\pi}_3)}{f_2(\pi_2, \pi_3)}$$  \hspace{1cm} (A-54)

Proceeding in a similar way while $\pi_2$ is kept constant leads to:
\[ f_2(\pi_2, \pi_3) = \frac{F(\pi_2, \pi_3)}{f_1(\pi_2, \pi_3)} \tag{A-55} \]

Values of \( f_1(\pi_2, \pi_3) \) and \( f_2(\pi_2, \pi_3) \) are substituted into eq. A-52 to give

\[ F(\pi_2, \pi_3) = \frac{F(\pi_2, \pi_3)F(\pi_2, \pi_3)}{f_2(\pi_2, \pi_3)f_1(\pi_2, \pi_3)} \tag{A-56} \]

The denominator of eq. A-56 is found from eq. A-52 with both \( \pi_2 \) and \( \pi_3 \) constant. Substituting into eq. A-56 results in:

\[ F(\pi_2, \pi_3) = \frac{F(\pi_2, \pi_3)F(\pi_2, \pi_3)}{F(\pi_2, \pi_3)} \tag{A-57} \]

From this equation we see that the value of \( C \) is:

\[ C = \frac{1}{F(\pi_2, \pi_3)} \tag{A-58} \]

and that the two component equations must have the same form.

To test the validity of the product combination we use a third set of experimental data in which one of the \( \Pi \) terms is held constant at a different value than in the preceding set of data. Eq. A-57 was obtained by holding \( \pi_2 \) at a value of \( \overline{\pi}_2 \). If this is valid it could have also been determined for a set of data in which \( \pi_2 = \overline{\pi}_2 \). Consequently

\[ F(\pi_2, \pi_3) = \frac{F(\pi_2, \pi_3)F(\pi_2, \pi_3)}{F(\pi_2, \pi_3)} \tag{A-59} \]

The right-hand side of eq. A-57 must be equal to the right-hand side of eq. A-59. Therefore:

\[ \frac{F(\pi_2, \pi_3)}{F(\pi_2, \pi_3)} = \frac{F(\overline{\pi}_2, \pi_3)}{F(\overline{\pi}_2, \pi_3)} \tag{A-60} \]

In a similar way, if \( \pi_3 \) had been held constant at a different value \( \overline{\pi}_3 \) we would obtain that

\[ \frac{F(\pi_2, \pi_3)}{F(\pi_2, \pi_3)} = \frac{F(\pi_2, \overline{\pi}_3)}{F(\pi_2, \overline{\pi}_3)} \tag{A-61} \]
Those last two equations are a test for the validity of eq. A-57. *If the third set of experimental data satisfies either eq. A-60 or A-61 the general equation can be formed by multiplying the component equations together and dividing it by the constant given by eq. A-58.*

### A.8.2 Conditions for $F$ to be a sum

In some cases a prediction equation containing three $\Pi$ terms may be of the form

\[ F(\pi_2, \pi_3) = f(\pi_2) + g(\pi_3) \quad \text{(A-62)} \]

If $\pi_2$ is held at the constant value $\overline{\pi}_2$ and rearranging the terms we get:

\[ g(\pi_3) = F(\overline{\pi}_2, \pi_3) - f(\overline{\pi}_2) \quad \text{(A-63)} \]

Similarly for $\pi_3 = \overline{\pi}_3$:

\[ f(\pi_2) = F(\pi_2, \overline{\pi}_3) - g(\overline{\pi}_3) \quad \text{(A-64)} \]

Substituting eqs. A-63 and A-64 into eq. A-62 gives:

\[ F(\pi_2, \pi_3) = F(\pi_2, \overline{\pi}_3) - g(\overline{\pi}_3) + F(\pi_2, \pi_3) - f(\overline{\pi}_2) \quad \text{(A-65)} \]

or

\[ F(\pi_2, \pi_3) = F(\pi_2, \overline{\pi}_3) + F(\pi_2, \pi_3) - F(\overline{\pi}_2, \overline{\pi}_3) \quad \text{(A-66)} \]

If the general prediction equation is formed by an addition of the component equations a constant must be subtracted.

A supplementary set of experimental work where one of the $\Pi$ terms is held constant at a value different from its magnitude in the original component equation is used to test the validity of eq. A-66. When $\pi_2$ is held at the value $\overline{\pi}_2$ eq. A-66 becomes

\[ F(\pi_2, \pi_3) = F(\pi_2, \overline{\pi}_3) + F(\overline{\pi}_2, \pi_3) - F(\overline{\pi}_2, \overline{\pi}_3) \quad \text{(A-67)} \]

Equating the right-hand sides of eqs. A-66 and A-67 yields:

\[ F(\overline{\pi}_2, \pi_3) - F(\overline{\pi}_2, \overline{\pi}_3) = F(\overline{\pi}_2, \pi_3) - F(\overline{\pi}_2, \overline{\pi}_3) \quad \text{(A-68)} \]
Therefore if the data of the third set satisfies this equation we may form the prediction equation by addition of the component equations.

Proceeding in a similar way for \( \pi_3 = \overline{\pi}_3 \) we obtain a second test for the validity of eq. A-62:

\[
F(\pi_2, \pi_3) - F(\overline{\pi}_2, \overline{\pi}_3) = F(\pi_2, \overline{\pi}_3) - F(\pi_2, \pi_3)
\]

A.8.3 Additional considerations

The methods discussed in the previous two sections dealt with problems involving only three \( \Pi \) terms. However they can be applied whatever the number of \( \Pi \) terms there are.

If the general prediction equation for a system involving \( s \) \( \Pi \) terms is formed by multiplication of the component equations, it can be shown that the form is:

\[
\pi_1 = \frac{F(\pi_2, \pi_3, \ldots, \pi_s) F(\pi_2, \pi_3, \ldots, \overline{\pi}_s) \ldots F(\pi_2, \overline{\pi}_3, \ldots, \pi_s)}{[F(\pi_2, \pi_3, \ldots, \overline{\pi}_s)]^{s-2}}
\]

A-70

In this equation the \( \overline{\pi}_i \) terms are the same in each component equation.

A general test for separating one \( \Pi \) term from the remainder and combining the two component equations by multiplication may be developed by using the technique employed for a system containing three \( \Pi \) terms as described in section A.8.1. The test becomes:

\[
\frac{F(\pi_2, \pi_3, \ldots, \pi_s)}{F(\overline{\pi}_2, \pi_3, \ldots, \pi_s)} = \frac{F(\pi_2, \overline{\pi}_3, \ldots, \pi_s)}{F(\overline{\pi}_2, \pi_3, \ldots, \pi_s)}
\]

A-71

If the test is satisfied, the equation can be written as:

\[
F(\pi_2, \pi_3, \ldots, \pi_s) = \frac{F(\pi_2, \pi_3, \ldots, \pi_s) F(\pi_2, \pi_3, \ldots, \pi_s)}{F(\pi_2, \overline{\pi}_3, \ldots, \pi_s)}
\]

A-72

The original prediction equation may be resolved step by step into a series of products of component equations of one variable as long as each component equation passes the test for combination as a product.

When the general prediction equation involving \( s \) \( \Pi \) terms is formed by addition of the component equations the form is:
\[ \pi_1 = F(\pi_2, \pi_3, \ldots, \pi_s) + F(\pi_2, \pi_3, \ldots, \pi_s) + \\
+ F(\pi_2, \pi_3, \ldots, \pi_s) - (s - 2)F(\pi_2, \pi_3, \ldots, \pi_s) \]  \hspace{1cm} A-73

The test for the validity of combining component equations by addition is developed term by term. If it is possible to isolate the term containing \( \pi_2 \) we obtain:

\[ \pi_1 = F(\pi_2, \pi_3, \ldots, \pi_s) + F(\pi_2, \pi_3, \ldots, \pi_s) - F(\pi_2, \pi_3, \ldots, \pi_s) \]  \hspace{1cm} A-74

An additional set of data in which one of the variables is given a different value (\( \pi_3 \) for example) should result in the same value of \( \pi_1 \). Therefore:

\[ F(\pi_2, \pi_3, \ldots, \pi_s) - F(\pi_2, \pi_3, \ldots, \pi_s) = \\
= F(\pi_2, \pi_3, \ldots, \pi_s) - F(\pi_2, \pi_3, \ldots, \pi_s) \]  \hspace{1cm} A-75

This equation is a test for the combination of the component equation containing \( \pi_2 \) with the component equation containing the other variables by addition. Separate tests may be run for each of the component equations containing one variable \( \pi_i \) term.

It may occur that a prediction equation be composed of component equations that combine by multiplication and by addition. In such cases the appropriate tests must be applied for each component.

**A.9 Similitude and Models**

Many engineering problems can be solved by direct application of well-known laws such as Newton's laws of motion. Other problems, however, involve too many variables, or the complexity is such that an analytical solution is not possible. It may also occur that an analytical solution can't be obtained since the laws that describe a particular condition of state are unknown.

When dealing with the last two groups of problems dimensional analysis together with experimental data can lead to the formulation of a prediction equation that describes the phenomenon under consideration.

This works as long as the number of primary quantities, and consequently the number of \( \pi \) terms, is small enough to allow for a reasonably limited amount of experimental work and more important, to be able to reduce the acquired data to a general formula. Unfortunately some engineering problems don't fulfill those requirements.
In many of those cases a general prediction equation is not necessary. An indication on how the different variables relate between them for a specific design, or within a small margin will yield sufficient information to allow for a proper design.

This valuable information can be obtained rapidly and relatively inexpensively by means of models.

A.9.1 Models

A model can be defined as a device, which is so related to a physical system, that observations on the model might be used to predict accurately the performance of the physical system in the desired respect. The physical system for which the predictions are to be made is called the prototype.

A.9.2 Theory of Models

The general prediction equation for a given phenomenon (eq. A-34) is entirely general. It can be used for any other system that is function of the same variables. Therefore it may be used for a specific system we will refer to as the model:

\[ \pi_{1m} = F(\pi_{2m}, \pi_{3m}, \ldots, \pi_{sm}) \]  

A-76

\[ \pi_1 \] can be predicted from \( \pi_{1m} \) by dividing eq. 34 by eq. 76:

\[ \frac{\pi_1}{\pi_{1m}} = \frac{F(\pi_2, \pi_3, \ldots, \pi_s)}{F(\pi_{2m}, \pi_{3m}, \ldots, \pi_{sm})} \]  

A-77

If the model is designed and run so that

\[ \pi_{2m} = \pi_2 \]

\[ \pi_{3m} = \pi_3 \]

\[ \ldots \]

\[ \pi_{sm} = \pi_s \]

A-78

then, from eq. A-77 we obtain:

\[ \pi_{1m} = \pi_1 \]  

A-79

Eqs. A-78 are referred to as design conditions, and when they are all satisfied the model is called a "true" model because it will give accurate information concerning the behavior of the prototype, provided that all the relevant parameters are included in the analysis from which the Pi terms are obtained.
As it is seen in several examples throughout this thesis it is not always possible to fulfill all the design requirements and the behavior of the model will be "distorted". The parameters involved in the distorted Pi term together with the experience and "feeling" of the engineer allow estimating the effect the distortion will have on the prediction equation. If it is small enough to be neglected that particular design condition may be relaxed, giving the designer more flexibility as far as the design and construction of the model is concerned.

Generally the design conditions include linear dimensions indicative of the size of model and prototype. The ratio of a linear dimension on the prototype to the corresponding one on the model is the length scale and is commonly designated by the letter $n$.

$$\frac{l}{l_m} = n$$

A.9.3 Types of models

According to the relation between prototype and model the latter can be classified in four different groups:

**True Models:** This group includes the models that are geometrically similar to the prototype, and in addition satisfy all other restrictions imposed by the design conditions.

**Adequate Models:** The models belonging to this group allow us to obtain accurate predictions of one characteristic of the prototype but will not necessarily provide us with useful information about other characteristics.

**Distorted Models:** All the models in which some design condition is altered sufficiently that modification on the prediction equation is required belong to this group. Those alterations must be made because in many cases it is not possible to satisfy all the design conditions if it's not a full-scale model.

**Dissimilar Models:** Those models have no physical resemblance with their prototype. It is a model that operates with a principle that is analogous to that of the model. For example the flow of water through a network can be modeled with an electric circuit consisting of a source and several resistors.
Appendix B: Review of Adhesion and Surface Forces

B.1 Research in Contact Forces

Historically, surface and adhesive forces have been studied because their comprehension was essential to understand phenomena such as wear and friction and to develop products such as adhesives.

The relatively new domain of micro-systems is in need of understanding those forces since their importance relative to other forces increases through miniaturisation (Fig. 5-1 in chapter 5). In particular effects like the one depicted in Fig. B-1 are to be avoided.

![Diagram of Pick & Place operation with a micro-part](image)

Figure B-1 Pick & Place operation with a micro-part. Surface forces attract the part to the gripper prior to gripping and during the release phase, resulting in a loss of positioning accuracy [FEA95].
At present we are not aware of any serious research on surface forces being carried out by micro-engineers.

Whenever two surfaces are in contact or brought together a series of forces of diverse origin appear. Burnham et al. [BUR97] classify the contact forces in two groups:

- **Surface forces**: forces that are present when two bodies are brought together;
- **Adhesive forces**: forces that keep two bodies in contact.

The same author subdivides the surface forces in the following way:

```
\begin{center}
\begin{tikzpicture}[->, level distance=1cm, level 1/.style={sibling distance=3cm}, level 2/.style={sibling distance=1cm}]
  \node {Surface Forces}
    child {node {\textbf{Long Range Forces}}
      child {node {Electrostatic}}
      child {node {Electrodynamic}}
      child {node {Electromagnetic}}
      child {node {Liquid forces}}
    }
    child {node {\textbf{Short Range Forces}}
      child {node {Chemical Bonding}}
      child {node {Metallic Bonding}}
    }
\end{tikzpicture}
\end{center}
```

**Figure B-2 Classification of Surface Forces**

Burnham et al. work in the field of atomic force microscopy. Micro-engineering oriented researchers state that the forces that appear when two surfaces are brought together belong to one of the following three groups [FEA95], [ARA96]:

- **Surface tension induced forces**: They arise from the interaction between the layers of adsorbed moisture in both surfaces.
- **Electrostatic related forces**: They appear due to charge generation or charges that are transferred during contact.
- **Van der Waals forces**: The weakest of the three are due to instantaneous polarization of atoms and molecules due to quantum mechanics effects.

No differentiation between surface and adhesive forces is made by micro-engineers.

Formulas that allow to estimate the magnitudes of those forces are given in [ARA96], [ARA98], [FEA95], [HEC90] and [BUR97].
The main problem encountered is that the formulae presented are valid for a very specific and simple geometry configuration and even for such cases certain parameters can’t be obtained and must be estimated.

While such simple models (like a plane and sphere) can be appropriate in the field of electronic microscopy, they can’t be applied to real geometries found in common micro-assembly operations.

Even though quantitative results can’t be obtained with the required degree of accuracy, qualitatively a few things can be learned from them. We should also add that it is difficult, or almost impossible, to isolate a given type of force in order to study it independently from the others.

Despite all the problems encountered a few directives that allow to minimize the magnitude of surface forces have been issued by the some of the authors mentioned.

- Minimize contact electrification by using materials with a small contact potential difference for the gripper and object;
- Use conductive materials which don’t easily form highly insulating native oxides;
- Gripper surfaces should be rough to minimize contact area;
- The high contact pressure from van der Waals and electrostatic forces can cause local deformation at the contact site. Hard materials are preferable to rubber or plastic;
- A dry atmosphere can help reduce surface tension effects. Surface tension can be used to help parts adhere better to the target location than the gripper.

![Figure B-3 Strategy used to minimize surface force effects during pick and release of a sphere with a flat-tipped tool. This strategy proofs effective when only van der Waals forces are present (no electrostatic forces) and assuming that the rolling contact that takes place between steps 4 and 5 poses no positioning accuracy problems [FED99].](image)
Figure B-4 Methods to reduce surface forces proposed by [ARA95]. A change in roughness can lead to a reduction of van der Waals forces. Static charges can, in principle, be modified if part can be connected to an electric circuit. Local heating can reduce humidity related forces if fluid evaporates.

The results presented up to now are results that have been obtained in recent years. The models on which these results are based were already published in the 70’s [TAB77] and [JOH71] as far as van der Waals related forces are concerned and earlier models for electrostatic and humidity related sources can be found. Since the release of those models no new models have been applied to this domain.

Hecht [HEC90] states that a relation between the microscopic level models and the macroscopic models has to be established before surface forces between micro-parts can be estimated.

Microscopic models have been studied thoroughly by different authors. A brief summary of the models found in literature is presented in the following sections.

### B.2 Adhesion Forces

Generally the adherence force between two solids will depend on the nature of the interfacial bonds, the geometry of the system, the rheological properties of the materials, the stiffness of the measuring apparatus and the method used (fixed grips, fixed load, or fixed velocity conditions).

A very common graph (appearing in almost every article on surface forces) that summarizes the magnitudes of the different surface forces is shown in Fig. B-5 below:
Figure B-5 Comparison of different surface forces as a function of object size. Based on models presented in this appendix [HEC90].

### B.2.1 Van der Waals Forces

The van der Waals force between two spheres is [FED99]:

\[
F_{vdwoo} = \frac{HCR_1R_2}{3} \left[ \frac{8R_1^2R_2^2 \left( C^2 - (R_1 - R_2)^2 \right) \left( C^2 - (R_1 + R_2)^2 \right)}{\left( C^2 - (R_1 + R_2)^2 \right)^2 \left( C^2 - (R_1 - R_2)^2 \right)^2} \right]
\]

where:

- \(R_1, R_2\) = radii of the two spheres [m],
- \(C\) = distance between centers [m],
- \(H = \pi^2n^2\lambda\) = Hamaker constant [J].

By letting \(R_2 \to \infty\) we obtain the van der Waals force between a sphere and a plane [FED99]:

\[
F_{vdwol} = \frac{2HR_1^3}{3d^2(d + 2R_1)^2}
\]
where:

\( d = \) distance from the wall to the edge of the sphere [m].

**Van der Waals force for a sphere and plane [FEA95]:**

\[
F_{vdw} = \frac{hr}{8\pi z^2}
\]

where:

\( h = \) Lifshitz-van der Waals constant [J],

\( r = \) radius of sphere [m],

\( z = \) atomic separation between surfaces [m].

This formula assumes atomically smooth surfaces. Important corrections have to be made for rough surfaces as the van der Waals forces decrease very rapidly with distance. For rough estimates Fearing assumes a true area of contact of 1% of the apparent area, or in other words a 1% of the maximum predicted for smooth surfaces.

The van der Waals force depends on the roughness of the surface [ARA96]:

\[
F_{vdbw} = \left( \frac{z}{z + b/2} \right)^2 F_{vdw}
\]

where:

\( b = \) surface roughness (average height of asperities) [m].

### B.2.2 Humidity Forces

In humid environments and when hydrophilic surfaces are in contact (or very close together) a liquid film may form between the two surfaces and a large capillary force may appear. This force is approximately given by [FEA95]:

\[
F_{tens} = \frac{\gamma(\cos \theta_1 + \cos \theta_2)A}{d}
\]

where:
\[ \gamma = \text{surface tension (73mN/m for water)} \ [\text{N/m}], \]

\[ A = \text{shared area} \ [\text{m}^2], \]

\[ d = \text{gap between surfaces} \ [\text{m}], \]

\[ \theta_1, \theta_2 = \text{contact angles between the liquid and the surfaces}. \]

Assuming hydrophilic surfaces and a separation distance much smaller than the object radius we have [FEA95]:

\[ F_{\text{tens}} = 4\pi r \gamma \] \hspace{1cm} \text{(B-6)}

where:

\[ r = \text{object radius} \ [\text{m}]. \]

Figure B-6 Different contact angles between fluid and surfaces. (a) shows a partial wetting that is stronger than that of (b). (c) corresponds to complete wetting, \( \alpha = 0 \). The surface tension of the fluids is smallest in case (c) and largest in case (b), assuming the surfaces are of the same material.

### B.2.3 Electrostatic Forces

Approximate force between a charged sphere and a conducting plane (Coulomb’s Law):

\[ F_{\text{elec}} = \frac{q^2}{4\pi \varepsilon (2r)^2} \] \hspace{1cm} \text{(B-7)}

where:

\[ q = \text{charge} \ [\text{C}], \]

\[ \varepsilon = \text{permittivity of dielectric} \ [\text{C}^2/\text{Nm}^2], \]
r = radius of sphere \([\text{m}]\).

The electrostatic force between a charged body and an uncharged wall (will occur when the wall is made of conductive material) [ARA96]:

\[
F_{ei} \approx \frac{\pi}{4\varepsilon_0} \frac{\varepsilon - \varepsilon_0}{\varepsilon + \varepsilon_0} d_r \sigma^2 \tag{B-8}
\]

where:

\(\sigma = \text{electric charge density of the surface} \,[\text{C/m}^2],\)

\(d_r = \text{reduced diameter}: d_r = d_1 d_2 / (d_1 + d_2)\)

For a plane and a sphere \(F_{ei}\) is obtained by substituting \(d\) into \(d_1\) and making \(d_2\) approach infinity. Thus:

\[
\lim_{d_2 \to \infty} \frac{d_1 d_2}{d_1 + d_2} = \lim_{d_2 \to \infty} \frac{d_2}{(d_1 + d_2)/d_2} = d_2 = d \tag{B-9}
\]

And so equation B-8 becomes:

\[
F_{ei} \approx \frac{\pi}{4\varepsilon_0} d^2 \sigma^2 \tag{B-10}
\]

The electrostatic force \(F_e\) between charged bodies is given by Coulomb's law. For two spheres separated by a distance \(z\) (see Figure B-7) small enough compared to the reduced diameter \(d\), Coulomb's equation may be written as [ARA96]:

\[
F_e \approx \frac{\pi \sigma_1 \sigma_2 d^2}{\varepsilon_0} \tag{B-11}
\]

using the relation:

\[
q = 4\pi r^2 \sigma \tag{B-12}
\]

where \(4\pi r^2\) is the surface of the sphere of radius \(r\).
It is difficult to ensure that parts and grippers remain at the same electrical potential, especially when the parts are very small because sound electrical contacts are difficult to obtain. Due to friction, small parts can be easily charged and potential differences appear creating the undesired attraction forces. Even if a good contact exists non-conducting objects would still remain charged maintaining a very high surface charge distribution.

Using Gauss's law we can estimate the intensity of an electric field in the proximity of a uniformly distributed surface charge. Not considering any field created by charges within the surface we have [FEA95]:

\[ \hat{z} \varepsilon_0 \vec{E}(z = 0) = \sigma_s \]  \hspace{1cm} (B-13)

where:

- \( \vec{E} \) = electric field \([\text{N/C}]\),
- \( \hat{z} \) = surface normal,
- \( \sigma_s \) = surface charge density \([\text{C/m}^2]\),
- \( \varepsilon_0 \) = permittivity of dielectric \([\text{C}^2/\text{Nm}^2]\).

For a point near the surface \( \vec{E} \) can be approximated by [FEA95]:

\[ \vec{E} = \frac{\sigma_s^2}{\varepsilon_0} \]  \hspace{1cm} (B-14)
The contact potential difference $U$ between two (semi-conducting) bodies is:

$$U = \varphi_1 - \varphi_2$$  \hspace{1cm} (B-15)

where:

$\varphi_1$, $\varphi_2$= electron affinities of contacting bodies [V].

The adhesive pressure due to contact charging (for the case of two half-spaces in contact with one another) may be found by differentiating the electrostatic field energy of the system after distance $z_0$ between the interacting bodies or, more simply, from field strength $E$ between them; thus, for the adhesive pressure $P_{ei}$ the conventional capacitor equations render [KRU67]:

$$P_{ei} = \frac{\varepsilon_0 E^2}{2} = \frac{\varepsilon_0 U^2}{2z_0^2} = \frac{\sigma^2}{2\varepsilon_0}$$  \hspace{1cm} (B-16)

where:

$\sigma$= surface charge density [C/m$^2$].

However equation B-16 applies only to metallic adherents where the charges are exclusively found on the surface, or to semi-conductors with a large surface charge density that practically shields the interior of the semi-conductor.

Electrostatic force between two contacting particles with a thin dielectric layer to insulate the two is considered next. The force between a sphere and a rectangular block is computed.
Figure B-8 Notation for computing electrostatic force between a sphere of radius $R_1$ and a rectangular block centered at $(x_0, y_0, z_0)$ and of dimensions $(a, b, c)$. The direction $y$ at the origin $O$ is into the page (from [FED99]).

Notation for computing electrostatic force between a sphere of radius $R_1$ and a rectangular block centered at $(x_0, y_0, z_0)$ and of dimensions $(a, b, c)$. The direction $y$ at the origin $O$ is into the page.

For a conductor the electric field inside the sphere and the block will be zero, and all the electrostatic charges will reside on the surface.

The electric field created by a sphere in a given point is [FED99]:

$$
\vec{E} = \frac{Q_1}{4\pi\varepsilon_0 r^2} \hat{r}
$$

for:

$r > R_1$
where:

\[ r = \text{distance from the center of the sphere to the point of interest} \]

\[ Q_1 = 4\pi R_1^2 \sigma_1 \]

Referring to Figure B-8 we have the relation:

\[ r^2 = x^2 + y^2 + z^2 \]

The infinitesimal force on a charge on the block's surface is given by:

\[ \text{d}\vec{F} = \vec{E}_L \sigma_2 \text{d}a_2 \]

where \( \sigma_2 \) is the charge density of the block and \( \text{d}a_2 \) is the infinitesimal area of the block's surface. The force in the \( x \) direction on the side of the block closest to the sphere is [FED99]:

\[ F_x \big|_{x=x_0-a/2} = \int_{z_0-a/2}^{z_0+a/2} \int_{y_0-b/2}^{y_0+b/2} \sigma_2 \vec{E}_1 \big|_{x=x_0-a/2} \frac{x_0 - a/2}{\sqrt{(x_0 - a/2)^2 + y^2 + z^2}} \text{dydz} = \]

\[ = \frac{R_1^2 \sigma_1 \sigma_2 (x_0 - a/2)}{\varepsilon_0} \int_{z_0-a/2}^{z_0+a/2} \int_{y_0-b/2}^{y_0+b/2} \frac{1}{\left[(x_0 - a/2)^2 + y^2 + z^2\right]^{3/2}} \text{dydz} \]

The forces in the \( y \) and \( z \) directions on the side closest to the sphere are [FED99]:

\[ F_y \big|_{x=x_0-a/2} = \frac{R_1^2 \sigma_1 \sigma_2}{\varepsilon_0} \int_{z_0-a/2}^{z_0+a/2} \int_{y_0-b/2}^{y_0+b/2} \frac{y}{\left[(x_0 - a/2)^2 + y^2 + z^2\right]^{3/2}} \text{dydz} \]

\[ F_z \big|_{x=x_0-a/2} = \frac{R_1^2 \sigma_1 \sigma_2}{\varepsilon_0} \int_{z_0-a/2}^{z_0+a/2} \int_{y_0-b/2}^{y_0+b/2} \frac{z}{\left[(x_0 - a/2)^2 + y^2 + z^2\right]^{3/2}} \text{dydz} \]
Figure B-9 Charge distribution on the surface of a rough surface.

Charges stored in this way are very difficult to remove. If we model the electric contact by means of two capacitors in series (one representing the air gap and the second representing the dielectric layer) (Fig. B-10) we see that by shorting the terminals we will not instantly remove the charge from both capacitors. In consequence there will be a residual attraction force between the gripper and the object. The stored charge (and the electric field) decays as a first order exponential with time constant \([\text{FEA95}]:\)

\[
\tau = \rho (\varepsilon + \varepsilon_0 \frac{d_1}{d_2}) \quad \text{B-22}
\]

where:

\(\rho=\) resistivity of dielectric (\(\Omega m\))

Figure B-10 Model of electric contact between two charged surfaces.
In these expressions the inner integral can easily be integrated analytically, but the outer integral is more complex, and is computed numerically. Similar expressions can be found for the forces exerted on the other five sides of the block. The total forces are given by:

$$F_L = F_L \big|_{x=x_0-a/2} + F_L \big|_{x=x_0+a/2} + F_L \big|_{y=y_0-b/2} + F_L \big|_{y=y_0+b/2} + F_L \big|_{z=z_0-c/2} + F_L \big|_{z=z_0+c/2}$$

### B.2.4 Contact Electrification

Electric charge flows between two materials having different contact potentials when they are brought in contact. In the case of metal to metal contact [LOW80] [KRU67], a rough approximation to the surface charge density is [FEA95]:

$$\sigma_s = \frac{\varepsilon_0 U}{z_0} \quad B-21$$

where:

- $U$ = contact potential difference (typically less than 0.5V),
- $z_0$ = gap for tunneling (about 1nm).

In laboratory experiments, contact electrification has been shown to generate significant charge density, which could cause adhesion.

### B.2.5 Charge storage in dielectrics

The use of conducting grippers can reduce static charging effects. A thin layer of oxide can cover however conducting objects. This is an insulating layer that can withstand electric fields of up to $3 \times 10^9$ Vm$^{-1}$. This means that significant amounts of charge can be stored in the oxide.

When a grounded gripper grasps an initially charged object, charge will be induced on the opposite surface of the regions that are not in contact (see Figure B-9).
B.2.5.1 Static charge elimination

Ion generation is required to reduce electrostatic force. Ions are generated by corona discharge. In consequence a classification of static charge elimination methods is equivalent to a classification of ion generation [ARA96].

There are three ways to do this [ELE81]:

1. Voltage impressed type charge elimination;
2. Self-discharge type charge elimination;
3. Radiation type charge elimination.

B.3 Author's Comments

The formulae that allow calculating the magnitude of the different forces presented in this appendix are all based on three physical phenomena that are relatively well-known. However the conditions in which they are applied are over simplified. Surfaces are not flat, charge distributions are not homogeneous, asperities are not equally shaped, the values of constants such as the surface tension of fluids must be revised before applying them to this domain, etc.

Obviously there is still much work left to do before the surface forces are understood, but from the engineering point of view we should ask ourselves: what do we really need to know about those forces in order to solve the problems they introduce in the world of microsystems?
Bibliography


[HUB97] "Huba Control AG Product Catalog": p. 4000EM, 1997


**Additional Bibliography**

The following references have provided a number of ideas and have helped to find new, original approaches to some of the problems described.


**List of Symbols**

- \(\lambda\): Linear force coefficient (N/m)
- \(A\): Cross-sectional area (m²)
- \(A_T\): Overall acceleration of bowl (m/s²)
- \(b\): Number of basic dimensions involved (-)
- \(B\): Magnetic flux density (T)
- \(C_b\): Bowl constant (-)
- \(C\): Propagation velocity of a wave in a fluid (m/s)
- \(D\): Diffusion coefficient (m/s²)
- \(D_o\): Diameter of drop (m)
- \(D_h\): Diameter of hole (m)
- \(d_r\): Reduced diameter \(d_1d_2/(d_1+d_2)\)
- \(E\): Young's modulus of elasticity (N/m²)
- \(e\): Elementary charge \(1.6022\times10^{-19}\) C
- \(E_e\): Electric field (V/m)
- \(f\): Friction factor (-)
- \(F_e/F_z\): Electrostatic forces (N)
- \(F_F\): Friction force (N)
- \(F_N\): Normal force (N)
- \(F_s\): Surface related forces (N)
- \(g\): Acceleration of gravity (m/s²)
- \(G\): Geometric dependant function (-)
- \(I\): Moment of inertia (m⁴)
- \(k_b\): Boltzmann’s constant \(1.3807\times10^{-23}\) Js
- \(l\): Linear dimension (m)
- \(L_h\): Length of hole (m)
- \(M\): Torque (Nm)
- \(m\): Mass (kg)
- \(M_0\): Friction torque independent from load (Nm)
- \(M_1\): Load dependent friction torque (Nm)
- \(n\): Number of quantities involved (-)
- \(n\): Scaling factor (-)
- \(N\): Number of conduction electrons per unit volume \(1/m³\)
- \(P\): Pressure (Pa)
- \(P_c\): Contact load (N)
- \(P_l\): Power transmitted to fluid (W)
- \(P_{nom}\): Nominal or target position (m)
- \(P_{obt}\): Obtained real position (m)
- \(Q\): Tangential (shearing) force (N)
- \(r\): Roughness of pipe (-)
\( R_0 \)-Size of particle (m)
\( s \)-Number of TT terms (-)
\( s_f \)-Shape factor (-)
\( T \)-Temperature (K)
\( t \)-Cross-sectional height of beam (m)
\( t_d \)-diffusion time (s)
\( t_{edge}/t_{feed}/etc... \)-Position tolerances (m)
\( U \)-Deformation energy (J)
\( V \)-Voltage (V)
\( V_b \)-Velocity of drop (m/s)
\( W \)-Work (J)
\( w \)-Cross-sectional width of beam (m)
\( y \)-distance from neutral axis (m)
\( \alpha \)-Angular displacement (rad)
\( \gamma \)-Surface force coefficient (N/m^2)
\( \delta \)-Offset/Linear displacement (m)
\( \Delta P \)-Pressure drop (Pa)
\( \varepsilon \)-Permittivity of dielectric (C^2/Nm^2)
\( \varepsilon_{max} \)-Maximum strain (-)
\( \eta \)-Kinematic viscosity (m^2/s)
\( \Theta \)-Angle of bowl feeder ramp (rad)
\( \mu \)-Friction coefficient (-)
\( \mu_l \)-Dynamic viscosity of fluid (kg/ms)
\( \rho \)-Density of fluid (kg/m^3)
\( \sigma \)-Electric charge density (C/m^2)
\( \sigma_x \)-Surface tension of fluid (N/m)
\( \sigma_{max} \)-Maximum stress (N/m^2)
\( \sigma_{res} \)-Variance of obtained position (m)
\( \sigma_{x} \)-Mechanical stress in the X direction (N/m^2)
\( \Psi \)-Angle between direction and motion of bowl feeder ramp (rad)
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