Solving a pricing problem under irrational behavior

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Outline

Decision rule

Homo economicus

Rational and narrowly self-interested economic actor who is optimizing her outcome

Utility

$$U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : a \rightsquigarrow U_n(a)$$

- captures the attractiveness of an alternative
- measure that the decision maker wants to optimize

Behavioral assumption

- the decision maker associates a utility with each alternative
- the decision maker is a perfect optimizer
- the alternative with the highest utility is chosen

Random utility model

Random utility

$$U_{in} = V_{in} + \varepsilon_{in} = \beta^T X_{in} + \varepsilon_{in}.$$

Choice model

$$P(i|\mathcal{C}_n) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n),$$

Logit model

Assumptions

ε_{in} are i.i.d. EV(0, μ).

Choice model

$$P_n(i|\mathcal{C}_n) = \frac{y_{in} e^{\mu V_{in}}}{\sum_{j=1}^{J} y_{jn} e^{\mu V_{jn}}}.$$

Outline

Beyond rationality

Motivation

- There is evidence that human beings are not necessarily rational in the way assumed by random utility models.
- We first review some experiments that illustrate that (apparent) irrationality.

Example: pain lovers

[Kahneman et al., 1993]

- Short trial: immerse one hand in water at 14° for 60 sec.
- Long trial: immerse the other hand at 14° for 60 sec, then keep the hand in the water 30 sec. longer as the temperature of the water is gradually raised to 15°.
- Outcome: most people prefer the long trial.
- Explanation: duration plays a small role, the peak and the final moments matter.

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EPFL

Example: The Economist

[Ariely, 2008]

Subscription to The Economist

Web only	@ \$59
Print only	@ \$125
Print and web	@ \$125

Example: The Economist

[Ariely, 2008]

Subscription to The Economist

Experiment 1	Experiment 2
Web only @ $59	Web only @ $59
Print only @ $125	
Print and web @ $125	Print and web @ $125

Example: The Economist

[Ariely, 2008]

Subscription to The Economist

	Experiment 1	Experiment 2	
16	Web only @ \$59	Web only @ \$59	68
0	Print only @ \$125		
84	Print and web @ \$125	Print and web @ \$125	32

The Economist: explanations

- Dominated alternative.
- According to utility maximization, should not affect the choice.
- But it affects the perception, which affects the choice.

Decoy effect

Decoy

High-price, low-value product compared to other items in the choice set.

Behavior

Consumers shift their choice to more expensive items.

Applications

- Travel and tourism. [Josiam and Hobson, 1995]
- Wine lists in restaurants. [Kimes et al., 2012]
- Tobacco treatment. [Rogers et al., 2020]
- Online diamond retail. [Wu and Cosguner, ta]

Another example of decoy

Example: good or bad wine?

Choose a bottle of wine...

	Experiment 1	Experiment 2
1	McFadden red at \$10	McFadden red at \$10
2	Nappa red at \$12	Nappa red at \$12
3		McFadden special reserve pinot noir at \$60
	Most would choose 2	Most would choose 1

- Context plays a role on perceptions.
- Here, perceived quality is increased.

Example: live and let die

[Kahneman and Tversky, 1986]

Population of 600 is threatened by a disease.

Two alternative treatments to combat the disease have been proposed.

	Experiment 1 # resp. = 152	Experiment 2 # resp. = 155	
72%	Treatment A: 200 people saved	Treatment C: 400 people die	22%
28%	Treatment B: 600 saved with prob. 1/3 0 saved with prob. 2/3	Treatment D: 0 die with prob. 1/3 600 die with prob. 2/3	78%

Example: to be free

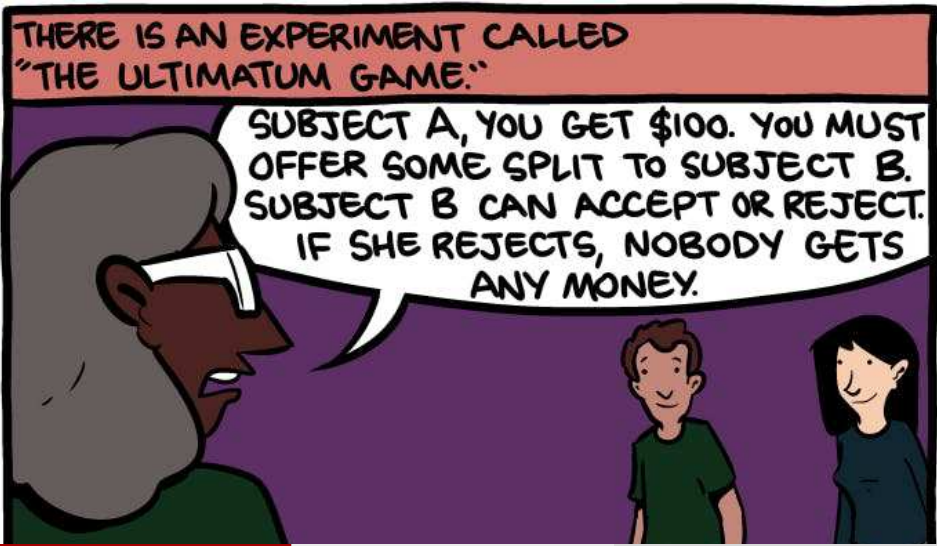
[Ariely, 2008]

Choice between a fine and a regular chocolate

	Experiment 1	Experiment 2
Lindt	\$0.15	\$0.14
Hershey	\$0.01	\$0.00
Lindt chosen	73%	31%
Hershey chosen	27%	69%

Discontinuity at 0

Ultimatum game



Ultimatum game

Ultimatum game

Ultimatum game

Optimal solution

Subject B should accept any offer.

In practice

Offers of less than 30% are often rejected.

Modeling latent concepts

Motivation

- Some observed behavior may appear irrational, and inconsistent with random utility.
- It is only apparent, as these behaviors can be explained by more complex formulations of the concept of utility.
- In particular, this may involve subjective and latent concepts such as perceptions and attitudes.
- Latent concepts can be introduced in choice models.

Indirect measurements of latent concepts

Attitude towards the environment

For each question, response on a scale: strongly agree, agree, neutral, disagree, strongly disagree, no idea.

- The price of oil should be increased to reduce congestion and pollution.
- More public transportation is necessary, even if it means additional taxes.
- Ecology is a threat to minorities and small companies.
- People and employment are more important than the environment.
- I feel concerned by the global warming.
- Decisions must be taken to reduce the greenhouse gas emission.

Indirect measurements of latent concepts

Psychometric indicators

- Usually easy to respond.
- Arbitrary units.
- Important to minimize framing.

Data

For each individual, we have

- Vector of independent variables: x.
- Choice: i.
- vector of psychometric indicators: I.

Prediction model

Latent variable

- Captures perceptions, attitudes, anchors, etc.
- Not observed.
- Modeled as a function of observed variables:

$$X^* = \text{EnvironmentalAttidude} = \text{f(Age, Education, etc.;}\ \theta) + \xi.$$

Random utility model

- Utility is also unobserved.
- Modeled as a function of observed variables, as well as the latent variable(s):

$$\text{Utility(PublicTransport)} =$$
$$\text{f(Price, Time, Frequency, EnvironmentalAttitude;}\ \theta) + \varepsilon$$

Prediction model

Choice model: mixture of logit models

$$P_n(i|x_n, X_n^*, \mathcal{C}_n) = \frac{y_{in} e^{\mu V_{in}(x_n, X_n^*)}}{\sum_{j=1}^{J} y_{jn} e^{\mu V_{jn}(x_n, X_n^*)}}.$$

$$\begin{aligned} P_n(i|x_n, \mathcal{C}_n) &= \int_t P_n(i|x_n, t, \mathcal{C}_n) f_{X_n^*}(t) dt \\ &= \int_t \frac{y_{in} e^{\mu V_{in}(x_n, t)}}{\sum_{j=1}^{J} y_{jn} e^{\mu V_{jn}(x_n, t)}} f_{X_n^*}(t) dt. \end{aligned}$$

Simulation

Main idea

- Simulate all random quantities involved.
- Work at the level of utilities, not probabilities.

Random utility model

$$P(i|p) = \Pr(U_{in}(p) \geq U_{jn}(p),\ \forall j).$$

Simulated random utility model

$$P(i|p) \approx \frac{1}{R} \sum_{r=1}^{R} \mathbb{1}\left[u_{inr}(p) \geq u_{jnr}(p),\ \forall j\right].$$

Outline

Motivation

Motivation

- Use advanced choice models to predict demand.
- Include the demand models in an optimization model.
- Example: a pricing problem.

Acknowledgments

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Context

The problem

- A competitive market with $J + K$ products.
- An operator who controls the price of J products.
- Customers freely choose their preferred product.
- Pricing problem: maximize the revenues of the operator.

The model

- The decisions of the competitors is known and considered fixed.
- Customers' choices are characterized by a random utility model.
- Customers may have different tastes and preferences.
- The utility of the controlled alternatives is a linear function of price.
- The utility is a random variable with a known distribution.

Utility

Controlled alternatives

$$U_{in}(p) = U_{in}(p_i) = \beta_{in} p_i + c'_{in} + \varepsilon_{in}, \; i = 1, \ldots, J$$

where

- p_i is the price of alternative i,
- $\beta_{in} < 0$ is the price coefficient for individual n (potentially a r.v.),
- c'_{in} is the fixed part of the utility observed by the analyst,
- ε_{in} is the fixed part of the utility unobserved by the analyst.

Uncontrolled alternatives

$$U_{jn}(p) = U_{jn} = c'_{jn} + \varepsilon_{jn}, \; j = 1 - K, \ldots, 0$$

Simulation

Draws from the distributions

For $r = 1, \ldots, R$, β_{inr} are draws from β_{in}, ε_{inr} are draws from ε_{in}, c'_{inr} are draws from c'_{in}.

Utility functions

$$\begin{aligned} u_{inr}(p) &= \beta_{inr} p_i + c'_{inr} + \varepsilon_{inr}, & i &= 1, \ldots, J, \\ u_{jnr}(p) &= c'_{jnr} + \varepsilon_{jnr}, & j &= 1-K, \ldots, 0, \end{aligned}$$

or

$$\begin{aligned} u_{inr}(p) &= \beta_{inr} p_i + c_{inr}, & i &= 1, \ldots, J, \\ u_{jnr}(p) &= c_{jnr}, & j &= 1-K, \ldots, 0. \end{aligned}$$

Outline

Choice model as a knapsack problem

For a given n and r:

Primal

$$\max_{w_{nr}} \sum_{i=1-K}^{J} w_{inr} u_{inr}(p)$$

subject to

$$\sum_{i=1-K}^{J} w_{inr} = 1,$$

$$w_{inr} \geq 0, \; i = 1-K, \ldots, J.$$

Dual

$$\min_{h_{nr}} h_{nr}$$

subject to

$$h_{nr} \geq u_{inr}, \; i = 1-K, \ldots, J.$$

h_{nr} is the largest utility.

Choice model as a knapsack problem

Integrality property

If $u_{inr} \neq u_{jnr}$, for each i, j, the solution of the knapsack problem is binary.

Optimality conditions: strong duality

$$h_{nr} = \sum_{i=1-K}^{J} w_{inr} u_{inr}(p),$$

$$h_{nr} \geq u_{inr},\ i = 1-K, \ldots, J,$$

$$\sum_{i=1-K}^{J} w_{inr} = 1,$$

$$w_{inr} \geq 0,\ i = 1-K, \ldots, J.$$

The pricing problem

$$\min_{p,w,u,h} -\frac{1}{R}\sum_{r=1}^{R}\sum_{n=1}^{N}\sum_{i=1}^{J} p_i w_{inr}$$

subject to

$$h_{nr} = \sum_{i=1-K}^{J} w_{inr} u_{inr}, \qquad \forall n, r,$$

$$h_{nr} \geq u_{inr}, \qquad \forall i, n, r,$$

$$\sum_{i=1-K}^{J} w_{inr} = 1, \qquad \forall n, r,$$

$$w_{inr} \geq 0, \qquad \forall i, n, r,$$

$$u_{inr} = \beta_{inr} p_i + c_{inr}, \qquad i = 1, \ldots, J, \forall n, r,$$

$$u_{jnr} = c_{jnr}, \qquad j = 1-K, \ldots, 0, \forall n, r.$$

The pricing problem

Two difficulties

- Nonlinearity: $p_i w_{inr}$.
- Number of constraints: order of $J \times N \times R$.

First simplification

$$\begin{aligned} h_{nr} &\geq u_{inr}, && \forall i, n, r, \\ u_{jnr} &= c_{jnr}, && j = 1-K, \ldots, 0, \forall n, r. \end{aligned}$$

- For each n and r, among the non controlled alternatives, only the best one matters.
- It can be safely assumed that there is only one "opt-out" alternative.

The pricing problem

Preprocessing

- For each n, r define

$$c_{0nr} = \max_{j=1-K,\dots,0} c_{jnr}$$

- We redefine the problem with $K = 1$.

Specification

- Substitute

$$\begin{aligned} u_{inr} &= \beta_{inr} p_i + c_{inr}, & i = 1, \dots, J, \forall n, r, \\ u_{0nr} &= c_{0nr}, & \forall n, r. \end{aligned}$$

- Define

$$\eta_{inr} = p_i w_{inr}.$$

The pricing problem

$$\min_{p,w,h,\eta} -\frac{1}{R}\sum_{r=1}^{R}\sum_{n=1}^{N}\sum_{i=1}^{J}\eta_{inr}$$

subject to

$$\begin{aligned}
h_{nr} &= c_{0nr}w_{0nr} + \sum_{i=1}^{J}\beta_{inr}\eta_{inr} + c_{inr}w_{inr}, && \forall n, r,\\
h_{nr} &\geq \beta_{inr}p_i + c_{inr}, && i = 1,\ldots,J, \forall n, r,\\
h_{nr} &\geq c_{0nr}, && \forall n, r,\\
\eta_{inr} &= p_i w_{inr}, && i = 1,\ldots,J, \forall n, r,\\
\sum_{i=0}^{J} w_{inr} &= 1, && \forall n, r,\\
w_{inr} &\geq 0, && \forall i, n, r,\\
p_i &\in [p_i^L, p_i^U] && i = 1,\ldots,J.
\end{aligned}$$

Outline

McCormick envelopes

$p_i \in [p_i^L, p_i^U]$, $w_{inr} \in [0, 1]$

$$\begin{aligned} a_L &= p_i - p_i^L \geq 0, \\ a_U &= p_i^U - p_i \geq 0, \\ b_L &= w_{inr} \geq 0, \\ b_U &= 1 - w_{inr} \geq 0. \end{aligned}$$

$$\begin{aligned} a_L b_L &= w_{inr} p_i - w_{inr} p_i^L = \eta_{inr} - w_{inr} p_i^L \geq 0, \\ a_L b_U &= p_i - p_i^L - w_{inr} p_i + w_{inr} p_i^L = p_i - p_i^L - \eta_{inr} + w_{inr} p_i^L \geq 0, \\ a_U b_L &= w_{inr} p_i^U - w_{inr} p_i = w_{inr} p_i^U - \eta_{inr} \geq 0, \\ a_U b_U &= p_i^U - p_i - w_{int} p_i^U + w_{inr} p_i = p_i^U - p_i - w_{int} p_i^U + \eta_{inr} \geq 0. \end{aligned}$$

McCormick envelopes

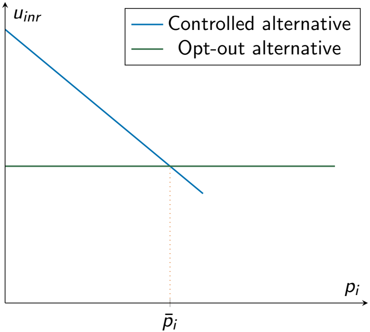
$$
\begin{aligned}
\eta_{inr} &\geq w_{inr} p_i^L, \\
\eta_{inr} &\leq p_i - p_i^L - w_{inr} p_i^L, \\
\eta_{inr} &\leq w_{inr} p_i^U, \\
\eta_{inr} &\geq -p_i^U + p_i + w_{int} p_i^U.
\end{aligned}
$$

Note

As we maximize on η_{inr}, the constraints setting lower bounds are not necessary.

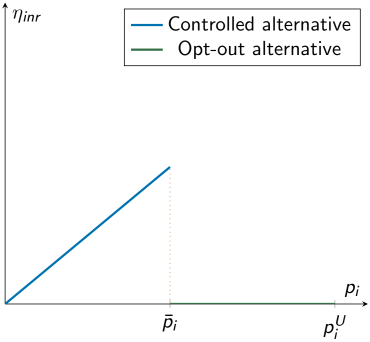
Break points

Competing with opt-out: utility



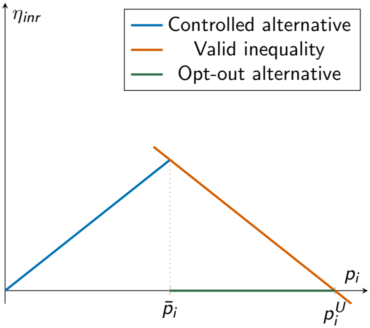
Break points

Competing with opt-out: revenue



Break points

Competing with opt-out: valid inequality



Valid inequalities based on break points

Competing with opt-out

$$\eta_{inr} \leq \frac{\bar{p}_i(p_i^U - p_i)}{p_i^U - \bar{p}_i}.$$

Competing with another controlled alternative

$$\eta_{inr} \leq \frac{\beta_j p_i^U p_j - c_i p_i^U + c_j p_i^U - p_i\left(\beta_j p_j^L - c_i + c_j\right)}{\beta_i p_i^U - \beta_j p_j^L + c_i - c_j}.$$

and

$$\eta_{inr} \leq \frac{\beta_j p_i^U p_j - c_i p_i^U + c_j p_i^U - p_i\left(\beta_j p_j^U - c_i + c_j\right)}{\beta_i p_i^U - \beta_j p_j^U + c_i - c_j}.$$

Outline

Dealing with nonlinearity

Spatial Branch & Bound

- Start with reasonable bounds on p_i: $[p_i^L, p_i^U]$.
- Relaxation: ignore the constraint $\eta_{inr} = p_i w_{inr}$.
- At each node, solve the relaxation: upper bound.
- Fix the price, identify the choices to obtain a feasible solution: lower bound.
- Split the price interval and branch.

Outline

Dealing with the large size

Observations

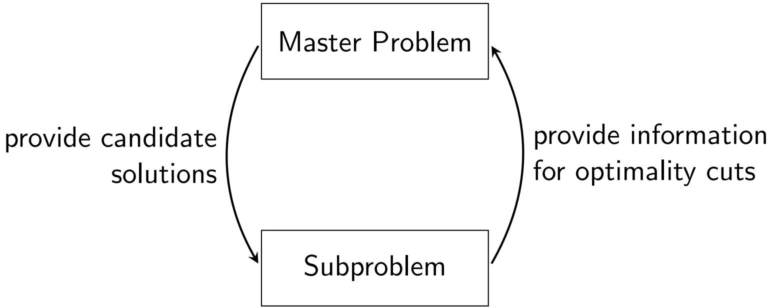
- The relaxation is a large LP.
- If the prices p are fixed, the problem is fully decomposed across n and r.
- Therefore, we consider Benders decomposition.

Benders decomposition

- Complicating variables: p.
- Benders subproblem: for each n and r.

Benders Decomposition

Decomposition scheme



- Iterative procedure.
- Candidate solutions provide upper bounds on the objective.
- Achieved objective values in the Master problem provide lower bounds.

Benders Decomposition

Subproblem(n, r)

$$
\begin{aligned}
\min_{p,w,h,\eta} \quad & -\frac{1}{R}\sum_{i=1}^{J}\eta_{inr} \\
\text{s.t.} \quad & h_{nr} = c_{0nr}w_{0nr} + \sum_{i=1}^{J}\beta_{inr}\eta_{inr} + c_{inr}w_{inr} \\
& h_{nr} \geq \beta_{inr}p_i + c_{inr}, & i = 1,\ldots,J, \\
& h_{nr} \geq c_{0nr}, \\
& \eta_{inr} \in \text{McCormick}[p_i, w_{inr}, p_i^L, p_i^U], & i = 1,\ldots,J, \\
& \sum_{i=0}^{J} w_{inr} = 1, \\
& w_{inr} \geq 0, & \forall i, \\
& p_i = p_i^c \quad (\varphi_{inr}^c) & i = 1,\ldots,J.
\end{aligned}
$$

- Computes dual values (φ_i^c) for optimality cuts.

Benders Decomposition

Master problem

$$\min_{\mathcal{P},p} \; -\sum_{nr} \mathcal{P}_{nr}$$

$$\begin{aligned}
\text{s.t.} \quad \mathcal{P}_{nr} &\leq \mathcal{P}^c_{nr} - \sum_{i=1}^{J} \varphi^c_{inr}(p_i - p^c_i), && \forall c \in \mathcal{C} \quad \forall n, r, \\
\mathcal{P}_{nr} &\leq \sum_{i=1}^{J} \mathrm{V}(\eta_{inr}), && \forall n, r, \\
\sum_{nr} \mathcal{P}_{nr} &\leq \mathcal{P}^{\text{best}}
\end{aligned}$$

- Computes candidate solutions for the price.
- Fully disaggregated optimality cuts $\mathcal{C}$.
- Includes valid inequalities (V).

Outline

Case Study

Parking space operator [Ibeas et al., 2014]

- **Alternatives**: Paid-Street-Parking (PSP), Paid-Underground-Parking (PUP) and Free-Street-Parking (FSP).
- Optimize prices for PSP and PUP, FSP is the **opt-out** alternative.
- **Socio-economic characteristics**: trip origin, vehicle age, driver income, residence area.
- **Product attributes**: access time to parking, access time to destination, and parking fee (price).
- Choice model is a **Mixed Logit**, $\beta_{\text{fee}}, \beta_{\text{time\_parking}} \sim \mathcal{N}(\mu, \sigma)$.

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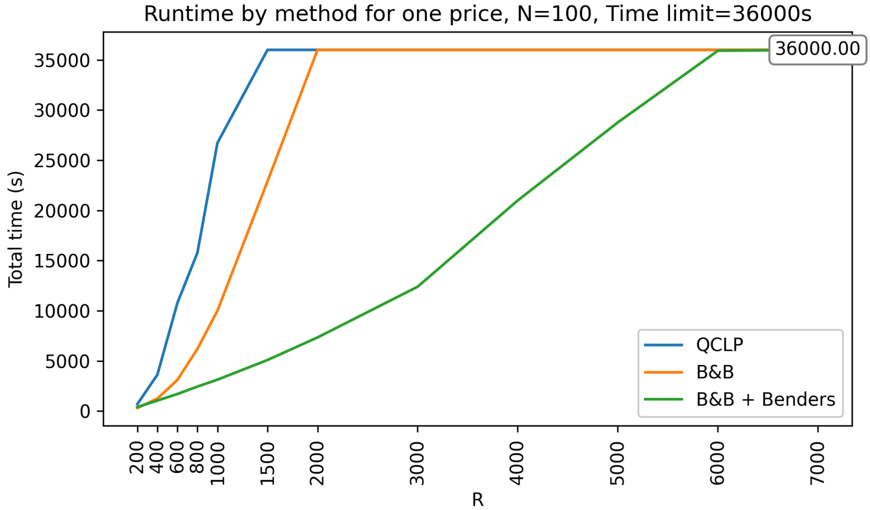
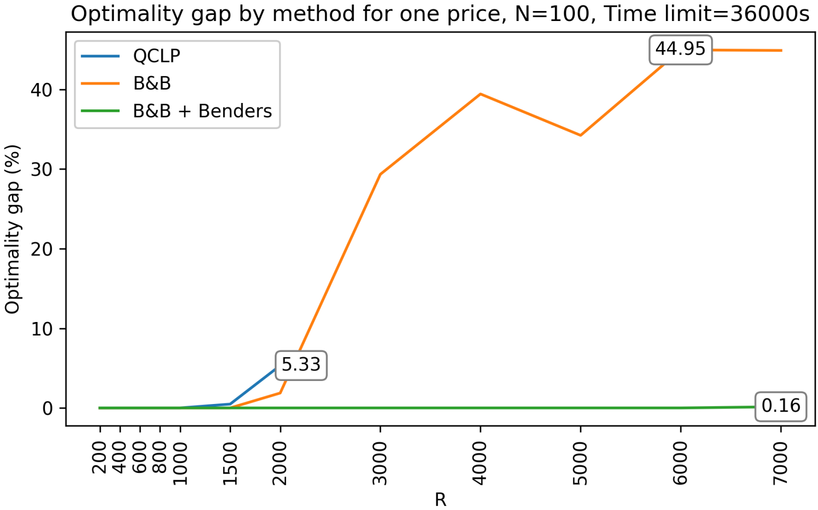
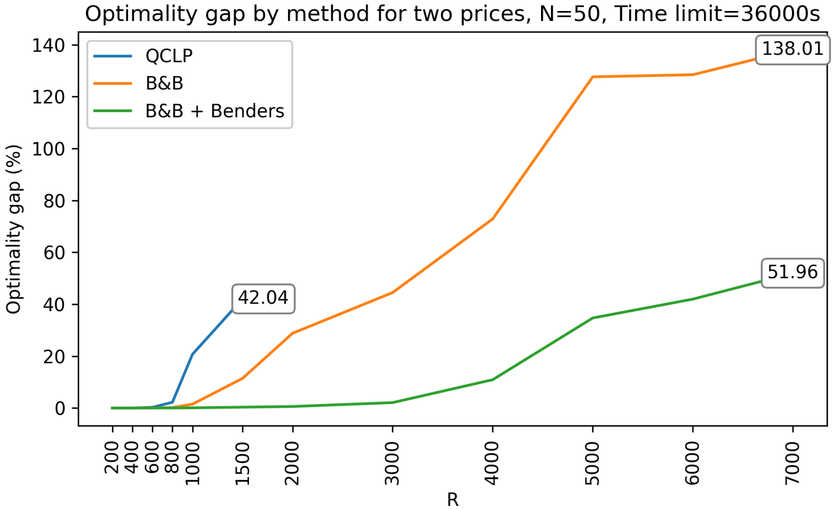
Computational results



Computational results



Computational results



Computational results

Table: Solve time (seconds) for single-price optimization (large-scale)

		QCLP		B&B		B&B + Benders	
N	R	Time	Gap (%)	Time	Gap (%)	Time	Gap (%)
100	200	698	0.01	310	0.00	409	0.01
100	400	3629	0.01	1255	0.01	1050	0.01
100	600	10775	0.01	3110	0.01	1707	0.01
100	800	15784	0.01	6206	0.01	2444	0.01
100	1000	26727	0.01	10007	0.01	3131	0.01
100	1500	36000	0.49	22892	0.01	5093	0.01
100	2000	36000	5.33	36000	1.88	7341	0.01
100	3000	36000	-	36000	29.33	12396	0.01
100	4000	36000	-	36000	39.42	20990	0.01
100	5000	36000	-	36000	34.22	28768	0.01
100	6000	36000	-	36000	44.95	35917	0.01
100	7000	36000	-	36000	44.88	36000	0.16

Computational results

Table: Solve time (seconds) for two-price optimization (large-scale)

		QCLP		B&B		B&B + Benders	
N	R	Time	Gap (%)	Time	Gap (%)	Time	Gap (%)
50	200	3338	0.01	2426	0.01	5498	0.01
50	400	23325	0.01	11746	0.01	21838	0.01
50	600	36000	0.26	26662	0.01	35367	0.01
50	800	36000	2.21	36000	0.16	35938	0.01
50	1000	36000	20.68	36000	1.48	36000	0.07
50	1500	36000	42.04	36000	11.41	36000	0.32
50	2000	36000	-	36000	28.79	36000	0.58
50	3000	36000	-	36000	44.48	36000	2.08
50	4000	36000	-	36000	72.86	36000	10.90
50	5000	36000	-	36000	127.64	36000	34.70
50	6000	36000	-	36000	128.44	36000	41.96
50	7000	36000	-	36000	138.01	36000	51.96

Simplifications + Valid inequalities

Table: One-price and two-price optimization runtime (seconds) when using simplifications (S) + valid inequalities (V1 and V2). Time limit = 36000s

N	R	QCLP	B&B	B&BD	B&BD+S	B&BD+S+V1	B&BD+S+V2
100	100	107	29	98	30	33	41
100	500	4739	625	851	252	673	519
100	1000	27586	10007	3387	1865	3329	2388
100	3000	-	25950	5606	3337	5019	3905

N	R	QCLP	B&B	B&BD	B&BD+S	B&BD+S+V1	B&BD+S+V2
50	100	840	660	1925	416	11253	18447
50	500	30600	16826	19904	4686	0.40%	1.01%
50	1000	20.68%	1.59%	0.07%	15066	1.87%	4.68%
50	3000	-	42.88%	2.07%	0.06%	3.54%	8.71%

Conclusions

- Individuals' behavior may appear irrational.
- It requires to generalize classical models, at the expense of more mathematical complexity.
- Integration of such models in optimization models is therefore challenging.
- We rely on first principles and simulation to obtain a mathematical optimization model.

Conclusions

- A pricing problem involving any choice model with utility linear in price.
- Exploiting the special structure of the problem helps a lot.
- Simplifications and valid inqualities.
- Branch & bound and Benders.
- We solve instances to optimality before GUROBI finds a first feasible solution.
- It is possible to solve problems with a large number of draws.

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