The stochastic multiple depot electric vehicle scheduling problem with recourse

Léa Ricard*[1], Guy Desaulniers[2], Andrea Lodi[3], and Louis-Martin Rousseau[2]

[1]Department of Computer Science, Université de Montréal, Montréal, Canada
[2]Department of Mathematics and Industrial Engineering, Polytechnique Montréal, Montréal, Canada
[3]Jacobs Technion-Cornell Institute, Cornell Tech and Technion - IIT, New York, USA

Short summary

Zero emission policies in urban centers are promoting the conversion of transit agencies fleets to battery electric buses (BEBs). This transition raises questions about battery management and more specifically about the best way to mathematically model this resource in order to respect energy feasibility constraints while being as little conservative as possible. In an attempt to partially answer these questions, this work presents a two-stage stochastic model with recourse for the multiple depot electric vehicle scheduling problem with stochastic travel time and energy consumption (S-MDEVSP). Vehicles are allowed to be partially recharged and a non-linear charging function is considered. Our model takes advantage of the full information on the current state of charge that is available in operation by allowing planned charge time to be extended when energy consumption deviations are observed. We propose a column-generation-based heuristic featuring stochastic pricing problems to solve a real-life instance from the city of Montréal, Canada. An analysis of the relevance of our approach for different commercially available BEBs is also provided.
Keywords: column generation, electric bus, two-stage stochastic program, vehicle scheduling

1 Introduction

The multiple depot electric vehicle scheduling problem (MDEVSP) is an extension of the multiple depot vehicle scheduling problem with additional limitations, including shorter driving range, longer refueling time, and special charging infrastructure. It aims at finding a set of vehicle routes that covers each timetabled trip exactly once while minimizing the operational costs and respecting energy feasibility and depot capacity constraints. These vehicle routes are subject to operational uncertainties (e.g., traffic jams, extreme weather conditions, or special happenings in the city) that impact travel time and energy consumption. Nevertheless, the MDEVSP is generally solved without taking these uncertainties into account. This strong assumption may compromise schedule adherence and lead to solutions with sub-optimal true costs (including recourse costs). A simple way to guarantee energy feasibility is to adopt a robust optimization approach, i.e., ensuring that energy feasibility is respected for the worst case energy consumption scenarios (see for example the work of Bie et al. (2021)). Some less conservative approaches, that we group into stochastic optimization (Li et al., 2021), robust optimization with cardinality constrained set (Jiang et al., 2021), and dynamic optimization (Tang et al., 2019), have been proposed in the literature to address the MDEVSP with uncertain travel time and/or energy consumption.
This work presents the first stochastic model for the MDEVSP with stochastic travel time and energy consumption (S-MDEVSP). We formulate the S-MDEVSP as a two-stage stochastic program and introduce a recourse policy to recover energy feasibility when the vehicle routes outputted a priori turn out to be infeasible. The main idea of our approach is to take advantage of the fact that charging time can be adjusted from day-to-day to cope with energy consumption deviations. This flexibility in the charging time could allow us to output less conservative vehicle routes than the robust optimization approach while guaranteeing energy feasibility. However, this flexibility may also induce delays. To control the build-up of delays, that can also be caused by travel time deviations, we add a penalty for delays in the objective function as in Ricard et al. (2022). Our objective is to access the relevance of our two-stage stochastic model for commercially available battery electric buses (BEBs). Precisely, we want to verify if a substantial reduction in the optimal

fleet size can be archived by introducing a recourse policy. We propose a branch-and-price algorithm to solve this challenging optimization problem and test our solution approach on a real-life instance of the city of Montréal.

This paper is organized as follows. Section 2 deals with the problem definition and a two-stage stochastic program is introduced. We devise a method compute the second stage cost analytically. A column generation-based solution approach is presented in Section 3. We present the results of computation in Section 4 and discuss the relevance of our approach for different commercially available BEBs. Our main conclusions are stated in Section 5.

2 Mathematical model

Let a timetable of trips $\mathcal{T}$, where trip $i \in \mathcal{T}$ is schedule to start at d_i, a set of depots $\mathcal{D}$, such that $|\mathcal{D}| \geq 2$, and a set of charging stations $\mathcal{Q}$. Each charging station $q \in \mathcal{Q}$ is time-expanded as $\tilde{q} = \{qs_1, \ldots, qs_k\}$, where $s_1, \ldots, s_k$ are k time intervals of ρ minutes. We denote the set of time-expanded charging stations $\tilde{Q}$. The S-MDEVSP is defined on the acyclic connection-based networks $G^d(V_d, A_d)$, for $d \in \mathcal{D}$, with node set $V_d = \mathcal{T} \cup \{n_0^d, n_1^d\} \cup \tilde{Q}$, where n_0^d and n_1^d represent depot d at the beginning and the end of the day, respectively, and arc set A_d. Given the probability mass function (PMF) with finite supports of the travel time $(h_i(t))$ and the PMF of the energy consumption $(e_i(\mu))$ of each timetabled trip $i \in \mathcal{T}$ as well as the travel time t_{ij} and the energy consumption e_{ij} between the end location of node i and the start location of node j, for all pairs of nodes $i, j \in V_d$, the first stage problem of the S-MDEVSP consists of finding an a priori set of vehicle routes $\mathcal{R}^*$ that covers exactly once each trip $i \in \mathcal{T}$ and respects the capacity b_d of each depot $d \in \mathcal{D}$. A vehicle route is defined as a sequence of timetabled trips and time-expanded charging nodes starting and ending at a depot $d \in \mathcal{D}$. The amount of energy recharged at each time-expanded charging node included in a vehicle route is derived from a piecewise linear function similar to the one used in Montoya et al. (2017). In the second stage, the travel time and energy consumption values are revealed and the a priori plan is modified with respect to a recourse policy to guarantee energy feasibility. A vehicle route is considered feasible if the state of charge (SoC) of the BEB never falls below σ^{min} (e.g., 0%), or if one or several recourse actions can be taken to regain energy feasibility. A recourse action is taken at the second stage if the SoC of a bus is under ω (e.g., 50%) after a charging activity. It consists in extending the charging activity by one or several time intervals in order to reach a SoC of at least ω.

Our model for the S-MDEVSP uses the following notation. Let $\mathcal{R}$ be the set of all feasible vehicle routes, $\mathcal{R}^d$ be the subset of these routes starting and ending at the depot d, y_r be a binary variable equal to 1 if vehicle route r is selected, and a_{ir} be a binary parameter equal to 1 if route r covers trip $i \in \mathcal{T}$. The S-MDEVSP can be formulated as the following integer linear program:

$$\min \quad \sum_{r \in \mathcal{R}} \bar{c}_r y_r \tag{1}$$

$$\text{s.t.} \quad \sum_{r \in \mathcal{R}} a_{ir} y_r = 1, \qquad \forall i \in \mathcal{T} \tag{2}$$

$$\sum_{r \in \mathcal{R}^d} y_r \leq b_d, \qquad \forall d \in \mathcal{D} \tag{3}$$

$$y_r \in \{0, 1\}, \qquad \forall r \in \mathcal{R}, \tag{4}$$

where $\bar{c}_r = c_r + \beta \mathbb{E}[W_r(t, \mu)]$ is the expected cost of vehicle route r, c_r is the operational costs of r, β is a weighting factor, and $\mathbb{E}[W_r(t, \mu)]$ is the expected second-stage cost or r. This latter cost penalizes the delay a passenger is likely to encounter in route r. Specifically, $\mathbb{E}[W_r(t, \mu)] = \sum_{i \in r \cap \mathcal{T}} \alpha_i \mathbb{E}(X_i^r)$, where α_i is the relative passenger flow (or demand volume) on timetabled trip i and X_i^r is the secondary delay of timetabled trip i covered by route r (in minutes). A vehicle route r may be delayed because the travel times of its trips deviate from the planned time, because buffer times before trips are not sufficient, or because recourse actions are required. By adjusting the weighting factor β, one can find solutions with different trade-offs between operational costs and the expected second-stage cost. In general, the larger the β the more reliable the S-MDEVSP solutions. Analytical equations to compute in the first stage $\mathbb{E}[W_r(t, \mu)]$, for all $r \in \mathcal{R}$ generated, are developed in the following two sections.

Probability of using a recourse action.

Consider a vehicle route $r = (1, 2, \ldots, i, i+1, \ldots, j-1, j, \ldots, n)$ with trips i and $j \in \mathcal{T}$ interspersed by a charging activity of $j-i$ time intervals (i.e., $i+1, i+2, \ldots, j-1$ are time-expanded charging nodes). Let $m_j^r(z)$ be the PMF with finite supports of the SoC of bus r just before trip j. The probability of not having to extend the charging time is

$$\Pr\{0 \text{ extra charge periods before } j \in r\} = \sum_{z=\omega}^{100} m_j^r(z), \tag{5}$$

and the probability of having to extend the charging time of ϕ charge periods is

$$\begin{aligned} \Pr\{\phi \text{ extra charge periods before } j \in r\} &= \Pr\{z | \Lambda_\omega(z) = \phi\} \\ &= \sum_{z=\sigma^{min}}^{\omega-1} m_j^r(z)[\Lambda_\omega(z) = \phi], \qquad \phi = 1, 2, \ldots, k, \end{aligned} \tag{6}$$

where $\Lambda_\omega(z)$ is a function outputting the minimum number of additional charge time periods to be performed when the initial SoC of a BEB is equal to z in order to get an updated SoC of at least ω. We use the Iverson bracket (Iverson, 1962) notation (i.e., $[P]$ is equal to 1 if P is true and 0 otherwise).

Delay propagation.

Let $f_i^r(y)$ be the PMF with finite supports of the actual start time of activity i assigned to route r and $g_i^r(x)$ be the PMF with finite supports of X_i^r, i.e., of the secondary delay of trip i, such that $g_i^r(x) = f_i^r(x + d_i)$ when $i \in \mathcal{T}$. For $i \notin \mathcal{T}$, $g_i^r(x)$ is not defined.
Consider a route $r = (1, 2 \ldots, n)$ and denote $P_{ir}^0 := \Pr\{0 \text{ extra charge periods before } i \in r\}$ and $P_{ir}^\phi := \Pr\{\phi \text{ extra charge periods before } i \in r\}$. We assume that the first timetabled trip of a vehicle route r is never delayed (i.e., $f_1^r(d_1)) = 1$). Consider a trip $j \in (2, 3, \ldots, n)$ preceded by a trip i. The distribution of the actual start time of trip j can be recursively computed as

$$f_j^r(y) = \begin{cases} \displaystyle\sum_{t=t_i^{min}}^{t_i^{max}} h_i(t) \sum_{y'=d_i}^{d_j - t - \Upsilon(i,j)} \left[f_i^r(y') P_{jr}^0 + \sum_{\phi=1}^{k} f_i^r(y' - \rho\phi) P_{jr}^\phi \right], & \text{if } y = d_j; \\ \displaystyle\sum_{t=t_i^{min}}^{t_i^{max}} h_i(t) \left[f_i^r(y - t - \Upsilon(i,j)) P_{jr}^0 + \sum_{\phi=1}^{k} f_i^r(y - t - \Upsilon(i,j) - \rho\phi) P_{jr}^\phi \right], & \text{if } y > d_j; \\ 0, & \text{otherwise}, \end{cases} \tag{7}$$

where t_i^{min} and t_i^{max} are the minimum and the maximum possible travel time values of timetabled trip $i \in \mathcal{T}$, respectively, and $\Upsilon(i,j)$ is equal to $t_{ij} + \tau$ if there is no charging activity between trips i and j, or $t_{iq} + t_{qj} + \tau + (j-i)\rho$ if there is a charging activity of $j-i$ time intervals at station $q \in \mathcal{Q}$ between trips i and j. Here, τ is the minimum layover time before each timetabled trip.
The expected secondary delay of a trip j assigned to r is expressed as $\mathbb{E}(X_j^r) = \sum_{x=0}^{x_{jr}^{max}} x \times f_j^r(x + d_j)$, where $x_{jr}^{max} = d_i + x_{ir}^{max} + t_{ij} + \tau + t_i^{max} - d_j$ is the maximum possible secondary delay of trip j when covered by vehicle route r. It should be observed that $f_j^r(y)$, $m_j^r(z)$, P_{ir}^0, and P_{ir}^ϕ are, by definition, route-dependent. Since the routes are not enumerated but rather generated in our algorithm, is it impossible to compute $f_j^r(y)$, $m_j^r(z)$, P_{ir}^0, and P_{ir}^ϕ for all $i \in \mathcal{T}$ and $r \in \mathcal{R}$ beforehand. Instead, the latter are dynamically generated throughout the solution process.
Every time a trip i is delayed of 1 minute, a penalty of $\beta\alpha_i$ is paid. Depending on the transport agency's level of delay aversion, the weighting factor β can be adjusted to find an appropriate trade-off between the operational costs and reliability. Generally speaking, the larger the β, the more reliable (or delay-tolerant) the S-MDEVSP solutions.

3 Heuristic branch-and-price algorithm for the S-MDEVSP

Since there is generally a very large number of feasible vehicle routes in the S-MDEVSP, we propose a branch-and-price solution approach that generates columns (i.e., vehicle routes) instead

of enumerating them. We use the same heuristic branching strategy as in Ricard et al. (2022) to obtain integer solutions in a reasonable amount of time.
To identify columns that could be potentially useful to add, we solve one pricing problem per depot $d \in \mathcal{D}$ at each iteration. These pricing problems are defined on the networks G^d, for $d \in \mathcal{D}$, with modified arc costs $\tilde{c}^r_{ij}$ defined as

$$\tilde{c}^r_{ij} = \begin{cases} \bar{c}^r_{ij} - \pi_d, & \text{if } i = n^d_o \\ \bar{c}^r_{ij} - u_i, & \text{if } i \in \mathcal{T}, \end{cases} \tag{8}$$

where $(u_i)_{i\in\mathcal{T}}$ and $(\pi_d)_{d\in\mathcal{D}}$ are dual variables associated with constraints (2) and (3), respectively. Since the cost of the arcs is stochastic and path-dependent in the S-MDEVSP, these pricing problems correspond to shortest path problems with stochasticity (Boland et al., 2015; Wellman et al., 2013) that can be solved by a modified version of the labeling algorithm (see Ahuja et al. (1993) for more details on this algorithm). Next, we specify the main characteristics of the dynamic programming algorithm, namely the labels, the extension functions, and the stochastic dominance criteria.

Labels.

Each label stores a representation of the actual start time cumulative distribution function (CDF), a representation of the SoC CDF, and the accumulated reduced cost. Let $F^p_j(y)$ be the CDF of $f^p_j(y)$ at node j defined as

$$F^p_j(y) = \sum_{y'=d_j}^{y} f^p_j(y'), \tag{9}$$

and let $M^p_j(z)$ be the CDF of $m^p_j(z)$ at node j defined as

$$M^p_j(z) = \sum_{z'=\sigma^{min}}^{z} m^p_j(z'). \tag{10}$$

The label L^p_j of path p at node j is defined as $L^p_j = (F^p_j(d_j), \ldots, F^p_j(y^{max}_{jp}), M^p_j(\sigma^{min}), \ldots, M^p_j(100), C^p_j)$, where y^{max}_{jp} is the maximum value of $F^p_j(y)$ and C^p_j is the accumulated reduced cost.

Extension functions.

We want to extend a label $L^{p'}_i = (F^{p'}_i(d_i), \ldots, F^{p'}_i(y^{max}_{ip'}), M^{p'}_i(\sigma^{min}), \ldots, M^{p'}_i(100), C^{p'}_i)$ associated with node i along arc (i, j) to create label L^p_j. The accumulated reduced cost C^p_j at node j is given by

$$C^p_j = C^{p'}_i + \tilde{c}^p_{ij}. \tag{11}$$

In Section 2, we devised a method to analytically compute the propagation of delays in a sequence of timetabled trips. Here, we specify this method in the form of an extension function. The PDF of the actual start time of trip j covered by path p is given by

$$f^p_j(y) = \begin{cases} \sum_{t=t^{min}_i}^{t^{max}_i} h_i(t) \sum_{y'=d_i}^{d_j-t-t_{ij}-\tau} f^{p'}_i(y'), & \text{if } i, j \in \mathcal{T} \text{ and } y = d_j \\ \sum_{t=t^{min}_i}^{t^{max}_i} h_i(t) f^{p'}_i(y - t - t_{ij} - \tau), & \text{if } i, j \in \mathcal{T} \text{ and } y > d_j \text{ or } i \in \mathcal{T} \text{ and } j \in \tilde{\mathcal{Q}} \\ f^{p'}_i(y - \rho), & \text{if } i, j \in \tilde{\mathcal{Q}} \\ \sum_{y'=d_i}^{d_j-t_{ij}-\tau} \left[f^{p'}_i(y') P^0_{jp} + \sum_{\phi=1}^{k} f^{p'}_i(y' - \rho\phi) P^\phi_{jp} \right], & \text{if } i \in \tilde{\mathcal{Q}}, j \in \mathcal{T}, \text{ and } y = d_j \\ f^{p'}_i(y - t_{ij} - \tau) P^0_{jp} + \sum_{\phi=1}^{k} f^{p'}_i(y - t_{ij} - \tau - \rho\phi) P^\phi_{jp}, & \text{if } i \in \tilde{\mathcal{Q}}, j \in \mathcal{T}, \text{ and } y > d_j \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

The components $M^p_j(\cdot)$ are computed as

$$m_j^p(z) = \begin{cases} \sum_{\mu=\sigma^{min}}^{100} e_i(\mu) m_i^{p'}(z+\mu+e_{ij}), & \text{if } i \in \mathcal{T} \\ \sum_{z'=\sigma^{min}}^{100} m_i^{p'}(z')[\lambda(z', \rho \times \Lambda_\omega(z')) = z], & \text{if } i,j \in \tilde{\mathcal{Q}} \\ m_i^{p'}(z+e_{ij}), & \text{if } i \in \tilde{\mathcal{Q}}, j \in \mathcal{T}, P_{jp}^0 = 1 \\ m_i^{p'}(z+e_{ij}) + \sum_{z'=0}^{\omega-1} m_i^{p'}(z')[\lambda(z', \rho \times \Lambda_\omega(z')) = z], & \text{if } i \in \tilde{\mathcal{Q}}, j \in \mathcal{T}, \\ & z \geq \omega, 1 - P_{jp}^0 > 0 \\ 0, & \text{otherwise,} \end{cases} \tag{13}$$

where $\lambda(z,t)$ is a piecewise linear function giving the final SoC of a battery after a charge of t minutes that started with an initial SoC of z. We assume all BEBs start the day fully charged.

Stochastic dominance criteria.

Consider two paths p^1 and p^2 with resident node i. Path p^1 dominates path p^2 when the following conditions hold:

1. $C_i^{p^1} \leq C_i^{p^1}$
2. $F_i^{p^1}(y) \geq F_i^{p^2}(y)$, for all $y \in \{d_i, d_i+1, \ldots, d_i + \max\{y_{ip^1}^{max}, y_{ip^2}^{max}\}\}$
3. $M_i^{p^1}(z) \leq M_i^{p^1}(z)$, for all $z \in \{\sigma^{min}, \sigma^{min}+1, \ldots, 100\}$

All dominated paths can be safely discarded because they are not part of the Pareto-optimal set of paths or will not be extended into Pareto-optimal paths.

4 Computational results

We tested our model on a real-life instance from the city of Montréal of 273 timetabled trips, 2 depots, and 2 charging stations. To minimize battery degradation, σ^{min} is set to $\sigma^{min} = 25\%$. We compared our approach for two different types of commercially available BEBs; an electric shuttle with battery capacity (C) of $C = 80$ kWh, charger power (W) of $W = 60$ kW, and an average consumption rate of 0.76 kWh/km (Gao et al., 2017) and a 35-foot transit bus with $C = 492$ kWh, $W = 221$ kW, and an average consumption rate of 1.57 kWh/km (Proterra, 2022). We assume the energy consumption distributions follow a normal distribution.
The heuristic performance of our solution approach and the quality of the solutions are reported in Table 1 and 2, for the first and second type of BEB, respectively, for β values ranging from 0 to η, where η is the cost per bus used, and ω values ranging from σ^{min} to 75% of the battery capacity. When $\omega = \sigma^{min}$, our approach is equivalent to a robust optimization approach (i.e., no corrective actions). The columns display the relative difference in percentage between the upper bound and the lower bound (Gap), the number of branching nodes explored (Nodes), the computing times (CPU time), including the total time in seconds (Total), the portion of the total time dedicated to solve the root node (Root) and the pricing problems (Pricing), the operational costs (Op. costs), the fleet size (bus) and the total penalty for delays ($\sum_{r \in \mathcal{R}^*} \mathbb{E}[W_r(t,z)]$).
For both vehicle types, all problems are solved in less than 2 hours with almost all the computing time spent on solving the pricing problems. Also, the solutions obtained with our approach are at most 0.16 % more expensive than their corresponding lower bound, suggesting that our heuristic can find near-optimal solutions. Generally speaking, when β increases, the operational costs increase and the reliability improves.
For the first type of BEB, namely the shuttle with $C = 80$ kWh and $W = 60$ kW, the introduction of the recourse policy provides significant cost savings. Indeed, the fleet size can be reduced from 30 BEBs to 29 BEBs by introducing a recourse policy with $\omega \geq 50$, which could be considered as a substantial reduction since the number of vehicles used constitutes the major part of the operational costs. Furthermore, the deterioration in reliability that the charging policy introduces can be compensated for by a higher weighting factor β.
For the second type of BEB, namely the 35-foot transit bus with $C = 492$ kWh and $W = 221$ kW, introducing a recourse policy does not improve the cost of the solutions found nor does it reduce the size of the fleet. Thus, for this second type of vehicle with larger battery capacity and higher charging power, our approach is not useful and a simple robust optimization approach should be used to find vehicle routes such that the vehicles never run out of energy. Indeed, for this type of

Table 1: S-MDEVSP heuristic performance and quality of the solutions, with $C = 80$ kWh and $W = 60$ kW

		Heuristic performance					Quality of the solutions		
				CPU time					
β	ω	Gap (%)	Nodes	Total (s)	Root (%)	Pricing(%)	Op. costs	Bus	$\sum_{r\in\mathcal{R}^*} \mathbb{E}[W_r(t,z)]$
0	σ^{min}	0.05	28	1,361	39.0	99.4	32,301.8	30	0.57
	35	0.05	28	1,362	39.4	99.5	32,301.8	30	0.57
	50	0.05	27	1,567	42.1	99.5	31,374.0	29	0.70
	75	0.09	27	2,054	38.5	99.6	31,438.4	29	0.67
	avg.	**0.06**	**28**	**1,586**	**39.8**	**99.5**	**31,854.0**	**30**	**0.63**
$\eta/2$	σ^{min}	0.13	28	3,264	34.7	99.8	32,364.0	30	0.24
	35	0.13	28	3,152	34.2	99.8	32,364.0	30	0.24
	50	0.14	28	5,170	24.9	99.9	31,451.6	29	0.26
	75	0.16	29	6,341	28.3	99.9	31,430.8	29	0.33
	avg.	**0.14**	**28**	**4,482**	**30.5**	**99.8**	**31,902.6**	**30**	**0.27**
η	σ^{min}	0.09	27	3,236	38.2	99.8	32,431.2	30	0.12
	35	0.09	27	3,226	37.8	99.8	32,431.2	30	0.12
	50	0.14	30	5,681	25.8	99.9	31,526.0	29	0.15
	75	0.08	25	6,032	36.3	99.9	31,471.0	29	0.22
	avg.	**0.10**	**27**	**4,544**	**34.5**	**99.9**	**31,964.8**	**30**	**0.15**

Table 2: S-MDEVSP heuristic performance and quality of the solutions, with $C = 492$ kWh and $W = 221$ kW

		Heuristic performance					Quality of the solutions		
				CPU time					
β	ω	Gap (%)	Nodes	Total (s)	Root (%)	Pricing(%)	Op. costs	Bus	$\sum_{r\in\mathcal{R}^*} \mathbb{E}[W_r(t,z)]$
0	σ^{min}	0.02	21	1,059	49.7	99.6	27,290.4	26	0.49
	35	0.02	21	1,434	53.8	99.6	27,290.4	26	0.49
	50	0.02	21	1,145	49.4	99.6	27,290.4	26	0.49
	75	0.01	19	1,106	53.3	99.6	27,286.4	26	0.41
	avg.	**0.02**	**21**	**1,186**	**51.5**	**99.6**	**27,289.4**	**26**	**0.47**
$\eta/2$	σ^{min}	0.01	19	2,025	49.2	100.0	27,325.2	26	0.16
	35	0.01	19	2,855	53.4	100.0	27,325.2	26	0.16
	50	0.01	19	2,051	49.1	100.0	27,325.2	26	0.16
	75	0.01	21	2,567	49.1	99.9	27,325.2	26	0.16
	avg.	**0.01**	**20**	**2,374**	**50.2**	**100.0**	**27,325.2**	**26**	**0.16**
η	σ^{min}	0.01	21	2,091	46.7	100.0	27,362.6	26	0.11
	35	0.01	21	3,002	50.1	100.0	27,362.6	26	0.11
	50	0.01	21	2,294	47.0	99.9	27,362.6	26	0.11
	75	0.01	22	2,051	47.5	99.9	27,364.0	26	0.11
	avg.	**0.01**	**21**	**2,360**	**47.8**	**99.9**	**27,363.0**	**26**	**0.11**

BEB, the battery capacity is large enough that vehicles only need to charge once or twice a day. Because timetables of trips typically include off-peak periods with fewer trips to make, charging activities can easily be scheduled during these periods and batteries are often charged to their maximal capacity. In this context, the recourse policy we introduced is never activated.

5 Conclusions

In this work, we introduced a stochastic model for the MDEVSP that we formulated as a two-stage stochastic program with a recourse action. We proposed an efficient branch-and-price algorithm to solve this challenging problem. Our results indicated that the use of recourse actions is beneficial for shuttle BEBs with relatively small battery capacity and charging power, but not for 35-foot transit BEBs with larger battery capacity and charging power. Medium- to large-scale transit agencies are typically equipped with up-to-date BEBs that resemble the second type of vehicle tested, so our approach is probably not relevant for them. However, our two-stage stochastic model may be relevant for smaller transit agencies or those with access to fewer resources. Future work includes translating our approach to other routing problems with smaller electric vehicles, for example the electric dial-a-ride problem.

Acknowledgements

This project was partially supported by GIRO Inc. and the Natural Sciences and Engineering Research Council of Canada under the grants RDCPJ 520349-17 and BESC D3-558645-2021.

References

Ahuja, R. K., Magnanti, T. L., & Orlin, J. B. (1993). *Network flows: Theory, algorithms, and applications.* Prentice Hall.

Bie, Y., Ji, J., Wang, X., & Qu, X. (2021). Optimization of electric bus scheduling considering stochastic volatilities in trip travel time and energy consumption. *Computer-Aided Civil and Infrastructure Engineering*, *36*(12), 1530-1548. doi: https://doi.org/10.1111/mice.12684

Boland, N., Dickson, S., Savelsbergh, M., & Smilowitz, K. (2015). *Dominance in pricing problems with stochasticity.* (accessed February 22, 2022)

Gao, Z., Lin, Z., LaClair, T. J., Liu, C., Li, J.-M., Birky, A. K., & Ward, J. (2017). Battery capacity and recharging needs for electric buses in city transit service. *Energy*, *122*, 588-600. doi: https://doi.org/10.1016/j.energy.2017.01.101

Iverson, K. E. (1962). *A programming language.* USA: John Wiley amp; Sons, Inc.

Jiang, M., Zhang, Y., & Zhang, Y. (2021). Optimal electric bus scheduling under travel time uncertainty: A robust model and solution method. *Journal of Advanced Transportation*, *2021*, 19. doi: https://doi.org/10.1155/2021/1191443

Li, L., Lo, H. K., Huang, W., & Xiao, F. (2021). Mixed bus fleet location-routing-scheduling under range uncertainty. *Transportation Research Part B: Methodological*, *146*, 155-179. doi: https://doi.org/10.1016/j.trb.2021.02.005

Montoya, A., Guéret, C., Mendoza, J. E., & Villegas, J. G. (2017). The electric vehicle routing problem with nonlinear charging function. *Transportation Research Part B: Methodological*, *103*, 87-110. doi: https://doi.org/10.1016/j.trb.2017.02.004

Proterra. (2022). *ZX5 35-foot battery-electric transit bus - platform specifications.* Retrieved 2023-03-01, from `https://www.proterra.com/wp-content/uploads/2022/09/SPEC_35_001_METRIC_Q4_2022_V1_09_09_22-1.pdf`

Ricard, L., Desaulniers, G., Lodi, A., & Rousseau, L.-M. (2022). Predicting the probability distribution of bus travel time to measure the reliability of public transport services. *Transportation Research Part C: Emerging Technologies*, *138*, 103619. doi: https://doi.org/10.1016/j.trc.2022.103619

Tang, X., Lin, X., & He, F. (2019). Robust scheduling strategies of electric buses under stochastic traffic conditions. *Transportation Research Part C: Emerging Technologies*, *105*, 163-182. doi: https://doi.org/10.1016/j.trc.2019.05.032

Wellman, M. P., Ford, M., & Larson, K. (2013). *Path planning under time-dependent uncertainty.*