Large-Scale Continuous Pricing With Discrete Choice Demand Modeling

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Introduction

Merging realistic demand modeling via advanced discrete choice models (DCMs, see box to the right) and optimization problems like continuous pricing has been a challenge for decades. This comes from the fact that, in general, the resulting probability of a costumer choosing a product does not have a closed-form expression. Paneque et. al [1] were the first to propose a general framework as a mixed integer linear program (MILP), dealing with stochasticity by the use of Monte-Carlo simulation. The consequence of this approach is having to deal with a large number of variables, making the MILP notoriously hard to solve and thus only capable of solving small instances.

We demonstrate first an equivalent, but more computationally tractable reformulation of the problem as a quadratically constrained linear program (QCLP) and propose an efficient Spatial Branch & Benders Decomposition algorithm to solve large problem instances.

Discrete Choice Models (DCMs) are used to predict choices made among any number of alternatives. Suppose N individuals, each characterized by some explanatory variables $\mathbf{x_n}$; a DCM computes their probability of choosing any alternative i as a function of $\mathbf{x_n}$ and β and p by first defining utilities $U_{in}(\beta, p)$ and then setting:

$$P_n(i \mid \mathbf{x_n}; \beta, p) \;=\; P_n(U_{in}(\beta, p) \geq U_{jn}(\beta, p) \; \forall j)$$

i.e. the probability of choosing a product is the probability that its utility is the highest among all alternatives.
Here β is a vector of parameters, estimated using for example maximum likelihood estimation, and p is the vector of prices set by the supplier.

Monte-Carlo Simulation generates R scenarios with deterministic utilities U_{inr} to be then compared directly.

Problem Formulation

$$\max_{p,\omega,\eta,U,H} \; \frac{1}{R} \sum_r \sum_n \sum_{i \geq 1} \eta_{inr}$$

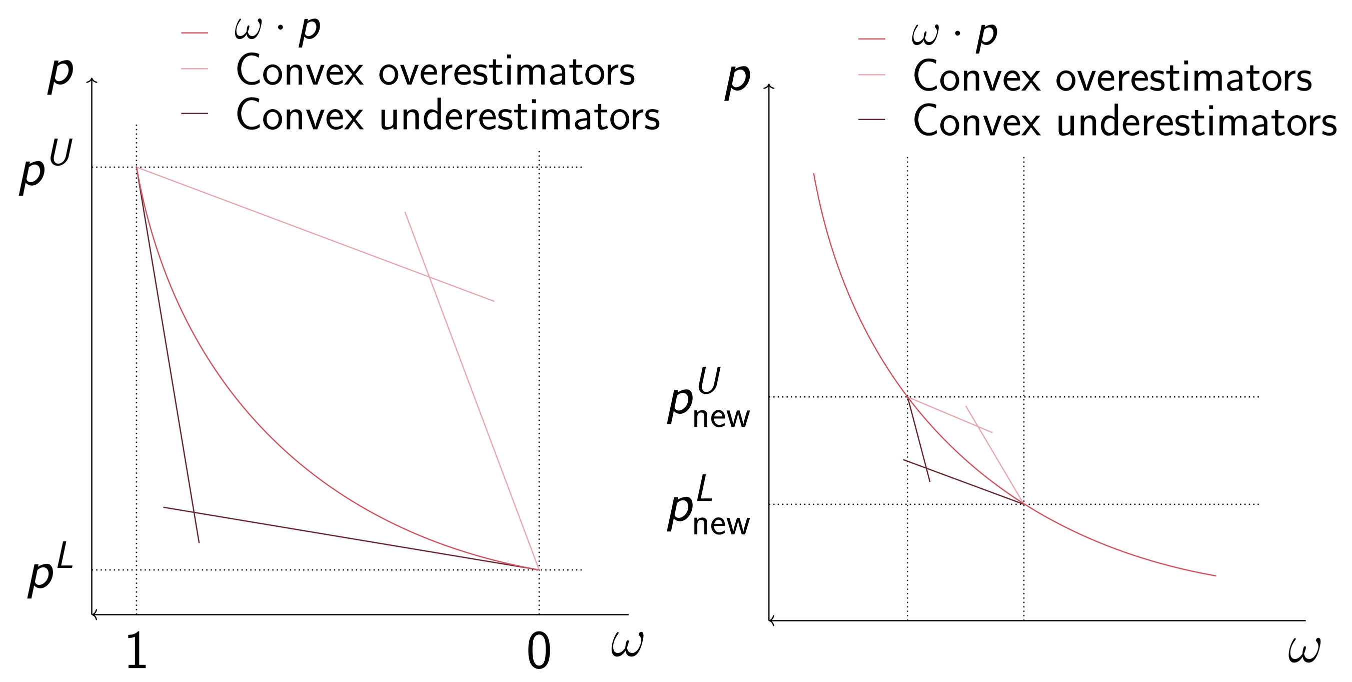
s.t.

$$
\begin{aligned}
\sum_i \omega_{inr} &= 1 & \forall n, r \quad (\mu_{nr}) \\
H_{nr} &= \sum_i c_{inr}\omega_{inr} + \beta_p^{in}\eta_{inr} & \forall n, r \quad (\zeta_{nr}) \\
H_{nr} &\geq c_{0nr} & \forall n, r \; (\alpha_{0nr}) \\
H_{nr} &\geq U_{inr} & \forall i \geq 1, n, r \; (\alpha_{inr}) \\
U_{inr} &= c_{inr} + \beta_p^{in} p_i & \forall i \geq 1, n, r \; (\kappa_{inr}) \\
\eta_{inr} &= p_i \omega_{inr} & \forall i \geq 1, n, r \; (\lambda_{inr}) \\
\omega &\in [0,1]^{(J+1)NR} \\
p, U, H &\in \mathbb{R}^J, \mathbb{R}^{JNR}, \mathbb{R}^{NR}
\end{aligned}
$$

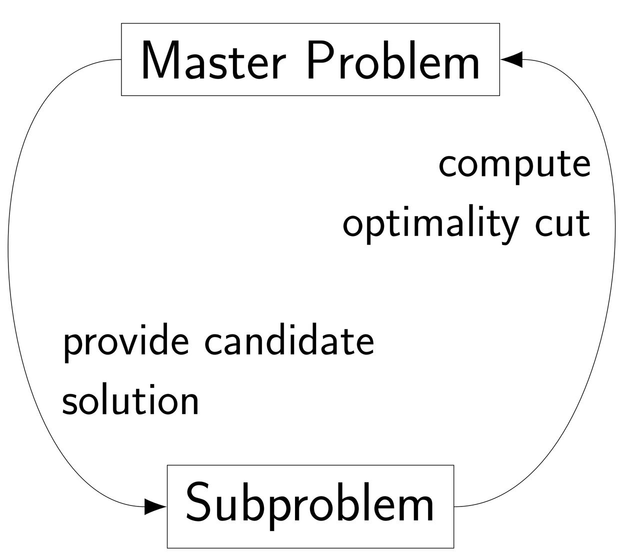
Formulation – Continuous pricing as a QCLP

Methodology

Spatial Branch & Bound with the McCormick Envelope :

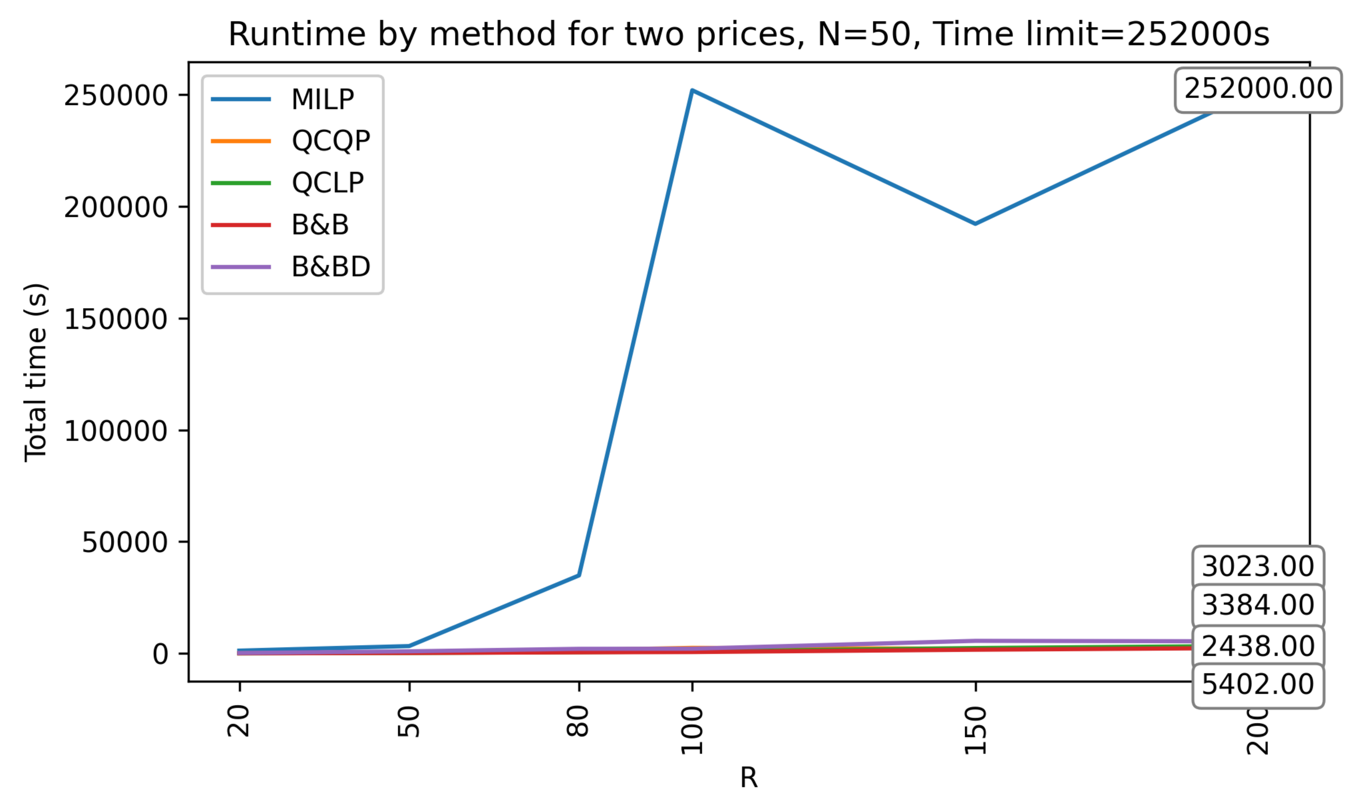


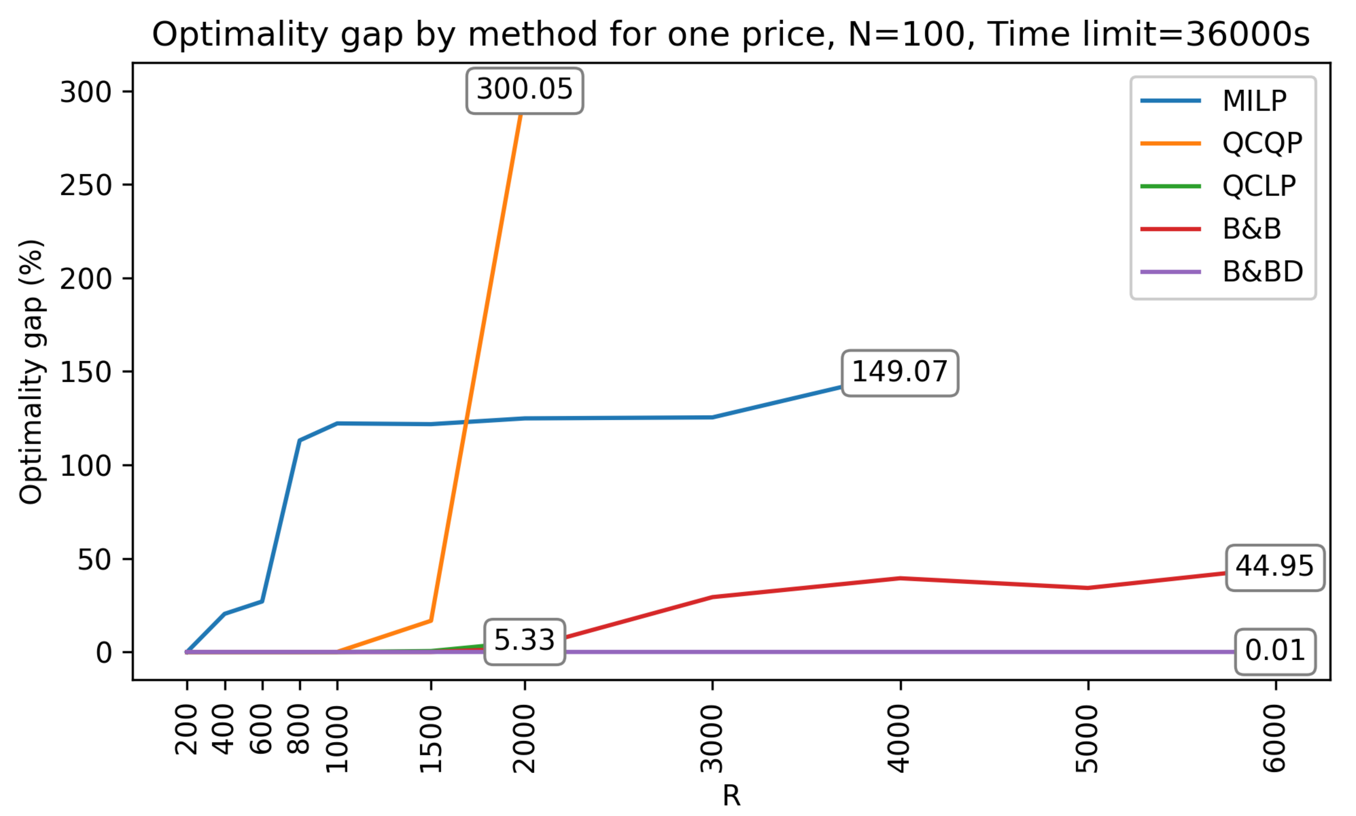
Benders Decomposition:



We relax the non-convex constraints $\eta_{inr} = p_i\omega_{inr}$ with the McCormick envelope, replacing it by two over- and two underestimators, and solve it using a Benders decomposition. We then branch on the bounds of the prices until our approximation reaches the desired resolution. For a fixed price, we can compute a feasible solution immediately, giving us fast upper bounds.

Results





Case study

We apply our method on a real-world dataset [2] taken directly from the literature, illustrating the fact that there is no need for any assumptions about the choice model to apply the methodology.

Conclusion

We developed a new formulation and solution approach for continuous pricing with DCM demand modeling. The gain in computational time mainly comes from the fact that we only branch on the prices, making use of the specific problem structure.
We show that already reformulating the MILP as a non-convex QCLP improves computational speed significantly. Our Branch & Bound approach substantially outperforms the state-of-the-art methods, solving large instances to optimality before GUROBI (the fastest publically available mathematical solver) finds a first feasible solution.

References

[1] Paneque, M. P., Bierlaire, M., Gendron, B., & Azadeh, S. S. (2021). Integrating advanced discrete choice models in mixed integer linear optimization. In: *Transportation Research Part B: Methodological, 146, 26–49*.

[2] Ibeas, A., Dell'Olio, L., Bordagaray, M., & Ortúzar, J. d. D. (2014). Modelling parking choices considering user heterogeneity. In: *Transportation Research Part A: Policy and Practice, 70, 41–49*.

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