

χ -Shell, a new spatial deployable lattice compared to traditional reticulated shells

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Abstract

The χ -Shells is a new kind of deployable reticulated shells that has features comparable to other traditional shells. Its deployment uses the mechanical properties of a beam grid to generate a three-dimensional shape. This new generation of reticulated shells differs in its typology, but its structural performance and construction process have points in common with other types of shells made in the past. We present the fundamental characteristics of this new generation of shells, comparing it to other existing families with criteria related to materials, span, self weight, load capacity, slenderness, construction process and joint detailing. The development of this technology is accompanied by small-scale physical models that validate the principle of deployment, and a full-scale pavilion to discuss constructive and static aspects.

Keywords: Deployable structure, grid-shells, scissors grid mechanism, reticulated shells.

1. χ -Shell concept

1.1. Definition

χ -Shells are a new class of deployable structure composed of ribbed linear elements connected together by hinges. The spacing between the hinges varies along each rib, creating a regular, but non-uniform pattern. The configuration of the χ -Shell ribs can be constructed and transported in a flat and compact state, then deployed into a double-curved spatial configuration, and finally used as a roof structure. The design of its ribs generates an elastic mechanism with only one degree of movement in the plane, which requires an expansion force to be activated. In theory, thanks to the property of the mechanism, the deployment can be actuated by pushing or pulling two opposite nodes of a quadrangle. In practice, because the actuating force depends on the return spring effect and the friction of the hinges, the actuation of several specific nodes reduces high local stresses and ensures uniform deployment.

1.1. Physical models

To apply the χ -Shell principle to surfaces with positive and negative curvature, we built two models, the first with hyperbolic geometry and the second with spherical geometry. The hyperboloid is composed of two crossed layers of nine straight ribs, and each rib is a 10 mm x 10 mm x 1 mm aluminum profile connected by a rivet that also serves as a pivot. The spherical model consists of two crossed layers of twelve identical straight flat ribs of 10 mm and 1 mm thick.

On both models, the deployment induces deformations and stresses in the ribs, the hyperboloid ribs only undergo torsion without bending, while the spherical ribs only undergo bending without torsion. The

cross-section of the ribs (angle for the hyperboloid and flat for the sphere) was chosen so that the ribs follow the deformations induced by the deployment while generating a minimum of parasitic stresses against this deployment.

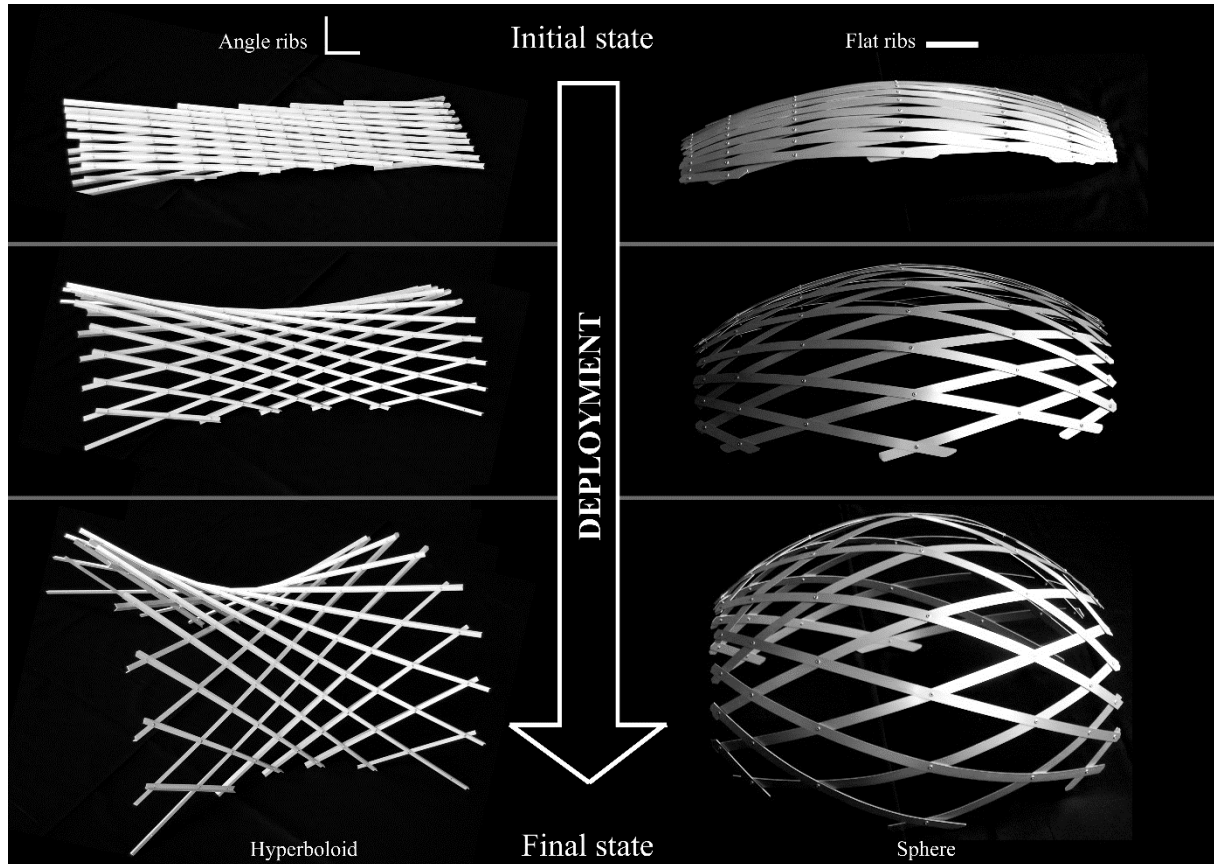


Figure 1: Left, one-meter long hyperboloid models. Right, one meter long spherical models where the flatness at the initial state is disturbed by collisions between the ribs.

The most important conclusion of these two models is that deployment leads to a single final geometry regardless of the nodes where the actuation is applied and the accuracy of the actuation. The deployment is robust in the sense that the actuation process can be completely independent of a geometric control process.

1.3. Shell like behavior

As mentioned above, deployment induces deformations and stresses in the ribs, which means that the actuation must counter a spring back effect, so that once deployment is achieved, the geometry must be locked. On the two previous models, the friction between the ribs at each node was sufficient to counter the spring back and lock the geometry, but other technical solutions must be devised on larger projects such as additional bars or ties, or even using a coating to reinforce the rib quads.

Once the geometry is locked, the χ -Shell becomes a solid unit independent of edge conditions and supports. In contrast to grid-shells where the support and edge conditions act strongly to maintain the geometry, the deployed shape of the χ -Shell is maintained by the rib pattern itself, once locked. The χ -Shell has no constraint at the boundary, which gives more freedom for shape design.

2. Structural typology

χ -Shells belong to the typology of deployable structures, but in terms of their structural properties and design process, a deployed χ -Shell could be linked to the diagrid family, as an irregular sequence of two diagonal generators, and could also be linked to the lattice shell family, as a variable quad pattern. As explained above, an χ -Shell differs from a grid-shell in terms of boundary conditions and design process (see also Panetta and al. [8]), but they have in common that the deformations induced at the ribs depend on the deployed configuration.

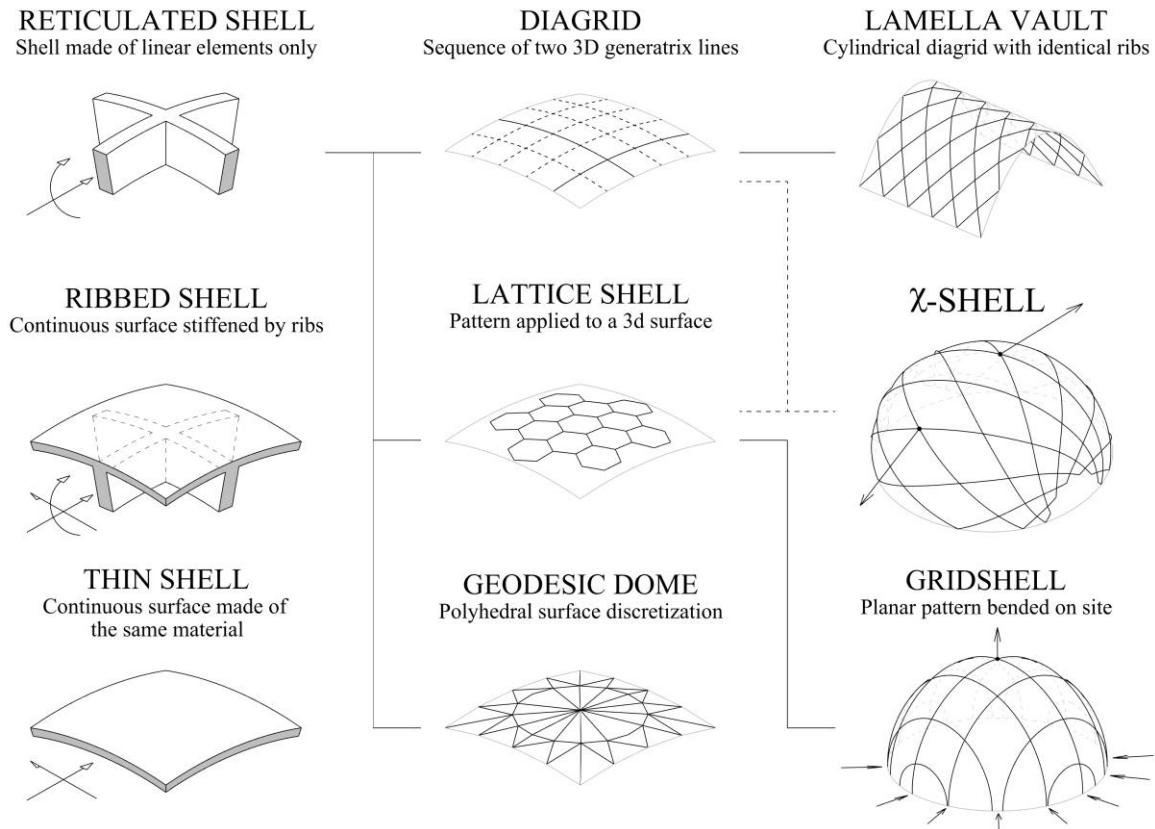


Figure 2: Diagram of shell families

As techniques progressed, new families of reticulated shells developed. Metal machining has favored the development of geodesic domes, new cement formulations have allowed the development of concrete shells, photogrammetry and digital tools have allowed the development of grid-shells. As shown on Fig. 3, until 1990 the development of shells was manifested by increasingly large spans. From the 2000s their geometries became more and more varied.

In this context, the future development of χ -Shells depends on architectural utility, performance, and economic reliability. We assume that their potential lies in short-span roof structures, for the main reason that their principal advantage is the rapid deployment on site. If the dimensions of the project require on-site assembly, on-site time increases and the use of a lifting device may be necessary, which limits the relevance of rapid deployment.

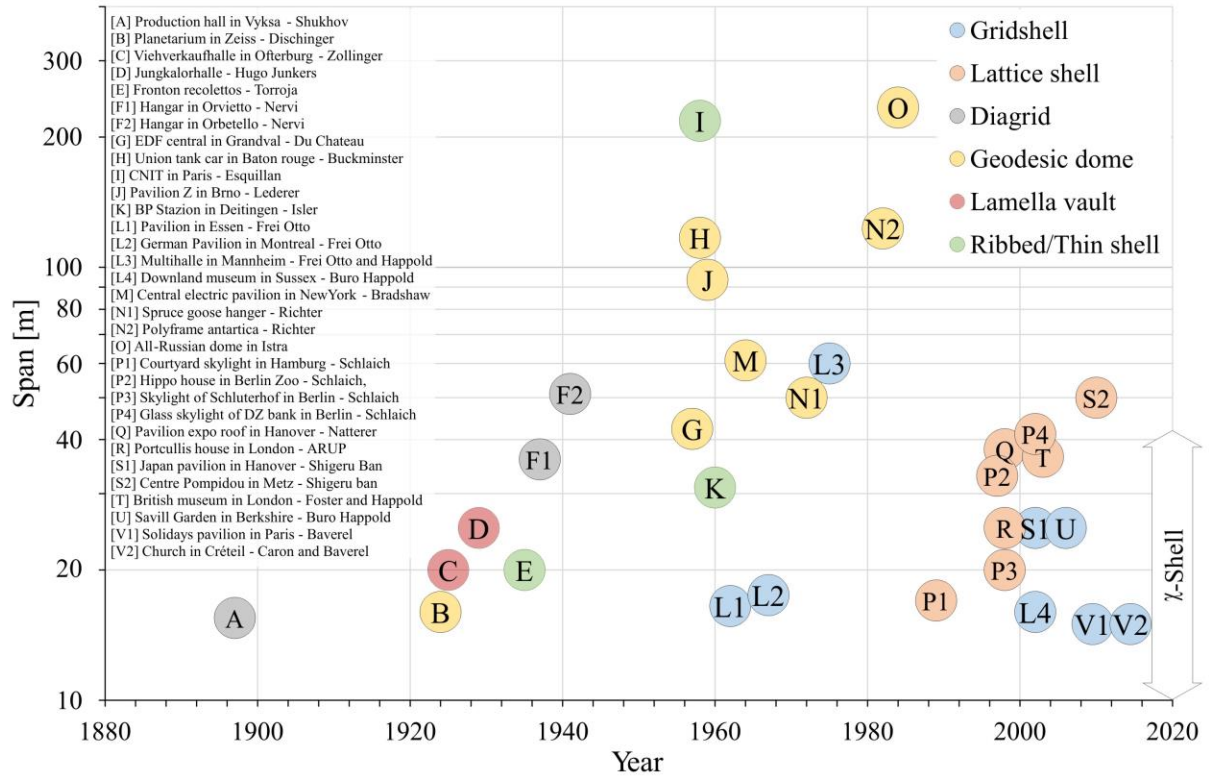
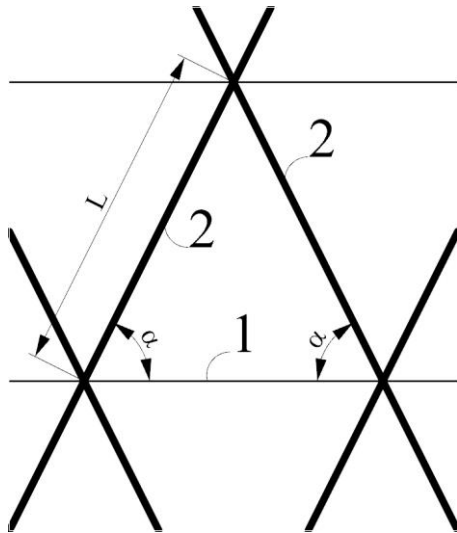


Figure 3: Few examples of shell by topology along time and span.

3. Performance

The assessment of the shell performance is based on two main criteria: relative capacity and slenderness. The first is related to the quantity of material used and the second to its relative size. To have an objective comparison with these two criteria, we estimate the equivalent using a method initiated by Wright [1] and extended here for a variable mesh geometry:



$$t'_x = \frac{24I_1 \sin \alpha}{\sqrt{3}A_1} \left(1 + \frac{I_2}{I_1} \cos^3 \alpha \right) (1 - v_x v_y) \quad (1)$$

$$t'_y = \frac{6I_2}{A_2 \cos \alpha} \left(1 + \frac{A_2}{A_1} \cos^3 \alpha \right) (1 - v_x v_y) \quad (2)$$

$$E'_x = \frac{A_1 E}{L t'_x \sin \alpha} \quad (3) \quad E'_y = \frac{2A_2 E \sin^3 \alpha}{L t'_y \left(1 + \frac{A_2}{A_1} \cos^3 \alpha \right)} \quad (4)$$

$$v_y = \frac{\cos \alpha}{2 \sin^2 \alpha} \quad (5) \quad v_x = \frac{2A_2 \cos^2 \alpha \sin^2 \alpha}{A_1 + A_2 \cos^3 \alpha} \quad (6)$$

$$p_c = 0,4 \frac{E' t'^2}{R^2} \quad (7)$$

Where E, A, I, L and R are respectively the elastic modulus, the area cross section, the inertia, the length and the radius of the rib elements, and t' , E' and p_c are respectively the effective thickness, the effective

modulus and the critical radial pressure. According to Mesnil and al [7], the buckling capacity of grid-shells must be reduced by 10%, due to the pre-bending stress, this reduction was applied to the concerned projects.

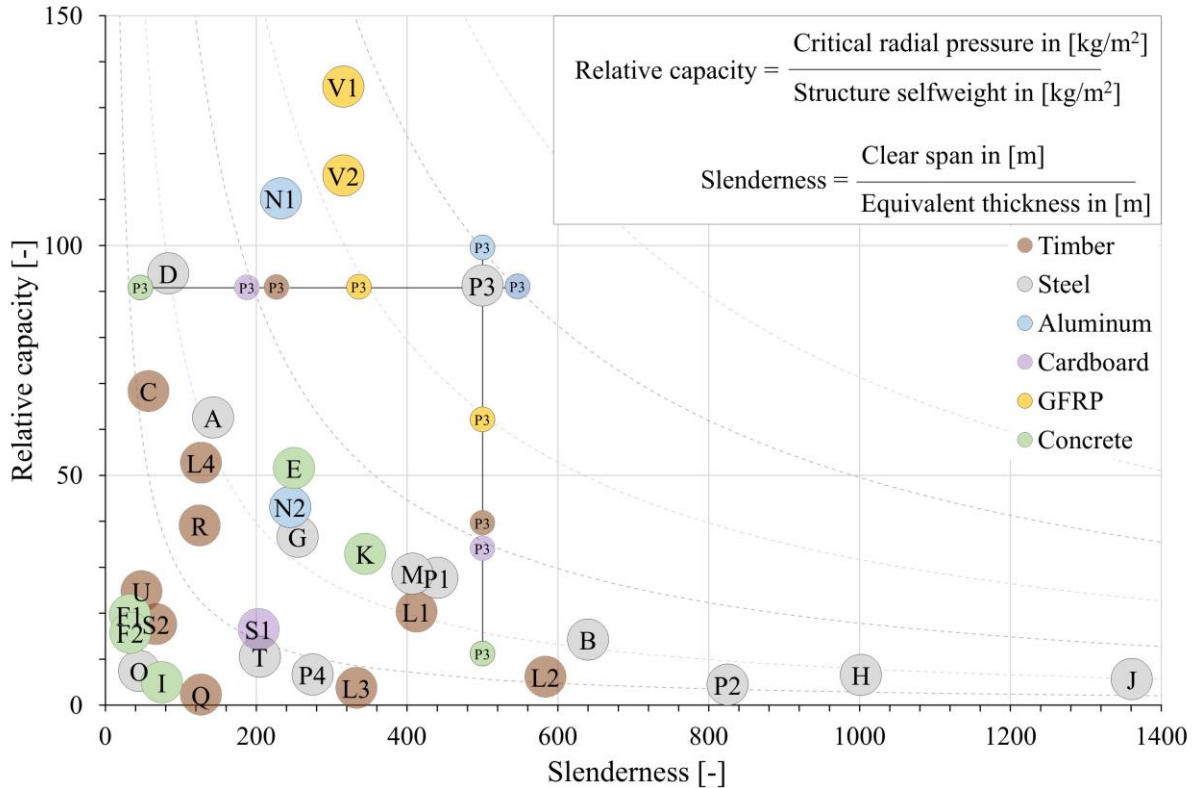


Figure 4: Diagram for shell examples comparison with the relative capacity and the slenderness. The dashed grey lines show the direction where the project tags move when the section property (A, I) change.

For all projects, the relative capacity evolves inversely to slenderness and vice versa. To increase the load capacity without changing the weight, it is necessary to increase the inertia and therefore reduce the slenderness. However, when one criterion is weak, its effect on the other is marginal. The risk of great slenderness with insufficient capacity is buckling instability as occurs for the project (J) - Pavilion Z in Brno which collapsed in 1963.

Changing the property of the material on a project such as (P3) leads to a variation in the relative capacity while maintaining the invariant slenderness. As shown in Fig. 4, if (P3) were made of aluminium instead of steel, the relative capacity would have increased by 9%, and if (P3) were made of GFRP its relative capacity would have decreased by 32%. In addition, with constant relative capacity, the variation in slenderness leads to the same order of material performance.

In general, the material modifies the performance of the shell, in order of importance: Aluminium, Steel, GFRP, Timber, Cardboard and last Concrete. For both criteria, it appears that the better the modulus/density (E/D) ratio of the material, the better the performance.

Obviously, an X-Shell needs an elastic material, because of the deformation induced at the ribs, but if the modulus is high, the stress induced by deployment would increase proportionally, so it is important to have a material with a long range of elasticity but not necessarily a high modulus E.

4. Construction process

Each reticulated shell has its own specific construction process, but we can distinguish three main types that are related to how geometry is managed during construction.

- Cast on mould: the geometry is nowhere in the shell elements and is entirely given by an associated temporary object which is both a mould and a support, which makes it difficult to remove afterward. This is the main type for thin concrete shells, masonry vaults, composite boat hulls, etc.
- Bricks stacking: the object is cut into small pieces for transport and assembly, then reassembled on site. The geometry is given by the elements themselves and the way they are subdivided. The accuracy of the final shape depends on the tolerances between elements, the assembly sets and the movements during the assembly sequence.
- Shaped on site: the elements are elastically shaped on site to obtain the spatial geometry, then anchored or fixed together to maintain this geometry. The accuracy of the geometry is difficult to manage due to the large number of parameters that influence this shaping. There is a trade-off between the need for rigidity to resist and the ease of shaping.

Each method presents particular difficulties that should not be underestimated in order to carry out the project correctly. Indeed, the Russian Istra dome (O) collapsed in 1984 due to a lack of geometric survey during construction, and several wooden ribs in the Mannheim Multihalle broke during lifting and had to be replaced.

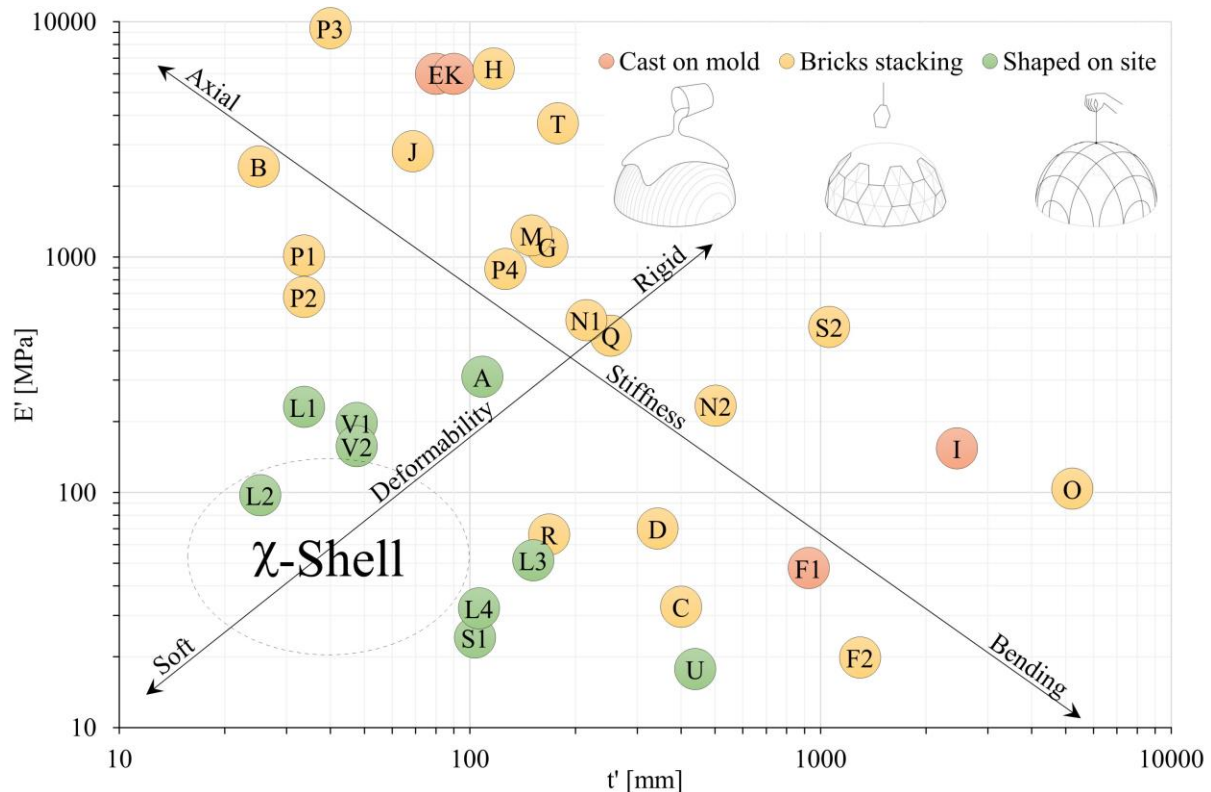


Figure 5: graph plotting the effective modulus E' with the effective thickness t' .

The χ -Shell belongs to the last type of construction process because the rib elements are bent or twisted in place to create the spatial shape, so the design consists of a compromise between the deformability of the ribs to deploy into a precise shape and the final stiffness to withstand once deployed.

The Wright method introduces the effective modulus of elasticity E' and the effective thickness t' which are related to the rigidity of an equivalent thin shell. As shown on fig. 5, projects that are shaped on site are those that combine a low E' value and a low t' value, therefore they require a high curvature to have an acceptable critical pressure. Usually, the bending strength requirement is determined by the local or global buckling requirement, which imposes a minimum thickness. The axial strength requirement could then be obtained by changing the density of the mesh or its axial section.

As with grid-shells, the χ -Shell design process is iterative. First, the need for strength defines a specific cross section of the rib, then the cross section of the ribs must allow deployment to achieve the desired geometry.

When the deformation of the ribs is torsional only, the need for buckling resistance is not in competition with the need for deployment. However, even for the hyperboloid model shown above, the irregular pattern implies an increase in the dimensions of the grid on the edge, which affects the effective module and reduces the loading capacity.

5. Joint detailing

Usually, the design of joint details comes after engineering calculations, but some joint details reflect the difficulties encountered during the design process and sometimes provide an opportunity to improve shell performance.

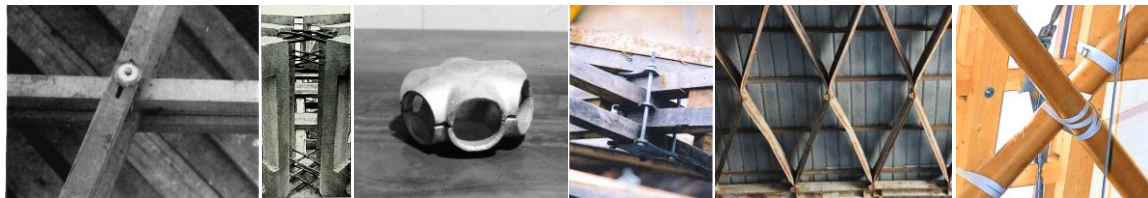


Figure 6: Few examples of joining, from left to right: Mannheim multihalle, Orbetello's hangar, Grandval Roof, Downland Museum, Oberschleißheim hangar, Hanover's Japan pavilion.

As shown on fig. 6, the slots in the Mannheim ribs allow sufficient sliding to limit stresses in the wooden ribs during lifting, on-site casting of connections between Orbetello's prefabricated beams eliminates differences in tolerance between each element, the prefabricated steel knot by Du Chateau makes repetition of detail affordable, the Junker joint system transmits bending stresses throughout the diagonal ribs, even the Japanese Pavilion nodes in Hanover is a solution to avoid drilling and weakening the cardboard pipes.

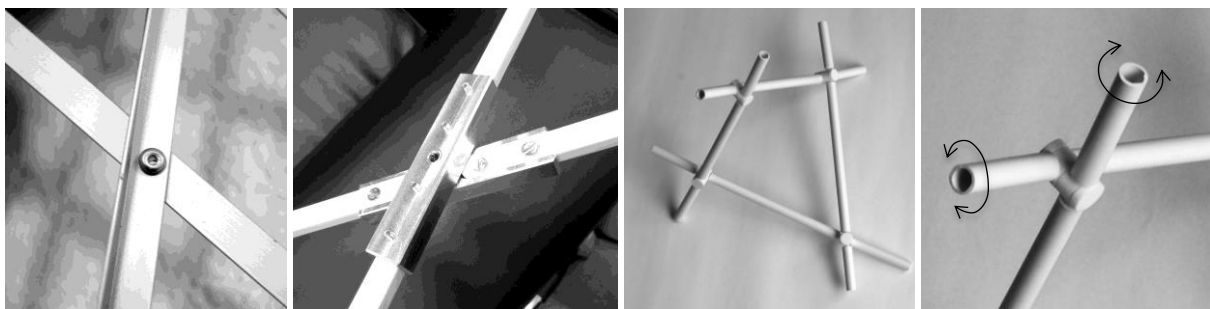


Figure 7: Few possibilities of rib joining for χ -Shell, from left to right: rib connected with a rivet, rib connected to a precast aluminum hinge, tubular rib connected with a 3D printed hinge, view of the 3d printed hinge with released rotation.

For an χ -Shell, the joints must transfer axial forces, bending and torsion moments and allow a rotational movement in the plane between the ribs. In the case of physical models, a single rivet between two ribs is sufficient, but for a larger project a more sophisticated joint that reduces friction between the ribs is required.

For the hyperboloid and other shapes where there is mainly torsion during deployment, it is even possible to design a node that does not transfer torsion, see fig.7. This will allow the use of a more rigid ribbed cross section like a tube without any disadvantages in terms of deployment.

Conclusion

The χ -Shell is completely new, it combines the performance of a reticulated shell with the advantage of a deployable mechanism. The scope of this technology is vast and it still needs to be further developed on several practical and technical aspects, such as joints, bracing and covering. The link between geometry and structure is so close that each project must have its own development, in terms of rib pattern, material, rib cross-section, assembly design and actuation system. The construction of the pavilion planned as part of the WG21 exhibition is an opportunity to show a concrete application of the χ -Shell on real scale, and to discuss the potential of the design method we have developed.

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