From Probability Graphical Models to Dynamic Networks

A Bayesian perspective on Smooth Best Estimate of Trajectory with applications in Geodetic Engineering

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A quick word about myself

- Background in computer vision, software engineering and statistical modelling.
- Working at EPFL, in the Environnement Sensing Observatory (ESO), a part of the CRYOS Lab in the ENAC faculty.
- Working on modelling new sensors in the factor graph optimization toolkit develloped by ESO : Dynamic Network.

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- Bayesian vs Frequentist, what is the difference
- Bayesian modelling – probability distributions as graphs
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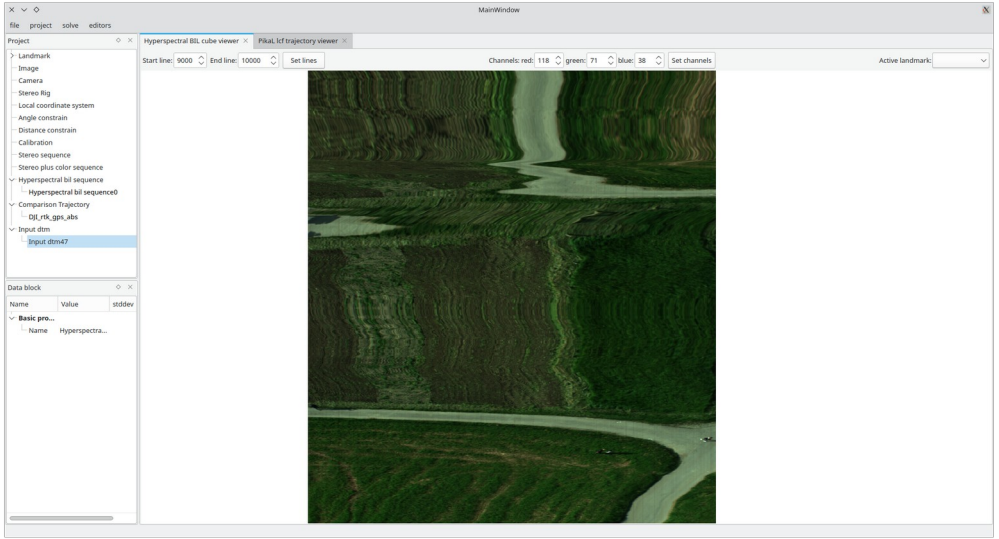
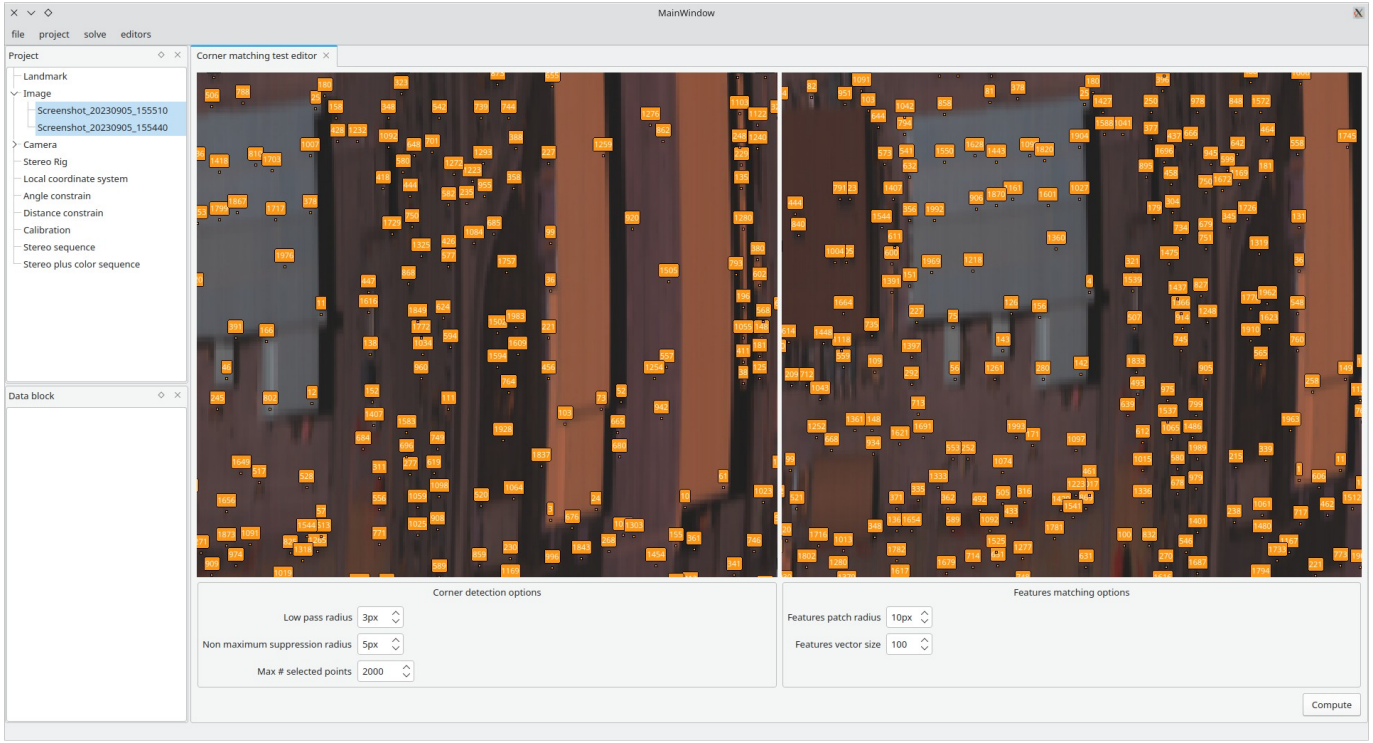
Challenges in mapping with lightweight hyperspectral sensors



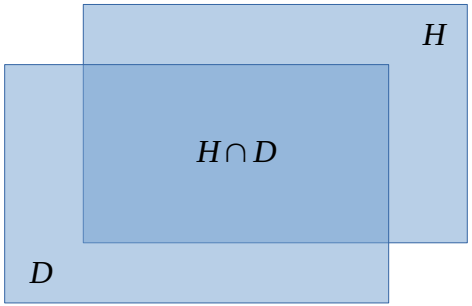
Hundreds of tie points, no redundancy



Bayesian vs Frequentist

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)} = \frac{P(D|H)P(H)}{\int P(D|H)P(H)dH} = \frac{P(D,H)}{P(D)}$$

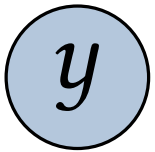
Frequentist interpretation
(set and measure theory)



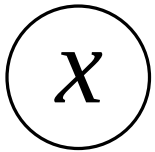
Bayesian interpretation
(information theory)

H	Initial information about hypothesis
D	Data measurements
$H\|D$	Refined knowledge with new evidences

Bayesian modelling – Probabilistic Graphical Models (PGM)



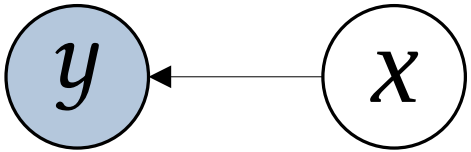
Observation



Latent (unobserved) variables

Function

Parameter



Conditional dependence of y on x in the prior

Conditional dependence of y on x and x on y in the prior

Building a probability distribution from a PGM

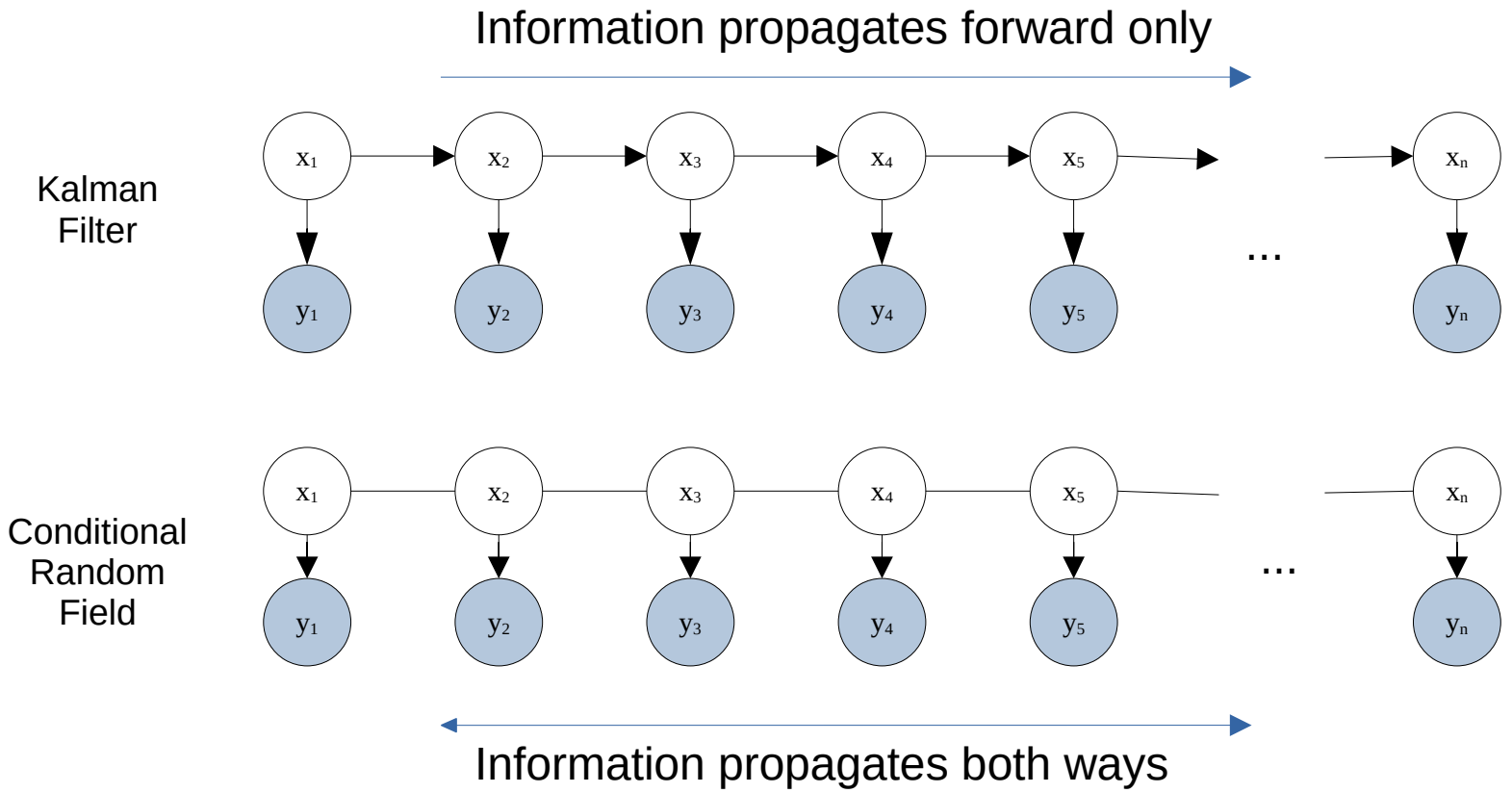
Directed PGM:

$$p(v_1, \ldots, v_n) = \prod_{i=1}^{n} p(v_i | parents(v_i))$$

Undirected PGM:

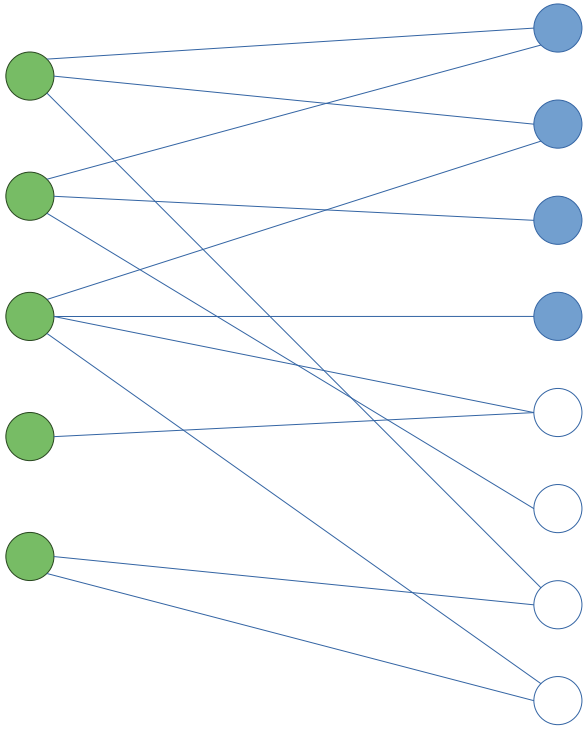
$$p(v_1, \ldots, v_n) = \prod_{i=1}^{n} f(v_i, connected(v_i))$$

The Kalman Filter vs Conditional Random Field



Factor graphs – modelling joint distributions

Functions



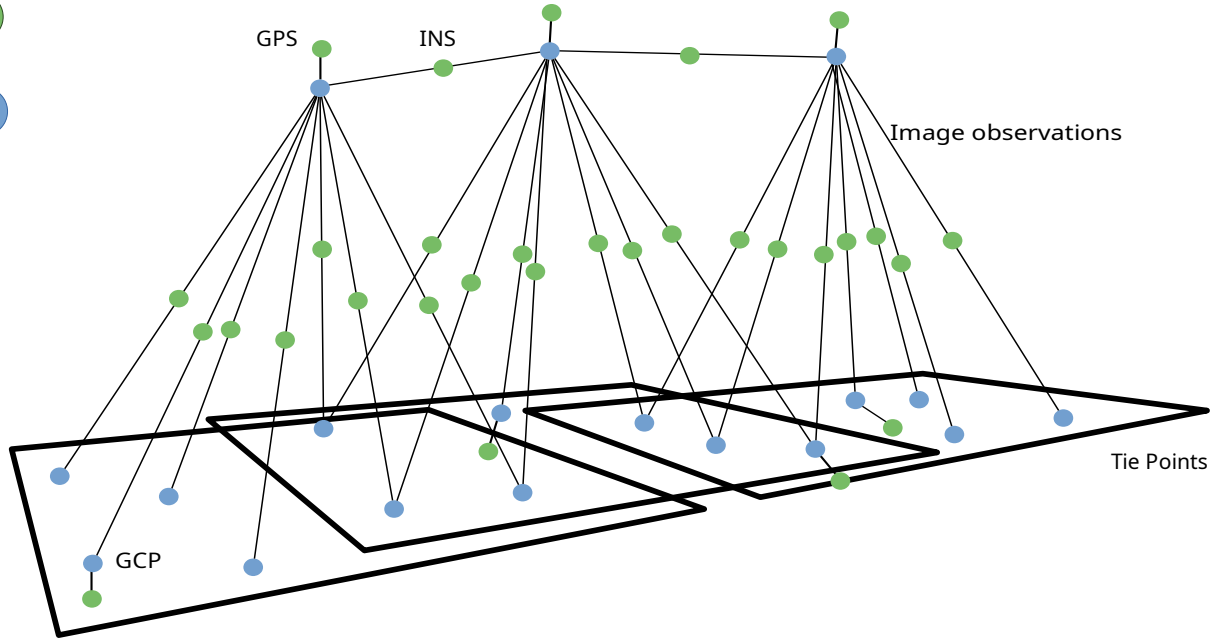
Observed variables

Latent variables

Modelling a factor graph – Bundle adjustment example

Functions

Variables



Modelling a push-broom hyperspectral camera

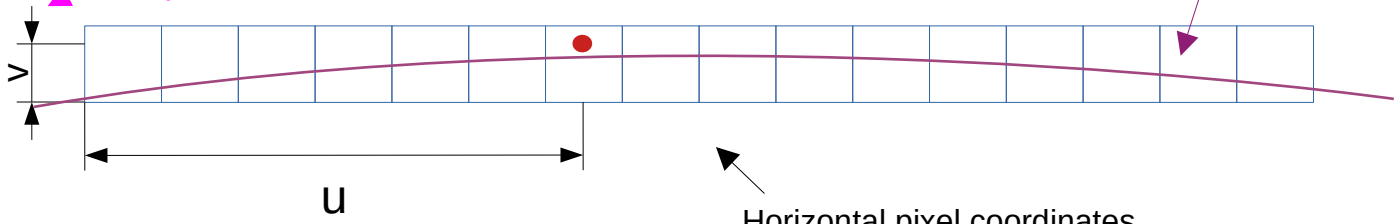
$$\Delta x_h = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5$$

$$\Delta y_h = b_0 + b_1 u + b_2 u^2 + b_3 u^3 + b_4 u^4 + b_5 u^5$$

$$\Gamma_t = \left(\nabla \Gamma_i\right)^v \Gamma_i$$

Vertical pixel coordinates v represent time

The line of pixel on the sensor might be distorted and not represent a perfectly straight line in the physical world



Horizontal pixel coordinates u represent a ray direction

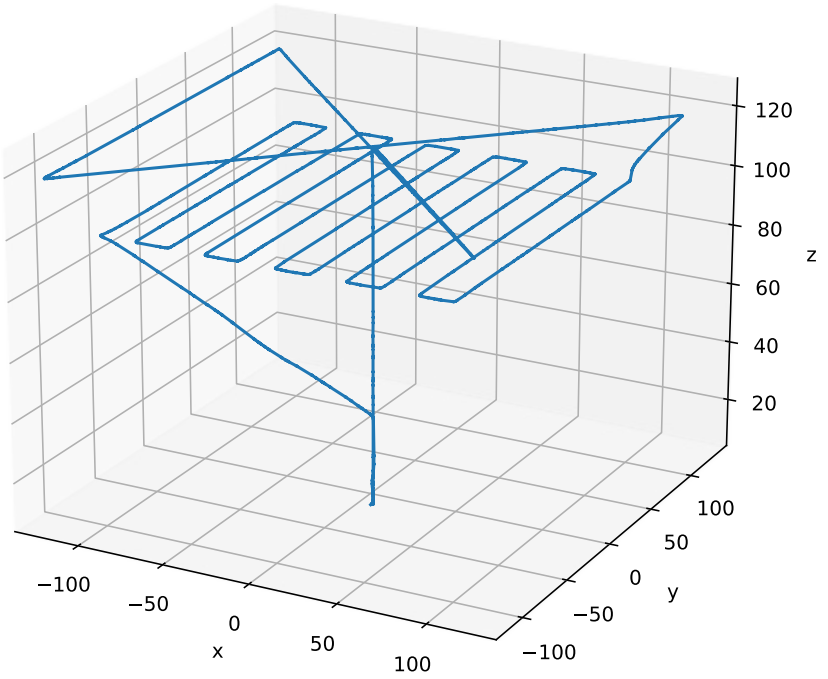
Error is assumed to follow a normal distribution:

$$\varepsilon \sim Normal\left(0, \sigma\right)$$

$$\varepsilon = \begin{bmatrix} u \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} f & 0 & pp_x \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\frac{1}{z_c} \Gamma_t \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + \begin{bmatrix} \Delta x_h \\ \Delta y_h \\ 0 \end{bmatrix} \right)$$

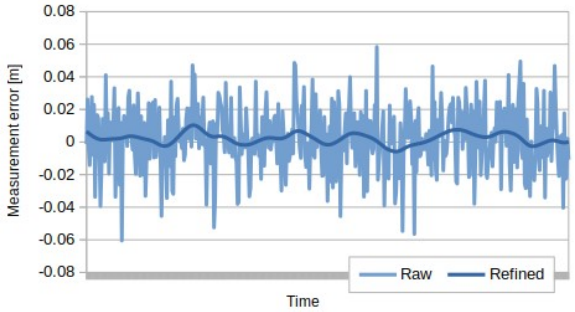
Simulated data for experiments

- Simulate a basic flight
- Based on quadcopter dynamic
- Basic normal noise models for sensors (GPS/INS)
- Simulated 1000 tie points distribution on random terrain
- Factor graph built with normal position prior based on GPS, normal speed priors based on INS and tie points observations

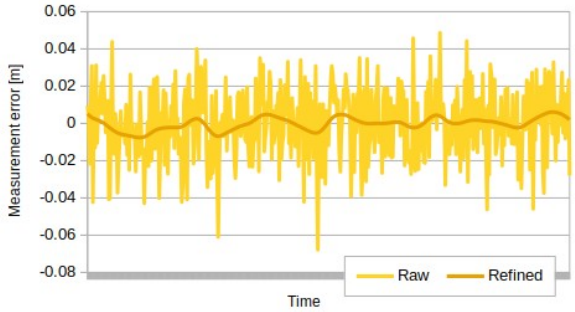


Some preliminary results

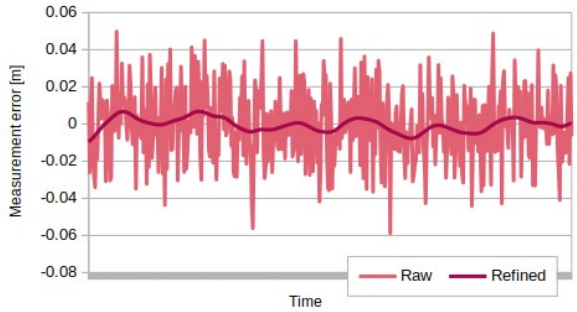
Position error X



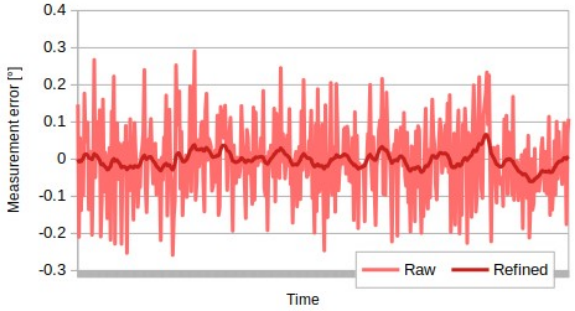
Position error Y



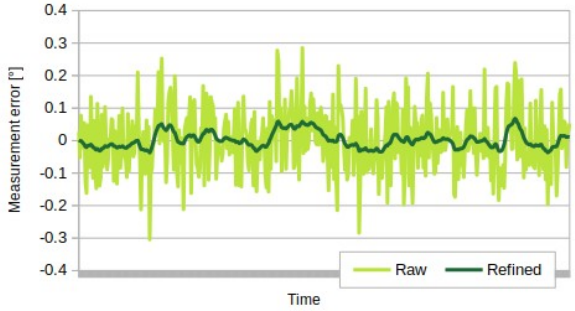
Position error Z



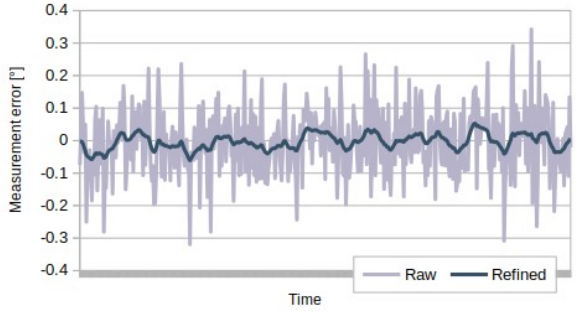
Orientation error X



Orientation error Y



Orientation error Z



Next steps

- Initial testing on simulated data supported the stability and accuracy of the proposed approach.
- Models hyperparameters and additional priors needs to be fine-tuned on real data.
- A flight campaign, with drone, helicopter and plane flights, is scheduled for 2024.
- We are open for collaborations if the project is relevant for your own research !

Trying out Dynamic Network (DN)

- DN is our implementation of factor graph optimization, based on the library ROAMFREE, for geodetic engineering applications (so far for photogrametry and lidar flights optimization, hyperspectral coming soon).
- Publicly available at https://odyn.epfl.ch/app_direct/odyn/

Thank you